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Spatial correlation of heat release rate and sound emission from turbulent premixed flames

Y. Liu*, A.P. Dowling, N. Swaminathan*, T.D. Dunstan

^aDepartment of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

Abstract

The two-point spatial correlation of the rate of change of fluctuating heat release rate is central to the sound emission from open turbulent flames, and a few attempts have been made to address this correlation in recent studies. In this paper, the two-point correlation and its role in combustion noise are studied by analysing direct numerical simulation (DNS) data of statistically multi-dimensional turbulent premixed flames. The results suggest that this correlation function depends on the separation distance and direction but, not on the positions inside the flame brush. This correlation can be modelled using a combination of Hermite-Gaussian functions of zero and second order, i.e. functions of the form $(1 - Ax^2)e^{-Bx^2}$ for constants A and B , to include its possible negative values. The integral correlation volume obtained using this model is about $0.2\delta_L^3$ with the length scale obtained from its cube root being about $0.6\delta_L$, where δ_L is the laminar flame thermal thickness. Both of the values are slightly larger than the values reported in an earlier study because

*Corresponding authors. Tel.: +44 1223 766063 (Y. Liu). Tel.: +44 1223 332586; fax: +44 1223 765311 (N. Swaminathan).

Email addresses: `yu.liu@eng.cam.ac.uk` (Y. Liu), `ns341@eng.cam.ac.uk` (N. Swaminathan)

of the anisotropy observed for the correlation. This model together with the turbulence-dependent parameter $\mathcal{K}$, the ratio of the root-mean-square (RMS) value of the rate of change of reaction rate to the mean reaction rate, derived from the DNS data are applied to predict the far-field sound emitted from open flames. The calculated noise levels agree well with recently reported measurements and show a sensitivity to $\mathcal{K}$ values.

Keywords: Combustion noise from open flames, Two-point correlation, Heat release rate fluctuation, Correlation volume and length scale, DNS V-flames

1. Introduction

Turbulent combustion produces sound emission due to its inherent unsteadiness. Many studies [1–6] have indicated that noise due to unsteady combustion may become a significant noise source particularly for lean burn gas turbines because lean burning generally involves highly unsteady flames. Combustion noise is also a signature of combustor health and performance because of its important role in the dynamics of the combustion system. Therefore, a thorough understanding of the source mechanisms of combustion noise is required at the design stage of gas turbines to help minimise the noise emissions. It is known that the combustion noise is generated by the fluctuating heat release rate, which causes unsteady expansion of reacting gases inside the turbulent flame brush and hence behaves locally as a distributed monopole source.

The prediction of combustion noise level for a practical burner is still a challenging issue [7] despite many efforts [2, 8–11] to develop a semi-empirical

16 correlation between the far-field acoustic power and burner geometry or
 17 operating flow conditions. It has been recently noted [5, 6] that the two-point
 18 spatial correlation of the rate of change of fluctuating heat release rate,
 19 and hence the associated correlation volume v_{cor} and its length scale, is
 20 crucial to predicting combustion noise. In previous studies, different length
 21 scales have been suggested to estimate v_{cor} empirically, ranging from the
 22 laminar flame thickness δ_L [12], turbulence integral length scale Λ [13] to the
 23 turbulent flame-brush thickness [14, 15] or a combination of them. However,
 24 the two-point correlation of heat release rate has not received sufficient
 25 attention primarily owing to the lack of availability of reliable numerical
 26 or experimental data for the fluctuating heat release rate required to directly
 27 investigate the correlation length scale.

28 Recently, it has become feasible to obtain high-fidelity information on the
 29 correlation of fluctuating heat release rate because of the advancement in
 30 computing technologies and techniques, and laser metrology. Swaminathan
 31 et al. [5, 6] have attempted to analyse and model this two-point correlation
 32 using DNS [16–18] and laser diagnostics [19] data of turbulent premixed
 33 flames. They found that the two-point correlations of heat release rate,
 34 Ω , and the rate of change of fluctuating heat release rate, Ω_1 , can be
 35 well represented by Gaussian-type functions commonly used in classical
 36 turbulence, and that the length scale of the correlation volume, v_{cor} , is
 37 $0.5\delta_L$. This model was then shown to give a good agreement with recent
 38 experimental measurements [20] of the far-field overall sound pressure level
 39 (OASPL) from open turbulent premixed flames.

40 The construction of Ω_1 requires the rate of change of fluctuating heat

41 release rate, which was calculated in Refs. [5, 6] indirectly by using a balance
 42 equation for a progress variable and taking the instantaneous reaction rate
 43 to be a function of the progress variable. In this way the time derivative
 44 is obtained using the spatial derivative of the progress variable field at one
 45 single time step from the DNS data [16–18]. These numerical data are,
 46 however, for statistically planar flames and the correlation length scale in
 47 the mean flame propagation direction (normal to the flame brush) was then
 48 used to estimate the correlation volume as $v_{\text{cor}} \sim \delta_L^3/8$ by assuming isotropy
 49 for the correlation. This value of v_{cor} may be underestimated, as one shall
 50 see in Section 4.3, since the heat release rate varies most dramatically in the
 51 flame normal direction and hence the correlation length is relatively short
 52 in this direction compared with the other directions in the turbulent flame.
 53 Moreover, the parameter $\mathcal{K}$ related to a time scale for the rate of change of
 54 fluctuating heat release rate, required to predict the combustion noise level,
 55 was estimated through combined analyses of the DNS and laser diagnostics
 56 data. Furthermore, this estimate strongly depends on the approximation
 57 used to obtain the time derivative of the fluctuating heat release rate.
 58 Nevertheless, the work of Swaminathan et al. [5, 6] makes an important
 59 contribution towards improved understanding of the two-point correlation
 60 of the rate of change of heat release rate fluctuation and its role in noise
 61 emission from open turbulent flames.

62 The prime objectives of this study are three folds, viz., (i) to assess
 63 the isotropy of the two-point spatial correlation function, Ω_1 , assumed in
 64 an earlier study [6], using DNS data [21] of statistically multi-dimensional
 65 flames; (ii) to evaluate the time derivative of the fluctuating heat release

rate directly in the DNS and its influence on the value of the parameter $\mathcal{K}$; and (iii) to study the sensitivity of the far-field OASPL values to the newly obtained information from the first two objectives. The results of Reynolds-Averaged-Navier-Stokes (RANS) calculations in Ref. [6] are used to recompute the far-field OASPL to address the third objective.

This paper is organised as follows. The theoretical background on the two-point correlation and its relation to the OASPL is reviewed briefly in the next section. The salient features of DNS data and the analyses are discussed in Section 3. The results are discussed in Section 4 and the conclusions are summarised in the final section.

2. Theoretical formulation

2.1. Combustion noise

The sound emission from a turbulent reacting flow has been derived by Dowling [4] using Lighthill's theory [22, 23] which includes a variety of sources associated with flow noise, viscous dissipation, heat and molecular transports, direct and indirect combustion noise. At low Mach number condition as in open flames, the direct noise from unsteady heat input is the dominant source and is about two orders of magnitude larger than other sources [24, 25]. This leads to a linear wave equation of the reduced form:

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial t} \left[\frac{\rho_0(\gamma - 1)}{\rho a^2} \dot{Q}'(\mathbf{y}, t) \right], \quad (1)$$

where p' is the pressure perturbation, γ is the ratio of specific heat capacities, and $\dot{Q}'$ is the fluctuating heat release rate per unit volume; ρ and a denote the fluid density and sound speed in the combustion zone and differ from the

88 ambient values ρ_0 and a_0 . When the turbulent combustion occurs at ambient
 89 pressure and γ is assumed to be independent of temperature,

$$\rho a^2 = \gamma p_0 = \rho_0 a_0^2, \quad (2)$$

90 and thus the wave equation (1) simplifies to

$$\frac{1}{a_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{(\gamma - 1)}{a_0^2} \frac{\partial \dot{Q}'(\mathbf{y}, t)}{\partial t}. \quad (3)$$

91 Readers are referred to Refs. [4, 6] for a complete derivation of the above
 92 expression starting from the Lighthill equation and a detailed analysis of the
 93 contributions of various source terms.

