Robust Multiuser Detection Using Kalman Filter and Windowed PAST Algorithm

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Abstract – We propose some robust adaptive multiuser detection schemes for direct-sequence code-division multiple-access (DS-CDMA) multipath frequency-selective fading channels. Multiple access interference (MAI) and intersymbol interference (ISI) are presented in an identical format using expanded signal subspace, which facilitates multiuser detection in a symbol-by-symbol fashion. This paper contributes to the theoretical aspect of adaptive multiuser detection by proving that the optimum linear multiuser detectors that achieve maximum signal-to-interference-plus-noise-ratio (SINR) must exist in the signal subspace, and the theoretic SINR upper bound is also derived. Another contribution of this paper is to propose the design of multiuser detectors in an expanded signal subspace, and introduce subspace estimation and Kalman filtering algorithms for their adaptive implementation. To robustify the adaptive detectors against subspace estimation and channel estimation errors, a modified projection approximation subspace tracking (PAST) algorithm is proposed for subspace tracking. It is demonstrated by simulations that these adaptive detectors effectively suppress both MAI and ISI and converge to the optimum SINR. They are robust against subspace estimation errors and channel estimation errors compared to the conventional Wiener minimum mean square error (MMSE) detector.

Index Terms: multiuser detection, Kalman filter, expanded signal subspace, optimum SINR detector, SINR bound, subspace tracking, multipath frequency-selective fading, MMSE, ISI, MAI, CDMA

I. INTRODUCTION

Multiple access interference (MAI) is a major impedance to achieve the promised advantages of code-division multiple-access (CDMA) technology in mobile communications. There have been intensive research interests in recent years to develop multiuser detection (MUD) technologies to overcome MAI [1]-[5]. Multipath frequency-selective fading channels cause further practical concerns for broadband CDMA communications, including the spreading waveform distortion and intersymbol interference (ISI). RAKE receiver [6] is a typical approach to tackle multipath problem, however, it is only optimum in single-user environments. For multiuser systems, the RAKE structure can still be adopted, but the matched filter bank needs to be replaced by a linear multiuser detector bank [7], [8], [9] [10] or parallel interference cancellation based nonlinear detectors [11],[12].

This paper addresses the multiuser detection problem for multipath channels with arbitrarily long delay and non-negligible ISI. The development effort begins with the derivation of an analytical one-shot signal model. Instead of suppressing the echo multipath components, an extended observation window and reformed user spreading codes are adopted so that all multipath components are used for the detection of one symbol. In this signal model, ISI and MAI are presented in a similar format and both can be suppressed by multiuser detectors.

In this paper, we propose some robust multiuser detection schemes in the signal subspace. In the literature, there exist various linear multiuser detectors designed in the full-rank space or in the signal subspace. For example, in [2], a canonical representation is introduced for the linear multiuser detector, and a blind adaptation is enabled by using the minimum output energy criteria and stochastic gradient-descent method. In [4], the detector adopts another type of canonical representation and the adaptation is based on Kalman filtering. Both detectors operate in the full-rank space. In contrast, signal subspace-based multiuser detectors are proposed in [3] and [5]. In [3], it is shown that the decorrelating detector and the MMSE detector can be obtained blindly based on signal subspace estimation. In [5], the detector is decomposed in the signal subspace and the weights are adaptively estimated by Kalman filtering. In this paper, we theoretically prove that the linear multiuser detectors that have optimum signal-to-interference-plus-noise-ratio (SINR) must exist in the signal subspace, which provides a guidance to design multiuser detector in the signal subspace or in the full space. This theoretical result

is the first contribution of this paper. The Wiener MMSE detector is well known for being optimum in the mean square error (MSE) sense. In this paper, we prove that it is also an optimum SINR detector and lies in the signal subspace. This provides another angle to appreciate the MMSE detector. For any system, there exists a SINR upper bound that can be expressed in a closed form. This motivates us to search for efficient multiuser detectors in the expanded signal subspace for multipath channel. If the channel and the autocorrelation matrix of the received signals are precisely known, then the Wiener MMSE detector can be applied [13]. However, in practical systems, accurate knowledge of the channel and autocorrelation matrix is hardly available. As will be shown by our simulations, such inaccuracy will seriously deteriorate the performance of the Wiener MMSE detector. Another contribution of this paper is to propose some adaptive strategies that are resilient to subspace and channel estimation errors. In the proposed schemes, the subspace-based detector is decomposed along all orthogonal directions in expanded signal subspace. Deflated batch processing method or the modified projection approximation subspace tracking (PAST) method [14],[15],[16] is used to estimate the signal subspace, and Kalman filter is applied to estimate the coefficients along each subspace basis. These adaptive detectors are shown to be efficient in suppressing both MAI and ISI, and approach the optimum SINR performance in multipath channels. More importantly, they are robust against subspace estimation errors and channel estimation errors.

Channel can be estimated using training sequences [17], [18], but this issue is not considered in this paper. The signal model and the detectors in this paper are derived for synchronous multipath channels, however, if the user delays are known or estimated, the asynchronous multipath channels can be directly modelled in a similar format as the synchronous channels, and hence the developed adaptive multiuser detectors can also be applied directly.

The rest of the paper is organized as follows. In Section II, the discrete one-shot chip-rate signal model for multipath channels is presented. In Section III, a proof is given for the fact that the optimum SINR detectors are in the signal subspace and the SINR bound of linear multiuser detectors is derived. The expanded signal subspace-based adaptive multiuser detectors are developed in Section IV. Two subspace estimation algorithms and a Kalman filter-based coefficient estimation algorithm are presented. Section V presents the simulation results, and conclusions are drawn in Section VI.

