Joint Complex Diversity Coding and Channel Coding¹

Over Space, Time and Frequency

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Abstract

This paper provides a general diversity analysis for joint CDC and channel coding based space-time-frequency codes (STFCs) is provided. The mapping designs from channel coding to CDC are crucial for efficient exploitation of the diversity potential. This paper provides and proves a sufficient condition of full diversity construction with joint 3-D CDC and channel coding, bit-interleaved coded complex diversity coding (BICCDC) and symbol-interleaved coded complex diversity coding (SICCDC). Both non-iterative and iterative detection of joint channel code and CDC transmission are investigated. The proposed minimum mean square error based iterative soft decoding achieves the performance of the soft sphere decoding with reduced complexity.

I. INTRODUCTION

A challenging problem in wideband MIMO system design is to develop new high rate coding schemes to efficiently exploit all the diversity available across space, time and frequency domains. To this end, the design of space-time-frequency coding (STFC) has been recently investigated in [4, 5]. We introduce a general terminology, complex diversity coding (CDC) which summarizes existing diversity coding approaches using complex conversion. Space-time-frequency codes may be categorized into different integrations of CDC and channel coding, such as error control coding (ECC). Note that CDC is also called signal space diversity [9] in single-input single-output communications and linear dispersion codes (LDC) in two-dimensional space time MIMO channels.

Unlike the previous analysis for pure 3-D CDC systems presented in [5, 23], this paper provides a general diversity analysis for systems with joint 3-D CDC and channel coding. Our diversity analysis also differs from the joint 1-D or 2-D CDC and channel coding related performance analysis (e.g., those conducted in [8, 11]), since we provide a clear construction of full diversity joint 3-D CDC and channel coding without assumption of infinite length of the channel code, and the physical dimensions used in our diversity analysis are different from those in [8, 11].

Unlike the computationally prohibitive maximum likelihood (ML) or sphere decoding based turbo decoding for joint ECC and 2-D CDC in [13], in this paper, a low complexity iterative MMSE inner decoding for high rate 3-D CDC based STFC and Log-MAP outer

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decoding for ECC is proposed, and is shown to have comparable performance to the non-iterative STFC near-ML sphere decoding.

Notations: $(\cdot)^T$ - matrix transpose, $(\cdot)^H$ - matrix transpose conjugate, $E[\cdot]$ - expectation operation, j - the square root of -1 , $\mathbf{I}_K$ - identity matrix of size $K \times K$, $\mathbf{0}_{M \times N}$ - zero matrix of size $M \times N$, $\mathbf{A} \otimes \mathbf{B}$ - Kronecker (tensor) product of matrices $\mathbf{A}$ and $\mathbf{B}$, $[\mathbf{A}]_{a,b}$ - the (a,b) entry of matrix $\mathbf{A}$, and $\text{diag}(\cdot)$ transforms the argument from a vector to a diagonal matrix, and $\text{vec}(\mathbf{X})$ - $\text{vec}(\mathbf{X}) = \left[[\mathbf{X}]_{:,1}^T, \dots, [\mathbf{X}]_{:,N}^T \right]^T$, where matrix $\mathbf{X}$ is of size $M \times N$.

II. PROPOSED SYSTEM MODEL

A. Space-time-frequency block (STFB) and space-time-frequency code (STFC)

The baseband received signal is formed as shown in Fig. 1. We consider a MIMO-OFDM system with N_t transmit antennas, N_r receive antennas, and a block of N_c OFDM subcarriers per antenna. Channel coefficients are assumed to be constant within one OFDM block. However, the channel coefficients change from block to block, and they are assumed to be statistically independent among different OFDM blocks. One 3-D CDC-based STFC codeword contains D space-time-frequency blocks (STFB), each of which is of size $N_t \times N_F \times T$, i.e. across N_t transmit antennas N_F subcarriers and T OFDM blocks, where $N_C = DN_F$. The data sequence is modulated using complex-valued symbols $\alpha_q + j\beta_q$, chosen from an arbitrary constellation (e.g., r-PSK or r-QAM). One STFB, denoted by $\mathbf{S}_{STFB}$, can be written in matrix form as

$$\mathbf{S}_{STFB} = \sum_{q=1}^Q (\alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q). \quad (1)$$

where $\mathbf{A}_q \in \mathbb{C}^{N_T \times N_F T}$ and $\mathbf{B}_q \in \mathbb{C}^{N_T \times N_F T}$ are dispersion matrices for the real and imaginary parts of source signals. Equation (1) may be considered as a 3-dimensional formulation of linear dispersion codes [10], and can be reformulated as follows:

1) if $\mathbf{A}_q \neq \mathbf{B}_q$,

$$\text{vec}(\mathbf{S}_{STFB}) = \mathbf{G}_{STFB}^{\text{vec}} \theta, \quad (2)$$

where

$$\begin{aligned} \mathbf{G}_{STFB}^{\text{vec}} &= [\text{vec}(\mathbf{A}_1), \dots, \text{vec}(\mathbf{A}_Q), j\text{vec}(\mathbf{B}_1), \dots, j\text{vec}(\mathbf{B}_Q)] \\ \theta &= [\alpha_1, \dots, \alpha_Q, \beta_1, \dots, \beta_Q]^T. \end{aligned}$$

2) if $\mathbf{A}_q = \mathbf{B}_q$,

$$\text{vec}(\mathbf{S}_{STFB}) = \mathbf{G}_{STFB} \mathbf{s}, \quad (3)$$

where

$$\begin{aligned} \mathbf{G}_{STFB} &= [\text{vec}(\mathbf{A}_1), \dots, \text{vec}(\mathbf{A}_Q)] \\ \mathbf{s} &= [s_1, \dots, s_Q]^T. \end{aligned}$$

We define the coding rate of CDC-based STFC as $R^{\text{sym}} = \sum_{i=1}^D Q_i / (N_t N_c T)$, where Q_i is the number of source symbols encoded in the i -th STFB. In our simulations, we apply rate-one full diversity CDC based STFCs proposed in [5], and these codes satisfy $\mathbf{A}_q = \mathbf{B}_q$.

B. Frequency domain system model and structure

Consider one joint CDC-ECC STFC block across K 3-D CDC based STFC codewords. The baseband frequency domain signal for the i -th STFB within the k -th STFC can be written as

$$\mathbf{y}^{(i,k)} = \sqrt{\frac{\rho}{N_t}} \mathbf{H}_{STFB}^{(i,k)} \mathbf{G}_{STFB} \mathbf{s}^{(i,k)} + \mathbf{n}^{(i,k)}, \quad (4)$$

where $\mathbf{H}_{STFB}^{(i,k)}$ is the corresponding frequency domain channel matrix of size $N_r N_F T \times N_t N_F T$. Both vectors $\mathbf{y}^{(i,k)}$ and $\mathbf{n}^{(i,k)}$ are of size $N_r N_F T$, and they are the frequency domain received signal and additive complex Gaussian noise vectors, respectively. The source signal vector $\mathbf{s}^{(i,k)}$ is of size $N_t N_F T$. The channel matrix $\mathbf{H}_{STFB}^{(i,k)}$ is formed as

$$\mathbf{H}_{STFB}^{(i,k)} = \text{diag}(\mathbf{H}_{STFB}^{(i,1,k)}, \dots, \mathbf{H}_{STFB}^{(i,T,k)}),$$

where $\mathbf{H}_{STFB}^{(i,t,k)} = \text{diag}(\mathbf{H}_{STFB(p_1)}^{(i,t,k)}, \dots, \mathbf{H}_{STFB(p_{N_F})}^{(i,t,k)})$, $\mathbf{H}_{STFB(p)}^{(i,t,k)}$ of size $N_r \times N_t$ is the frequency domain MIMO channel matrix for the p -th subcarrier, t -th OFDM block, i -th STFB, k -th CDC based STFC ($i = 1 \dots D, t = 1 \dots T, n_F = 1 \dots N_F, k = 1 \dots K$). $\{p_1^{(i)}, \dots, p_{N_F}^{(i)}\}$ is the subcarrier set chosen for i -th CDC encoded STF block. As shown in Fig. 2, the ECC coded streams are first interleaved with random interleaver, and mapped into complex source symbols, which are subsequently encoded into CDC based STFCs. One set of ECC streams is across K STFCs and N_a STFBs within one STFC.