94 The solution of Eq. (3) can be written as

$$p'(\mathbf{x}, t) = \frac{(\gamma - 1)}{4\pi r a_0^2} \frac{\partial}{\partial t} \int_{v_f} \dot{Q}'(\mathbf{y}, t - r/a_0) d^3 \mathbf{y} \quad (4)$$

95 by using a free-space Green's function, where $\mathbf{x}$ denotes the observer point
 96 in far field, $\mathbf{y}$ is the source position inside the acoustically compact flame
 97 brush of volume v_f , and $r = |\mathbf{x}|$ is the observer distance from an origin
 98 within v_f . The effects of convection and refraction of sound [26, 27] caused
 99 by the variations of γ and a within the flame brush due to temperature
 100 inhomogeneities are neglected as noted earlier in order to focus on the role
 101 of the dominant source from the unsteady heat addition. More importantly,
 102 it is clearly shown in Eq. (4) that the far-field sound pressure is expressed
 103 in terms of a volume integral of the unsteady heat release rate over the
 104 turbulent flame brush at a retarded time $t - r/a_0$. The source of combustion
 105 noise originates from the rate of change of this integral and behaves as an
 106 acoustic monopole. The far-field OASPL characterised by $\overline{p'^2}$, a measurable

107 quantity in experiments, can be simply obtained from Eq. (4) as

$$\overline{p'^2(\mathbf{x}, t)} = \frac{(\gamma - 1)^2}{16\pi^2 r^2 a_0^4} \int_{v_f} \int_{v_{\text{cor}}} \overline{\ddot{Q}'(\mathbf{y}, t) \ddot{Q}'(\mathbf{y} + \mathbf{\Delta}, t)} d^3\mathbf{\Delta} d^3\mathbf{y}, \quad (5)$$

108 where $\ddot{Q}'$ is the time derivative of the fluctuating heat release rate, $\mathbf{\Delta}$ is
 109 the separation vector, and the overbar refers to an averaging process. The
 110 volume over which $\ddot{Q}'$ is correlated is denoted as the correlation volume
 111 v_{cor} that has originated from Bragg's theory [12]. The two-point correlation
 112 $\overline{\ddot{Q}'(\mathbf{y}, t) \ddot{Q}'(\mathbf{y} + \mathbf{\Delta}, t)}$ and its volume v_{cor} are the focus of this paper and will
 113 be discussed in detail subsequently.

114 2.2. Two-point correlation

115 Following the common practice in the analysis of turbulent premixed
 116 flames [5, 6], one can use a progress variable, c , which varies from zero in
 117 reactants to unity in products and define it using the fuel mass fraction [28].
 118 The instantaneous heat release rate $\dot{Q}$ is then related to the instantaneous
 119 chemical reaction rate $\dot{w}$ as $\dot{Q} = Y_{f,u} H \dot{w}$, where $Y_{f,u}$ is the fuel mass fraction
 120 in the unburnt reactants, and H is the lower heating value of the fuel. This
 121 approximation holds good even when a complex chemical kinetics is used to
 122 model combustion [28]. In doing so, $\dot{Q}$ can be replaced by $\dot{w}$ in the analysis.
 123 Noting the simple relation between the heat release rate and the reaction
 124 rate, the two-point correlation in Eq. (5) is expressed as

$$\overline{\ddot{Q}'(\mathbf{y}, t) \ddot{Q}'(\mathbf{y} + \mathbf{\Delta}, t)} = Y_{f,u}^2 H^2 \overline{\ddot{w}'(\mathbf{y}, t) \ddot{w}'(\mathbf{y} + \mathbf{\Delta}, t)}, \quad (6a)$$

$$\overline{\ddot{w}'(\mathbf{y} - \mathbf{\Delta}/2, t) \ddot{w}'(\mathbf{y} + \mathbf{\Delta}/2, t)} = \Omega_1(\mathbf{y}, \mathbf{\Delta}) \overline{\ddot{w}'^2(\mathbf{y}, t)}, \quad (6b)$$

125 where Ω_1 is the correlation function for the rate of change of heat release rate
 126 fluctuation. All terms with overbar are independent of time t . By definition

127 the correlation function Ω_1 depends on both the spatial location, $\mathbf{y}$, and
 128 separation, Δ , in the flame brush. In spite of this, the former dependence is
 129 nearly negligible as will be observed in Section 4.2, and hence the correlation
 130 function can be denoted as $\Omega_1(\Delta)$. Further discussion on the assumptions of
 131 Eq. (6) can be found in Ref. [6].

132 For the purpose of computing OASPL, it is helpful to express the
 133 two-point correlation $\overline{\ddot{Q}(\mathbf{y}, t)\ddot{Q}(\mathbf{y} + \Delta, t)}$ in Eq. (6) in terms of the square of
 134 the local mean heat release rate $\overline{\dot{Q}(\mathbf{y}, t)}^2$. This is achieved by relating $\overline{\ddot{w}^2}$ to
 135 the mean reaction rate $\bar{w}$ as

$$\overline{\ddot{w}^2(\mathbf{y}, t)} = \mathcal{K}^2 \overline{\dot{w}(\mathbf{y}, t)}^2, \quad (7)$$

136 where the parameter $\mathcal{K}$ was decomposed into two terms as in Refs. [5, 6]:

$$\mathcal{K} = \left(\overline{\ddot{w}^2}/\overline{\dot{w}^2}\right)^{1/2} \cdot \left(\overline{\dot{w}^2}/\bar{w}^2\right)^{1/2} = \mathcal{B}_1 \mathcal{B} \quad (8)$$

137 with $\mathcal{B}_1$ as the inverse of an average time scale for the rate of change of
 138 the fluctuating reaction rate, and $\mathcal{B}$ the ratio of the RMS of the reaction
 139 rate fluctuation to the mean value. Using the relationship (7) the two-point
 140 correlation in Eq. (6) becomes

$$\overline{\ddot{Q}'(\mathbf{y}, t)\ddot{Q}'(\mathbf{y} + \Delta, t)} = Y_{f,u}^2 H^2 \mathcal{K}^2 \Omega_1(\Delta) \overline{\dot{w}(\mathbf{y}, t)}^2. \quad (9)$$

141 By substituting the above expression into Eq. (5), one obtains the far-field
 142 OASPL as:

$$\overline{p'^2(\mathbf{x}, t)} = \frac{(\gamma - 1)^2}{16\pi^2 r^2 a_0^4} Y_{f,u}^2 H^2 \int_{v_f} \mathcal{K}^2 \overline{\dot{w}(\mathbf{y}, t)}^2 d^3 \mathbf{y} \int_{v_{\text{cor}}} \Omega_1(\Delta) d^3 \Delta, \quad (10)$$

