Reimagining Mathematics: The Role of Mental Imagery in Explaining Mathematical Calculation Skills in Childhood

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ABSTRACT—Evidence for associations between spatial skills and mathematics has led to the argument that spatial visualization plays a role in mathematical calculation. However, there is no single accepted definition of what spatial visualization encompasses. Here, we investigated spatial visualization in the context of a mental imagery framework. We applied a component model of mental imagery, involving image generation, image maintenance, image transformation (measured using mental rotation), and image scanning, to determine associations between each component and mathematical calculation ability in primary school children ($N = 92$, age 6–11 years). We found that, after accounting for age, only mental rotation explained significant variation in mathematical calculation. Our findings advance theoretical understanding by demonstrating that spatial visualization definitions, applied to mathematics, should be refined to focus on transformation. This highlights the practical implication that transformation strategies are promising targets for future intervention work, rather than broad visualization strategies.

Spatial Thinking and Mathematics

Tasks used to capture spatial visualization ability involve spatial transformations of mental representations, for example, mental rotation or mental paper folding (Hawes & Ansari, 2020). However, definitions of spatial visualization sometimes include a reference to *visual* representations, as well

as spatial transformations. For example, Linn and Peterson (1985) define spatial visualization as the ability to mentally represent and manipulate visual–spatial information. Yet other definitions specifically refer to transformations of spatial properties only (Lowrie, Logan, & Hegarty, 2019). Moreover, many suggest a specific role of visual representations in their interpretations of an association between spatial visualization tasks and mathematics (Gilligan, Thomas, & Farran, 2019; Mix, Hambrick, Levine, & Young, 2016). However, no known studies in this domain include explicit measures of visual representation to support this. Therefore, a key contribution of this article is to delineate the role of *spatial* transformations and *visual* representations in mathematical calculation. Furthermore, on account of these ambiguities as to what spatial visualization refers to, for the remainder of the introduction, when referring to previous studies we will refer to the components that were measured rather than the terminology used to describe them.

Many studies investigating such relations have focused on one specific sub-domain of mathematics: numerical competencies. As described by Hawes and Ansari (2020), numerical competencies relate to an understanding of number symbols, their relations and applications, for example, number ordering, arithmetic, and numerical word problems. Beyond correlational evidence, there is evidence that spatial visualization has a causal effect on these numerical skills. Cognitive training studies have demonstrated transfer of gains from spatial visualization training (measured using spatial transformation tasks) to numerical competencies including: calculation performance (Cheng & Mix, 2014; Gilligan, Thomas, & Farran, 2019; Mix, Levine, Cheng, Stockton, & Bower, 2020); symbolic number comparison (Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017); word problems (Lowrie et al., 2019; Mix et al., 2020); and place

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value (Mix et al., 2020). For example, it has been found children who were trained in spatial transformation demonstrated significant improvements in accuracy on a range of number-based measures including Missing Term Problems (Mix et al., 2020). In short, several studies show that spatial transformation contributes causally to numerical skills.

One explanation for associations between spatial visualization and numerical competencies is that these skills have shared requirements for visualizing, including the mental representation, organization, and manipulation of information (Lourenco, Cheung, & Aulet, 2018; Mix et al., 2016). In the spatial modeling account described by Hawes and Ansari (2020), spatial visualization may provide “a mental blackboard on which numerical relations and operations can be modeled and visualized.” However, few studies have investigated this theory, and little is known about the underlying processes involved in spatial visualization. That is, what is spatial visualization, and what makes it so important for mathematics? By drawing on theoretical perspectives from the mental imagery literature, this study provides novel insights into the role of spatial visualization for mathematics. In our novel approach, we investigate spatial visualization in the context of Kosslyn’s (1980) model of MI.

Mental Imagery

Mental imagery (MI) is defined as the ability to create and manipulate images in the mind in the absence of sensory input (Kosslyn, 1980; Kosslyn, Thompson, & Ganis, 2006). Despite parallels between these fields, spatial visualization has never been investigated in alignment with what is known about MI. Therefore, a significant contribution of this article to the literature is to investigate spatial visualization within an established theory of MI. Several decades of behavioral and neuroimaging research support a depictive theory of MI, that is, that individuals are capable of generating visually depictive representations that are distinct in format from abstract, symbolic representations of general thought (Dijkstra, Bosch, & van Gerven, 2019; Kosslyn et al., 2006; Pearson & Kosslyn, 2015). Further investigation has refined our understanding to demonstrate that the degree to which early visual areas are recruited in MI (as measured using functional magnetic resonance imaging) is positively associated with the visual quality of images reported by participants (Dijkstra et al., 2019; Lee, Kravitz, & Baker, 2012). This suggests that while we can “see” with spatial and visual accuracy in the mind’s eye, the visual quality of MI differs across individuals. In this study, we investigate how these individual differences in MI are associated with mathematics outcomes in primary school children.

