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Horizontal collapse propagation of concrete flat slabs supported on columns

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Abstract

The progressive collapse of the World Trade Centre showed the devastating consequences of pancake-type collapses, triggering significant research on failed floor impacts for different forms of construction. In current high-rise construction, concrete flat slabs supported on columns are widely used and, in this case, the fall of the slab could be prevented depending on the detailing and the horizontal propagation of the collapse from one support to adjacent supports. The activation and interaction of different phenomena during horizontal propagation governed by the slab-column response is investigated in this study for cases of flat slabs with and without integrity reinforcement. The paper focuses on slabs without punching reinforcement which are more critical. A slab system analytical model is presented based on a column removal scenario considering the dynamic response of the column-slab connections before and after punching including membrane effects. The model was verified using finite element models with solid elements at the connections. The results highlighted the key role of integrity reinforcement towards preventing slabs from falling by means of activating tensile membrane action concentrated around the columns.

Introduction

Twenty years ago the tragic events leading to the progressive collapse of the World Trade Centre in New York showed the dramatic consequences of pancake-type collapses, reigniting the discussion internationally about design for structural robustness. Different well-known building collapses from World War II, Ronan Point, Alfred P. Murrah Federal Building, Sampoong Department Store described in the literature (e.g. Ellingwood and Dusenberry 2005, Byfield et al. 2014, Adam et al. 2018, Russell et al. 2019) and more recent underground car park failures in Europe (Fernández Ruiz et al. 2013) show that the response to accidental events can vary significantly depending on the form of construction. One of the main reasons behind the broad range of responses is the different response of the connections in the structure which has a critical role on the propagation of failure.

Current forms of construction include traditional frame structures, modular construction and flat slab construction amongst others. In current high-rise building schemes for residential and commercial purposes, flat slab construction (slabs supported on columns) are highly popular due to their reduced story height, elimination of clashes with suspended services and flexibility for partition layouts. In this form of construction the response of slab-column connections in accidental events is governed by flexure until punching occurs at low deformations followed by a sudden reduction in the shear transfer at the connection and stiffness (post-punching response) which can be mitigated using integrity reinforcement. Fig. (1) shows this type of failure in a car park structure in the UK due to poor detailing and unchecked deterioration and in an underground car park in Spain due to overloading after drainage modifications in the earth cover.

Fig. 1. Collapse of flat slab structures: (a) Pipers Row Car Park, Wolverhampton (Wood, 2001); (b) Nueva Montaña car park, Santander (photo by Joaquín Gomez Sastre).

Designing for the unforeseen is challenging and building codes reflect this through different proposed strategies for robustness design (Ellingwood and Dusenberry 2005, Starossek et al. 2011, IStructE 2010). These strategies follow design philosophies introducing different lines of defense or layers of protection (LOP). A first LOP is to prevent the failure initiation. This can be achieved as described in Starossek et al. (2011) externally from the structure through protection or event control and internally to the structure through increased local resistance of key elements (e.g. slab-column connections in flat slabs) although the latter can be uneconomic or difficult to achieve for unknown actions. A second LOP is to prevent disproportionate propagation of the failure through the analysis of alternative load paths (ALP) which is the main focus of this paper; segmentation can be also adopted alternatively as suggested by Ellingwood and Dusenberry (2005) and Starossek et al. (2011) amongst others.

In flat slabs, the local failure of one column (failure initiation) can propagate horizontally to immediately opposite columns shown in Fig. (2) as reported by Hawkins and Mitchel (1979), Regan (1981), Fernández Ruiz et al. (2013), Sagasetta et al. (2017) and Garzón-Roca et al. (2021). Further horizontal propagation of failure to other slab-column connections can result into the slab falling onto the floor below. This horizontal propagation and interaction between post-punching in one connection and punching in the adjacent connection is not well known; tests on slab system assemblies subject to column removal (Yi et al. 2014, Russell et al. 2015,

Peng 2015, Xue et al. 2018) provided useful information on dynamic effects in the slab although the information on the slab-column connections is limited due to the boundary conditions adopted. Research on slab impacting on the floor below showed that this often triggered the vertical progression of the collapse due to the high dynamic shear demands introduced in the connections (Vlassis et al. 2009, Olmati et al. 2017, Yankelevsky et al. 2020) and therefore this scenario should be prevented, especially when the size of the debris falling is significant.

This paper focuses on the horizontal propagation of failure at a slab system level to provide a better understanding of the interaction between the phenomena preventing or leading to slab falling. Considerations are made in the proposed model on flexure, compressive membrane action, punching and post-punching leading to tensile membrane action. An analytical slab progressive collapse model is proposed which was verified against advanced numerical models. Dynamic considerations are made in the analytical model using the pseudo-static response of the slab system based on energy balance as proposed in the Ductility-Centre approach by Izzuddin et al. (2018) and punching is assessed through the Critical Shear Crack Theory (CSCT) rotation-based approach by Muttoni (2008). The work contributes to previous research on progressive collapse analysis at slab system level (Keyvani et al. 2014, Liu et al. 2015) where post-punching modelling was simplified significantly. In this work the asymmetric post-punching model for progressive collapse analyses was adopted which was derived and validated in Ulaeto and Sagaseta (2021). This model is based on the early work on post-punching on isolated connections from Melo and Regan (1998), Fernández-Ruiz et al. (2013) and constitutive models from Mirzaei (2010) and Habibi et al. (2014). The results in this paper provide relevant information on the role of integrity reinforcement for the critical case of flat slabs without punching reinforcement and the activation of tensile membrane action after punching.

Slab progressive collapse model

An analytical model is proposed for the slab system to provide information on the dynamic horizontal propagation of collapse after sudden internal column removal. This section presents the main equations and fundamentals of this approach. The model calculates the nonlinear static response of the system considering: (i) gravity load redistribution, (ii) flexural and compressive membrane action, (iii) punching around the connections and (iv) asymmetric post-punching shear at the connections resulting into tensile membrane action. Once the nonlinear static response is obtained, the pseudo-static response is calculated from energy considerations as proposed by Izzuddin et al. (2008) in order to assess the dynamic peak response. The model was used to assess how post-punching and detailing influences the horizontal propagation of failure.