II. SIGNAL MODEL FOR MULTIPATH CHANNEL

Consider a synchronous K -user DS-CDMA system employing binary phase-shift keying (BPSK) modulation to transmit signals through multipath channels. The symbol interval and chip interval are denoted by T and T_c respectively ($NT_c = T$, where N is the spreading factor). The k th user's spreading waveform is

$$c_k(t) = \sum_{n=0}^{N-1} c_k(n) \psi(t - nT_c), \quad t \in [0, T], k = 1, \dots, K \quad (1)$$

where $\{c_k(n)\}_{n=0}^{N-1}$ is the signature code assigned to the k th user, it is normalized such that

$\sum_{n=0}^{N-1} c_k^2(n) = 1$; $\psi(t)$ is a normalized chip waveform defined in $[0, T_c]$, $\int_0^{T_c} \psi^2(t) dt = 1$. The multipath

channel is modeled by a tapped delay line with tap spacing $1/W$ and tap coefficients $\{h_k(l)\}_{l=0}^{L-1}$, where

L is the number of resolvable paths for each user. We assume $W = 1/T_c$, $L = \lceil T_m/T_c \rceil$, where W is the

bandwidth of the spread-spectrum signals and T_m is the multipath delay spread. Transmitting the

signals through the multipath channel, the received signal due to the k th user is given by

$$y_k(t) = \sum_{i=-\infty}^{\infty} A_k b_k(i) \sum_{l=0}^{L-1} h_k(l) c_k(t - iT - \frac{l}{W}) \quad (2)$$

where A_k and $b_k(i)$ are the amplitude and the i th transmitted information bit of the k th user,

respectively; $b_k(i) \in \{+1, -1\}$ follows identical independent distribution (i.i.d.). Finally, the total

received signal is the superposition of the information-bearing data signals of K users plus the additive

white Gaussian noise (AWGN), i.e.,

$$r(t) = \sum_{k=1}^K y_k(t) + v(t) \quad (3)$$

where $v(t)$ is the zero-mean white Gaussian noise. After a chip-rate filtering followed by a chip-rate

sampling, the discrete-time format of the received signal is given by

$$r(j) = \sum_{k=1}^K y_k(j) + v(j) \quad (4)$$

where

$$y_k(j) = \sum_{i=-\infty}^{\infty} A_k b_k(i) s_k(j - iN) \quad (5)$$

$$s_k(n) = \sum_{l=0}^{\Delta} h_k(l) c_k(n-l) \quad (6)$$

$$v(j) = \int_{jT_c}^{(j+1)T_c} v(t) \psi(t - jT_c) dt \quad (7)$$

On the transmitter side, the energy of each symbol is limited to a duration of NT_c . However, on the receiver side, the energy is spread over an extended interval $(N+L-1)T_c$ due to the channel dispersion, which can be observed from (6). We aim to design a multiuser detector that detects the signals in a symbol-by-symbol fashion without detection delays. To this end, we employ a processing window of length $N+L-1$ to model the received signal in a vector form. The windowing scheme is illustrated in Figure 1. The windows for neighbouring symbols overlap for $L-1$ chips. Within the window are not only the information symbols at the current time instant, but also those before and after the current time.

To describe it quantitatively, we define $P \stackrel{\text{def}}{=} \lceil (L-1)/N \rceil$. Then, there are $2P+1$ symbols of the desired user involved in the processing window of length $N+L-1$, including the current symbol, P symbols before and P symbols after the current symbol. Taking into account of all the K users, there are $(2P+1)K$ symbols involved in the detection of one symbol from the desired user. To summarize, the $(N+L-1) \times 1$ -dimensional received signal vector for the system under study can be expressed as

$$\mathbf{r}(i) = \sum_{k=1}^K \mathbf{y}_k(i) + \mathbf{v}(i) \quad (8)$$

where

$$\mathbf{y}_k(i) = A_k b_k(i) \mathbf{s}_k + \sum_{\tau=1}^P A_k b_k(i-\tau) \bar{\mathbf{s}}_{k,\tau} + \sum_{\tau=1}^P A_k b_k(i+\tau) \bar{\bar{\mathbf{s}}}_{k,\tau} \quad (9)$$

$$\mathbf{s}_k = [s_k(0) \quad \dots \quad s_k(N+L-2)]^T \quad (10)$$

$$\bar{\mathbf{s}}_{k,\tau} = [s_k(\tau N) \quad s_k(\tau N+1) \quad \dots \quad s_k(N+L-2) \quad 0 \quad \dots \quad 0]^T, \tau=1 \dots P \quad (11)$$

$$\bar{\bar{\mathbf{s}}}_{k,\tau} = [0 \quad \dots \quad 0 \quad s_k(0) \quad s_k(1) \quad \dots \quad s_k(N+L-2-\tau N)]^T, \tau=1 \dots P \quad (12)$$

$$\mathbf{v}(i) = [v(iN) \quad \dots \quad v(iN+N+L-2)]^T \quad (13)$$

Each element of the $(N+L-1) \times 1$ noise vector $\mathbf{v}(i)$ has variance σ^2 . Define

$$\bar{\mathbf{s}}_k = [\bar{\mathbf{s}}_{k,1} \quad \dots \quad \bar{\mathbf{s}}_{k,P}]_{(N+L-1) \times P}^T, k=1 \dots K \quad (14)$$

$$\bar{\bar{\mathbf{s}}}_k = \begin{bmatrix} \bar{\bar{\mathbf{s}}}_{k,1} & \dots & \bar{\bar{\mathbf{s}}}_{k,P} \end{bmatrix}_{(N+L-1) \times P}, k=1 \dots K \quad (15)$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \bar{\mathbf{s}}_1 & \bar{\bar{\mathbf{s}}}_1 & \dots & \mathbf{s}_K & \bar{\mathbf{s}}_K & \bar{\bar{\mathbf{s}}}_K \end{bmatrix}_{(N+L-1) \times (2P+1)K} \quad (16)$$

$$\mathbf{A}_{(2P+1)K \times (2P+1)K} = \text{diag} \{ A_1 \dots A_1 \dots \dots A_K \dots A_K \} \quad (17)$$