While one branch of MI research has focused on deciphering the format of representations, another branch has

applied Kosslyn’s component model of MI to investigate how MI develops throughout childhood (Bates & Farran, 2020; Kosslyn, Margolis, Barrett, Goldknopf, & Daly, 1990; Wimmer, Maras, Robinson, Doherty, & Pugeault, 2015; Wimmer, Robinson, & Doherty, 2017). Kosslyn’s separable-component model proposes that MI is comprised of distinct components, namely: image generation, image maintenance, image transformation, and image scanning. Image generation and image maintenance tasks commonly involve memorizing an object or abstract shape, generating an image to discriminate between the reference object/shape and a target shape after a distractor task (image generation) or holding an image in mind in order to discriminate between the subsequent target object/shape and the reference held in mind (image maintenance). Image transformation involves the manipulation of an imagined object or anticipating what would be seen if an external force manipulated an object (Kosslyn et al., 2006). It is most commonly measured using a mental rotation task. Image scanning is defined as the ability to shift attention across an image and involves a participant visualizing a map, for example, and mentally traversing between landmarks on the map (Bates & Farran, 2020; Borst & Kosslyn, 2010; Kosslyn, Ball, & Reiser, 1978; Wimmer, Maras, Robinson, & Thomas, 2016a, 2016b). This theoretical model is supported by computational evidence suggesting that the sub-components are computationally distinct (Kosslyn, Brunn, Cave, & Wallach, 1984), as well as neurological evidence that these sub-components are associated with activation in different brain regions (Kosslyn et al., 2004).

With reference to the development of MI, research has shown competence in each sub-component of MI from 4 years with adult-like levels reached at approximately 8 years (4, 6, 8, 10 years and adults tested) (Wimmer et al., 2015, 2017). Recent research has extended previous findings by adapting a battery of tasks to investigate individual differences in the visual precision of MI developmentally throughout the primary school years, that is, from what age can children generate and manipulate images of high visual precision (Bates & Farran, 2020). In this study, it was found that children from the youngest age tested (age 6 years) can generate and maintain highly visual mental images and these skills reach adult-like levels at approximately age 8 years in Image Generation and approximately age 10 years in Image Maintenance. Moreover, the ability to accurately represent the spatial location of objects within a visual representation (image scanning) and how efficiently representations are transformed (mental rotation) were present from the youngest age tested (age 6 years) (Bates & Farran, 2020). Evidence has largely found that the sub-components of MI are not correlated in childhood (Bates & Farran, 2020; Kosslyn et al., 1990; Wimmer et al., 2017) supporting Kosslyn’s separable-component model. To summarize,

evidence shows that primary school children from at least 6 years can generate and maintain visually precise images and transform representations. However, the extent to which these MI skills are related to mathematics is unknown.

To date, inconsistencies in terminology between the MI and spatial literatures have hindered communication between these two related fields. The term spatial visualization is often used in the spatial literature to solely encompass the ability to transform representations (e.g., mental rotation) and many spatial visualization tasks explicitly measure transformations only (e.g., Mix, 2019). However, interpretations of findings demonstrating a relationship between spatial visualization and mathematics often go broader than the components measured by the tasks, and refer to spatial visualization as also encompassing representing and manipulating visual information. For example, it has been argued that children may be visualizing the numbers in a calculation problem as blocks and counting the difference to solve the equation (Gilligan, Thomas, & Farran, 2019; Lourenco et al., 2018), which implies that visual representations beyond transformations play a role in mathematical calculation performance. Moreover, further confusion arises in the selection of spatial visualization tasks. A recent longitudinal study assessing the role of spatial visualization in mathematics found that spatial visualization at 7 years was associated with mathematics performance at age 8 years (Geer, Quinn, & Ganley, 2019). Interestingly, the spatial visualization task employed in this study was the jigsaw-puzzle task; a task originally designed to measure MI (a composite of image generation, image maintenance, and image transformation) (Casey, Dearing, Vasilyeva, Ganley, & Tine, 2011; Richardson & Vecchi, 2002); however, the authors only refer to spatial transformations in their interpretations of the positive relationship between performance on this task and mathematics performance. Taken together, in the context of interpreting the cognitive mechanisms underpinning mathematics performance, there is a lack of clarity as to whether it is dependent on the spatial transformation of representations, or whether there is a specific role of a visual representation or whether both are important. Furthermore, no known studies investigate the role of different components of MI (beyond transformation) on mathematics. To address this gap in the literature, this study investigates how the individual components of MI, that is, generating, maintaining, and manipulating information in mind, underpins performance in mathematical calculation in childhood.