III. DIVERSITY ANALYSIS

We assume that one channel coding stream is encoded across K STFCs and N_a STFBs per STFC with indices $i = i_1, \dots, i_{N_a}$. Denote the i -th STFB within the k -th STFC as $\mathbf{C}^{(i,k)}$, which is formed as

$$\mathbf{C}^{(i,k)} = \begin{bmatrix} [\mathbf{C}^{(1,i,k)}]^T & [\mathbf{C}^{(2,i,k)}]^T & \dots & [\mathbf{C}^{(T,i,k)}]^T \end{bmatrix}^T,$$

where $\mathbf{C}^{(t,i,k)}$ has entries $[\mathbf{C}^{(t,i,k)}]_{a,b} = c_{b,p_a^{(i)}}^{(t,i,k)}$, and $c_{m,p_a^{(i)}}^{(t,i,k)}$ is the channel symbol of the k -th OFDM block, the t -th OFDM block, the $p_a^{(i)}$ -th subcarrier from the m -th transmit antenna, and $p_a^{(i)} = p_1^{(i)}, \dots, p_{N_F}^{(i)}$ is the subcarrier index for the i -th STFB. The received signal corresponding to STFB $\mathbf{C}^{(i,k)}$ can be reformed as

$$\bar{\mathbf{y}}^{(i,k)} = \sqrt{\frac{\rho}{N_T}} \mathbf{M}^{(i,k)} \bar{\mathbf{H}}^{(i,k)} + \bar{\mathbf{v}}^{(i,k)},$$

where $\bar{\mathbf{y}}^{(i,k)}$ and $\bar{\mathbf{v}}^{(i,k)}$ are the receive signal vector and noise vector, respectively, $\mathbf{M}^{(i,k)} = I_{N_r} \otimes [\mathbf{M}_1^{(i,k)}, \dots, \mathbf{M}_{N_t}^{(i,k)}]$, and

$$\mathbf{M}_m^{(i,k)} = \text{diag} \left(c_{m,p_1^{(i)}}^{(1,i,k)}, \dots, c_{m,p_{N_F}^{(i)}}^{(1,i,k)}, \dots, c_{m,p_1^{(i)}}^{(T,i,k)}, \dots, c_{m,p_{N_F}^{(i)}}^{(T,i,k)} \right).$$

The equivalent frequency domain channel vector of size $N_F N_t N_r T \times 1$ can be expressed as

$$\bar{\mathbf{H}}^{(i,k)} = \begin{bmatrix} [\bar{\mathbf{H}}_{1,1}^{(i,k)}]^T, \dots, [\bar{\mathbf{H}}_{N_t,1}^{(i,k)}]^T, \dots, [\bar{\mathbf{H}}_{1,2}^{(i,k)}]^T, \dots, [\bar{\mathbf{H}}_{N_t,2}^{(i,k)}]^T, \dots, [\bar{\mathbf{H}}_{1,N_r}^{(i,k)}]^T, \dots, [\bar{\mathbf{H}}_{N_t,N_r}^{(i,k)}]^T \end{bmatrix}^T,$$

where

$$\bar{\mathbf{H}}_{m,n}^{(i,k)} = \begin{bmatrix} H_{m,n,p_1^{(i)}}^{(1,k)}, H_{m,n,p_2^{(i)}}^{(1,k)}, \dots, H_{m,n,p_{N_F}^{(i)}}^{(1,k)}, \dots, \\ H_{m,n,p_1^{(i)}}^{(T,k)}, H_{m,n,p_2^{(i)}}^{(T,k)}, \dots, H_{m,n,p_{N_F}^{(i)}}^{(T,k)} \end{bmatrix}^T,$$

and $H_{m,n,p_{n_F}^{(i)}}^{(k)}$ is the frequency domain channel gain of the k -th OFDM block, the $p_{n_F}^{(i)}$ -th subcarrier for block between the m -th transmit antenna and the n -th receive antenna, where $m = 1, \dots, N_t$ and $n = 1, \dots, N_r$.

