A Tight Upper Bound for Enhanced DCT-OFDM with Index Modulation

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Abstract—Orthogonal frequency division multiplexing (OFDM) with index modulation (IM) (OFDM-IM), which employs the activated sub-carrier indices to convey information, exhibits higher energy efficiency and lower peak-to-average power ratio (PAPR) than conventional OFDM systems. To further improve the throughput of discrete Fourier transform (DFT) based OFDM-IM (DFT-OFDM-IM), discrete cosine transform (DCT) based OFDM-IM (DCT-OFDM-IM) can be employed with double subcarriers given the same bandwidth. However, one of the main disadvantage of DCT-OFDM-IM is its lack of circular convolutional property over a dispersive channel. To address this issue, an enhanced DCT-OFDM-IM (EDCT-OFDM-IM) system has been proposed by introducing symmetric prefix and suffix at the transmitter and a pre-filter at the receiver leading to better performance than DFT-OFDM-IM in terms of bit error rate (BER). However, due to its special structure, it is difficult to derive the accurate average bit error probability (ABEP) upper bound, which is essential for the performance evaluation. In this paper, a tight ABEP upper bound is derived using the moment-generating-function (MGF). Our theoretical analysis is validated by simulation results and proven to be very accurate. Consequently the advantages of the EDCT-OFDM-IM system over the classic OFDM-IM system are further demonstrated analytically.

Index Terms—Discrete Cosine Transform (DCT), Inverse Discrete Cosine Transform (IDCT), Orthogonal Frequency Division Multiplexing (OFDM), Index Modulation (IM), Averaged Bit Error Probability (ABEP), Pair-wised Error Probability (PEP)

I. INTRODUCTION

Orthogonal frequency division multiplexing with index modulation (OFDM-IM), which employs the activated subcarriers indices as a completed new degree of freedom to convey information, is a novel modulation technique and has attracted increasing attentions. Specifically, in the OFDM-IM scheme, parts of the subcarriers are activated to transmit conventional amplitude phase modulation (APM) symbols, so that the information bits are conveyed both by the activated subcarrier indices as well as APM symbols. Since some subcarriers remain dormant, the OFDM-IM scheme is capable of exhibiting better bit error ratio (BER) as well as lower peak-to-average power ratio (PAPR) over its classic OFDM counterpart [1] - [7]. Several works have demonstrated that OFDM-IM is a promising candidate for vehicle-to-vehicle (V2V) or vehicle-to-

infrastructure (V2X) [8], [9], as well as 5G and beyond wireless communication scenarios [10].

However, the current research on OFDM-IM mainly focuses on the discrete Fourier transform (DFT) based OFDM-IM systems. Inspired by the merits of discrete cosine transform (DCT) based OFDM system, we proposed a novel enhanced DCT-OFDM-IM (EDCT-OFDM-IM) scheme to further exploit the benefits of OFDM-IM and DCT techniques, where the DFT operation is replaced by the DCT [2]. By contrast to the classic DFT-OFDM-IM scheme, the EDCT-OFDM-IM scheme employs the symmetric prefix and suffix at the transmitter, and prefiltering is used at the receiver to efficiently solve the circular convolutional problem encountered in the DCT-OFDM-IM of [12]. EDCT-OFDM-IM is capable of doubling the number of subcarriers compared to OFDM-IM given the same bandwidth, resulting in higher spectral efficiency and better bit error rate (BER) performance. We demonstrated the advantages of the EDCT-OFDM-IM scheme in [2] by simulations. Although a semi-analytical BER performance has been studied in [2], it relies on generating randomly channels using a closed form of Q-function. Due to its special structure, it is difficult to derive the closed form expression of the pair-wised error probability (PEP).

To tackle this issue, a tight upper bound of average bit error probability (ABEP) is proposed for EDCT-OFDM-IM using moment-generating-function (MGF) in this paper. More specifically, the correlation coefficients of the adjacent subcarriers of the EDCT-OFDM-IM is obtained with the aid of derived effective channel diagonal matrix, to facilitate the performance analysis. The proposed theoretical ABEP upper bound is further validated by the simulation results. Both simulation and theoretical results further demonstrate the advantages of the EDCT-OFDM-IM scheme.