The analytical model is based on a static push-down analysis of an interior column in which different deformation modes are identified sequentially as the deflection at the point of the removed column is increased and punching occurs at the opposite and adjacent columns as shown in Fig. (2) for one quarter of the slab segment. Three deformation modes are considered which absorb (by means of strain energy) most of the potential energy introduced by the gravity loads after sudden column removal. The formulation is written in terms of a single degree of freedom, i.e. total vertical displacement at the point of column removal (w_{ctr}), to calculate the reactions at the supports. Vertical deflections near the opposite and adjacent columns are derived at different control points from compatibility assumptions as described in more detail in subsequent sections.

The first deformation mode considers the slab system response after internal column removal prior to punching failure of the opposite column. Flexure is the primary response mode around the adjoining connections as the slab deforms to establish a new state of equilibrium. As cracking develops in the bottom of the slab at the point of column removal, the slab dilates and

compressive membrane action (CMA) is activated if the slab is confined laterally (in-plane confinement); this can be the case for interior columns. At the second deformation mode the opposite connection has failed in punching with the activation of post-punching shear. The total vertical displacement of the slab at the opposite connection is $w_{ctr.opp.2}$ while at the point of column removal is $w_{ctr.2}$ at this stage of the response. The third deformation mode is when both the opposite and adjacent connections have failed in punching shear. In this deformation mode the opposite connection is at an advanced stage of post-punching whereas the adjacent connection is at the early stages of post-punching and gravity load is redistributed to other adjoining columns. The overall modelling and analysis stages are summarised in Fig. (2b) which makes references to equations described in the following sections.

(a)

(b)

Fig. 2. Horizontal failure propagation model after internal column removal (quarter flat slab system model): (a) deformations modes and (b) analysis flow-chart.

Deformation mode 1 after column removal

Different flexural models can be used for deformation mode 1 until punching occurs around the opposite connections, as shown by Muttoni (2008) with different levels of approximation as proposed in Model Code 2010 (fib 2013). Refined considerations can be made to take into account CMA for fully restrained slabs as shown by Einpaul et al. (2016), introducing a factor f_{CMA} which is further discussed in this section for column removal situations. Punching at the opposite connection and the flexural response was assessed in this work using Level of Approximation II from Model Code 2010 given by Eq. (1) which is valid until punching; in Model Code 2010 CMA is neglected ($f_{CMA} = 1$).

$$\psi = 1.5 \frac{r_s}{d} \frac{f_y}{E_s} \left(\frac{V}{km_R} \right)^{\frac{3}{2}} f_{CMA} \quad \dots (1)$$

where ψ is the slab rotation near the opposite column along the direction of the removed column; r_s is the distance from the axis of the opposite column to the point of counter-flexure measured towards the removed column (for internal column removal $r_s = 0.44L_0$ where L_0 is the span length between column centrelines before column removal); d is the effective depth of the slab measured to the centroid of the reinforcement layers in tension; f_y and E_s are the steel yield strength and Young's modulus, respectively; m_R is the average flexural strength per unit width in the column strip and k is the ratio between the shear and the average moment per unit width which is approximately equal to 8 for small eccentricities in the column reaction such as those in internal column removal situations (Sagasetta et al. 2017).

Deflection mode 1 ends at $w_{ctr.1} = w_{ctr.1-2}$ when punching in the opposite column occurs. Eq. (2) is the punching failure criterion from the CSCT (Muttoni 2008) which was used in the case study shown in subsequent section for a slab without punching reinforcement. The punching strength V_R and slab rotation at punching ψ_R are obtained solving for $V = V_R$ and $\psi = \psi_R$ in Eq. (1) and Eq. (2). In Eq. (2), f_c is the concrete compression strength, d_g is the

maximum aggregate size (in mm) and b_0 is the length of the shear-resisting control perimeter as defined in Model Code 2010 (fib 2013).

$$\frac{V_R}{b_0 d \sqrt{f_c}} = \frac{3/4}{1 + 15 \frac{\psi \cdot d}{16 + d_g}} \quad \dots (2)$$

In order to apply dynamic considerations in the slab system, the proposed model is formulated based on a single degree of freedom w_{ctr} and therefore a compatibility condition is needed between ψ and w_{ctr} . This compatibility relationship allows to calculate bending moments, shear and ultimately slab reactions for different values of w_{ctr} . For general purposes of column removal in slab-column subsystems, Sagaseta et al. (2017) proposed a compatibility relationship between slab displacement at the point of column removal (w_{ctr}) (sagging) and the slab rotation close to the column (ψ) as $w_{ctr} \approx (2/3)L_0\psi$ based on an elastic deformation profile. This relationship is influenced by the amount of hogging reinforcement at the opposite column (ρ_{hog}) and the amount of sagging reinforcement where the column is removed (ρ_{sag}), which is normally low. However, using the elastic estimate can result in the underestimation of w_{ctr} for high values of ρ_{hog} and ρ_{hog}/ρ_{sag} .

In order to address this, a more refined expression was proposed by Ulaeto (2018) given by Eq. (3). This expression was derived based on the observation that in flat slabs the slab rotation increases significantly close to the column and it remains constant at distances of about $2d$ to $4d$ from the column face (Guandalini 2005, Tassinari 2011) before decreasing to zero at the point of maximum sagging moments. A control deflection point is adopted in this work at a distance of $3.5d$ from the column face; this distance is around $0.28L_0$ for a standard slab with span-to-depth ratio of around 25 and column length-to-depth ratio of around 1.3.

$$w_{ctr} = \zeta(0.28L_0)\psi \quad \dots (3)$$

In Eq. (3), ζ is the ratio of the displacement at the removed column to that at a control point at $0.28L_0$ from the centre of the column where rotation remains constant. The relationship between ζ and ρ_{hog}/ρ_{sag} obtained by Ulaeto (2018) is given in Eq. (4) where f is a reduction

factor which varied with the level of loading ($f = V/V_{flex}$), although the variation for loads above the cracking load was negligible and a constant value ($f = 1$) was adopted.

$$\zeta = 0.32f \left[2.0 \left(\frac{\rho_{hog}}{\rho_{sag}} \right) + 6.5 \right] \dots (4)$$

Fig. (3) shows that Eq. (3) gave consistent results with a parametric analysis using nonlinear FE analyses with shell elements conducted by Tassinari (2011) as well as laboratory results from Yi et al. (2014) of an internal column removal static test. In these analyses, ρ_{hog} varied from 1.5% to 0.75% and ρ_{hog}/ρ_{sag} varied from 5 to 1.5 covering a wide range of values typically found in practice.