143 where the original double integral is split into two single integrals
 144 to divide up the respective contributions from the flame brush v_f

145 (turbulence, thermo-chemistry and their interaction) and correlation
 146 volume v_{cor} (thermo-chemistry). The first integral contains the
 147 turbulence-thermo-chemistry dependent terms $\mathcal{K}$, $\overline{w}$ and v_f . The spatially
 148 varying $\mathcal{K}$ can be derived from the V-flame DNS data of Dunstan et al. [21] as
 149 mentioned before. The second integral over the correlation volume is denoted
 150 as the integral correlation volume V_{cor} and its length scale is defined as the
 151 cube root of V_{cor} . This correlation integral can be evaluated separately since
 152 the correlation function Ω_1 is found to be insensitive to source positions inside
 153 the flame brush. Also, the spatial correlation of $\ddot{Q}$ is dictated predominantly
 154 by the thermo-chemical processes and the influence of turbulence on Ω_1 is
 155 negligibly small [6]. The DNS data from turbulent premixed V-flames [21]
 156 will be analysed to study the correlation function Ω_1 and a length scale for
 157 V_{cor} to address the first two objectives of this study. The mean reaction rate
 158 $\overline{\dot{w}(\mathbf{y}, t)}$ available from steady RANS calculations [6] is used to predict the
 159 far-field OASPL of open flames [20] to address the third objective.

160 **3. DNS V-flames**

161 The single V-flame, one of several canonical configurations for premixed
 162 turbulent flames, has been studied extensively in previous experimental
 163 work [29–33] and numerical simulations [34–36]. Recently, DNS for
 164 laboratory-scale V-flames has become available and much progress has been
 165 made [21, 35, 36]. Dunstan et al. [21] carried out three-dimensional (3D)
 166 fully compressible DNS of premixed turbulent V-flames with particular
 167 emphasis on evaluation of turbulent flame speed. They used a cubic
 168 computation domain with the Navier-Stokes Characteristic Boundary

169 Conditions (NSCBC) [37] in the streamwise, x , direction and the transverse,
 170 y , direction. The standard NSCBC has been modified to accommodate the
 171 steep thermal and compositional gradients generated when the flame crosses
 172 the boundary. The spanwise, z , direction was specified to be periodic. The
 173 flame holder is aligned in the spanwise direction and the flame gradually
 174 evolves into two statistically symmetric branches that become progressively
 175 thicker as they propagate downstream of the flame holder. The general
 176 features of these V-flames will be discussed in more detail in Section 4.1.

177 3.1. Flame conditions

178 Three cases of weak, moderate and high intensity turbulence were used
 179 by Dunstan et al. [21] and all of these cases will be analysed in this
 180 paper. Their fluid dynamic conditions are summarised in Table 1 and
 181 a single-step chemical mechanism was used for preheated reactants with
 182 unity Lewis numbers. The unstrained laminar flame thermal thickness is
 183 $\delta_L = 0.43$ mm and the flame speed is $S_L = 0.60$ m s⁻¹ giving a flame time of
 184 $\tau_f = \delta_L/S_L = 0.71$ ms. These thermo-chemical parameters are representative
 185 of a premixed methane-air flame with an equivalence ratio of $\phi \approx 0.6$. The

Table 1: Combustion conditions of the DNS V-flames [21].

Case	u_{rms}^+	$\bar{u}^+$	Λ^+	Re	Da	τ_D^+
I	1.0	16.57	3.57	18	3.57	1.79
II	2.0	16.57	3.62	37	1.81	1.79
III	6.0	24.86	3.43	92	0.57	1.19

quantities with a superscript $+$ in the following discussion are normalised appropriately using the laminar flame thickness, its speed and the density of unburnt reactant ρ_u . In Table 1, u_{rms} is the inlet turbulence intensity, i.e. the RMS value of turbulence velocity fluctuation, $\bar{u}$ is the inlet mean streamwise velocity, and Λ is the inlet turbulence integral length scale. The turbulence Reynolds number Re is based on u_{rms} and Λ , and the Damköhler number is defined as $\text{Da} = \Lambda^+ / u_{\text{rms}}^+$. The flame conditions in Table 1 suggest that Cases I to III respectively correspond to the wrinkled flamelets, corrugated flamelets and thin reaction zones, regimes of turbulent combustion, according to the Borghi-Peters classification [38].

With respect to the spatial resolution, the computational domain has dimensions of 12.77 mm ($29.7\delta_L$) in each direction and is discretised using $512 \times 512 \times 512$ uniform grid, ensuring a minimum of about 10 grid points inside the laminar flame thickness δ_L . The flame holder was positioned at $3.48\delta_L$ from the inlet to ensure stable combustion whilst minimising its effect on the downstream flame statistics. While running the DNS, data were collected consecutively during one flow-through time τ_D (the mean convection time from the inlet to outlet boundaries) after the turbulence had reached a fully developed state and one time length of τ_D had been run to ensure the decay of initial transients. The data of Cases I and II have 254 uniform time steps ($\Delta t = 5 \times 10^{-6}$ s), whereas for Case III only 85 time steps ($\Delta t = 10^{-5}$ s) were taken due to the shorter flow-through and turbulence time scales. More details of the DNS V-flames can be found in Ref. [21].

209 3.2. Data processing

210 The rate of change of fluctuating reaction rate, $\dot{w}'$, required to construct
 211 the two-point correlation, $\Omega_1(\Delta)$, is calculated using the $\dot{w}$ fields saved during
 212 the DNS. First, the mean reaction rate is obtained using

$$\bar{\dot{w}}(x, y) = \frac{1}{N_t N_z} \sum_{n=1}^{N_t} \sum_{k=1}^{N_z} \dot{w}(x, y; t_n, z_k), \quad (11)$$

213 where N_t is the number of sample fields collected over the second flow-through
 214 time in the DNS, and N_z is the number of grid points in the spanwise,
 215 periodic direction. This type of averaging helps us to improve the statistical
 216 convergence for the mean values. The fluctuating reaction rate is then
 217 obtained using $\dot{w}' = \dot{w} - \bar{\dot{w}}$. Since the averaging is done over the time
 218 also one gets $\ddot{w}' = \ddot{w}$.

219 The construction of the correlation function Ω_1 is then straightforward
 220 following Eq. (6b). A second-order central difference scheme is used to obtain
 221 the time derivative, $\ddot{w}$, and the results have been verified using a higher-order
 222 numerical scheme and a finer time resolution. The mean flow field of the DNS
 223 flames is predominantly two-dimensional (2D) and thus the Favre-averaged
 224 (density weighted) progress variable $\tilde{c}$ is related to the (x, y) plane only. The
 225 progress variable $\tilde{c}$ can be used to denote the transverse position, y , inside
 226 the flame brush for a given streamwise location. Figure 1 illustrates the 2D
 227 contours of $\tilde{c}$ for Cases I to III and this figure will be discussed further later.

228 The samples for analysis are collected at points located at least $10\delta_L$
 229 downstream of the flame holder and at least δ_L apart from any boundaries to
 230 avoid any possible influences from the flame holder and boundary conditions.
 231 Moreover, sample points are restricted in the range $0.1 < \tilde{c} < 0.9$ to ensure

232 meaningful statistics by avoiding regions with $\bar{w}$ close to zero [5, 6]. Three
 233 sample positions, $x_o^+ = 16.7, 20.4$ and 24.2 (see Fig. 1) as in Ref. [21],
 234 are selected for analysis. For each x_o^+ position, a series of $\tilde{c}$ values,
 235 $0.2, 0.3, \dots, 0.8$, distributed along the transverse direction are chosen in each
 236 of the two flame branches. The separation distances in all three directions
 237 Δ_x, Δ_y and Δ_z are taken from these sample positions and the correlation
 238 function $\Omega_1(\Delta)$ is finally averaged between the two flame branches.