Considering a pair of matrices $\mathbf{M}_{(a)}^{(i,k)}$ and $\mathbf{M}_{(b)}^{(i,k)}$ which correspond to two different blocks $\mathbf{C}_a^{(i,k)}$ and $\mathbf{C}_b^{(i,k)}$, the upper bound for the pairwise error probability between $\mathbf{M}_{(a)}^{(i,k)}$ and $\mathbf{M}_{(b)}^{(i,k)}$ is [14]

$$\mathbf{P}_r\{\mathbf{M}_a^{(i,k)} - \mathbf{M}_b^{(i,k)}\} \leq \binom{2r_{(i,k)} - 1}{r_{(i,k)}} \left(\prod_{c=1}^{r_{(i,k)}} \gamma_c^{(i,k)} \right)^{-1} \left(\frac{\rho}{N_t} \right)^{-r_{(i,k)}}, \quad (5)$$

where $r_{(i,k)}$ has the rank of

$$\Lambda_{(a,b)}^{(i,k)} = \left(\mathbf{M}_{(a)}^{(i,k)} - \mathbf{M}_{(b)}^{(i,k)} \right) \mathbf{R}_{\bar{\mathbf{H}}^{(i,k)}} \left(\mathbf{M}_{(a)}^{(i,k)} - \mathbf{M}_{(b)}^{(i,k)} \right)^{\mathcal{H}},$$

and $\mathbf{R}_{\bar{\mathbf{H}}^{(i,k)}} = \mathbb{E} \left\{ \mathbf{H}^{(i,k)} \left[\mathbf{H}^{(i,k)} \right]^{\mathcal{H}} \right\}$ is correlation matrix of $\mathbf{H}^{(i,k)}$, and $\gamma_c^{(i,k)}$, $c = 1, \dots, r_{(i,k)}$ are the non-zero eigenvalues of

$\Lambda_{(a,b)}^{(i,k)}$. Denote $\psi_{(b)}^{(i,k)} = \mathbf{M}_{(b)}^{(i,k)} - \mathbf{M}_{(a)}^{(i,k)}$, then $\Lambda_{(a,b)}^{(i,k)} = \psi_{(a,b)}^{(i,k)} \mathbf{R}_{\bar{\mathbf{H}}^{(i,k)}} \left[\psi_{(a,b)}^{(i,k)} \right]^{\mathcal{H}}$. Also denote $\psi_{(a,b)} = \text{diag}(\psi_{(a,b)}^{(1)}, \dots, \psi_{(a,b)}^{(K)})$,

$\psi_{(a,b)}^{(k)} = \text{diag}(\psi_{(a,b)}^{(i_1,k)}, \dots, \psi_{(a,b)}^{(i_{N_a},k)})$,

$\bar{\mathbf{H}} = \left[\left[\bar{\mathbf{H}}^{(1)} \right]^{\mathcal{T}}, \dots, \left[\bar{\mathbf{H}}^{(K)} \right]^{\mathcal{T}} \right]^{\mathcal{T}}$, $\bar{\mathbf{H}}^{(k)} = \left[\left[\bar{\mathbf{H}}^{(i_1,k)} \right]^{\mathcal{T}}, \dots, \left[\bar{\mathbf{H}}^{(i_{N_a},k)} \right]^{\mathcal{T}} \right]^{\mathcal{T}}$, $\mathbf{R}_{\bar{\mathbf{H}}} = \mathbb{E} \left\{ \bar{\mathbf{H}} \left[\bar{\mathbf{H}} \right]^{\mathcal{H}} \right\}$, $\mathbf{M}_{(a)} = \text{diag}(\mathbf{M}_{(a)}^{(k)}, \dots, \mathbf{M}_{(a)}^{(K)})$,

and $\mathbf{M}_{(a)}^{(k)} = \text{diag}(\mathbf{M}_{(a)}^{(i_1,k)}, \dots, \mathbf{M}_{(a)}^{(i_{N_a},k)})$.