Notation: $E[\cdot]$ calculates the expected value of any random variable. $\|\cdot\|_F$ denotes the Frobenious norm of a matrix. $|\cdot|$ represents the magnitude of a complex quantity or the cardinality of a given set. $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for the transpose, conjugate and Hermitian transpose of a vector/matrix, respectively. The dimension $N \times N$ matrix $\mathbf{I}_N$, $\mathbf{J}_N$ and $\mathbf{0}_{\Omega \times N}$, are the identity matrix, reversal matrix and zero matrix, respectively. (Ω, K, N) denotes an IM scheme where K subcarriers are activated within a group of Ω subcarriers, and N is the total number of available subcarriers.

II. SYSTEM MODEL OF EDCT-OFDM-IM

The proposed enhanced DCT-OFDM with index modulation (EDCT-OFDM-IM) scheme consists of an inverse DCT (IDCT) module followed by a symmetric prefix and suffix insertion at transmitter side, and a corresponding pre-filtering at the front-end of the receiver side, in order to obtain the circular convolu-

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Fig. 1. System model of EDCT-OFDM with index modulation.

tional property as DFT-OFDM. It not only enables simply one-tap equalization, but also provides robustness against inherent inter-channel interference (ICI) and inter-symbol interference (ISI).

Moreover, the EDCT-OFDM-IM can further reap the benefits of the DCT-OFDM and IM techniques, and achieves better BER performance over its conventional DFT-OFDM-IM counterparts [9]. Fig. 1 shows a block diagram of an EDCT-OFDM-IM system.

A. Transmitter

In the EDCT-OFDM-IM system, information bits stream to be transmitted is firstly divided into G groups with each group containing B bits. In each group, B bits are further split into P_1 and P_2 bits for an index selector and an ordinary M -ary ASK modulator respectively. Index modulator selects K active subcarriers among Ω subcarriers according to the index selector to accommodate K M -ary modulated symbols. The output of G index modulators which contains $N = G \times \Omega$ subcarriers in total is then fed into an inverse DCT (IDCT) module followed by the prefix and suffix insertion.

We denote the output of each index modulator by $\mathbf{x}_g = [x_{(g,0)}, x_{(g,1)}, \dots, x_{(g,\Omega-1)}]^T$, $g = 0, 1, \dots, G-1$, in which there are $\Omega - K$ zeros. Then the data symbol vector fed into IDCT can be expressed as:

$$\mathbf{X} = [X_0, X_1, \dots, X_{N-1}] = [\mathbf{x}_0^T, \mathbf{x}_1^T, \dots, \mathbf{x}_{G-1}^T]^T. \quad (1)$$

For each group, K out of Ω available subcarriers are activated to map $p_1 = \lfloor \log_2 C_\Omega^K \rfloor$ bits into a set of subcarrier indices combinations. On the other hand, $p_2 = K \log_2(M)$ bits are modulated to M -ary ASK data symbols with a set size of M , and subsequently transmitted by the K active subcarriers. Accordingly, the total number of bits transmitted per EDCT-OFDM-IM block is $G \cdot (p_1 + p_2) = GB$. After subcarrier IM, the g -th group signal can be written as

$$\mathbf{x}_g = [0, \dots, s_{(g,0)}, 0, \dots, s_{(g,1)}, 0, \dots, s_{(g,K-1)}, 0, \dots]^T, \quad (2)$$

where $s_{(g,k)}$ ($k = 0, 1, \dots, K-1$) represents the M -ary ASK constellation point, and $\mathbf{x}_g \in \Lambda$, where Λ is the set of all possible symbol vector combinations. In general, the symmetric extended prefix and suffix are assumed to have the same length

v . Therefore, the total block length is $L_1 = N + 2v$ and the transmitted signal vector is represented by

$$\mathbf{u} = \mathbf{T}_{PS} \mathbf{D}^H \mathbf{x}, \quad (3)$$

where $\mathbf{D} \in \mathbb{R}^{N \times N}$ is a power normalised type-II DCT [11] matrix with its (l, m) entry given by:

$$C(l, m) = \begin{cases} \sqrt{\frac{2}{N}} \cos\left(\frac{(l-1)(2m-1)\pi}{2N}\right) & l > 1 \\ \sqrt{\frac{1}{N}} & l = 1, \end{cases} \quad (4)$$

and $\mathbf{T}_{PS} = [\mathbf{J}_v, \mathbf{0}_{v \times (N-v)}; \mathbf{I}_N, \mathbf{0}_{v \times (N-v)}, \mathbf{J}_v]$ is the $L_1 \times N$ matrix that inserts the prefix and suffix at both sides of a data symbol block.