239 4. Results and discussion

240 4.1. Flame features

241 First the contour plots of $\tilde{c}$ in the (x^+, y^+) plane are shown in Fig. 1 for the
 242 three cases, together with the three streamwise positions used in the analysis.
 243 As in typical V-flames, the orientation angle of the mean flame brush to the
 244 reactant stream increases with downstream position resulting in a slightly
 245 curved mean flame. The effects of the turbulence intensity, u_{rms}^+ , and the
 246 mean inlet velocity, $\bar{u}^+$, can also be observed from Fig. 1. The thickness of
 247 the turbulent flame brush increases with u_{rms}^+ and the separation between
 248 the flame branches is further reduced by the larger value of $\bar{u}^+$ in Case III,
 249 which is in agreement with earlier observation [39].

250 Figures 2 and 3 illustrate typical contours of the instantaneous reaction
 251 rate, $\dot{w}$, and its time derivative, $\ddot{w}$, respectively in the mid (x^+, y^+) plane.
 252 The values of $\dot{w}$ and $\ddot{w}$ are normalised by $\rho_u S_L / \delta_L$ and $\rho_u S_L^2 / \delta_L^2$, respectively.
 253 The $\dot{w}$ contours are confined to thin regions and typical thickness of the
 254 reaction zone, $\delta_{\dot{w}}$, is about one laminar flame thickness δ_L . This thickness
 255 remains almost unchanged along the flame front except in regions with large

256 curvature where $\delta_{\dot{w}}$ becomes larger up to $2\delta_L$ and this is clear in Case III.
 257 The influence of turbulence is also observed as the curvature of the contours
 258 increases from Case I to Case III with increasing turbulence intensities,
 259 generating gradually more sinuous contours.

260 The contours of $\ddot{w}$ in Fig. 3 retain the same overall wrinkling patterns as
 261 the $\dot{w}$ contours. Profiles through the local flame normal exhibit two adjacent
 262 positive and negative peaks due to the transit of the reaction zone. The
 263 instantaneous direction of displacement of the flame front is along a line
 264 going from the negative to positive peaks of $\ddot{w}$, and is predominantly in the
 265 streamwise direction due to the strong mean flow velocity. The thickness of
 266 the $\ddot{w}$ contours, $\delta_{\ddot{w}}$, is similar between Cases I and II, and the single positive
 267 or negative peak of the $\ddot{w}$ contours is confined to a thinner region compared
 268 to the thickness $\delta_{\dot{w}}$ of the $\dot{w}$ contours in Fig. 2. The thickness $\delta_{\ddot{w}}$ for Case
 269 III, however, increases due to the higher mean streamwise velocity, resulting
 270 in the thicker $\delta_{\ddot{w}}$ for a single peak than the thickness $\delta_{\dot{w}}$.

271 4.2. Correlation function and model

272 The results of the two-point correlation function $\Omega_1(\Delta^+)$ for the rate
 273 of change of reaction rate fluctuation $\ddot{w}$ are shown in Figs. 4–6 for Cases I
 274 to III, respectively. In these figures, the separation distances are taken in
 275 the streamwise, Δ_x , transverse, Δ_y , and spanwise, Δ_z , directions; three x_o^+
 276 positions and seven transverse positions, denoted by $\tilde{c}$ inside the flame brush,
 277 are shown giving 21 groups of data in total for each flame case and each
 278 direction. As can be seen from Figs. 4–6, the correlation function Ω_1 is
 279 symmetric about $\Delta^+ = 0$ in all cases and directions. The value of Ω_1 drops
 280 quickly from one within a short distance and negative values are observed,

281 i.e. Ω_1 is as small as -0.4 at $|\Delta_y^+| \approx 0.6$ for Cases I and II and at $|\Delta_x^+| \approx 0.9$
 282 for Cases III. For Cases I and II with low turbulence levels, the negative zones
 283 for $\Omega_1(\Delta_y^+)$ arise due to the adjacent positive and negative peaks in the $\ddot{w}$
 284 contours (see Fig. 3) in which the flame normal is predominantly aligned in
 285 the transverse, y , direction. The highly turbulent Case III is associated with
 286 highly intermittent reaction rate signals in space as well as in time. The
 287 large increase in the intermittency of the reaction zone makes it obvious a
 288 strong convection in the streamwise direction which accounts for the negative
 289 zones of $\Omega_1(\Delta_x^+)$, as will be explained below. The mean velocities in other
 290 directions are much smaller and the respective convection effect is negligible.

291 The results from the V-flame DNS data enables the development of
 292 a model for Ω_1 , plotted as solid lines in Figs. 4–6. Previous studies
 293 on combustion noise [5, 6] and jet noise [40] have indicated that the
 294 correlation of the noise sources can be represented reasonably well by a
 295 Gaussian-type function. Small negative values of the correlation function
 296 Ω_1 were also observed by Swaminathan et al. [5, 6] but were ignored in
 297 the model of a standard Gaussian function. In this work, a combination of
 298 Hermite-Gaussian functions of zero and second order is used to model the
 299 negative zones in the Δ_x and Δ_y directions as well as a standard Gaussian
 300 function (zero-order Hermite-Gaussian function) in the Δ_z direction. These
 301 models developed from the V-flame DNS data are:

$$\Omega_1(\Delta^+) = \Omega_1(\Delta_x^+) \cdot \Omega_1(\Delta_y^+) \cdot \Omega_1(\Delta_z^+), \quad (12a)$$

$$\text{and} \quad \Omega_1(\Delta_x^+) = \left(1 - \epsilon_x \pi \Delta_x^{+2}\right) \exp\left(-\sigma_x \pi \Delta_x^{+2}\right), \quad (12b)$$

$$\Omega_1(\Delta_y^+) = \left(1 - \epsilon_y \pi \Delta_y^{+2}\right) \exp\left(-\sigma_y \pi \Delta_y^{+2}\right), \quad (12c)$$

$$\Omega_1(\Delta_z^+) = \exp\left(-\sigma_z \pi \Delta_z^{+2}\right). \quad (12d)$$

302 The Hermite-Gaussian functions for $\Omega_1(\Delta_x^+)$ and $\Omega_1(\Delta_y^+)$ are inspired by a
 303 correlation model of the reaction rate fluctuation, $\dot{w}'$, in an earlier study [41].
 304 In that work, the convection effect in the streamwise direction led to a coupled
 305 two-point space-time correlation function for $\dot{w}'$ as

$$\Omega(\Delta_x^+, \tau^+) = \exp\left[-\sigma_1 \pi (\Delta_x^+ - \bar{u}^+ \tau^+)^2 - \sigma_2 \pi (\Delta_x^+ + \bar{u}^+ \tau^+)^2\right], \quad (13)$$

306 where the constants $\sigma_1 \gg \sigma_2$. Taking a double time derivative, $\partial^2 \Omega / \partial \tau^{+2}$,
 307 of Eq. (13) at $\tau^+ = 0$ and recognising that $\Omega(0, 0) = 1$, one gets the form of
 308 Hermite-Gaussian functions as in Eq. (12b).