The upper bound of the pairwise error probability between $\mathbf{M}_{(a)}$ and $\mathbf{M}_{(b)}$ can now be expressed as

$$\mathbf{P}_r\{\mathbf{M}_{(a)} - \mathbf{M}_{(b)}\} \leq \binom{2r - 1}{r} \left(\prod_{c=1}^r \gamma_c \right)^{-1} \left(\frac{\rho}{N_t} \right)^{-r}, \quad (6)$$

where r is the rank of $\Lambda_{(a,b)}$, and γ_c , $c = 1, \dots, r$ are the non-zero eigenvalues of $\Lambda_{(a,b)}$. Note that the upper limit diversity order of this system is

$$\min \{ \text{rank}(\Lambda_{(a,b)}) \} \leq K \min \{ N_t N_r T(L+1), N_r T N_C \} \leq \text{rank}(\mathbf{R}_{\bar{\mathbf{H}}}). \quad (7)$$

For the system under investigation, the rank r is actually a function of the Hamming or free distance d of ECC, the mapping τ of the ECC coded bit stream into different STFBs across the whole block, and the mapping σ of the ECC coded bit stream into constellation symbols. The system diversity order is further bounded by

$$\min \{ \text{rank}(\Lambda_{(a,b)}) \} \leq \min \{ K, d_{\min} \} \min \{ N_t N_r T(L+1), N_r T N_C \} \leq \text{rank}(\mathbf{R}_{\bar{\mathbf{H}}}), \quad (8)$$

where $d_{\min}$ is the minimum distance of the employed channel code. For block ECC, it refers to Hamming distance; for convolutional codes, it refers to free distance. Let us denote $r = f_{(d,\tau,\sigma)}$. If r and $\prod_{c=1}^r \gamma_c$ are approximately the same for the same (d, τ, σ) , the union bound for the average symbol error rate can be simplified as

$$\begin{aligned} \mathbf{P}_e &\leq \sum_a \mathbf{P}_r(a) \sum_{b \neq a} \mathbf{P}_r\{\mathbf{M}_{(a)} - \mathbf{M}_{(b)}\} \\ &\approx \sum_{(\tau,d,\sigma)} \frac{W_{(d,\tau,\sigma)}}{N_B} \binom{2f_{(d,\tau,\sigma)} - 1}{f_{(d,\tau,\sigma)}} \left(\prod_{a=1}^{f_{(d,\tau,\sigma)}} \gamma_c^{(d,\tau,\sigma)} \right)^{-1} \left(\frac{\rho}{N_t} \right)^{-f_{(d,\tau,\sigma)}}, \end{aligned} \quad (9)$$

where $W_{(d,\tau,\sigma)}$ is the number of pairs of $\mathbf{M}_{(a)}$ and $\mathbf{M}_{(b)}$ with the same (d, τ, σ) .

In order to demonstrate the relation between the diversity performance and the mapping τ more precisely, let us assume that the channels are independent over different CDC based STFCs, then

$$\mathbf{\Lambda}_{(a,b)} = \psi_{(a,b)} \text{diag}(\mathbf{R}_{\mathbf{H}^{(1)}}, \dots, \mathbf{R}_{\mathbf{H}^{(K)}}) [\psi_{(a,b)}]^{\mathcal{H}} = \text{diag}(\mathbf{\Lambda}_{(a,b)}^{(1)}, \dots, \mathbf{\Lambda}_{(a,b)}^{(K)}),$$

where $\mathbf{\Lambda}_{(a,b)}^{(k)} = \psi_{(a,b)}^{(k)} \mathbf{R}_{\mathbf{H}^{(k)}} [\psi_{(a,b)}^{(k)}]^{\mathcal{H}}$. In what follows, we discuss the mapping τ of the ECC coded bit stream into different STFBs across the whole block.

- 1) Case 1: Assume $N_F \geq N_T(L+1)$, and full diversity space-time-frequency CDC, which achieves the upper bound of $\text{rank}(\mathbf{\Lambda}_{(a,b)}^{(i,k)})$ for any pairs of channel coded streams, is chosen, for each STFB.

Note that

$$\text{rank}(\mathbf{\Lambda}_{(a,b)}^{(i,k)}) \leq \min \{N_t N_r T(L+1), N_r T N_C\} \leq \text{rank}(\mathbf{R}_{\mathbf{H}^{(i,k)}}).$$

In this case,

$$\min \{ \text{rank}(\mathbf{\Lambda}_{(a,b)}^{(k)}) \} = \min \{ \text{rank}(\mathbf{\Lambda}_{(a,b)}^{(i,k)}) \}.$$

Apparently, increasing the number of STFBs per CDC based STFC to $N_a > 1$ will not increase the diversity order, which is $\min \{ \text{rank}(\mathbf{\Lambda}_{(a,b)}^{(k)}) \}$ for the k -th CDC based STFC. However, there might be some coding gain through channel coding. In this case, $N_a = 1$ is the best choice for exploiting diversity, i.e., one channel code stream is across multiple STFCs, and the part of the stream with one CDC based STFC is only encoded in one STFB. However, since this may introduce long delay for long channel codes, $N_a > 1$ may still be a practical choice.