B. Receiver

Denote the vector for a L -tap Rayleigh fading channel by $\mathbf{h} = [h_1, h_2, \dots, h_L]$. Its corresponding time-reverse pre-filter vector is thus in the reverse form as $\mathbf{p} = [h_L, h_{L-1}, \dots, h_1]$. Correspondingly, the matrix form of the multipath channel $\mathbf{H} \in \mathbb{R}^{L_1 \times L_1}$ is represented as a Toeplitz matrix with its first row and first column being $[h_L, h_{L-1}, \dots, h_1, \mathbf{0}_{1 \times (L_1-L)}]$ and $[h_L, \mathbf{0}_{1 \times (L_1-1)}]^T$ respectively, while the matrix form of the pre-filter $\mathbf{P} \in \mathbb{R}^{L_1 \times L_1}$ is represented also by a Toeplitz matrix with its first row and column being $[h_L, \mathbf{0}_{1 \times (L_1-1)}]$ and $[h_L, h_{L-1}, \dots, h_1, \mathbf{0}_{1 \times (L_1-L)}]^T$, respectively.

Thus, at the receiver side, by performing the time-reverse pre-filtering and the DCT demultiplexing, the received frequency-domain sample vector obtained after guard sequence removal can be expressed as

$$\mathbf{z} = \mathbf{D} \mathbf{R}_{PS} \mathbf{P} (\mathbf{H} \mathbf{u} + \mathbf{n}). \quad (5)$$

The guard sequence removal operation is denoted by the matrix $\mathbf{R}_{PS} = [\mathbf{0}_{N \times v}, \mathbf{I}_N, \mathbf{0}_{N \times v}]$. $\mathbf{n} \in \mathbb{R}^{L_1 \times 1}$ is the additive white Gaussian noise (AWGN) noise vector with its elements subject to Gaussian distribution with zero mean and variance N_0 .

III. PERFORMANCE ANALYSIS OF EDCT-OFDM-IM

Recalling the aforementioned one-tap equalization mechanism, with the introduction of a pre-filter and the symmetric guard sequence, the EDCT-OFDM-IM is now applicable for ICI and ISI free transmission. The selection of $\mathbf{T}_{PS}$ and $\mathbf{R}_{PS}$ diagonalizes the output of $\mathbf{D} \mathbf{R}_{PS} \mathbf{P} \mathbf{H} \mathbf{T}_{PS} \mathbf{D}^H$ into an effective diagonal matrix $\mathbf{H}_{eff}$ and Eq. (5) thus can be simplified to

$$\mathbf{z} = \mathbf{H}_{eff} \mathbf{x} + \mathbf{G}_{eff} \mathbf{n}, \quad (6)$$

where $\mathbf{H}_{eff} \in \mathbb{R}^{N \times N}$ is the effective channel diagonal matrix with its elements expressed by $\text{diag}(\mathbf{H}_{eff}) = [H_0, H_1, \dots, H_{N-1}]$, and $\mathbf{G}_{eff} = \mathbf{D} \mathbf{R}_{PS} \mathbf{P}$ represents the effective noise correlation matrix. Furthermore we can derive the effective channel diagonal matrix with its k -th diagonal elements (i.e., the channel frequency response for the k -th subcarrier) as follows:

$$H_k = \frac{\sum_{m=1}^{L-1} C(k, m) (\bar{h}_{m+1} + \bar{h}_m) + C(k, L) \bar{h}_L}{C(k, 1)}. \quad (7)$$

Where $\bar{h}_m$ is the m -th element of convolution of $[h_1, h_2, \dots, h_L]$ and $[h_L, h_{L-1}, \dots, h_1]$, i.e., $\bar{h}_m = \sum_{i=1}^m h_i * h_{L-m+i}$. Due to the pre-filtering procedure, the noise vector becomes correlated and subject to different kinds of correlation gains. In order to calculate the effective noise variance for each subcarrier, we

take a scalar form of (6), as expressed by

$$z_k = H_k x_k + \sum_{i=1}^N g_{k,i} n_i, \quad (8)$$

where x_k is the transmitted symbol at k -th subcarrier, and z_k and n_i are the received observation and AWGN sample at the k -th subcarrier and i -th subcarrier, respectively, whereas $g_{k,i}$ is the entry in the k -th row and i -th column of $\mathbf{G}_{eff}$.