$$\mathbf{b}_k(i) = [b_k(i) \ b_k(i-1) \ \dots \ b_k(i-P) \ b_k(i+1) \ \dots \ b_k(i+P)]_{1 \times (2P+1)}, \quad k=1 \dots K \quad (18)$$

$$\mathbf{b}(i) = [\mathbf{b}_1(i) \ \dots \ \mathbf{b}_K(i)]^T \quad (19)$$

then (8) can be written in a compact vector/matrix format as

$$\mathbf{r}(i) = \mathbf{S}\mathbf{A}\mathbf{b}(i) + \mathbf{v}(i) \quad (20)$$

The signal model in (20) is developed for detecting the symbol at time instant i . In (20), $b_k(i)$, $\{b_k(i-\tau)\}_{\tau=1}^P$, $\{b_k(i+\tau)\}_{\tau=1}^P$ are all from the same user (the k th user), but can be effectively deemed as statistically independent signals from different users which include the k th user with modified signature codes $\mathbf{s}_k$, and $(2P+1)$ virtual users with signature codes $\{\bar{\mathbf{s}}_{k,\tau}\}_{\tau=1}^P$ and $\{\bar{\bar{\mathbf{s}}}_{k,\tau}\}_{\tau=1}^P$. Performing eigendecomposition of the autocorrelation matrix $\mathbf{R} = E\{\mathbf{r}\mathbf{r}^H\}$ yields

$$\mathbf{R} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s + \sigma^2 \mathbf{I}_s & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{I} \quad (21)$$

where $\mathbf{\Lambda}_s$ is a diagonal matrix with positive diagonal elements; $\mathbf{\Lambda}_s + \sigma^2 \mathbf{I}_s$ contains the most significant eigenvalues of $\mathbf{R}$ that are not equal to σ^2 and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors; the columns of $\mathbf{U}_n$ are the orthonormal eigenvectors corresponding to the eigenvalues that are equal to σ^2 . The columns of $\mathbf{U}_s$ span the signal subspace and the columns of $\mathbf{U}_n$ span the noise subspace. It can be easily shown that $\mathbf{S}$ and $\mathbf{U}_s$ have the same rank and span the same subspace. If $\mathbf{s}_k$, $\{\bar{\mathbf{s}}_{k,\tau}\}_{\tau=1}^P$, $\{\bar{\bar{\mathbf{s}}}_{k,\tau}\}_{\tau=1}^P$, $k=1 \dots K$ are linearly independent, the signal subspace has a dimension of $(N+L-1) \times (2P+1)K$. It is an expanded signal subspace compared with the signal subspace of dimension $N \times K$ for the AWGN channel.

III. OPTIMUM MULTIUSER DETECTOR

Throughout this paper, we assume that user 1 is the user of interest. Any linear multiuser detector for user 1 can be expressed as

$$\begin{aligned}\mathbf{d}_1 &= \mathbf{U}_s \mathbf{a} + \mathbf{U}_n \mathbf{b} \\ &= \mathbf{d}_{1s} + \mathbf{d}_{1n}\end{aligned}\quad (22)$$

where $\mathbf{a}$ is a $(2P+1)K \times 1$ weight vector and $\mathbf{b}$ is an $[N+L-1-(2P+1)K] \times 1$ weight vector, $\mathbf{d}_{1s} = \mathbf{U}_s \mathbf{a}$ is the projection of the detector in the signal subspace and $\mathbf{d}_{1n} = \mathbf{U}_n \mathbf{b}$ is the projection in the noise subspace. The decision for its i th symbol is obtained as $\hat{b}(i) = \text{sign}\{\text{Re}[\mathbf{d}_1^H(i) \mathbf{r}(i)]\}$. Now, we have the following theorem:

Theorem 1: The maximum SINR linear detector must exist in the signal subspace.

Proof: For user 1, SINR in the soft output of the linear multiuser detector is defined as

$$\text{SINR} = \frac{E\left\{\left|\mathbf{d}_1^H A_1 b_1(i) \mathbf{s}_1\right|^2\right\}}{E\left\{\left|\mathbf{d}_1^H (\mathbf{r} - A_1 b_1(i) \mathbf{s}_1)\right|^2\right\}} \quad (23)$$

Substituting (8), (9) and (22) into (23) results in

$$\begin{aligned}\text{SINR} &= \frac{A_1^2 \left|\mathbf{d}_1^H \mathbf{s}_1\right|^2}{E\left\{\left|\mathbf{d}_1^H \left(\sum_{k=2}^K \mathbf{y}_k(i) + \sum_{\tau=1}^P A_1 b_1(i-\tau) \bar{\mathbf{s}}_{1,\tau} + \sum_{\tau=1}^P A_1 b_1(i+\tau) \bar{\bar{\mathbf{s}}}_{1,\tau} + \mathbf{v}\right)\right|^2\right\}} \\ &= \frac{A_1^2 \left|\mathbf{d}_1^H \mathbf{s}_1\right|^2}{\sum_{k=2}^K A_k^2 \left|\mathbf{d}_1^H \left(\mathbf{s}_k + \sum_{\tau=1}^P (\bar{\mathbf{s}}_{k,\tau} + \bar{\bar{\mathbf{s}}}_{k,\tau})\right)\right|^2 + A_1^2 \left|\mathbf{d}_1^H \left(\sum_{\tau=1}^P (\bar{\mathbf{s}}_{1,\tau} + \bar{\bar{\mathbf{s}}}_{1,\tau})\right)\right|^2 + \sigma^2 \mathbf{d}_1^H \mathbf{d}_1} \\ &= \frac{A_1^2 \left|\mathbf{d}_{1s}^H \mathbf{s}_1\right|^2}{\sum_{k=2}^K A_k^2 \left|\mathbf{d}_{1s}^H \left(\mathbf{s}_k + \sum_{\tau=1}^P (\bar{\mathbf{s}}_{k,\tau} + \bar{\bar{\mathbf{s}}}_{k,\tau})\right)\right|^2 + A_1^2 \left|\mathbf{d}_{1s}^H \left(\sum_{\tau=1}^P (\bar{\mathbf{s}}_{1,\tau} + \bar{\bar{\mathbf{s}}}_{1,\tau})\right)\right|^2 + \sigma^2 (\mathbf{d}_{1s}^H \mathbf{d}_{1s} + \mathbf{d}_{1n}^H \mathbf{d}_{1n})} \quad (24)\end{aligned}$$