Fig. 3. Comparison between numerical prediction of displacement at removed column location and Eq. (3): (a) to (c) cases in Tassinari (2011) with $\rho_{hog} = 1.5\%$, 1.05% and 0.75% in PD150, PD105 and PD075 respectively; (d) comparison with test from Yi et al. (2014).

Regarding the CMA component, if the slab is confined laterally as shown in Fig. (4), the response in the deformation mode 1 can be significantly stiffer. This can be considered by using factor, f_{CMA} in Eq. (5) proposed by Einpaul et al. (2016)

$$f_{CMA} = k_{\psi} \left(1 - \frac{m_{cr}}{\eta_R m_{R.hog}} \right) \quad \dots (5)$$

where parameter k_{ψ} is the factor of proportionality for rotation at the onset of yielding in a continuous slab relative to the corresponding rotation in an isolated specimen, η_R is the ratio between sagging and hogging moment capacities, m_{cr} is the cracking moment per unit width of slab, $m_{R.hog}$ is the maximum hogging moment per unit width of slab. Eq. (5) was originally conceived for design of continuous flat slabs, looking at internal columns with equal orthogonal spans and without consideration of column removal; hence k_{ψ} and η_R were taken as 1 and 0.5 respectively for common cases in practice. For the column removal situations investigated in this work, k_{ψ} was 2.5 and η_R was lower than 0.5 since the sagging reinforcement at the slab where the column was removed was minimal. For the case study shown in subsequent sections η_R was 0.28 although in most practical cases η_R is around 0.15 (ACI 352.1R-89 1997, The Concrete Society 2007). Eq. (1) was verified for the case study described in the subsequent section against shear-rotation curves obtained from FE models using shell elements with lateral restraint; Fig. (4) shows that the curves considering CMA were comparable.

Fig. 4. (a) Compressive membrane action after column removal and (b) shear-rotation relationship using Eq. (1) and (5) compared to FE predictions using shell elements (case study shown in Fig. 7).

Deformation modes 2 and 3 after punching

Deformation mode 2 of the slab system response occurs when the opposite columns have punched and the slab at the adjacent columns behaves predominantly in flexure. As shown in Fig. (2), the main kinematic parameters considered in this deformation mode were the vertical displacement at the point of column removal ($w_{ctr.2}$), the vertical displacement at the opposite column ($w_{ctr.opp.2}$) and the slab rotation around the adjacent column at the side towards the opposite column ($\psi_{adj.2}$). These parameters are related to each other through different compatibility relationships. The slab rotation $\psi_{adj.2}$ is calculated in terms of $w_{ctr.opp.2}$ using Eq. (3). The vertical displacement at the opposite column $w_{ctr.opp.2}$ is related to the column removal displacement $w_{ctr.2}$ according to $w_{ctr.opp.2} \approx 0.43w_{ctr.2}$. This relationship was obtained based on the deflected shape of a continuous plate (with equal spans) in deformation mode 2 in Fig. (2) which results in an almost linear deflection profile along a line drawn from the removed column at $w_{ctr.2}$ to zero deflection at around a distance of $0.25L_0$ from the back columns. A similar relationship was adopted for deformation mode 3, i.e. $w_{ctr.adj.3} \approx$

0.43 $w_{ctr.opp.3}$. A different value could be derived in a slab with different spans using the linear deflection assumption.

The reaction at the adjacent column was obtained from flexural considerations using Eq. (1) with the limit at $w_{ctr.2} = w_{ctr.2-3}$, given by punching in Eq. (2) from the CSCT formulation similar to the opposite column reaction in deformation mode 1. The reaction developed at the opposite column in deformation mode 2 was estimated using the post-punching response model in Ulaeto and Sagaseta (2021) which is summarised in Appendix A. As shown in Ulaeto and Sagaseta (2021) this model is also applicable to asymmetric cases shown in Fig. (5) by carrying out the calculation for each sector separately corresponding to each column face. The four sectors corresponded to (i) the critical side between the opposite and removed column, (ii) the opposite side between the opposite and back column and (iii) the two adjacent sides between the opposite and adjacent columns. The control points for the vertical deflections at each sector were positioned at $3.5d$ from the column face for consistency with the flexural control points in deformation mode 1 and deflection ratios between them were adopted.

Fig. 5. Post-punching of continuous flat slab at opposite column.

Ulaeto and Sagaseta (2021) showed that in asymmetric post-punching, damage (including fracture of the reinforcement) localises at the critical side. This response is captured by the model through refinements in the punching cone geometry adopted and through the displacement factors at the different sectors around the column. In the asymmetric model the inclination of the punching cone at the critical sides was $\theta = 45^\circ$ whereas for the opposite and

adjacent sides a flatter angle was adopted $\theta = 33^\circ$ consisting of a first segment at 45° to the horizontal until a height of $2d/3$ and 25° to the horizontal until the centroid of the hogging reinforcement (Ulaeto and Sagaseta, 2021). For adding the shear contribution from each sector the following displacement factors were adopted at the opposite column between the critical, opposite and adjacent sides: $w_{opp,3.5d}/w_{crit,3.5d} = 0.65$ and $w_{adj,3.5d}/w_{crit,3.5d} = 0.75$ as shown in Ulaeto and Sagaseta (2021) for the geometry considered. For the post-punching calculation in the adjacent column in deformation mode 3, only one displacement factor was required, i.e. deflection ratio between the critical and opposite sides $w_{opp,3.5d}/w_{crit,3.5d} = 0.65$ (in this case the critical side was between the adjacent and opposite column and the opposite side was between the adjacent and back column).