In this study, we identify how each of the separable skills included in Kosslyn's model of MI contribute to mathematics performance in childhood. Mathematical calculation was selected as the mathematics outcome. This was chosen given the evidence suggesting training spatial visualization leads

to gains in mathematical calculation (Cheng & Mix, 2014; Gilligan, Thomas, & Farran, 2019), although it should be noted that not all studies have found causal effects between training spatial skills and mathematics (Hawes, Moss, Caswell, & Poliszczuk, 2015; Lowrie, Logan, & Ramful, 2017; Xu & LeFevre, 2016). In order to understand the relationship between MI components and mathematical calculation in children, and ultimately contribute to the literature aiming to produce impactful findings that could have implications in mathematics learning in the classroom, primary aged children (6–11 years) were recruited. We hypothesize that all sub-components of MI will be associated with mathematical calculation. However, we propose that there are different underlying reasons for these associations. We predict associations between image generation and image maintenance and mathematics due to requirements for visual precision of representations during mathematics calculation, that is, visual representations of symbolic numbers and quantities. We predict associations between mental rotation and our mathematics task, due to shared requirements for spatial transformation, that is, the requirement to mentally manipulate numbers in an equation (or other abstract representations of quantity) to solve missing term problems. We predict associations between image scanning and mathematics because accurately representing the spatial location of an object within a scene, that is, the relative positions of different numbers and symbols within an equation, is required for calculation particularly for missing term problems. We have no predefined hypotheses as to which sub-component will explain the most variation in mathematics performance. By comparing the role of these individual sub-components of MI in explaining calculation outcomes, we will provide novel insights into the relationship between spatial visualization and mathematics, beyond mental transformation tasks.

METHODS

Participants

A priori power analysis using GPower was used to determine the sample size required for multiple regression with five predictors. Power was set at 80%, alpha level was 0.5, and effect size was set at medium ($f^2 = .15$). This was based on previous estimates of mental rotation-math relations (Gilligan, Hodgkiss, Thomas, & Farran, 2019). The suggested sample size was $N = 92$. The final sample consisted of 92 primary school children aged 6–11 years recruited from two primary schools. Participant details are described in Table 1. Both parents and children provided written informed consent. The study was approved by the University Ethics Committee and was conducted in accordance with the Declaration of Helsinki.

Table 1
Demographics for Sample

Age group	N (female)	Age in years, mean (SD)
6–7-year-olds	31 (20)	6.94 (.54)
8–9-year-olds	26 (16)	8.47 (.54)
10–11-year-olds	35 (14)	10.63 (.36)

Materials and Procedure

The data presented here was collected as part of a larger study. Each participant completed three 30-min sessions. All participants completed the missing number problems first to avoid possible mathematics improvements due to spatial training effects. Task order of all other tasks was counterbalanced. All tasks were computer-based and were presented on a 23-inch ASUS LCD monitor running with MATLAB v2018a and PsychToolbox v3.0.14.

Image generation

The image generation task was adapted from Wimmer et al. (2015) and is as used in Bates and Farran (2020). In this task, participants are required to memorize an abstract shape, which they later have to recall in the form of a visual image in order to detect a subtle difference in a target shape. The purpose of the distractor task, which is completed immediately after the memorization of the shape, is to prevent participants from being able to maintain the image in mind. Instead, they have to subsequently retrieve information from long-term memory in order to generate the visual image. Each trial was comprised of four steps (see Figure 1). First, an abstract shape was presented on screen for 15 s. Participants were instructed to try and remember this shape (the reference shape). Second, a checkerboard mask was presented onscreen for 500 ms to remove after-effects. Third, participants completed a paper and pencil letter cancellation task for 30 s, that is, a distractor task. In this task, participants were required to search a 16×20 grid of letters to find the letters T and G (5 Ts and 5 Gs). Fourth, participants were presented with another abstract shape (target shape) and were asked to identify whether the shape was the same or different to the first shape they had seen (reference shape). Participants responded by pressing a labeled key on the keyboard: either, “YES” (the shape looked different) or “NO” (the shape did not look different). If the participant pressed the “YES” key, they had to point to the part of the shape they thought looked different and the experimenter clicked it with the mouse to record accuracy.