As long as the OFDM symbol duration is far less than the coherence time, $g_{k,i}^2$ can be treated as a constant noise coefficient and the overall coloured noise coefficient on arbitrary subcarrier index is in the summation of the noise coefficient from all subcarriers. Accordingly, the coloured noise variance v_k at the k -th subcarrier can be obtained as

$$v_k = \frac{N_0}{2} \sum_{i=1}^N g_{k,i}^2. \quad (9)$$

With the instantaneous coloured noise variance v_k , an optimum maximum likelihood detector for the g -th group can be formulated accordingly. Denoting by $z_{\epsilon+gG} = z_{(g,\epsilon)}$, $H_{\epsilon+gG} = H_{(g,\epsilon)}$ and $v_{\epsilon+gG} = v_{(g,\epsilon)}$, where ϵ is the subcarrier offset to the first subcarrier index within group g , i.e., $\epsilon \in 0, 1, 2, \dots, \Omega - 1$, and g is the group index, $0, 1, \dots, G - 1$, then the transmit symbols in the g -th group can be estimated as

$$\mathbf{x}_t = \arg \min_{\mathbf{x}_g \in \Lambda} \sum_{\epsilon=0}^{\Omega-1} \frac{|z_{(g,\epsilon)} - H_{(g,\epsilon)} x_{(g,\epsilon)}|^2}{v_{(g,\epsilon)}}. \quad (10)$$

Eq. (10) can be further simplified:

$$\mathbf{x}_t = \arg \min_{\mathbf{x}_g \in \Lambda} \sum_{\epsilon=0}^{\Omega-1} w_{g,\epsilon} |y_{(g,\epsilon)} - x_{(g,\epsilon)}|^2, \quad (11)$$

where $y_{g,\epsilon}$ is the output of the one-tap equalizer at subcarrier ϵ in the g -th group, and $w_{g,\epsilon}$ is a weight only related to the channel:

$$w_{g,\epsilon} = \frac{|H_{g,\epsilon}|^2}{v_{(g,\epsilon)}}. \quad (12)$$

A. Average bit error probability

With the above clarification, we can obtain the averaged bit error probability (ABEP) of the i -th OFDM symbol. For the sake of simplicity, we drop i in all equations. ABEP of $\mathbf{x}_g$ detected as $\mathbf{x}_t$ then can be expressed as [13]:

$$P_{b,g} = \frac{1}{B \cdot 2^B} \sum_{\forall \mathbf{x}_g} \sum_{\forall \mathbf{x}_t \neq \mathbf{x}_g} P(\mathbf{x}_g \rightarrow \mathbf{x}_t) d(\mathbf{x}_g, \mathbf{x}_t), \quad (13)$$

where $d(\mathbf{x}_g, \mathbf{x}_t)$ is the Hamming distance (HD) of $\mathbf{x}_g$ and $\mathbf{x}_t$, and $\mathbf{H}_g = [H_{0+gG}, H_{1+gG}, \dots, H_{(\Omega-1)+gG}]$ is the effective channel coefficients in the g -th group shown in Eq. (7). Hence, the calculation of the pair-wised error probability (PEP) event $P(\mathbf{x}_g \rightarrow \mathbf{x}_t)$ becomes pivotal. Specifically, we first derive $P(\mathbf{x}_g \rightarrow \mathbf{x}_t | \mathbf{H}_g)$ as

$$\begin{aligned} P(\mathbf{x}_g \rightarrow \mathbf{x}_t | \mathbf{H}_g) &= P(\|\mathbf{z}_g - \mathbf{H}_g \mathbf{x}_g\|^2 \geq \|\mathbf{z}_g - \mathbf{H}_g \mathbf{x}_t\|^2) \\ &= P(\|\mathbf{n}_g\|^2 \geq \|\mathbf{H}_g(\mathbf{x}_g - \mathbf{x}_t) + \mathbf{n}_g\|^2) \approx Q\left(\sqrt{\frac{\|\mathbf{H}_g(\mathbf{x}_g - \mathbf{x}_t)\|^2}{2\sigma_e^2}}\right). \end{aligned} \quad (14)$$

Furthermore, if considering non-perfect channel estimation at the receiver, which means the channel estimates used in the demodulation may contain errors shown below:

$$\tilde{\mathbf{H}}_g = \mathbf{H}_g + \mathbf{H}_e^{(g)} \quad (15)$$

where $\mathbf{H}_e^{(g)} \in \mathcal{CN}(0, \sigma_e^2)$ and σ_e^2 corresponds to accuracy of pragmatic channel estimates, it is inversely proportional to the

signal-to-noise-ratio (SNR). Then the PEP can be rewritten as:

$$\begin{aligned} P(\mathbf{x}_g \rightarrow \mathbf{x}_t | \tilde{\mathbf{H}}_g) &= P(\|\mathbf{z}_g - \tilde{\mathbf{H}}_g \mathbf{x}_g\|^2 \geq \|\mathbf{z}_g - \tilde{\mathbf{H}}_g \mathbf{x}_t\|^2) \\ &= P(\|\mathbf{n}_g - \mathbf{H}_e^{(g)} \mathbf{x}_t\|^2 \geq \|\mathbf{H}_g(\mathbf{x}_g - \mathbf{x}_t) + \mathbf{n}_g - \mathbf{H}_e^{(g)} \mathbf{x}_t\|^2) \\ &\approx Q\left(\sqrt{\frac{\|\mathbf{H}_g(\mathbf{x}_g - \mathbf{x}_t)\|^2}{2(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2)}}\right). \end{aligned} \quad (16)$$

Assuming that $\beta = \|\mathbf{H}_g(\mathbf{x}_g - \mathbf{x}_t)\|^2 = \mathbf{H}_g \mathbf{A} \mathbf{A}^H (\mathbf{H}_g)^H$ and $\mathbf{A} = (\mathbf{x}_g - \mathbf{x}_t)$, the average PEP event in general can be obtained by

$$P(\mathbf{x}_g \rightarrow \mathbf{x}_t) = \int_{\beta} Q\left(\sqrt{\frac{\beta}{2(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2)}}\right) f_{\beta}(\beta) d\beta. \quad (17)$$

B. PEP based on accurate Q-function

Before proceeding further, we use another equivalent form of Q-function:

$$Q(t) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{t^2}{2 \sin^2 \theta}\right) d\theta. \quad (18)$$

Then Eq. (17) can be rewritten as

$$\begin{aligned} P(\mathbf{x}_g \rightarrow \mathbf{x}_t) &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_{\beta} \exp\left(-\frac{\beta}{4(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2) \sin^2 \theta}\right) f_{\beta}(\beta) d\beta d\theta \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Gamma_{\beta}\left(-\frac{1}{4(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2) \sin^2 \theta}\right) d\theta, \end{aligned} \quad (19)$$

where $\Gamma_{\beta}(s) = \int_{\beta} e^s f_{\beta}(\beta) d\beta$ and $f_{\beta}(\beta)$ is the MGF and the probability density function (PDF) of β respectively.

According to [16] [17], the generic MGF is further calculated as

$$\Gamma_{\beta}(s) = \frac{\exp(s \bar{\mathbf{H}}_g \mathbf{A} \mathbf{A}^H (\mathbf{I} - s \mathbf{Cov}(\mathbf{H}_g) \mathbf{A} \mathbf{A}^H)^{-1} (\bar{\mathbf{H}}_g)^H)}{|\mathbf{I} - s \mathbf{Cov}(\mathbf{H}_g) \mathbf{A} \mathbf{A}^H|}, \quad (20)$$

where $\bar{\mathbf{H}}_g = \mathbf{E}(\mathbf{H}_g)$ and $\mathbf{Cov}(\mathbf{H}_g) = \mathbf{E}[\mathbf{H}_g(\mathbf{H}_g)^H]$.

For Rayleigh channel, the average matrix $\bar{\mathbf{H}}_g = 0$, and the covariance matrix $\mathbf{Cov}(\mathbf{H}_g)$ can be obtained by Eq. (29) with $\sigma_h^2 = 1/L$ in Appendix, thus $\Gamma_{\beta}(s)$ is simplified as:

$$\Gamma_{\beta}(s) = |\mathbf{I} - s \mathbf{Cov}(\mathbf{H}_g) \mathbf{A} \mathbf{A}^H|^{-1}. \quad (21)$$

Suppose there are $Q_{\mathbf{x}_g, \mathbf{x}_t}$ non-zero values labeled as $\{f_1, f_2, \dots, f_{Q_{\mathbf{x}_g, \mathbf{x}_t}}\}$ in $\mathbf{A} = (\mathbf{x}_g - \mathbf{x}_t)$, and the corresponding positions of these non-zero positions are $\{e_1, e_2, \dots, e_{Q_{\mathbf{x}_g, \mathbf{x}_t}}\}$, then the non-zero eigenvalues of $\mathbf{Cov}(\mathbf{H}_g) \mathbf{A} \mathbf{A}^H$ are

$$\lambda_{q, \mathbf{x}_g, \mathbf{x}_t} = f_q^2 \times r_{e_q, g}. \quad (22)$$