The total vertical deflection at the control point $w_{crit,3.5d}$ has a component due to flexure and a component due to post-punching. The flexural component is equal to $(w_{ctr.1-2}/\zeta)$ from deformation mode 1 where $w_{ctr.1-2}$ is the deflection at the removed column when punching of the opposite column occurs and ζ is given in Eq. (4). The post-punching component is the increment of the vertical deflection at the removed column with respect to the deflection when punching occurs ($w_{ctr.2} - w_{ctr.1-2}$). The vertical deflection due to post-punching at $3.5d$ from the column face was similar to that at the point where the reinforcement crossed the punching cone after local damage ($w_{pp,crit.2}$) shown in Fig. (5) due to the small angles $\theta_{ctr,2}$ and $\theta_{opp,2}$. Therefore, the displacement ($w_{ctr.2} - w_{ctr.1-2}$) was directly taken in the post-punching model to estimate the residual shear transferred through the connection.

Static and dynamic load considerations

The removal of an internal column results in a static load redistribution; for slabs with equal spans it is generally assumed in deformation mode 1 that the load carried by the removed column is redistributed equally to the opposite columns (Regan 1981) as shown in Fig. (6).

278 This results in a ratio of the reactions between the adjacent and opposite columns of 0.8 and
279 neglects any variation in the reactions in the back columns. These simplifying assumptions
280 neglect slab continuity and eccentricity in the loading. These static considerations were adopted
281 in deformation mode 1 to calculate the reactions at the adjacent columns from the load at the
282 opposite column, which was estimated from flexure using Eq. (1) for a given value of $w_{ctr.1}$.
283 For deformation modes 2 and 3 the reactions at the opposite and adjacent columns were
284 obtained from flexural and post-punching considerations, as described in previous sections.
285 The reactions at the back columns were not checked in this work since further modes of
286 deflections were not considered.

287 Once the nonlinear static response was obtained, the peak dynamic response (pseudo-static
288 response) was calculated using energy balance as proposed in the Ductility-Centred approach
289 by Izzuddin et al. (2008). Fig. (6) shows the portion of the gravity loads activated dynamically
290 considered in deformation modes 1, 2 and 3 which correspond to the area of the slab mobilised
291 after column removal, punching of opposite column and adjacent columns respectively. For
292 example, in mode 1 only 75% of the load carried by the opposite column contributed to the
293 dynamic response and this fraction was 25% for the adjacent column. These approximated
294 values estimated from geometrical considerations provided reasonable predictions of the peak
295 deformations compared with FE results shown in the subsequent section. Deformation modes
296 1, 2 and 3 were applied only to flat slab systems with integrity reinforcement where the opposite
297 connection provided some significant resistance in post-punching. Deformation modes 1 and
298 2 were applied to flat slab systems without integrity reinforcement where the slab failed
299 progressively after the end of mode 2.

Fig. 6. Gravity load distribution at three phases of response after removal of internal column.

The sum of the various load contributions to the dynamic response gave the total load (P_{stat}) used in the calculation of the pseudo-static response of the flat slab system. Calculation of the pseudo-static load (P_d) for deformation modes 1, 2 and 3 was carried out using Eq. (6), Eq. (7) and Eq. (8), respectively, where $w_{ctr.1d}$, $w_{ctr.2d}$ and $w_{ctr.3d}$ are the dynamic vertical displacement at the removed column in mode 1, 2 and 3 respectively. The energy balance approach adopted takes into account the sudden drop of the total load P_{stat} between the different deformation modes resulting into a significant reduction of the slope of the pseudo-static curve (large increase in dynamic peak deflection) as shown in subsequent sections.

$$P_{d.1} = \frac{1}{w_{ctr.1.d}} \int_0^{w_{ctr.1.d}} P_{stat.1} dw_{ctr.1.d} \quad \dots (6)$$

$$P_{d.2} = \frac{1}{w_{ctr.2.d}} \left(\int_0^{w_{ctr.1-2}} P_{stat.1} dw_{ctr.1.d} + \int_{w_{ctr.1-2}}^{w_{ctr.2.d}} P_{stat.2} dw_{ctr.2.d} \right) \quad \dots (7)$$

$$P_{d.3} = \frac{1}{w_{ctr.3.d}} \left(\int_0^{w_{ctr.1-2}} P_{stat.1} dw_{ctr.1.d} + \int_{w_{ctr.1-2}}^{w_{ctr.2-3}} P_{stat.2} dw_{ctr.2.d} + \int_{w_{ctr.2-3}}^{w_{ctr.3.d}} P_{stat.3} dw_{ctr.3.d} \right) \quad \dots (8)$$

Verification using advanced numerical models

Flat slab case study

The proposed slab progressive collapse model was applied to a quarter sub-system representing the internal column region of a continuous flat slab shown in Fig. (7). Edge boundary restraints were modelled accordingly, including confinement restraints leading to compressive and tensile action. The slab was designed according to Eurocode 2 (CEN 2004) for flexure and punching for non-seismic regions. The span lengths, column size and overall slab depth were adopted following guidelines in the UK (The Concrete Society, 2007); the parameters are shown in Table 1. The gravity loading corresponded to a high occupancy building category (C1, C2 or C3 in Eurocode 0 CEN 2010a) with a superimposed dead load of 2 kN/m² and live load of 3 kN/m². For the design, load factors of 1.35 and 1.5 were used for the permanent and live loads whereas for accidental loading (column removal) the load factor was 1 with permanent, quasi-permanent and frequent factors applied to the characteristic value of the live load of 0, 0.3 and 0.5 respectively.

Fig. 7. Geometry and reinforcement layout of flat slab case study (Notation key: “H” reinforcement grade B500A, B500B or B500C followed by diameter size and spacing in mm, “B”-“T” refer to bottom-top layers and “1”-“2” outer-inner layers).