The second equality follows from the fact that $b_k(i)$, $\{b_k(i-\tau)\}_{\tau=1}^P$, $\{b_k(i+\tau)\}_{\tau=1}^P$, $k=1\dots K$ and $\mathbf{v}$ are

statistically independent, and $E[\mathbf{v}\mathbf{v}^H] = \sigma^2 \mathbf{I}$. The last equality follows from the fact that $\mathbf{s}_k$, $\{\bar{\mathbf{s}}_{k,\tau}\}_{\tau=1}^P$,

$\{\bar{\bar{\mathbf{s}}}_{k,\tau}\}_{\tau=1}^P$, $k=1\dots K$, and $\mathbf{d}_{1s}$ are orthogonal to $\mathbf{d}_{1n}$. Because

$$\mathbf{d}_{1s}^H \mathbf{d}_{1s} \geq 0, \quad \mathbf{d}_{1n}^H \mathbf{d}_{1n} \geq 0, \quad (25)$$

thus

$$SINR \leq \frac{A_1^2 |\mathbf{d}_{1s}^H \mathbf{s}_1|^2}{\sum_{k=2}^K A_k^2 \left| \mathbf{d}_{1s}^H \left(\mathbf{s}_k + \sum_{\tau=1}^P (\bar{\mathbf{s}}_{k,\tau} + \bar{\bar{\mathbf{s}}}_{k,\tau}) \right) \right|^2 + A_1^2 \left| \mathbf{d}_{1s}^H \left(\sum_{\tau=1}^P (\bar{\mathbf{s}}_{1,\tau} + \bar{\bar{\mathbf{s}}}_{1,\tau}) \right) \right|^2 + \sigma^2 (\mathbf{d}_{1s}^H \mathbf{d}_{1s})} \quad (26)$$

Eq. (26) indicates that the linear multiuser detectors that achieve maximum SINR must be in the signal subspace since equality holds only when $\mathbf{d}_{1n} = \mathbf{0}$. The question would arise whether the well-known MMSE detector is in the signal subspace. Propositions below answer this question.

Proposition 1: If $\mathbf{d}_1$ is an optimum detector with maximum SINR, then $\alpha \mathbf{d}_1$ is also an optimum detector, where α is an arbitrary real nonzero scalar.

Proof:

$$SINR(\alpha \mathbf{d}_1) = \frac{E \left\{ |(\alpha \mathbf{d}_1)^H A_1 b_1(i) \mathbf{s}_1|^2 \right\}}{E \left\{ |(\alpha \mathbf{d}_1)^H (\mathbf{r} - A_1 b_1(i) \mathbf{s}_1)|^2 \right\}} \quad (27)$$

$$\begin{aligned} &= \frac{\alpha^2 E \left\{ |\mathbf{d}_1^H A_1 b_1(i) \mathbf{s}_1|^2 \right\}}{\alpha^2 E \left\{ |\mathbf{d}_1^H (\mathbf{r} - A_1 b_1(i) \mathbf{s}_1)|^2 \right\}} \\ &= \frac{E \left\{ |\mathbf{d}_1^H A_1 b_1(i) \mathbf{s}_1|^2 \right\}}{E \left\{ |\mathbf{d}_1^H (\mathbf{r} - A_1 b_1(i) \mathbf{s}_1)|^2 \right\}} \\ &= SINR(\mathbf{d}_1) \end{aligned} \quad (28)$$

Discussion:

1. The proof shown above is generic and the conclusions in Theorem 1 and Proposition 1 are applicable to any linear multiuser detectors.
2. The conclusions in Theorem 1 and Proposition 1 are applicable to the AWGN channel which can be considered as a special case of multipath channels.
3. The optimum SINR detectors refer to a family of detectors, rather than just one.

Proposition 2: The MMSE detector is also a maximum SINR detector that lies in the signal subspace.

Proof: Assume $\mathbf{d}_1$ is an optimum detector with maximum SINR. Let

$$\alpha = \frac{1}{\mathbf{d}_1^H \mathbf{s}_1} \quad (29)$$

According to Proposition 1, $\alpha \mathbf{d}_1$ is also an optimum detector and has the same maximum SINR:

$$\text{SINR}(\alpha \mathbf{d}_1) = \frac{E \left\{ \left| (\alpha \mathbf{d}_1)^H A_1 b_1(i) \mathbf{s}_1 \right|^2 \right\}}{E \left\{ \left| (\alpha \mathbf{d}_1)^H (\mathbf{r} - A_1 b_1(i) \mathbf{s}_1) \right|^2 \right\}} \quad (30)$$

$$\begin{aligned} &= \frac{E \left\{ \left| A_1 b_1(i) \alpha \mathbf{d}_1^H \mathbf{s}_1 \right|^2 \right\}}{E \left\{ \left| \alpha \mathbf{d}_1^H \mathbf{r} - A_1 b_1(i) \alpha \mathbf{d}_1^H \mathbf{s}_1 \right|^2 \right\}} \\ &= \frac{A_1^2}{E \left\{ \left| \alpha \mathbf{d}_1^H \mathbf{r} - A_1 b_1(i) \right|^2 \right\}} \end{aligned} \quad (31)$$

Define

$$\mathbf{d}_{MMSE} = \alpha \mathbf{d}_1 \quad (32)$$

It is straightforward to see that

$$\mathbf{d}_{MMSE} = \arg \max_{\mathbf{d}} \frac{A_1^2}{E \left\{ \left| \mathbf{d}^H \mathbf{r} - A_1 b_1(i) \right|^2 \right\}} \quad (33)$$

$$= \arg \min_{\mathbf{d}} E \left\{ \left| \mathbf{d}^H \mathbf{r} - A_1 b_1(i) \right|^2 \right\} \quad (34)$$

According to (29) and (32), an implicit constraint in (33) and (34) is

$$\mathbf{d}^H \mathbf{s}_1 = 1 \quad (35)$$

The solution to (34) is the MMSE detector. Note that, according to (33), this MMSE detector also achieves the maximum SINR, i.e., it is optimum in terms of both maximum SINR and minimum mean square error. According to Theorem 1, the MMSE detector is in the signal subspace.