With the above results, Eq. (21) can be simplified further as following:

$$\Gamma_{\beta}(s) = |\mathbf{I} - s \mathbf{Cov}(\mathbf{H}_g) \mathbf{A} \mathbf{A}^H|^{-1} = \prod_{q=1}^{Q_{\mathbf{x}_g, \mathbf{x}_t}} (1 - s \lambda_{q, \mathbf{x}_g, \mathbf{x}_t})^{-1}. \quad (23)$$

Accordingly PEP is calculated as:

$$P(\mathbf{x}_g \rightarrow \mathbf{x}_t) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{q=1}^{Q_{\mathbf{x}_g, \mathbf{x}_t}} \left(1 + \frac{\lambda_{q, \mathbf{x}_g, \mathbf{x}_t}}{4(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2) \sin^2 \theta}\right)^{-1} d\theta. \quad (24)$$

Substituting (24) in to (13) yields the final expression of the average bit error probability as

$$\begin{aligned} P_{b,g} &= \frac{1}{\pi B \cdot 2^B} \sum_{\forall \mathbf{x}_g} \sum_{\forall \mathbf{x}_t \neq \mathbf{x}_g} d(\mathbf{x}_g, \mathbf{x}_t) \\ &\quad \int_0^{\frac{\pi}{2}} \prod_{q=1}^{Q_{\mathbf{x}_g, \mathbf{x}_t}} \left(1 + \frac{\lambda_{q, \mathbf{x}_g, \mathbf{x}_t}}{4(\sigma^2 + \|\mathbf{x}_g\|^2 \sigma_e^2) \sin^2 \theta}\right)^{-1} d\theta. \end{aligned} \quad (25)$$

C. PEP based on approximated Q -function

It is noteworthy that there may be different approximation to the $Q(\cdot)$ function in Eq. (17), e.g., the following approximation is used in [1]:

$$Q(x) \approx \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2x^2}{3}}. \quad (26)$$

Accordingly, the PEP can be reformed as

$$P(\mathbf{x}_g \rightarrow \mathbf{x}_t) = \frac{1}{12}\Gamma_\beta\left(-\frac{\beta}{4(\sigma^2 + \|\mathbf{x}_g\|^2\sigma_e^2)}\right) + \frac{1}{4}\Gamma_\beta\left(-\frac{\beta}{4(\sigma^2 + \|\mathbf{x}_g\|^2\sigma_e^2)}\right). \quad (27)$$

The average bit error can be derived accordingly as

$$P_{b,g} = \frac{1}{B \cdot 2^B} \sum_{\forall \mathbf{x}_g} \sum_{\forall \mathbf{x}_t \neq \mathbf{x}_g} d(\mathbf{x}_g, \mathbf{x}_t) \left(\frac{1}{12}\Gamma_\beta\left(-\frac{1}{4(\sigma^2 + \|\mathbf{x}_g\|^2\sigma_e^2)}\right) + \frac{1}{4}\Gamma_\beta\left(-\frac{1}{3(\sigma^2 + \|\mathbf{x}_g\|^2\sigma_e^2)}\right) \right). \quad (28)$$

The approximation is valid for all types of channels with their own MGF applied in the equation.

IV. NUMERICAL RESULTS

Simulation are conducted in order to compare the simulation results of the EDCT-OFDM-IM system, and the theoretical analysis with different (Ω, K, N) configurations and two $Q(x)$ approximations described in this paper. Furthermore, the performance of DFT-OFDM-IM is simulated by assuming the same spectrum efficiency as that of the EDCT-OFDM-IM system. The detailed simulation setup is shown in Table I for both EDCT-OFDM-IM and DFT-OFDM-IM. In addition to simulation setup, we also compare the complexity of these two

TABLE I
SIMULATION SETUP FOR EDCT-OFDM-IM AND DFT-OFDM-IM

Parameter	EDCT-OFDM-IM	DFT-OFDM-IM
Spectrum Efficiency(SE)	1.5 bps/Hz	
DCT/DFT size (N)	256	128
suffix length (v)	16	-
prefix length (v)	16	
channel	L-tap Rayleigh	
tap number (L)	10	
PDP (Power Delay Profile)	uniformly distribution	
modulation	BPSK	QPSK
group size (Ω)	4	2
active SC number (K)	1	1
ML Rx complexity /symbol	2048RM+1536 RA	4608RM+3072RA
Notes	RM: real multiplication; RA: real addition	

Fig. 2 shows the comparative results for systems with ideal channel knowledge under a Rayleigh channel. It can be seen that EDCT-OFDM-IM is superior to DFT-OFDM-IM with the same spectrum efficiency. The performance gain is greater than 3dB at 1% bit error rate. It can also be observed from the figure that analytical results obtained using Eqs. (25) and (28) are quite close and both serve as a very tight upper bound to the average BER performance of the EDCT-OFDM-IM system. The bound becomes even tighter in the high SNR region of practical interest, where it actually provides a very accurate prediction of the real BER performance. Fig. 3 illustrates the corresponding case with imperfect channel estimation. The simulation and theory curves are still matched

Fig. 2. Simulation Vs analytical performance in the case with ideal channel estimation.