In all trials, the target shape presented was either the same as the reference shape (33% of trials) or different to the reference shape (67% of trials). In difference trials, the target shape was different to the reference shape based on

Fig. 1. (a) Image generation difference trial: presentation of the reference shape for 15 s, presentation of checkerboard mask for 500 ms, letter cancellation distractor task for 30 s, presentation of target shape. (b) Image maintenance difference trial: presentation of the reference shape for 15 s, presentation of checkerboard mask for 500 ms, fixation cross for 3,000 ms, presentation of target shape.

one alteration: either the concave or the convex attribute of a curved line was increased or the angle between two straight lines was reduced/increased (see Figure 1a). Reference shapes were presented in a different location for each trial to avoid expectancy and target shapes were presented in the same location of the reference shape of that trial. The order of trial presentation was randomized per participant. Participants completed four practice trials (two same and two different trials randomized). If participants answered two or more practice trials incorrectly, the practice block was repeated. Practice trials were followed by 15 experimental trials.

Participants could score 0, 1, or 2 on a trial. Incorrect responses on same or difference trials were scored as 0, correct responses on same trials were scored as 1. Correct responses on difference trials were scored as 1 if the participant correctly identified that the shape was different but did not correctly identify the location of the difference on the shape, which reflects a visual image of low precision. Correct responses on difference trials were scored as 2 if the participant correctly identified both a different shape and the exact location of the difference on the shape, which reflects a visual image of high precision. Therefore, the overall accuracy dependent variable had a range of 0–25.

Image maintenance

The procedure for the image maintenance task was almost identical to the image generation task described above. It differed only in that participants were not presented with the paper and pencil cancellation task. Instead, participants

Fig. 2. Example stimulus from the Mental Rotation task (45° anti-clockwise trial).

were instructed to hold the shape in mind and look at the fixation cross presented in the center of the screen for 3,000 ms. Participants were then presented with the target shape and responded in the same manner as in the image generation task (see Figure 1b). A different set of abstract shapes were used in the image maintenance task to avoid practice effects. All other details of the task were identical to the image generation Task, including type and number of trials, practice trials, and scoring system.

Mental rotation

Participants were required to identify which of two mirror-images presented above a horizontal line matched a target image below the line (Gilligan, Thomas, & Farran, 2019). Participants used the left and right arrow keys on the computer keyboard to respond. Degree of rotation differed across trials such that the target image below the line was presented at 0°, 45°, 90°, 135°, or 180° (eight trials per degree of rotation; see Figure 2). Equal numbers of clockwise and anti-clockwise trials were presented. The presentation order of the 40 experimental trials was randomized per participant. Trials were displayed onscreen until participants responded. Trials were separated by a fixation point visible for 500 ms. Four practice trials at 0° were completed, the first two with feedback. If participants scored <75% the practice was repeated. Five animal stimuli were used in the task including an elephant (practice trials), tiger, zebra, dog, and horse. Each stimulus occupied approximately the same surface area. Percentage accuracy was recorded.

Image scanning

The image scanning task used here was adapted from Wimmer et al. (2016a, 2016b) and is as used in Bates and Farran (2020). Participants first viewed a map of an island containing a lighthouse, volcano, hut, pond, and tree for

45 s whereby they were instructed to memorize the island and the locations of the landmarks (see Figure 3). To ensure that participants correctly remembered the locations of the landmarks, they were then shown an empty island and were required to click on the location of each landmark. Following the participant click, the corresponding landmark appeared in the correct location and participants could compare their prediction.