309 In the correlation model given in Eq. (12), the coefficients $\sigma_x, \epsilon_x, \sigma_y, \epsilon_y, \sigma_z$
 310 represent the rate of decay for Ω_1 from unity to zero. These coefficients
 311 are given in Table 2. They take different values in different directions and

Table 2: Coefficient constants of decay rate in the correlation model (12) for Ω_1 .

Case	σ_x	ϵ_x	σ_y	ϵ_y	σ_z
I	0.8	0.0	2.0	3.4	0.2
II	2.0	0.0	1.2	1.8	0.5
III	0.6	1.0	1.5	0.0	0.9

312 vary from case to case. For the streamwise Δ_x direction, the value of the
 313 correlation function Ω_1 drops rapidly from 1 to about 0.05 within about half
 314 to one laminar flame thickness δ_L . The dynamics and fluctuation levels of the
 315 chemical reactions occurring in thin regions are predominantly controlled by
 316 the dynamics of small scale turbulence [5, 6], and this accounts for the sharp
 317 fall of the correlation of the fluctuating $\ddot{w}$ in space. The decay rate in the
 318 Δ_y direction is close to that in the Δ_x direction for Case III, but it becomes
 319 greatly faster for Cases I and II because of the large transverse gradients of
 320 fluctuating quantities within the relatively thinner flame brush of these two
 321 cases as shown in Fig. 1. The correlation in the spanwise direction $\Omega_1(\Delta_z^+)$,
 322 however, drops slowly in comparison with the other two directions. The
 323 smallest value of $\sigma_z = 0.2$ occurs for Case I and $\Omega_1(\Delta_z^+)$ reaches zero within
 324 about $3\delta_L$.

325 As can be seen from Figs. 4–6, the Hermite-Gaussian function given in
 326 Eq. (12) represents a reasonable approximation to the DNS results including
 327 the negative values. A very small variation of Ω_1 with x^+ and $\tilde{c}$ is seen
 328 within the flame brush, which confirms the independence of the correlation
 329 function on spatial position as has been claimed in Section 2.2. In addition,
 330 small oscillations of Ω_1 at large values of $|\Delta_x^+|$ are seen in Cases II & III, and
 331 small negative values of $\Omega_1(\Delta_z^+)$ occur at some spatial positions for all cases.
 332 These are due to the limited statistical samples available for averaging since
 333 these oscillations increase when the sample size is halved. Nevertheless, the
 334 level of agreement between the DNS and modelled values of the correlation
 335 function Ω_1 in Figs. 4–6 is acceptable.

336 Similar to the two-point correlation for $\ddot{w}$, one can write another

337 correlation function, Ω , for the reaction rate fluctuation as

$$\overline{\dot{w}'(\mathbf{y} - \Delta/2, t) \dot{w}'(\mathbf{y} + \Delta/2, t)} = \Omega(\mathbf{y}, \Delta) \overline{\dot{w}'^2(\mathbf{y}, t)}. \quad (14)$$

338 This correlation has no explicit effect on the OASPL of combustion noise
 339 as it does not appear in Eq. (10). The results of Ω are presented in this
 340 paper merely for comparison with the correlation function Ω_1 . Figure 7
 341 shows the correlation function Ω for Case III exhibiting very similar features
 342 as those of Ω_1 in Figs. 4–6 except for the broader curves and less distinct
 343 negative zones. It can be seen that Ω can be approximately modelled as the
 344 standard Gaussian function $\exp(-\sigma\pi\Delta^{+2})$ but with slower decay coefficients
 345 $\sigma_x = 0.6$, $\sigma_y = 1.0$ and $\sigma_z = 0.5$ as expected. The value of Ω reaches
 346 zero around $|\Delta_x^+| = 2$, $|\Delta_y^+| = 1$ and $|\Delta_z^+| = 3$, larger than the respective
 347 values of 0.5, 0.5 and 1.5 for Ω_1 . This is because the time derivative of
 348 the fluctuating reaction rate, $\ddot{w}$, is associated with stronger fluctuations and
 349 hence is correlated within a shorter length than the reaction rate fluctuation,
 350 $\dot{w}'$, itself. Note that the decay rate σ_x remains the same for both correlation
 351 functions Ω and Ω_1 because Ω_1 retains the exponential form of Ω after the
 352 double time derivative.

353 4.3. Correlation volume and length scale

354 The integral length scale for $\ddot{w}$, normalised by the laminar flame thickness
 355 δ_L , is defined as

$$\ell_1^+ = \int_0^\infty \Omega_1(\Delta^+) d\Delta^+. \quad (15)$$

356 By using the Hermite-Gaussian function model for Ω_1 described by Eq. (12),
 357 the integral length scales in the streamwise, transverse and spanwise
 358 directions are calculated as

$$\ell_{1,x}^+ = \frac{1}{2} \left(\sigma_x^{-1/2} - \frac{1}{2} \epsilon_x \sigma_x^{-3/2} \right), \quad (16a)$$

$$\ell_{1,y}^+ = \frac{1}{2} \left(\sigma_y^{-1/2} - \frac{1}{2} \epsilon_y \sigma_y^{-3/2} \right), \quad (16b)$$

$$\ell_{1,z}^+ = \frac{1}{2} \sigma_z^{-1/2}, \quad (16c)$$

359 because

$$\int_{-\infty}^{\infty} \exp \left(-\sigma \pi \Delta^{+2} \right) d\Delta^+ = \sigma^{-1/2} \quad (17)$$

360 and

$$\int_{-\infty}^{\infty} \Delta^{+2} \exp \left(-\sigma \pi \Delta^{+2} \right) d\Delta^+ = \frac{1}{2\pi} \sigma^{-3/2} \quad (18)$$

361 through analytical integration. The calculated values of $\ell_{1,x}^+, \ell_{1,y}^+, \ell_{1,z}^+$ are
 362 given in Table 3 by applying the constants of decay rate in Table 2, and
 363 they show more clearly the anisotropy of the correlation function $\Omega_1(\Delta)$.

364 Therefore, the integral of Ω_1 over the correlation volume in Eq. (10) can
 365 be evaluated as

$$\int_{v_{\text{cor}}} \Omega_1(\Delta) d^3\Delta = V_{\text{cor}}^+ \delta_L^3, \quad (19)$$

Table 3: Integral length scales ℓ_1^+ in the x, y, z -directions, integral correlation volume V_{cor}^+ and its length scale ℓ_{cor}^+ .

Case	$\ell_{1,x}^+$	$\ell_{1,y}^+$	$\ell_{1,z}^+$	V_{cor}^+	ℓ_{cor}^+
I	0.56	0.05	1.12	0.27	0.64
II	0.35	0.11	0.71	0.23	0.61
III	0.11	0.41	0.53	0.19	0.57

366 where the non-dimensional integral correlation volume

$$V_{\text{cor}}^+ = 8\ell_{1,x}^+\ell_{1,y}^+\ell_{1,z}^+ = \left(1 - \frac{1}{2}\epsilon_x\sigma_x^{-1}\right)\left(1 - \frac{1}{2}\epsilon_y\sigma_y^{-1}\right)(\sigma_x\sigma_y\sigma_z)^{-1/2}. \quad (20)$$

367 The values of V_{cor}^+ are 0.27, 0.23 and 0.19 for Cases I, II and III respectively,
 368 and they are all around 0.2 despite the variation of ℓ_1^+ in different directions
 369 and cases. This can be accounted for by the similar thermo-chemical
 370 processes of the three cases that dictate the correlation function Ω_1 and hence
 371 the correlation integral of Eq. (19), and it confirms that the correlation length
 372 scale does not depend on turbulence [5, 6]. Note that this correlation volume
 373 evaluated in all three directions and with the negative zones considered is
 374 about 1.5 to 2 times larger than the value $\delta_L^3/8$ estimated in Refs. [5, 6]
 375 using the correlation in the mean flame propagation direction only which
 376 was modelled by a standard Gaussian function $\exp(-4\pi\Delta^{+2})$.