Discussion:

1. The solution to the constrained minimization problem in (34) and (35) is

$$\mathbf{d}_{CMMSE} = \frac{\mathbf{R}^{-1} \mathbf{s}_1}{\mathbf{s}_1^H \mathbf{R}^{-1} \mathbf{s}_1} \quad (36)$$

where $\mathbf{d}_{CMMSE}$ is the constrained MMSE detector.

2. Without the constraint in (35), solving (34) leads to the Wiener solution

$$\mathbf{d}_{WMMSE} = A_1^2 \mathbf{R}^{-1} \mathbf{s}_1 \quad (37)$$

Comparing (36) and (37), one can see that the Wiener detector is a scaled version of the constrained MMSE detector. According to Proposition 1, they should achieve the same maximum SINR which is

$$SINR(\mathbf{d}_{CMMSE}) = SINR(\mathbf{d}_{WMMSE}) = \frac{A_1^2}{\frac{1}{\mathbf{s}_1^H \mathbf{R}^{-1} \mathbf{s}_1} - A_1^2} \quad (38)$$

But $\mathbf{d}_{CMMSE}$ and $\mathbf{d}_{WMMSE}$ have different mean square errors given as follows

$$MSE(\mathbf{d}_{CMMSE}) = \frac{1}{\mathbf{s}_1^H \mathbf{R}^{-1} \mathbf{s}_1} - A_1^2 \quad (39)$$

$$MSE(\mathbf{d}_{WMMSE}) = A_1^2 - A_1^4 \mathbf{s}_1^H \mathbf{R}^{-1} \mathbf{s}_1, \quad (40)$$

where the autocorrelation matrix can be computed analytically as

$$\mathbf{R} = \mathbf{S} \mathbf{A}^2 \mathbf{S}^H + \sigma^2 \mathbf{I}_{(N+L-1) \times (N+L-1)} \quad (41)$$

where

$$\mathbf{A}_{(2P+1)K \times (2P+1)K}^2 = \text{diag} \{ A_1^2 \quad \dots \quad A_1^2 \quad \dots \quad A_K^2 \quad \dots \quad A_K^2 \} \quad (42)$$

3. The proof for the fact that the MMSE detector is in the signal subspace is given in the Appendix.

IV. ADAPTIVE MULTIUSER DETECTORS FOR MULTIPATH CHANNELS

It can be seen from (36) and (37) that in order to calculate the MMSE detector, we first need to know the autocorrelation matrix and then perform matrix inversion. In practice, the autocorrelation matrix can be estimated by:

$$\mathbf{R} = E(\mathbf{r} \mathbf{r}^H) \quad (43)$$

$$\approx \frac{1}{M} \sum_{i=1}^M \mathbf{r}(i) \mathbf{r}(i)^H \quad (44)$$

The larger M , the more precise estimate of the autocorrelation matrix. However, a larger M also means a longer detection delay, and the matrix inversion imposes a higher computational complexity especially when $N+L-1$ is large. The performance of the direct MMSE implementation is largely affected by the estimation errors as will be shown by simulations.

Alternatively, we propose some adaptive schemes to achieve reduced complexity and robustify the multiuser detectors. Similar to (22), the desired detector can be decomposed in the signal subspace of dimension $(N+L-1) \times (2P+1)K$ as

$$\mathbf{d}_1 = \mathbf{U}_s \mathbf{a} = \mathbf{s}_1 + \mathbf{U}_{s,\perp} \mathbf{w} \quad (45)$$

where $\mathbf{U}_{s,\perp}$ is a $(N+L-1) \times [(2P+1)K-1]$ matrix and all of its column vectors are in the expanded signal subspace and are orthogonal to $\mathbf{s}_1$; $\mathbf{w}$ is a $[(2P+1)K-1] \times 1$ weight vector. To pursue a MMSE solution, the constraint in (35) is applied, and is integrated into the expression of $\mathbf{d}_1$ as follows

$$\mathbf{d}_1 = \frac{\mathbf{s}_1}{\|\mathbf{s}_1\|^2} + \mathbf{U}_{s,\perp} \mathbf{w} \quad (46)$$

Based on (46), it can be easily verified that $\mathbf{d}_1^H \mathbf{s}_1 = 1$. Now the tasks are to estimate the signal subspace basis $\mathbf{U}_{s,\perp}$ and the coefficient vector $\mathbf{w}$. We propose to use Kalman filter to estimate $\mathbf{w}$ and then present two methods to estimate $\mathbf{U}_{s,\perp}$.