Next the island disappeared, and participants had to close their eyes and imagine the island. For the experimental trials, the experimenter read a list of five pairs of landmarks in the following order: Lighthouse-Tree (262 mm), Lighthouse-Volcano (81 mm), Hut-Pond (100 mm), Hut-Lighthouse (70 mm), and Lighthouse-Pond (154 mm). When the experimenter said “Start,” the participant had to imagine the parrot walking from the first landmark to the second landmark and they were instructed to say “Stop” when he reached the second landmark. For all trials, response time (RT) was recorded from when the experimenter said “Start” until the participants’ “Stop” command. The practice was first completed with a map of a park. Participants watched a video of the parrot walking between the park landmarks. The experimenter said three lists of pairs (e.g., Bench-Swing), and the participant had to imagine the parrot walking between the landmarks. Responses were recorded as in the experimental trials.

Recent research has shown that an index of metric properties score can be calculated based on RT ratios to capture participants’ ability to internally represent distance (Bates & Farran, 2020). Moreover, research has demonstrated a linear time–distance relationship between RT and distance, whereby RT increases as a function of distance in Image Scanning (Borst & Kosslyn, 2010; Wimmer et al., 2016a); therefore the RT ratios are derived based on this assumption. To calculate this score, two values are required. First, actual distance ratios were calculated by dividing each actual distance on the map (81 mm, 100 mm, 154 mm, 262 mm) by the shortest distance on the map (70 mm; e.g., actual ratio 1: 81/70 = 1.16). Second, imaged distance ratios were calculated for each participant by dividing their RT for each distance by their RT for the first distance. Participants’ metric properties score was then calculated by subtracting each imaged distance ratio from each actual distance ratio and squaring the result. Thus, the closer the score to 0, the lower the deviation in imaged distance ratio from actual distance ratio, that is, the higher the ability to accurately represent varying spatial locations.

Missing number problems

The missing number problems were taken from Gilligan, Thomas, and Farran (2019) who adapted them from Cheng and Mix (2014). In each trial, participants were required to complete a simple mathematics equation by identifying what

Fig. 3. Island scanning task with distances: Hut-Lighthouse (70 mm), Lighthouse-Volcano (81 mm), Hut-Pond (100 mm), Lighthouse-Pond (154 mm), Lighthouse-Tree (262 mm). Lines between landmarks and distance labels added for reference and not presented during task.

$$3 + \square = 4$$

Fig. 4. Example stimulus from Missing Number Problems (presented in the center of the screen).

number should go in the empty box to complete the equation (see Figure 4). Twenty-one items were presented in order of increasing difficulty with a 25 s time limit per item. The type of item, addition versus subtraction and single versus multi-digit numbers, and the position of the missing box, was balanced across items. Participants completed two practice items with feedback. Percentage accuracy was recorded.

Analysis strategy

Analyses were preregistered (<https://osf.io/m2s5q/>). A one-way analysis of variance (ANOVA) investigating development of missing number problems ability across age groups is additional to the preregistration.

Tests of normality revealed that the image generation, image maintenance, and image scanning were normally distributed (Kolmogorov-Smirnov tests, p 's > .05), whereas mental rotation and missing number problems were not normally distributed (p 's < .05). However, the sample size was large enough for the central limit theorem to apply, and thus

parametric tests were used (Field, 2013). Adjusted R^2 are reported for all regression analyses.

RESULTS

Descriptive statistics are detailed in Table 2. One sample t -tests per age group were conducted between missing number problems ability and 0% and missing number problems ability and 100% to check for floor and ceiling effects, respectively. This revealed no floor or ceiling effects for any age groups (p 's < .05). Despite unequal proportions of males and females, there were no gender differences for any measures (p 's > .05). Partial correlations controlling for age were conducted between all variables to inform the subsequent regression model (see Table 3). After controlling for age, only mental rotation was positively associated with missing number problems, $r(90) = .255$, $p = .015$ (Figure 5).

To determine developmental differences in performance on missing number problems, we conducted a one-way ANOVA between age groups (6–7-year-olds, 8–9-year-olds, 10–11-year-olds) on missing number problems accuracy. There was a significant main effect of age, Welch's $F(2,48.18) = 12.49$, $p < .001$, $\eta^2 = .21$. Post-hoc comparisons revealed a significant increase in accuracy between 6–7-year-olds and 10–11-year-olds ($p < .001$) only (all other comparisons: $p > .05$). Due to the age-based differences in missing number problems, we controlled for