377 The length scale of V_{cor}^+ is obtained as $\ell_{\text{cor}}^+ = V_{\text{cor}}^{+1/3}$ and its values are
 378 0.64, 0.61 and 0.57, as given in Table 3, for Cases I, II and III respectively.
 379 This length scale is about $0.6\delta_L$ which is slightly larger than $0.5\delta_L$ reported
 380 in Refs. [5, 6]. Similarly, for the correlation function Ω shown in Fig. 7 for
 381 Case III, the integral volume is $1.8\delta_L^3$ and its length scale is $1.2\delta_L$, almost
 382 twice the length scale ℓ_{cor} for Ω_1 . This relationship between the length scales
 383 of the correlation functions Ω_1 and Ω is in agreement with the finding in
 384 Refs. [5, 6] that the integral length scale of Ω_1 is half the integral length scale
 385 of Ω .

386 4.4. Prediction of combustion noise level

387 With the second integral over v_{cor} in Eq. (10) evaluated separately, the
388 far-field OASPL of combustion noise can be rewritten as

$$\overline{p'^2(\mathbf{x}, t)} = \frac{(\gamma - 1)^2}{16\pi^2 r^2 a_0^4} Y_{f,u}^2 H^2 V_{\text{cor}}^+ \delta_L S_L^2 \int_{v_f} \mathcal{K}^{+2} \overline{\dot{w}(\mathbf{y}, t)^2} d^3\mathbf{y}, \quad (21)$$

389 where the non-dimensional parameter $\mathcal{K}^+$ has been normalised using the
390 inverse of the laminar flame time, $1/\tau_f = S_L/\delta_L$. The remaining integral
391 represents the contribution from the turbulence through the parameter $\mathcal{K}$,
392 mean reaction rate, $\bar{w}$, and flame-brush size, v_f .

393 First, the parameter $\mathcal{K}$ can be calculated directly from the DNS data of
394 the V-flames using the definition in Eq. (7), i.e. $\mathcal{K} = (\overline{\dot{w}'^2})^{1/2}/\bar{w}$, for the
395 three cases. This approach of obtaining $\mathcal{K}$ is inherently more consistent than
396 that used by Swaminathan et al. [5, 6] in which the parameter $\mathcal{B}_1$ in Eq. (8)
397 was derived indirectly from the DNS flames [16–18] whereas $\mathcal{B}$ was obtained
398 from the laser diagnostics data [19]. In addition, both $\mathcal{K}$ and $\mathcal{B}_1$ were treated
399 in Refs. [5, 6] as a constant from all the numerical and experimental flames,
400 and a direct measurement of the parameter $\mathcal{B}_1^+$ was therefore suggested for
401 a more rigorous modelling.

402 Figure 8 shows the variation of $\mathcal{K}^+$ and $\mathcal{B}_1^+$ within the flame brushes for
403 Cases I to III. It can be seen that both $\mathcal{K}^+$ and $\mathcal{B}_1^+$ vary with the x_o^+ and
404 $\tilde{c}$ positions and they increase at the lower bound of $\tilde{c}$ due to the decreases
405 in the mean reaction rate $\bar{w}$ and the RMS value of reaction rate fluctuation
406 $\sqrt{\overline{\dot{w}'^2}}$ in which $\bar{w}$ drops more rapidly than $\sqrt{\overline{\dot{w}'^2}}$. A rise of $\mathcal{K}^+$ at the upper
407 bound of $\tilde{c}$ can also be seen for Case III. In order to simplify the prediction
408 of combustion noise level, $\mathcal{K}^+$ is approximated to be constants of 18, 40 and

Table 4: Operating conditions of the experimental flames for OASPL prediction. The fuel type is natural gas for No. 1–8 and propane for the other flames.

No.	D (mm)	U_b (m s ⁻¹)	ϕ	u_{rms}^+	S_L (m s ⁻¹)	δ_L (mm)
1	10.9	21.8	1.02	1.80	0.40	0.42
2	10.9	19.0	0.82	1.77	0.31	0.48
3	10.9	21.8	1.02	1.31	0.40	0.42
4	6.4	24.1	0.90	0.54	0.36	0.44
5	6.4	24.1	1.08	0.48	0.40	0.41
6	17.3	17.4	1.02	1.74	0.40	0.42
7	34.8	8.6	1.02	2.69	0.40	0.42
8	10.9	21.8	0.95	0.81	0.38	0.43
9	10.9	16.3	0.67	1.84	0.19	0.61
10	6.4	32.2	0.80	0.74	0.30	0.45
11	17.3	17.4	1.03	4.67	0.43	0.37
12	17.3	17.4	1.03	0.97	0.43	0.37
13	17.3	8.7	0.99	0.83	0.43	0.37

68, marked as dashed lines in Fig. 8, for Cases I, II and III respectively.
The increasing values of $\mathcal{K}^+$ reflect the corresponding turbulence levels in
the three cases (see Table 1).

Recently, Rajaram [20] and Rajaram & Lieuwen [42] reported combustion
noise measurements of the far-field OASPL from statistically stationary,
pilot stabilised, turbulent premixed flames. These axisymmetric flames with

415 burner diameter D cover a wide range of turbulence and thermo-chemical
 416 conditions in terms of turbulence intensity, u_{rms} , mean burner exit velocity,
 417 U_b , fuel type, equivalence ratio, ϕ , etc. Following Swaminathan et al. [5, 6],
 418 thirteen natural gas- and propane-air flames are selected arbitrarily out of
 419 the large database of flame conditions obtained by Rajaram [20], and their
 420 flame conditions as listed in Table 4 will be used for the OASPL prediction
 421 in this study. The combustion noise level was measured in the far field
 422 at $r = 1.02$ m in an anechoic facility with an estimate of the maximum
 423 error $\sim \pm 2$ dB. Further details of these experimental flames can be found
 424 in Ref. [20]. These flames are specifically chosen since one of the objectives
 425 of this study is to investigate the sensitivity of the far-field OASPL to the
 426 changes in $\mathcal{K}^+$ values and correlation volume.

427 To predict the far-field OASPL for the thirteen flames in Table 4, RANS
 428 data of the mean reaction rate field, $\overline{\dot{w}(\mathbf{y}, t)}$, is required for the volume
 429 integral in Eq. (21). Swaminathan et al. [5, 6] performed RANS calculations
 430 of the flames in Table 4 using a standard $\tilde{k}$ - $\tilde{\epsilon}$ turbulence model. The mean
 431 reaction rate, $\overline{\dot{w}}$, was calculated using an algebraic closure [43] and a recent
 432 scalar dissipation rate model [44]. The spatial distribution of $\overline{\dot{w}}(x, R)$ from
 433 the RANS results [5, 6] and the values of $\mathcal{K}^+$ obtained from the DNS of
 434 V-flames [21] are employed to compute the integral over the flame brush
 435 in Eq. (21). The axisymmetry of the Bunsen flames allows the differential
 436 volume to be written as $d^3\mathbf{y} = 2\pi R dR dx$ where x and R denote the axial
 437 and radial coordinates.