Fig. 3. Simulation Vs analytical performance in the case with imperfect channel estimation.

well. Both validate our analyses conducted shown in this paper. Fig. 4 shows the analytical and simulated performance for a case with high order modulation. Similar observations can be drawn whereas the gap between the theoretical calculation and simulation results is slightly larger than that with lower order modulation at the same E_b/N_0 . In addition, given the same spectrum efficiency, the maximum likelihood receiver for DFT-OFDM-IM has more than twice amount of real operations than EDCT-OFDM-IM, as shown in Table I.

V. CONCLUSION

The performance of an EDCT-OFDM-IM system was analyzed theoretically. First the closed-form expression of the effective channels was derived by taking into account the insertion of prefix and suffix and prefiltering, and then its correlation coefficients were obtained and applied in MGF. In addition to original analytical expression, we also derived its approximate version. Both have been proven to be very accurate upon comparing with the simulation results under uniformly distributed Rayleigh channels, for both low and high order

Fig. 4. Simulated performance Vs theoretic analysis for high order modulation.

modulation schemes. Our theoretical framework provides an effective means of evaluating the performance of the EDCT-OFDM-IM system and a very useful tool to guide the practical design of such a system. For the case with Nakagami-m fading channels, we plan to investigate in our future work.

VI. APPENDIX

The element ε -th row and the l -th column of the correlation matrix $\text{Cov}(\mathbf{H}_g)$, i.e., $r_{\varepsilon,l}^{(g)}$ is calculated as:

$$\begin{aligned} r_{\varepsilon,l}^{(g)} &= \mathbf{E}[\mathbf{H}_{\varepsilon+g\Omega}(\mathbf{H}_{l+g\Omega})^H] \\ &= \mathbf{E}\left\{ \frac{\sum_{m=1}^{L-1} C(\varepsilon+g\Omega, m)(\bar{h}_{m+1} + \bar{h}_m) + C(\varepsilon+g\Omega, L)\bar{h}_L}{C(\varepsilon+g\Omega, 1)} \times \right. \\ &\quad \left. \frac{\sum_{m=1}^{L-1} C(l+g\Omega, m)(\bar{h}_{m+1}^* + \bar{h}_m^*) + C(l+g\Omega, L)\bar{h}_L^*}{C(l+g\Omega, 1)} \right\} \end{aligned} \quad (29)$$

And it can be further simplified as:

$$\begin{aligned} r_{\varepsilon,l}^{(g)} &= \sum_{m=1}^{L-2} \left\{ \frac{C(\varepsilon+g\Omega, m)}{C(\varepsilon+g\Omega, 1)} \frac{C(l+g\Omega, m)}{C(l+g\Omega, 1)} \xi_{m+1} \right. \\ &\quad + \frac{C(\varepsilon+g\Omega, m)}{C(\varepsilon+g\Omega, 1)} \frac{C(l+g\Omega, m+1)}{C(l+g\Omega, 1)} \xi_{m+1} \\ &\quad + \frac{C(\varepsilon+g\Omega, m+1)}{C(\varepsilon+g\Omega, 1)} \frac{C(l+g\Omega, m)}{C(l+g\Omega, 1)} \xi_{m+1} \\ &\quad + \frac{C(\varepsilon+g\Omega, m)}{C(\varepsilon+g\Omega, 1)} \frac{C(l+g\Omega, m)}{C(l+g\Omega, 1)} \xi_m \} \\ &\quad + \frac{C(\varepsilon+g\Omega, L-1)}{C(\varepsilon+g\Omega, 1)} \frac{C(l+g\Omega, L-1)}{C(l+g\Omega, 1)} (\xi_{L-1} + \xi_L) \\ &\quad + \left\{ \frac{C(\varepsilon+g\Omega, L-1)}{C(\varepsilon+g\Omega, 1)} \cdot \frac{C(l+g\Omega, L)}{C(l+g\Omega, 1)} \right. \\ &\quad + \frac{C(\varepsilon+g\Omega, L)}{C(\varepsilon+g\Omega, 1)} \cdot \frac{C(l+g\Omega, L-1)}{C(l+g\Omega, 1)} \\ &\quad + \frac{C(\varepsilon+g\Omega, L)}{C(\varepsilon+g\Omega, 1)} \cdot \frac{C(l+g\Omega, L)}{C(l+g\Omega, 1)} \} \xi_L \end{aligned} \quad (30)$$

where ξ_m is the average power of $\bar{h}_m$, i.e., $\xi_m = E\{|\bar{h}_m|^2\}$, $m = 1, \dots, L$.