Table 2
Descriptive Statistics for All Variables

	<i>Missing number problems</i>			<i>Image generation</i>			<i>Image maintenance</i>			<i>Mental rotation</i>			<i>Image scanning</i>		
	<i>6–7 years</i>	<i>8–9 years</i>	<i>10–11 years</i>	<i>6–7 years</i>	<i>8–9 years</i>	<i>10–11 years</i>	<i>6–7 years</i>	<i>8–9 years</i>	<i>10–11 years</i>	<i>6–7 years</i>	<i>8–9 years</i>	<i>10–11 years</i>	<i>6–7 years</i>	<i>8–9 years</i>	<i>10–11 years</i>
Mean	65.89	76.74	84.76	13.00	14.19	15.97	14.36	14.89	16.54	74.30	73.27	85.43	1.56	1.50	1.39
Standard deviation	19.52	17.58	9.65	3.26	3.20	3.19	3.61	3.08	2.67	13.88	14.09	14.47	0.76	0.92	0.72
Skewness	−1.23	−4.02	−0.92	−0.21	0.49	−0.57	−0.18	−1.76	0.13	0.67	0.09	−2.88	1.35	0.39	0.83
Min.	19.05	19.05	66.67	8.00	9.00	7.00	7.00	7.00	11.00	52.50	50.00	55.00	0.29	0.13	0.06
Max.	100.00	100.00	100.00	19.00	21.00	22.00	22.00	20.00	22.00	100.00	97.50	100.00	3.52	3.16	2.84

Table 3
Pearson's Correlation Matrix Between All Variables Controlling for Age

	1.	2.	3.	4.	5.
1. Missing number problems	—				
2. Image generation	.169	—			
3. Image maintenance	.152	.196	—		
4. Mental rotation	.255*	.053	−.053	—	
5. Image scanning	.053	−.052	.086	−.216*	—

* $p < .05$.

Fig. 5. Partial correlation between missing number problems and mental rotation controlling for age.

age in the subsequent regression model, thus enabling us to investigate the contribution of MI ability in explaining variance in missing number problems ability over and above age.

A stepwise multiple regression was carried out. In step 1, age was added. In step 2, all MI measures were added: image generation accuracy, image maintenance accuracy, mental rotation accuracy, and image scanning index of metric properties. Collinearity statistics indicated appropriate

Tolerance and VIF scores, where the cutoff was set at >0.2 and <10 , respectively (Menard, 1995; Myers, 1990). In step 1, age accounted for 24% of the variance in missing number problems. The MI measures added in step 2 accounted for an additional 5.9% of the variance. Age and mental rotation were significant predictors in the final model, which together accounted for 29.9% of the variance (see Table 4).

DISCUSSION

The current study examined the contribution of each component of MI (image generation, image maintenance, mental rotation, and image scanning) to mathematics calculation. We found that age explained a substantial amount of variation in calculation skill (24%). However, both age and mental rotation were significant variables in the final model, with mental rotation accounting for 5.9% unique variance in calculation skill. In this study, we found no significant associations between other sub-components of MI and calculation performance. Our findings suggest that there may be something particularly important about the ability to spatially transform mental representations in the mathematics classroom.

From a theoretical perspective, we found a distinction between mental rotation, a measure of spatial transformation, and other sub-components of MI in their relative contribution to mathematics ability. Thus, we propose that future mechanistic explanations of the role of spatial visualization in mathematics should be refined to a role for spatial transformations, as opposed to generating, maintaining, or scanning visual representations. Our findings of differential associations are supported by evidence of neural activation differences that also show dissociations between spatially transforming information and visually representing information. Evidence shows that early visual areas support depiction of visual representations in MI and visual perception (e.g., Dijkstra et al., 2019), while activation in the parietal cortices is implicated in spatial transformation and numerical operations (Hawes, Sokolowski, Ononye, & Ansari, 2019; Zacks, 2008). Thus, the evidence suggests that

Table 4
Factors Predicting Performance on the Missing Number Problems

	b	SE b	β	t	p	F(df)	p	Adj. R ²	Δ Adj. R ²
Step 1									
Age	5.307	.974	.498***	5.451	<.001	29.72(1,90)	<.001	.240	
Step 2									
Age	3.645	1.087	.342**	3.352	.001	8.78(5,86)	<.001	.299	.059
Image generation	.644	.498	.125	1.292	.200				
Image maintenance	.656	.521	.121	1.259	.211				
Mental rotation	.295	.110	.253**	2.685	.009				
Image scanning	2.128	2.034	.095	1.046	.298				

** $p < .01$, *** $p < .001$.

spatial transformation may have a closer neural link to number skills. Future research could also investigate whether this association between spatial transformation and mathematics is exclusive to mental rotation, or whether it extends to other spatial transformations, including, for example, translation and folding. Evidence from previous studies of maths (Cheng & Mix, 2014) and science (Hodgkiss, Gilligan, Tolmie, Thomas, & Farran, 2018) suggest that the role of transformation may not be limited to rotation.