Table 1. Slab physical characteristics and material properties

Characteristic	Value	Unit
Span, L	6.8	m
Column strip width	3.4	m
Slab overall depth, h	0.275	m
Nominal cover, c_1	0.03	m
Flexural reinforcement bar diameter adopted, ϕ	0.012	m
Average effective depth, d	0.233	m
Concrete compressive strength, f_c	30	MPa
Concrete tensile strength, f_{ct}	2.355	MPa
Yield strength of steel reinforcement, f_y	500	MPa
Ultimate strength of steel reinforcement, f_u	540	MPa

Fig. (7) shows the flexural reinforcement layout adopted with a bottom reinforcement ratio of 0.44% across the entire slab and top reinforcement ratio of 0.53% at the columns. Punching shear reinforcement around internal columns was avoided in the design by providing sufficient

slab thickness. The demand capacity ratio of the connections at ULS ($V_{Ed}/V_{Rd,c}$) was 0.9 using Eurocode 2 and Model Code 2010 (LoAI) and 0.8 using ACI 318 (2014). The demand capacity ratio was consistent with FE predictions of the nonlinear static response of the isolated column-slab connection in Fig. (8) using solid elements described in the next section. Fig. (8) shows that the response before and after punching (i.e. deformation mode 1 and 2) obtained numerically was consistent with the analytical response which was based on the shear-rotation curve from Model Code 2010 and post-punching model in Ulaeto and Sagaseta (2021). Fig. (8) shows a case without integrity reinforcement except for two 10 mm bars at the bottom of the slab crossing the column provided from the nominal basic reinforcement mesh. The static FE and analytical post-punching models predicted that the bottom bars fractured at a similar vertical deflection shown in Fig. (8). In subsequent analyses of the flat slab with integrity reinforcement, two bars of 25 mm were provided crossing the column (at both orthogonal directions); the diameter was estimated using different formulae available in Mitchell and Cook (1984), ACI 352.1R-11, CSA A23.3-04 and Model Code 2010. Variations of the amount of integrity reinforcement is discussed in the following sections based on the slab progressive collapse model considering also dynamic effects.

Fig. 8. Nonlinear static response of isolated column-slab connection in the case study.

Numerical modelling of slab system using solid and shell elements

The analysis of isolated column-slab connections provides limited information regarding the effects of local failure in flat slab structures since slab continuity influences significantly the horizontal propagation of failure in the slab. Modelling slab systems using shell and frame elements (e.g. Olmati et al. 2017, Liu et al. 2015) can provide useful additional information of the response up to punching of the connections which is estimated through the post-processing analysis of the shear and rotations around the column using the punching failure criteria from the CSCT (Muttoni 2008). After punching, these models need further refinement by means of connector elements (Keyvani et al. 2014, Liu et al. 2015) which can be problematic since the connector stiffness and capacity varies with the deflection and local damage of the concrete around the reinforcement during post-punching. Therefore, in this study, a 3D FE explicit modelling was adopted with solid elements for the column-slab connections, as shown in Ulaeto and Sagaseta (2021). To reduce the computational demand of the slab model, the portion of the slab outside the column region was modelled using layered shell elements whereas the slab-column region was modelled using 3D solid elements shown in Fig. (9). The solid

elements were 8-noded hexagonal with Winfrith material for the concrete; embedded Hughes-Liu beam elements with plastic kinematic model for the reinforcement; and Constrained Lagrange solid model for the bond between beam and solid elements. For the layered shell elements, the Concrete_EC2 material model was adopted (Olmati et al. 2017).

Fig. 9. (a) Quarter flat slab system model using solid and shell elements, (b) section through connection region and (c) solid-shell interface.

Rigid body constraints were used at the solid-shell interface to impose compatibility of rotations and displacements at the coupling plane shown in Fig. (9) for the transfer of shear and bending moments. This interface was verified against isolated punching tests in the literature providing similar predictions to models with only solid elements (Ulaeto 2018). In the model shown in Fig. (9), the solid modelled areas extended to a distance of $6d$ from the column face

to capture the peak in punching and post-punching. This distance was estimated based on post-punching tests (Habibi 2012 and Fernández-Ruiz et al. 2013). Columns were modelled using 0.1 m thick stubs fully restrained at the bottom as shown in Fig. (9).

Nonlinear static response of slab system

The nonlinear static response of the slab system described above obtained numerically was consistent with the prediction from the analytical model. This is shown in Fig. (10) for the case where CMA was considered, the same was true for cases where CMA was neglected. The load and deflection points at which punching was predicted at the opposite and adjacent columns were also predicted consistently, as well as the post-punching response. Fig. (10) shows that the drop in loading after punching of adjacent connections was not as sudden in the numerical response as that predicted analytically. This was due to progressive development of punching and shear redistribution along the perimeter in the FE model.

Fig. 10. Slab response: (a) without and (b) with integrity reinforcement.

Dynamic peak response from pseudo-static curve

Fig. (11) shows the pseudo-static curves obtained from energy balance (Izzuddin et al. 2008) from the nonlinear static response obtained analytically in previous section. The first two peaks in the nonlinear static response in Fig. (11) corresponded to punching of the opposite and

adjacent connections; subsequent peaks were due to fracture of the flexural reinforcement and the integrity reinforcement (in this order) at the critical side of the opposite column. Fig. (11) shows the loads corresponding to cases of permanent (g_k), quasi-permanent ($g_k + \psi_2 q_k$), frequent ($g_k + \psi_1 q_k$) and a load combination $1.214(g_k + q_k)$. The dynamic peak for a specific load is obtained from the intersection with the pseudo-static curve.

The results suggest that in the case study with permanent loading, sudden column removal does not result in punching of the opposite connection, i.e. only deformation mode 1 occurs and horizontal propagation of failure is arrested. This result is irrespective of whether integrity reinforcement is provided or not. For the case of quasi-permanent loading, punching is predicted to occur at the opposite connection and not at the adjacent connection (deflection mode 2 only). In this case, the provision of integrity reinforcement is critical as shown in Fig. (11) since the peak (point of dynamic equilibrium) is unstable for cases without integrity reinforcement leading to horizontal propagation of failure and slab falling after fracture of the integrity reinforcement. As shown in Fig. (11), this can be avoided with the provision of integrity reinforcement which enables the development of deformation mode 2.