REFERENCES

- [1] E. Basar, U. Aygolu, E. Panayici and H. V. poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536-5549, Nov. 2013.
- [2] C. He, A. Cao, L. Xiao, L. Zhang, P. Xiao and K. Nikitopoulos, "Enhanced DCT-OFDM system with index modulation", *IEEE Trans. Veh. Technol.*, vol. 68, no. 5, pp. 5134-5138, May 2019.
- [3] M. Au, et al, "Joint Code-Frequency Index Modulation for IoT and Multi-User Communications", in *IEEE J. of Sel. Topics in Signal Processing*, vol. 13, no. 6, pp. 1223-1236, Oct. 2019.
- [4] E. Soujeri, et al, "Frequency Index Modulation for Low Complexity Low Energy Communication Networks", in *IEEE Access*, vol. 5, pp. 23276-23287, 2017.
- [5] M. Wen, X. Cheng, M. Ma, B. Jiao, and H. V. Poor, "On the achievable rate of OFDM with index modulation", *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 1919-1932, Apr. 2016.
- [6] Y. Xiao, S. Wang, L. Dan, X. Lei, P. Yang and W. Xiang, "OFDM with interleaved subcarrier-index modulation", *IEEE Commun. Lett.*, vol. 18, no. 8, pp. 1447-1450, Aug. 2014
- [7] T. Jiang, M. Guizani, H. Chen, W. Xiang, and Y. Wu, "Derivation of PAPR distribution for OFDM wireless systems based on extreme value theory", *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1298-1305, Apr. 2008.
- [8] M. Wen, X. Cheng, et al., "Index modulated OFDM for underwater acoustic communications", *IEEE Commun. Mag.*, vol. 54, no. 4, pp. 132-137, May 2016.
- [9] T. Mao, Q. Wang, Z. Wang and S. Chen, "Novel index modulation techniques: A survey", *IEEE Commun. Surveys Tuts.*, vol. 21, no. 1, pp. 315-348, First Quart. 2019.
- [10] X. Cheng, M. Zhang, M. Wen, and L. Yang, "Index modulation for 5G: Striving to do more with less", *IEEE Wireless Commun. Mag.*, vol. 25, no. 2, pp. 126-132, Apr. 2018
- [11] V. Sanchez, P. Garcia, A. M. Peinado, J. C. Segura and A. J. Rubio, "Diagonalizing properties of the discrete cosine transforms", *IEEE Trans. on Signal Processing*, vol. 43, no. 11, pp. 2631-2641, Nov. 1995.
- [12] N. Al-Dhahir H. Minn S. Satish, "Optimum DCT-based multicarrier transceivers for frequency-selective channels", *IEEE Trans. on Commun.* vol. 54, no. 4, pp. 760-760, Apr. 2006.
- [13] L. Xiao, P. Yang, Y. Xiao, S. Fan, M. Renzo, and S. Q. Li, "Efficient compressive sensing detection for generalized spatial modulation", *IEEE Trans. Veh. Technol.*, vol. 66, no. 2, pp. 1284-1298, Feb. 2017.
- [14] G. D. Mandyam, "Sinusoidal transforms in OFDM systems", *IEEE Trans. Broadcast.*, vol. 50, no. 2, pp. 172-184, Jun. 2004.
- [15] W.H Chen, C. Smith and S. Fralick, "A fast computational algorithm for the discrete cosine transform", *IEEE Trans. Commun.*, vol. 25, no. 9, pp. 1004-1009, Sep. 1977.
- [16] S. Althunibat, R. Mesleh and E. Basar, "Differential subcarrier index modulation", *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7429-7436, May 2018.
- [17] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables", *Biometrika*, vol. 47, no. 1/2, pp. 199-201, Jun. 1960. [Online]. Available: <http://www.jstor.org/stable/2332977>.
- [18] Robert T. Short, Computation of Rice and probabilities, PhaseLocked noncen-trals Technical Report PHS0254, <http://www.phaselockedsystems.com/NoncentralChiSquared.pdf>.