Transfer effects from mental rotation training to improvements in mathematics calculation can be interpreted in the context of spatial transformation strategies and visualizing strategies, respectively (Cheng & Mix, 2014; Gilligan, Thomas, & Farran, 2019; Lourenco et al., 2018). The spatial transformation strategy suggests that children complete calculation problems by mentally manipulating and restructuring equations, for example, $4 + _ = 9$, could become $9 - 4 = _$ (Cheng & Mix, 2014). Given that the rules associated with this transformation are advanced (i.e., changing a plus to a minus when moving across an equal sign), this strategy is somewhat unlikely for young children. However, other transformations like retrieving common number facts $5 + 5 = 10$ and transforming them to match the required equation (e.g., take one from each side to get: $4 + 5 = 9$), would also be effective in solving missing number problems. Conversely, the visualization strategy suggests for example that children visualize the numbers as blocks; e.g., four blocks and nine blocks and count the difference (Gilligan, Thomas, & Farran, 2019; Lourenco et al., 2018). The evidence presented here supports the former strategy. It also appears to be the best explanation for difficult equations, for example, $_ + 9 = 25$, where it is unlikely that visualizing 9 and 25 blocks would be efficient.

It might be argued that working memory underlies relationships between the MI components and mathematical calculation in this study, given evidence for associations between MI and working memory in adults (e.g., Keogh & Pearson, 2011, 2014) and evidence for a relationship between working memory and mathematics in children

(see Friso-Van Den Bos, Van Der Ven, Kroesbergen, & Van Luit, 2013 for review). However, the goal of the current article is to derive the MI components (as opposed to any working memory components) of our MI tasks. Furthermore, in another recent article using the same sample of children included in the current study, we investigated the relationship between these MI components and forward and backward spatial span in working memory. We found no significant correlations between the MI components and working memory measures (Bates & Farran, 2020). Thus, we are confident that the dependent variables derived from the MI component tasks are distinct from working memory and are sensitive to generating and maintaining a visual image (image generation and image maintenance), transforming a mental image (mental rotation) and internally representing distance (image scanning).

A limitation of this study is that we only included one measure of mathematics ability. While we investigated mathematics calculation as it has been shown previously to involve spatial visualization, future studies should extend this research to other domains of mathematics. For example, previous research has speculated that visual representations are needed for solving mathematical word problems (Hegarty & Kozhevnikov, 1999; van Garderen, 2006). Within the domain of mathematics calculation, future research should also investigate whether strategies vary depending on difficulty of the equation at hand. Specifically, visualizing numbers (i.e., image generation) as blocks as suggested previously (Lourenco et al., 2018) might be most efficient in simpler equations using single-digit numbers, whereas transformation strategies may be more efficient in more difficult equations with double-digit numbers. Finally, this study was not designed to investigate the differential relations between sub-components of MI and calculation skills in different age groups, and thus we did not have the power to do so. There is evidence that spatial-mathematical relations change through development (Gilligan, Hodgkiss, et al., 2019; Mix et al., 2016). Future studies could therefore investigate the sub-components of

MI and mathematics at different developmental ages to identify whether associations between sub-components of MI and mathematics are developmentally sensitive.

To conclude, our study is the first to examine how sub-components of MI underpin mathematical calculation. We supported previous evidence by showing that mental rotation explains unique variation in mathematics calculation, and we extended these findings by demonstrating that image generation, image maintenance, and image scanning do not uniquely contribute to mathematical calculation. Until now, spatial visualization strategies have been proposed as the mechanistic explanation for the relationship between mental rotation and mathematical calculation. This study provides evidence to demonstrate that this assumption is incorrect and that the association is specific to spatial transformations only. This novel evidence is of theoretical importance in that it shows that when describing cognitive mechanisms of mathematical calculation, the definition of spatial visualization used should emphasize spatial transformations, as opposed to generating visual representations.

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