A. Estimate $\mathbf{w}$ by Kalman Filter

Assume that $\mathbf{U}_{s,\perp}$ has been estimated. For a stationary system, the state space model is given by

$$\mathbf{w}(i) = \mathbf{w}(i-1) \quad (\text{state transition equation}) \quad (47)$$

$$y(i) = \mathbf{H}^H(i) \mathbf{w}(i) + e(i) \quad (\text{measurement equation}) \quad (48)$$

where

$$y(i) \triangleq \frac{\mathbf{s}_1^H \mathbf{r}(i)}{\|\mathbf{s}_1\|^2} \quad (49)$$

$$\mathbf{H}^H(i) \triangleq -\mathbf{r}^H(i) \mathbf{U}_{s,\perp} \quad (50)$$

$$e(i) = \mathbf{d}_1^H(i) \mathbf{r}(i) = \frac{\mathbf{s}_1^H \mathbf{r}(i)}{\|\mathbf{s}_1\|^2} + \mathbf{r}^H(i) \mathbf{U}_{s,\perp} \mathbf{w}(i) \quad (51)$$

$e(n)$ is the measurement noise. Its mean and variance are

$$\mu = E\{e(i)\} = E\{\mathbf{d}_1^H(i) \mathbf{r}(i)\} = \mathbf{d}_1^H(i) E\{\mathbf{r}(i)\} = 0 \quad (52)$$

$$\varphi = \text{cov}\{e(i)\} = E\{|e(i) - \mu|^2\} = E\{|e(i)|^2\} = E\left[\left|\mathbf{d}_1^H \mathbf{r}(i)\right|^2\right] \quad (53)$$

Because the mean square error of the detector is

$$\varepsilon = E[|\mathbf{d}_1^H \mathbf{r}(i) - A_1 b_1(i)|^2] = E[|\mathbf{d}_1^H \mathbf{r}(i)|^2] - A_1^2 \quad (54)$$

Hence

$$\varphi = A_1^2 + \varepsilon \quad (55)$$

When the detector converges to the MMSE detector

$$\varphi_{\min} = A_1^2 + \varepsilon_{\min} \quad (56)$$

In high SNR scenario, the minimum mean square error $\varepsilon_{\min} = 1/\mathbf{s}_1^H \mathbf{R}^{-1} \mathbf{s}_1 - A_1^2 \approx 0$, therefore

$$\varphi_{\min} \approx A_1^2 \quad (57)$$

B. Estimate $\mathbf{U}_{s,\perp}$ by Batch Processing Method

Because $\mathbf{U}_{s,\perp}$ contains only subspace basis that are orthogonal to $\mathbf{s}_1$, the received signal can be deflated by projecting the received signal onto $\mathbf{s}_1$ and then subtracting the projection from the received signal as follows

$$\mathbf{r}_{\perp}(i) = \mathbf{r}(i) - \mathbf{s}_1 \mathbf{s}_1^H \mathbf{r}(i) / \|\mathbf{s}_1\|^2 \quad (58)$$

It can be easily shown that $\mathbf{r}_{\perp}(i)$ and $\mathbf{s}_1$ are orthogonal, therefore the subspace basis extracted from $\mathbf{r}_{\perp}(i)$ will be orthogonal to $\mathbf{s}_1$. The autocorrelation matrix can be estimated by time average as follows

$$\mathbf{R}_{\perp} \approx \frac{1}{M} \sum_{i=1}^M \mathbf{r}_{\perp}(i) \mathbf{r}_{\perp}(i)^H \quad (59)$$

By performing eigendecomposition of $\mathbf{R}_{\perp}$, the matrix $\mathbf{U}_{s,\perp}$ of size $(N+L-1) \times [(2P+1)K-1]$ is obtained. Here we also have the problem of inaccurate estimation of $\mathbf{R}_{\perp}$, however, as will be shown by the simulations, the Kalman filter-based multiuser detector is robust against subspace estimation errors. Therefore, a large batch M is not necessary. The multiuser detector using the batch-processing subspace estimation and Kalman filter-based weight estimation is summarised in Figure 2.

C. Estimate $\mathbf{U}_{s,\perp}$ by Windowed PAST Algorithm

The batch processing-based subspace estimation method causes long delay in signal detection, and the required batch eigenvalue decomposition (ED) or singular value decomposition (SVD) operation has

computational complexity $O((N+L-1)^3)$. Projection approximation subspace tracking (PAST) is a different approach based on a novel interpretation that the signal subspace is the solution to an unconstrained minimisation problem. This lends itself to adaptive implementation. PAST algorithm guarantees global convergence and has linear complexity.

We made three modifications to the original PAST subspace tracking algorithm in order to integrate it into the proposed multiuser detector. First, the received signal is pre-processed by the deflation operation shown in (58) for the same reason as stated earlier. Second, the recursive least square (RLS) version of PAST does not guarantee that the output eigenvectors are orthonormal, hence one explicit orthonormalization step is introduced to tackle this problem. Finally, a tracking window is imposed on the PAST algorithm, that is, the subspace tracking process only lasts for a limited time duration. Intuitive belief is that the more data used for subspace tracking, the more accurate estimate of subspace, and consequently the better detection performance can be achieved. However, because Kalman filtering and the subspace tracking are two separate dynamic processes, and the Kalman filtering tracking is based on the subspace tracking output, frequent change of the subspace basis affects the convergence of the Kalman filter. The windowing scheme enables initial subspace tracking and then stops it. This avoids the error propagation in the Kalman filter tracking and allows it to converge. Kalman filter has strong tracking ability based on imperfect estimate of subspace basis. This windowing scheme also reduces the computational complexity. The diagram of the multiuser detector using this subspace tracking scheme and Kalman filter estimation is illustrated in Figure 3, and the algorithm is summarized in Figure 4, where the subspace and Kalman filter parameters $\mathbf{G}(0)$, $\mathbf{U}_{s,\perp}(0)$, $\mathbf{P}(0)$ are initialized to identity matrices and $\mathbf{w}(0)$ is initialized to zero vector.