For the frequent load combination, horizontal propagation of failure and slab falling was not avoided in the case without integrity reinforcement whereas for the case with integrity reinforcement the slab found a point of unstable dynamic equilibrium after punching of the adjacent column. For higher load combinations, the horizontal propagation of failure and slab falling was not arrested for the amount of integrity reinforcement provided; the results using different amounts of integrity reinforcement are discussed in subsequent section.

The dynamic peak predictions described above using the pseudo-static curve were verified with dynamic FE analysis using the slab system model with solid-shell elements described in previous sections. The results are summarised in the next section.

Fig. 11. System non-linear static and pseudo-static response: (a) without integrity reinforcement and (b) with integrity reinforcement.

Dynamic response from nonlinear FE dynamic analysis

In order to verify the results in previous section, a nonlinear explicit dynamic FE analysis was carried out using the slab system model with solid and shell elements described previously. The gravity load was applied gradually (quasi-statically) followed by a column removal at $t = 6$ s where the elements at the support were removed, resulting in a removal time of 10^{-6} s which is below recommended values of $T/10$ in GSA (2013) where T is the vibration period. Fig. (12) shows the results for the frequent load combination with and without integrity reinforcement. In the case with integrity reinforcement progressive collapse was arrested; punching of the opposite columns was predicted at $t = 11$ s which was reflected in the sudden increase of vertical reaction in the adjacent column (Fig. (12)) which punched at around $t = 13$ s. The system became stable at around $t = 18$ s with yielding of the flexural reinforcement near the opposite columns and no fracture of the integrity reinforcement. In the case without integrity reinforcement (Fig. (12)), punching of the opposite column occurred at $t = 8.5$ s and adjacent column punched at $t = 9.5$ s which resulted in the slab falling as shown by the sudden increase in the deflection at around 10 s. The variations in the vertical reactions in the columns in the slab without integrity reinforcement followed a similar trend to the case without integrity

reinforcement shown in Fig. (12) except that punching around the opposite and adjacent columns occurred sooner and after punching around the adjacent column the reactions became unstable and dropped to zero. The dynamic amplification factor of the shear around opposite column after column removal was 1.2 which is consistent with previous studies of internal column removal (Olmati et al. 2017, Liu et al. 2015, Sagaseta et al. 2017).

Fig. 12. Results from the dynamic FE (frequent load case): (a) deflection in the slab with and without integrity reinforcement, (b) damage in the connection at $t=10$ s without integrity reinforcement and (c) vertical reactions in the slab with integrity reinforcement.

Overall, the response for the frequent load case was consistent with the prediction from the pseudo-static curve in Fig. (13). The same was true for other load cases as shown in Fig. (13). In the slab without integrity reinforcement, horizontal propagation was arrested only in the permanent load case and the time at which punching first occurred varied with the gravity load

applied as expected. In the slab with integrity reinforcement, slab falling was observed only for loads higher than the frequent load case as show in Fig. (13).

Fig. 13. Results from the dynamic FE for different load cases: deflections in the slab (a) without integrity reinforcement and (b) with integrity reinforcement.

The FE model captured the development of membrane forces at the connections which was consistent with the membrane behaviour adopted in the analytical model for deformation modes 1 to 3. The member forces in the slab were determined from the FE model at the edge of the opposite column at the critical side towards the removed column with strip widths of 1 m and 1.8 m. The strip width of 1 m corresponded roughly to the diameter of the punching cone estimated in the post-punching model used in the deformation mode 2 and 3 prior to local damage of top concrete at the edges of the cone (Ulaeto and Sagaseta 2021). The membrane force in the strip was estimated as the integral of the average normal stresses of each element across the depth (solid elements for concrete and beam elements for the reinforcement).

Fig. 14. Membrane forces (frequent load): (a) without and (b) with integrity reinforcement.

Fig. (14) shows that compressive membrane action CMA increased significantly after column removal; the membrane forces were around 850 kN/m which are of similar magnitude to those obtained in previous studies for internal columns Olmati et al. (2017). Fig. (14) also shows that the membrane forces distributed more widely across the column strip whereas tensile membrane action TMA that developed at larger deflections concentrated predominantly over the punching cone strip. The transition from compressive to tensile membrane action was progressive from column removal until punching around the opposite column which triggered the sudden change from compression to tensile membrane forces. The membrane forces were close to zero when the vertical deflection w_{ctr} was around 70% of the slab total depth; which occurred at around $t = 9$ s and 12 s for the slab without and with integrity reinforcement, respectively. The limit deflection obtained for CMA is reasonably consistent with theoretical considerations by Rankin and Long (1997). It can be concluded that tensile membrane action was activated in the slab with integrity reinforcement after large deflections. TMA concentrated mainly above the column and the peak average axial tensile stress in the integrity reinforcement was about 90% of the yield stress.

Influence of amount of integrity reinforcement and gravity load on the response

Sensitivity analyses were carried out using the analytical model including variations of the amount integrity reinforcement within acceptable limits in current Eurocode 2 and ACI 318 recommendations. For the case study considered in this work, two additional cases were investigated, one with two bars of 20 mm diameter and one with three bars of 25 mm diameter which cover an intermediate and unusually high value case respectively.

Fig. (15) shows the nonlinear static and corresponding pseudo-static response for these two cases. For the case of two 20 mm bars, the frequent load combination resulted in horizontal propagation of failure and slab falling whereas for quasi-permanent loading the point of dynamic equilibrium was unstable, similarly as when using two 10 mm bars although in this

case the response reached a plateau which is more favourable. For the case with three 25 mm bars, for both quasi-permanent and frequent load combinations the slab resisted column removal within deformation mode 3 well below the peak in the post-punching response. It is interesting to note in this case that increasing the amount of integrity reinforcement was not sufficient to arrest the horizontal propagation to the adjacent column.

Fig. 15: Influence of amount of integrity reinforcement on static and pseudo-static response.