438 The integral correlation volume V_{cor}^+ obtained from Cases I to III in
 439 Section 4.2 and the laminar flame speed and its thickness given in Table 4 are

440 used in Eq. (21) to predict the far-field OASPL for all 13 flames. Figure 9
 441 shows the comparison between the predicted OASPL, the calculations by
 442 Swaminathan et al. [5, 6] and the measured results of Rajaram [20] together
 443 with the error bar of ± 2 dB. Note that the OASPL obtained by Swaminathan
 444 et al. [5, 6] would somewhat decrease if the negative values were considered
 445 in their correlation model. As shown in Fig. 9, the agreement between the
 446 predicted and measured noise levels are observed to be very good for Cases II
 447 and III. When compared to Swaminathan et al.'s calculations, the predictions
 448 of Case III exhibit even better degree of agreement with measurements in
 449 5 flames (No. 1 & 8–11), while Case II overestimates the OASPL by about
 450 0.3 dB resulting in slightly better agreement in most of the flames. For Case I,
 451 however, the predicted noise levels are too low and the underprediction is
 452 about 6 dB compared to the predictions of Case II.

453 The difference in the predictions from Cases I to III can be explained
 454 by their different turbulence levels and correlation volumes. As noted
 455 by Dunstan et al. [21], the turbulence intensities evaluated locally on the
 456 $\tilde{c} = 0.05$ isosurface, i.e. $u_{\text{rms}}^+ = 0.6, 1.1$ and 3.3 for the three cases, are
 457 considerably lower than the nominal inlet intensities and only decay slightly
 458 with downstream position. On the other hand, in the experimental flames
 459 the majority of the values of u_{rms}^+ vary between 0.5 to 2.0 (see Table 4)
 460 and their average is about 1.3 if flame No. 11 ($u_{\text{rms}}^+ = 4.67$) is excluded.
 461 In terms of turbulence intensity, Case II is very close to the experimental
 462 flames and this explains its best agreement with the measured noise data
 463 among all predictions. The underprediction of 6 dB for Case I is due to
 464 its small turbulence intensity which is less than half the average u_{rms}^+ for

465 the experimental flames. With regard to Case III, its high turbulence level
 466 gives the highest predicted noise levels among the three cases, which also
 467 accounts for the closest prediction to measurement for flame No. 11 with the
 468 highest u_{rms}^+ value. This very high turbulence level of Case III ($u_{\text{rms}}^+ = 3.3$),
 469 however, is somewhat compensated by the relatively small correlation volume
 470 $V_{\text{cor}}^+ = 0.19$. The resulting predictions of Case III are only 3.5 dB higher than
 471 the Case II predictions, and still exhibit good agreement with measurements
 472 particularly for the propane flames. The results of the predicted combustion
 473 noise levels shown in Fig. 9 indicate that turbulence plays a significant role
 474 as well as the two-point correlation of $\ddot{w}$ in sound emission from turbulent
 475 premixed flames.

476 5. Concluding remarks

477 It has been suggested in very recent studies [5, 6] that the sound emission
 478 from open turbulent flames is dictated by the two-point spatial correlation of
 479 the rate of change of heat release rate fluctuation. In this work, recent data
 480 of 3D DNS of turbulent premixed V-flames [21] are analysed to investigate
 481 this two-point correlation and its role in the production of combustion noise,
 482 specifically to address the isotropy of the correlation volume suggested in
 483 Refs. [5, 6] and its influence on the far-field OASPL prediction. The three
 484 DNS cases of V-flames, with inlet turbulence intensities of $u_{\text{rms}}^+ = 1, 2$
 485 and 6, represent turbulent combustion in the regimes of wrinkled flamelets,
 486 corrugated flamelets and thin reaction zones. The correlations of the rate
 487 of change of fluctuating heat release rate in the streamwise, transverse
 488 and spanwise directions within the flame brush are studied, and the time

489 derivative of the instantaneous reaction rate is calculated using the DNS
 490 data stored at discrete time steps to directly construct the correlations.
 491 Furthermore, the turbulence controlled parameter $\mathcal{K}$, which affects the noise
 492 prediction, is directly obtained from the V-flame DNS data.

493 The two-point correlation function, Ω_1 , constructed at a number of
 494 positions inside the flame brush has shown that the value of Ω_1 drops quickly
 495 from one resulting in a short correlation length scale and negative values. It
 496 is also observed that the decay rates of the correlation functions vary in
 497 different directions, a conclusion that differs from Swaminathan et al. [6]
 498 who suggested that these correlation functions can be isotropic. Also, the
 499 correlation is observed to be different for the three flames studied. However
 500 there is no spatial variation of this correlation function inside the flame brush.
 501 More importantly, the correlation function Ω_1 including the negative values
 502 can be well approximated by a Hermite-Gaussian function, which gives the
 503 non-dimensional integral correlation volume to be $V_{\text{cor}}^+ \simeq 0.2$ for all three
 504 flames and it is about 1.5 to 2 times larger than the value reported in earlier
 505 studies [5, 6]. The length scale, ℓ_{cor} , is about 60% of the laminar flame
 506 thickness, slightly longer than $0.5\delta_L$ in Refs. [5, 6] as well. The correlation
 507 function of the reaction rate fluctuation is very similar to Ω_1 but with
 508 slower decay rates leading to the enlarged correlation volume and length
 509 scale. Although an anisotropic behaviour of the correlation function, Ω_1 ,
 510 is observed for statistically multi-dimensional flames, its influence on the
 511 correlation volume and its length scale is seen to be small.

512 The correlation model and the parameter $\mathcal{K}^+$ have been applied to predict
 513 the far-field OASPL of open turbulent flames measured by Rajaram [20]. The

514 noise levels calculated using the information obtained from the Cases II and
 515 III flames agree very well with the measured values and show slightly better
 516 agreement than the calculations reported earlier [5, 6]. The predicted OASPL
 517 using Case I DNS data appears too low. The far-field OASPL is found to
 518 be sensitive to $\mathcal{K}^+$, since it has a direct influence as shown by Eq. (21).
 519 Also, the value of $\mathcal{K}^+$ is observed to strongly depend on turbulence and
 520 thermo-chemical conditions, and their interactions. Further investigations on
 521 DNS flames are therefore suggested: i) more conditions for a possible scaling
 522 of the parameter $\mathcal{K}^+$; and ii) more time steps to inspect how an enlarged
 523 sample size would improve the correlation model. A sufficiently long DNS
 524 signal is needed to study the spectral characteristics of combustion noise
 525 which will be addressed in future.

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References

- [1] I. R. Hurle, R. B. Price, T. M. Sugden, A. Thomas, Proc. R. Soc. Lond. A 303 (1968) 409–427.
- [2] W. C. Strahle, Prog. Energy Combust. Sci. 4 (1978) 157–176.
- [3] H. Jones, Proc. R. Soc. Lond. A 367 (1969) 291–309.