D. Complexity Consideration

The computational complexity and latency of the proposed Kalman filter-based algorithms and the MMSE detector are compared in Table 1. The MMSE detector is formed by batch processing and remains unchanged in the signal detection stage. The batch processing procedure causes a delay of M symbols. The computational complexity in forming the detector is $O(N+L-1)^3$ due to matrix inversion; it is $O(N+L-1)$ during signal detection. The ExpSubKF detector with batch processing uses the Kalman filter in the expanded signal subspace. It has a delay of M symbols due to the batch

processing to estimate the subspace basis $\mathbf{U}_{s,\perp}$. But as will be shown by simulation, it requires much smaller batch than the MMSE detector, and hence much shorter latency. The complexity in batch processing is $O(N+L-1)^3$ due to the SVD operation in estimating $\mathbf{U}_{s,\perp}$; it is $O[(N+L-1)\times((2P+1)K-1)]$ in signal detection. The ExpSubKF detector with windowed PAST also uses the Kalman filter in the expanded signal subspace and applies windowed PAST algorithm to estimate $\mathbf{U}_{s,\perp}$. It causes no delay. Detector formation and signal detection proceed simultaneously. The algorithm complexity varies with time. When performing the windowed PAST algorithm, the complexity is $O[((2P+1)K-2)^2 \times (N+L-1)]$. Outside the window, it becomes $O[((2P+1)K-1)\times(N+L-1)]$. As will be shown by simulation, the window can be small to achieve satisfactory performance and hence the overall complexity is kept low.

Table 1: Comparison of complexity and latency between the proposed subspace-based detectors and the MMSE detector.

Algorithm	Delay	Complexity in Batch Processing	Complexity in Signal Detection
MMSE with batch processing	M symbols	$O(N+L-1)^3$	$O(N+L-1)$
ExpSubKF with batch processing	M symbols	$O(N+L-1)^3$	$O[(N+L-1)\times((2P+1)K-1)]$
ExpSubKF with windowed PAST	0	-	$O[((2P+1)K-2)^2 \times (N+L-1)]$

V. SIMULATION

A $K=10$ -user system is simulated. User 1 is assumed to be the desired one and has fixed SNR of 20dB. Compared with user 1, users 2 to 6 are 10dB stronger, users 7 to 9 are 20dB stronger, and users 10 is 30dB stronger, respectively. The spreading codes are randomly generated and normalized, and the spreading gain is $N=31$. The multipath channel coefficients of each user are randomly generated and normalized and have exponential decay profile. Signal demodulation is assumed to be coherent, so that the fading coefficients can be modeled after phase elimination as real random variables. The presented results are averaged over 1000 Monte Carlo simulations.

Example 1: This example examines the effect of the column rank of $\mathbf{U}_{s,\perp}$ on the expanded subspace-based Kalman filter. In AWGN channel, it is well-known that the rank of the signal subspace is equal to the number of active users. In multipath channels, as analysed in Section II, there are real users and virtual users, and the rank of the expanded signal subspace is $(2P+1)K$. It is expected that for best signal detection performance in multipath channels, $K-1$ columns of $\mathbf{U}_{s,\perp}$ are insufficient and no more than $(2P+1)K-1$ columns are needed. The result in Figure 5 verifies this prediction. In this experiment, the multipath channel spread is 25 (compared to the symbol duration 31). Batch processing method (Figure 2) is adopted for subspace estimation and the batch length is 100 symbols. The plotted SINR is the converged SINR at various $\mathbf{U}_{s,\perp}$ ranks. It can be seen that the detection performance is improved as the rank of $\mathbf{U}_{s,\perp}$ increases because more MAI and ISI are suppressed; after the rank is bigger than 29, the SINR of the detector stops increasing because nearly all MAI and ISI have been cancelled.

Example 2: This example studies the effect of the batch length in the batch processing-based subspace estimation. The expanded subspace and Kalman filter-based multiuser detector in Figure 2 is compared with the Wiener filter in (37). Batch processing method is used to estimate $\mathbf{U}_{s,\perp}$ for the former and is used to estimate the correlation matrix $\mathbf{R}$ for the latter. Figure 6(a) shows the results when the multipath channel spread is 25, while Figure 6(b) corresponds to the channel spread 56 (longer than one symbol duration). Batches of $M=100, 500$ and 10000 symbols are tested for both detectors. It can be seen that the Wiener filter is very sensitive to batch length. The longer batch length, the more accurate estimate of the autocorrelation matrix. When the batch length reaches 10000, the SINR performance nearly approaches the SINR upper bound of linear multiuser detectors. The SINR upper bound is calculated according to (38). Since the SINR bound is specific to the system settings (channel coefficients and user spreading codes) which vary in each Monte Carlo run, the plotted SINR bound is averaged over all Monte Carlo simulations. One can also see that the expanded subspace and Kalman filter-based detector is almost not affected by the batch length, in other words, it is robust against subspace estimation errors.

Example 3: this example examines the effect of the window length on the expanded subspace and Kalman filter-based multiuser detector described in Figure 4. In Figure 7, windows of 50, 150 and 500 symbols are examined for subspace tracking. The batch processing-based adaptive multiuser detector in Figure 2 inevitably causes detection delays, but it only does Kalman filtering once the signal detection phase begins. The windowed PAST-based adaptive multiuser detector in Figure 4 introduces no delays, but during the window interval, it needs to carry out both subspace tracking and Kalman filter tracking at each signal detecting iteration. Longer window demands computation for longer period. The simulation results in Figure 7 show that windows of 50 and 500 achieve similar performance, which reflects the robustness of the adaptive detector.