Alternative notional damage scenario

An alternative notional accidental scenario was considered where all the connections in the slab were assumed to have punched previously. Similar to column removal, the cause leading to this scenario (e.g. local or general overloading, inappropriate punching design or construction) was not specified in any detail though notionally it could represent an upper bound situation where punching occurs at several connections in the slab with sudden propagation to other connections. Under this notional scenario, the slab system analysis is simplified to that of a single connection subject to symmetric post-punching shown in Fig. (16) for the two connections investigated in the case study (without and with integrity reinforcement). The connection needs to resist the gravity loads in the slab when punching occurs which are estimated using accidental load combination equations in design codes for permanent, frequent and quasi-permanent cases. The sudden drop in the slab after punching due to the significant reduction in stiffness induces a motion resulting in a dynamic

amplification of the loads and deformations. Therefore, the pseudo-static response was calculated for the post-punching curve in Fig. (16) to obtain the peak dynamic response. The overall results were similar to those obtained in the column removal case, i.e. the case without integrity reinforcement resulted into slab falling for all gravity loads considered whereas the slab hanged from the column in the slab with integrity reinforcement. The dynamic load and displacement factors obtained analytically in Fig. (16) were around 1.6 and 2.1 respectively; damping was neglected. The demand capacity ratio in the slab with integrity reinforcement was 0.95 whereas this was 0.72 for the same gravity loading in the column removal case. This suggests that the alternative notional damage scenario investigated was more detrimental since the progressive propagation of failure is neglected. This alternative scenario could be used in design for medium consequence class structures. However, the approach does not consider explicitly the cause of the local damage and therefore more refined analyses (scenario-based) are recommended for the highest consequence class structures.

Fig. 16. Static and pseudo-static response of punched connection in notional accidental case.

Conclusions

The collapse of the World Trade Centre amongst other building failures has shown the catastrophic effects of pancake-type of collapses triggered by slabs falling onto the floors below. This has led to significant research on the overload introduced in the slab and connections during these events. This paper shows that in flat slab construction, slab falling could be prevented during the horizontal propagation of failure from the point where the support is lost to other connections. An analytical slab progressive collapse model is presented and applied to the critical case of slabs without punching reinforcement with different gravity loadings. The analyses considered punching and post-punching behaviour at the connections including dynamic and membrane effects. The analyses included cases of connections with and without integrity reinforcement which is generally used in prescribed detailing for robustness. The main conclusions from this work are summarised below:

- 1) In order to model accurately the horizontal progressive collapse in flat slabs, a higher level of detail is required in the modelling of the slab-column connections compared to normal design situations. The work shows that compressive membrane action and flexure leading to punching are relevant at the initial mode of deformation after column removal. If punching occurs at the opposite columns, refined post-punching models are required which consider the local damage of the concrete near at the punching cone and at slab near the column. The model should also consider the asymmetry in the deflections as well as the dynamic amplification after sudden column removal.
- 2) The analyses from the proposed analytical model and numerical FE models showed both cases of slab falling and slab hanging from the columns depending on the gravity load considered and the amount of integrity reinforcement. The results showed that integrity reinforcement was effective in preventing slab falling except when using small diameter bars due to bar fracture at the critical face of the column as predicted by the

model. The results in cases with significant amount of integrity reinforcement showed that the slab could hang from the columns although horizontal propagation of failure (i.e. punching of the adjacent columns) could not be prevented.

3) The dynamic numerical models using solid finite elements for the slab-column region provided consistent predictions of the damage and overall response to that predicted with the analytical approach. The numerical model indicated that the gradual transition from compression to tensile membrane action observed before punching had a snap-through at punching leading to a high concentration of tensile membrane forces above the punching cone. Local damage around the punching cone predicted numerically and analytically influenced the amount of flexural reinforcement activated in tensile membrane action whereas integrity reinforcement was shown to be more effective as expected.

4) An alternative notional accidental scenario was investigated in which the slab was assumed to be supported on slab-column connections which had already punched. The gravity frequent and quasi-permanent loads were assumed to be activated dynamically, which resulted into a dynamic displacement and load factor of 2.1 and 1.6 respectively. The demand on the connections due to this scenario was higher than in the column removal approach which suggest that this scenario could be used in design. However, the potential application of this scenario should be investigated further since the cause of punching around the connections is not considered explicitly and therefore it might not be suitable for highest consequence class structures.

5) The work presented here shows that the phenomena and interactions between these effects (compression membrane action, punching, post-punching and tensile membrane action) is complex and therefore the study was limited to internal column removal. The principles of the proposed slab model can be extrapolated to other geometries although

the compatibility relationships should be reviewed accordingly. The numerical model using solid finite elements is applicable to general cases although the models require a high computational demand (running each simulation took about 300 hours in a High Performance Computer).

6) Further experimental research at a building system level will be required to fully validate the proposed approaches, which have been verified numerically and validated experimentally at a connection level where test data was available. Further research is also needed on slabs with punching reinforcement and the effects on the post-punching response.

7) The development of analytical progressive collapse models is an essential step in allowing sensitivity studies and probabilistic performance-based assessment of specific building typologies to be undertaken; especially given the computational demands and multi-dimensional design parameter domains that need to be studied in order to produce design guidance on robustness.

Data availability statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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Appendix A: Overview of post-punching shear model

The main equations in the post-punching shear model by Ulaeto and Sagaseta (2021) are summarised here for completeness. Fig. (A1) shows the definition of the main variables and the shape of the punching shear cone adopted. Angles of inclination of the punching shear cone to the horizontal were 25° and 45° , for θ_1 and θ_2 respectively.

Fig. A11. Geometric assumptions in proposed analytical modelling of post-punching shear mechanism (Ulaeto and Sagaseta 2021)

The load-deflection response V - w is obtained for the flexural reinforcement and for the integrity reinforcement separately using the approach in Fig. (A2) where the distance of local damage in the punching cone (p_i) and in the slab (q_i) are increased gradually to obtain the shear transferred V and the vertical deflection w . Subsequently both curves are superimposed at points with the same vertical deflection w . The equations used for the modelling of the progression of the local damage and the equations used for the activation of flexural and integrity reinforcement are summarised in the following sections.