- [4] D. G. Crighton, A. P. Dowling, J. E. Ffowcs Williams, M. Heckl, F. G. Leppington, *Modern Methods in Analytical Acoustics: Lecture Notes*, Springer-Verlag, London, 1992.
- [5] N. Swaminathan, R. Balachandran, G. Xu, A. P. Dowling, *Proc. Combust. Inst.* 33 (2011) 1533–1541.
- [6] N. Swaminathan, G. Xu, A. P. Dowling, R. Balachandran, *J. Fluid Mech.* 681 (2011) 80–115.
- [7] J. R. Mahan, *A critical review of noise production models for turbulent, gas-fueled burners*, NASA CR-3803, NASA Lewis Research Center, Ohio, USA, 1984.
- [8] W. C. Strahle, B. N. Shivashankara, *Proc. Combust. Inst.* 15 (1975) 1379–1385.
- [9] J. H. Kilham, N. Kirmani, *Proc. Combust. Inst.* 17 (1979) 327–336.
- [10] S. Kotake, K. Takamoto, *J. Sound Vib.* 112 (1987) 345–354.
- [11] S. Kotake, K. Takamoto, *J. Sound Vib.* 139 (1990) 9–20.
- [12] S. L. Bragg, *J. Inst. Fuel* 36 (1963) 12–16.
- [13] W. C. Strahle, *J. Fluid Mech.* 49 (1971) 399–414.
- [14] J. Wäsle, A. Winkler, T. Sattlemayer, *Flow Turbul. Combust.* 75 (2005) 29–50.
- [15] C. Hirsch, J. Wäsle, A. Winkler, T. Sattlemayer, *Proc. Combust. Inst.* 31 (2007) 1435–1441.

- [16] C. J. Rutland, R. S. Cant, Turbulent Transport in Premixed Flames, Proceedings of Summer Program, Centre for Turbulence Research, NASA Ames/Stanford University, California, USA, 1994.
- [17] Y. Nada, M. Tanahashi., T. Miyauchi, J. Turbul. 5 (2004) 1–15.
- [18] Y. Nada, N. Shiwaku, M. Tanahashi., T. Miyauchi, in: 5th Asia-Pacific Conference on Combustion, 2005, The University of Adelaide, Australia.
- [19] R. Balachandran, B. O. Ayoola, C. F. Kaminski, A. P. Dowling, E. Mastorakos, Combust. Flame 143 (2005) 37–55.
- [20] R. Rajaram, Characteristics of sound radiation from turbulent premixed flames, Ph.D. thesis, Department of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA, 2007.
- [21] T. D. Dunstan, N. Swaminathan, K. N. C. Bray, R. S. Cant, Flow Turbul. Combust. (2010).
- [22] M. J. Lighthill, Proc. R. Soc. Lond. A 211 (1952) 564–587.
- [23] M. J. Lighthill, Proc. R. Soc. Lond. A 222 (1954) 1–32.
- [24] F. Flemming, A. Sadiki, J. Janicka, Proc. Combust. Inst. 31 (2007) 3189–3196.
- [25] M. Ihme, H. Pitsch, D. Bodony, Proc. Combust. Inst. 32 (2009) 1545–1553.
- [26] A. P. Dowling, Mean temperature and flow effects on combustion noise, AIAA Paper 1979-0590, 1979.

- [27] W. C. Strahle, *Proc. Combust. Inst.* 13 (1973) 527–535.
- [28] T. J. Poinso, D. Veynante, *Theoretical and Numerical Combustion*, Edwards Inc., Philadelphia, PA, USA, 2nd edition, 2005.
- [29] K. O. Smith, F. C. Gouldin, *Progr. Astronaut. Aero.* 58 (1978) 37–54.
- [30] D. Veynante, J. M. Duclos, J. Piana, in: *Twenty-Fifth Symposium (International) on Combustion*, pp. 1249–1256.
- [31] F. C. Gouldin, in: *Twenty-Sixth Symposium (International) on Combustion*, pp. 381–388.
- [32] I. G. Shepherd, in: *Twenty-Sixth Symposium (International) on Combustion*, pp. 373–379.
- [33] S. S. Sattler, D. A. Knaus, F. C. Gouldin, *Proc. Combust. Inst.* 29 (2002) 1785–1792.
- [34] T. M. Alshaalan, C. J. Rutland, in: *Twenty-Seventh Symposium (International) on Combustion*, pp. 793–799.
- [35] P. Domingo, L. Vervisch, S. Payet, R. Hauguel, *Combust. Flame* 143 (2005) 566–586.
- [36] J. B. Bell, M. S. Day, I. G. Shepherd, M. R. Johnson, R. K. Cheng, J. F. Grcar, V. E. Beckner, M. Lijewski, *Proc. Natl. Acad. Sci.* 102 (2005) 10006–10011.
- [37] T. J. Poinso, S. K. Lele, *J. Comp. Phys.* 101 (1992) 104–129.

- [38] N. Peters, Turbulent Combustion, Cambridge University Press, Cambridge, UK, 2000.
- [39] A. N. Lipatnikov, J. Chomaik, Prog. Energy Combust. Sci. 28 (2002) 1–74.
- [40] S. A. Karabasov, M. Z. Afsar, T. P. Hynes, A. P. Dowling, W. A. McMullan, C. D. Pokora, G. J. Page, J. J. McGuirk, AIAA J. 48 (2010) 1312–1325.
- [41] Y. Liu, A. P. Dowling, N. Swaminathan, T. D. Dunstan, R. Morvant, M. A. Macquisten, L. Caracciolo, Prediction of noise source for an aeroengine combustor, AIAA Paper 2011-2913, 2011.
- [42] R. Rajaram, T. Lieuwen, J. Fluid Mech. 637 (2009) 357–385.
- [43] K. N. C. Bray, Proc. Combust. Inst. 17 (1979) 223–233.
- [44] H. Kolla, J. W. Rogerson, N. Chakraborty, N. Swaminathan, Combust. Sci. Tech. 181 (2009) 518–535.

Figure 1: Contours of $\tilde{c}$ in the (x^+, y^+) plane for Cases I to III. Streamwise sample positions 1, 2 and 3 are located at $x_o^+ = 16.7, 20.4$ and 24.2 , respectively.

Figure 2: Contours of the instantaneous reaction rate, $\dot{w}^+$ (normalised by $\rho_u S_L / \delta_L$), in the mid (x^+, y^+) plane for Cases I to III.

Figure 3: Contours of the instantaneous time derivative of reaction rate, $\dot{w}^+$ (normalised by $\rho_u S_L^2 / \delta_L^2$), in the mid (x^+, y^+) plane for Cases I to III.

Figure 4: Correlation function Ω_1 for Case I. Separation distances in $\Delta_x, \Delta_y, \Delta_z$ directions, streamwise sample positions at $x_o^+ = 16.7, 20.4, 24.2$.

Figure 5: Correlation function Ω_1 for Case II. Separation distances in $\Delta_x, \Delta_y, \Delta_z$ directions, streamwise sample positions at $x_o^+ = 16.7, 20.4, 24.2$.

Figure 6: Correlation function Ω_1 for Case III. Separation distances in Δ_x , Δ_y , Δ_z directions, streamwise sample positions at $x_o^+ = 16.7, 20.4, 24.2$.

Figure 7: Correlation function Ω for Case III. Separation distances in $\Delta_x, \Delta_y, \Delta_z$ directions, streamwise sample positions at $x_o^+ = 16.7, 20.4, 24.2$.

Figure 8: Parameters $\mathcal{K}^+$ and B_1^+ within the flame brushes for Cases I to III. Streamwise sample positions at $x_{o1}^+ = 16.7$, $x_{o2}^+ = 20.4$ and $x_{o3}^+ = 24.2$.

Figure 9: Comparison of OASPL between the experimental data [20], previous calculations [5, 6] and current predictions from Cases I, II and III.