Example 4: this example examines the robustness of the adaptive detector against the channel estimation error. In (46), $\mathbf{U}_{s,\perp}$ and $\mathbf{w}$ can be estimated by using the methods in Figure 2 and Figure 4. The channel can be estimated, e.g., using training sequence, and then $\mathbf{s}_1$ can be calculated according to (6). Are the adaptive detectors also robust against the channel estimation error? In Figure 8, $\mathbf{h}_1$ represents the real channel of user 1, and $\Delta\mathbf{h}_1$ represents the channel estimation error, it is random variable independent of $\mathbf{h}_1$ [19]. Figure 8 (a) shows the SINR performance of the adaptive detectors and the Wiener filter when $\|\Delta\mathbf{h}_1\|/\|\mathbf{h}_1\| = 0.1$; Figure 8 (b) shows their SINR performance when $\|\Delta\mathbf{h}_1\|/\|\mathbf{h}_1\| = 0.25$. One can see that the performance of the Wiener filter is severely deteriorated by channel estimation errors, while the Kalman filter-based adaptive detectors are only slightly affected. As indicated by Figure 8 (b), the windowed PAST-based adaptive detector is more robust to the large channel estimation errors ($\|\Delta\mathbf{h}_1\|/\|\mathbf{h}_1\| = 0.25$) than the batched processing based adaptive detector. According to our analysis in Section IV-D, the windowed PAST-based detector has the lowest overall computational complexity and processing latency, it is therefore a preferred solution for practical CDMA systems, especially when the channel cannot be accurately estimated.

VI. CONCLUSION

Multiple access interference and intersymbol interference caused by frequency-selective fading are the two main impedances in high-speed CDMA communications. In this paper, we have proven that the

maximum SINR detectors are in the signal subspace, and there exists a SINR performance upper bound for linear multiuser detectors. Two adaptive multiuser detectors are developed in an expanded signal subspace by using subspace estimation and Kalman filtering estimation algorithms. They effectively suppress both MAI and ISI simultaneously and approach the optimum SINR performance bound, more importantly, they are much more robust against subspace estimation errors and channel estimation errors than the conventional Wiener MMSE detector.

APPENDIX

DIRECT PROOF THAT MMSE DETECTOR IS IN THE SIGNAL SUBSPACE

Equation (37) can be rewritten as

$$\mathbf{R}\mathbf{d}_{WMMSE} = A_1^2 \mathbf{s}_1 \quad (60)$$

Substituting (41) into (60) leads to

$$\sigma^2 \mathbf{d}_{WMMSE} = A_1^2 \mathbf{s}_1 - \mathbf{S}\mathbf{A}^2 \mathbf{S}^H \mathbf{d}_{WMMSE} \quad (61)$$

In (61), $\mathbf{S}$, $\mathbf{A}^2$ and $\mathbf{d}_{WMMSE}$ are matrices of sizes $(N+L-1) \times (2P+1)K$, $(2P+1)K \times (2P+1)K$ and $(N+L-1) \times 1$, respectively, and the columns of $\mathbf{S}$ are all in the expanded signal subspace. Denote $\boldsymbol{\theta}_{(2P+1)K \times 1} = \mathbf{A}^2 \mathbf{S}^H \mathbf{d}_{WMMSE}$, (61) can be written as

$$\sigma^2 \mathbf{d}_{WMMSE} = A_1^2 \mathbf{s}_1 - \mathbf{S}\boldsymbol{\theta} \quad (62)$$

$$= A_1^2 \mathbf{s}_1 - \sum_{k=1}^K \left[\theta_k \mathbf{s}_k + \sum_{\tau=1}^P \bar{\theta}_{k,\tau} \bar{\mathbf{s}}_{k,\tau} + \sum_{\tau=1}^P \bar{\bar{\theta}}_{k,\tau} \bar{\bar{\mathbf{s}}}_{k,\tau} \right] \quad (63)$$

where θ_k , $\bar{\theta}_{k,\tau}$ and $\bar{\bar{\theta}}_{k,\tau}$ are all scalars. They are the elements of vector $\boldsymbol{\theta}$ corresponding to $\mathbf{s}_k$, $\bar{\mathbf{s}}_{k,\tau}$ and $\bar{\bar{\mathbf{s}}}_{k,\tau}$, respectively. Equality (63) follows from (14)-(16). Apparently, the right side of (63) is a linear combination of $\mathbf{s}_k$, $\bar{\mathbf{s}}_{k,\tau}$ and $\bar{\bar{\mathbf{s}}}_{k,\tau}$, which are all in the expanded signal subspace, therefore $\mathbf{d}_{WMMSE}$ is in the expanded signal subspace, and so is $\mathbf{d}_{CMMSE}$ since $\mathbf{d}_{CMMSE}$ and $\mathbf{d}_{WMMSE}$ differ only by a scaling factor.

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FIGURES

Figure 1: Observation window for symbol detection.

Figure 2: Flowchart for the multiuser detector for multipath channels using the batch-process subspace estimation and Kalman filter-based weight estimation.

Figure 3: Block diagram of multiuser detector based on Kalman filter and windowed PAST subspace tracking algorithm.

Figure 4: Flowchart for the multiuser detector for multipath channels using the windowed PAST subspace tracking and Kalman filter-based weight tracking algorithms (β is the forgetting factor)

Figure 5: ($K=10$, $N=31$, $\text{SNR}=20\text{dB}$, $L=25$) SINR performance of the subspace Kalman filter versus the column ranks of $\mathbf{U}_{s,\perp}$. Batch processing method is adopted for subspace estimation.

(a)

(b)

Figure 6: ($K=10$, $N=31$, $\text{SNR}=20\text{dB}$) SINR performance of the expanded subspace Kalman filter-based adaptive detector and the Wiener MMSE detector in multipath channels. Subspace and autocorrelation matrix are estimated by batch processing. (a) Channel spread=25; (b) channel spread =56.

(a)

(b)

Figure 7: ($K=10$, $N=31$, $\text{SNR}=20\text{dB}$) SINR performance of expanded subspace Kalman filter-based adaptive detector in multipath channels. Subspace is estimated by the windowed PAST algorithm. (a) Channel spread=25; (b) channel spread =56.

(a)

(b)

Figure 8: ($K=10$, $N=31$, $\text{SNR}=20\text{dB}$) SINR performance of expanded subspace multiuser detectors with channel estimation error in multipath channels, Channel spread=25. (a) $\|\Delta\mathbf{h}_1\|/\|\mathbf{h}_1\| = 0.1$

(b) $\|\Delta\mathbf{h}_1\|/\|\mathbf{h}_1\| = 0.25$.