750 **Modelling of progression of local damage**

751 As p_i and q_i increases, the depth of concrete above the steel bars, d_{res} , and the break out
 752 strength of the steel bars, $V_{con.}$, also increases. Equilibrium is ensured using the relationships;

$$753 \quad V_{con.break} > V_{con.spall} : \begin{cases} q_i \text{ increases by } \delta \\ p_i \text{ remains constant} \end{cases}$$

$$754 \quad V_{con.break} \leq V_{con.spall} : \begin{cases} q_i \text{ remains constant} \\ p_i \text{ increases by } \delta. \end{cases}$$

$$755 \quad V_{con.} = A_{ch} \cdot f_{ct.eff} \quad \dots (A.1)$$

756 where $f_{ct.eff} = 0.33\sqrt{f_c}$ and A_{ch} is defined in Fig. (A3) for a single bar and a group of n
 757 number of bars ($s < 2d_{res}$) is given by Eq. (A.2) and (A.3) respectively,

$$758 \quad A_{ch} = \frac{\pi}{2} d_{res}^2 \cot^2 \gamma \quad \dots (A.2)$$

$$759 \quad A_{ch} = \left\{ \left[\Delta + \frac{n}{2} (\pi - 2\Delta) \right] d_{res}^2 \cot^2 \gamma + \frac{n-1}{2} s d_{res} \cot \gamma \sin \Delta \right\} \quad \dots (A.3)$$

760 where, $\Delta = \arccos \left(\frac{s}{2d_{res} \cot \gamma} \right)$ and $\gamma = 35^\circ$. To model the damage progression, the depth of
 761 concrete above bars (d_{res}) was written in terms of p_i or q_i .

$$762 \quad d_{res}(p_i) = p_i \tan \theta_i \cot \gamma \quad \dots (A.4)$$

763 Based on geometrical considerations in Fig. (A1) and Fig. (A3).

$$764 \quad d_{res} = \begin{cases} p_i \cdot \tan \theta_1 & : 0 \leq d_{res} \leq d/3 \\ \left[p_i - \frac{d}{3} \cot \theta_1 \right] \tan \theta_2 + \frac{d}{3} & : d/3 \leq d_{res} \leq d \end{cases} \quad \dots (A.5)$$

$$765 \quad d_{res.int} = \begin{cases} q_{int.i} \tan \theta_2 & : 0 \leq d_{res.int} \leq \left(\frac{2d}{3} - g \right) \\ \left[q_{int.i} - \left(\frac{2}{3}d - g \right) \cot \theta_2 \right] \tan \theta_1 + \left(\frac{2d}{3} - g \right) & : \left(\frac{2d}{3} - g \right) \leq d_{res.int} < (d - g) \dots (A.6) \\ d - g & : d_{res.int} = (d - g) \end{cases}$$

766

Fig. A3. Basis of analytical modelling of progression of concrete breakage

Activation of flexural and integrity reinforcement

The width of the punching cone, b_{flex} is given by;

$$b_{flex} = c + 2d \cot \theta - 2p_i \quad \dots (A.7)$$

The total number of flexural reinforcement bars contributing to the concrete breakage of the punching cone, n_{break} , was calculated using Eq. (A.8). A minimum number of bars, $n_{break.min}$ was calculated using Eq. (A.9).

$$n_{break} = 2 \left[\frac{b_{flex.dir.1}}{s_{b.dir.1}} + \frac{b_{flex.dir.2}}{s_{b.dir.2}} \right] \geq n_{break.min} \quad \dots (A.8)$$

$$n_{break.min} = 2c \left(\frac{1}{s_{b.dir.1}} + \frac{1}{s_{b.dir.2}} \right) \quad \dots (A.9)$$

where s_b is the spacing of the reinforcement bars considered. The number of top reinforcement bars activated outside the punching cone by the underlying bottom layer of flexural reinforcement, is $4q_i/s_{b.dir.1}$. Hence, the total number of bars contributing to the spalling of concrete outside the punching cone is as given in Eq. (A.10).

$$n_{spa.total} = \left[2 \frac{b_{flex.dir.1}}{s_{b.dir.1}} + 2 \frac{b_{flex.dir.2}}{s_{b.dir.2}} + 4 \frac{q_i}{s_{b.dir.1}} \right] \quad \dots (A.10)$$

Fig. (A1) shows the length of flexural reinforcement activated around the post-punching shear transfer zone, l , given in Eq. (A.11)

$$l = z\phi + p_i + q_i + z\phi \quad \dots (A.11)$$

where ϕ is the reinforcement diameter and z is a factor which takes into consideration the curvature influenced zone for the embedded length of the reinforcement (equal to 6 for the flexural deformation and 4 for the membrane deformation based on Dei Poli et al. 1993).

Eq. (A.12) and (A.13) give the rotation and end-to-end displacement at the embedded rebar crossing the crack; w_{flex} is equal to the slab displacement. In Eq. (A.12) and (A.13), V is the shear, E_s is the Young modulus and I_s is the second moment of area of the rebar.

$$\psi_{flex} = -\frac{Vl^2}{8n_{break}E_sI_s} \quad \dots (A.12)$$

$$w_{flex} = \frac{Vl^3}{12n_{break}E_sI_s} \quad \dots (A.13)$$

Eq. (A.14) provides the transition from flexure to membrane behaviour, given by the strain $\varepsilon_{s,f-m}$ at the tensile edge of the bar for a given displacement.

$$\varepsilon_{s,f-m} = \frac{3w_{flex}}{l^2}\phi \quad \dots (A.14)$$

The membrane stress in the reinforcement was calculated using Eq. (A.15) from vertical equilibrium assuming that all the shear was carried through membrane action. The inclination of the reinforcement at each step of the membrane phase is taken as the sum of the initial flexural rotation, ψ_{flex} , and the rotation due the strain in the membrane phase $\varepsilon_{s,mem}$,

$$f_{s,mem.} = \frac{4V}{n_{break}\pi\phi^2 \sin \psi_{mem.}} \quad \dots (A.15)$$

where, $\psi_{mem.} = \left(\arccos \frac{1}{1+\varepsilon_{s,mem.}} \right) + \psi_{flex} \quad \dots (A.16)$

The vertical displacement of each reinforcement bar during the membrane phase, $w_{mem.}$, was calculated using the following geometric relationship

$$w_{mem.} = l \tan \psi_{mem.} \quad \dots (A.17)$$