- 2) Case 2: Assume $N_F < N_T(L+1)$, and a non-full-diversity space-time-frequency CDC is chosen for each STFB. In this case, $N_a > 1$ will increase the diversity order of the k -th CDC based STFC.

One further issue is to choose the number of units (such as symbols or bits) of one channel code stream to be allocated to each STFB.

Now we have the following proposition.

Proposition 1: One STF communications channel is of full rank over space, time, and frequency, and is independent over different STFBs in time. Consider a joint 3-D CDC and channel coding system. The physical dimensions of 3-D CDC STFBs are sufficient to achieve full diversity over space, time, and frequency. The channel coding sequences operate in units (either bits or symbols). There are N_u channel coding sequences, and each of them is of length K units and with minimum pairwise distance $d_{\min} \leq K$ units to be encoded into K STFBs. If one STFB only encodes a single unit of each channel coding sequence, the system achieves the diversity order upper bound

$$\min \{ \text{rank}(\mathbf{\Lambda}_{(a,b)}) \} = d_{\min} \min \{N_t N_r T(L+1), N_r T N_C\}.$$

Proof: Note that

$$\begin{aligned} \mathbf{\Lambda}_{(a,b)} &= \phi_{(a,b)} \text{diag}(\mathbf{R}_{\mathbf{H}^{(1)}}, \dots, \mathbf{R}_{\mathbf{H}^{(K)}}) [\phi_{(a,b)}]^{\mathcal{H}} = \text{diag}(\mathbf{\Lambda}_{(a,b)}^{(1)}, \dots, \mathbf{\Lambda}_{(a,b)}^{(K)}), \\ &= \text{diag}(\psi_{(a,b)}^{(1)} \mathbf{R}_{\mathbf{H}^{(1)}} [\psi_{(a,b)}^{(1)}]^{\mathcal{H}}, \dots, \psi_{(a,b)}^{(K)} \mathbf{R}_{\mathbf{H}^{(K)}} [\psi_{(a,b)}^{(K)}]^{\mathcal{H}}). \end{aligned}$$

Thus

$$\text{rank}(\mathbf{\Lambda}_{(a,b)}) = \sum_{k=1}^K \psi_{(a,b)}^{(k)} \mathbf{R}_{\mathbf{H}^{(k)}} [\psi_{(a,b)}^{(k)}]^{\mathcal{H}}.$$

Because each channel coding sequence has minimum pairwise distance $d_{\min} \leq K$ units, there are differences of $d_{\min}$ units for any two different information sequences. Note that one STFB only encodes a single unit of each channel coding sequence, so that there are at least $d_{\min}$ STFBs with different channel coded input for any two different information sequences. Hence,

$$\min \{rank(\mathbf{\Lambda}_{(a,b)})\} = d_{\min} rank \left\{ \psi_{(a,b)}^{(1)} \mathbf{R}_{\mathbf{H}^{(1)}} \left[\psi_{(a,b)}^{(1)} \right]^H \right\} = d_{\min} \min \{N_t N_r T(L+1), N_r T N_C\} \quad (10)$$

■

The coded diversity system described in Proposition 1 actually encodes N_u channel coded streams of K units in parallel. If the unit is one bit, we call it bit-interleaved coded complex diversity coding (BICCDC) based approach. Bit-interleaved coded modulation (BICM) is different from BICCDC, since in BICM, bits are interleaved simply across different constellation symbols. If the unit is one symbol, we call the corresponding approach symbol-interleaved coded complex diversity coding (SICCDC).

IV. ITERATIVE DECODING OF CDC-ECC STFC

Fig. 4 depicts the iterative CDC-ECC STFC decoding scheme. The STFC decoder takes channel observation vector $\mathbf{y}$ and a priori information $\lambda(c'; I)$ on the coded and interleaved bits c' and computes its extrinsic information $\lambda(c'; O)$, which is subsequently de-interleaved to $\lambda(c; I)$. With a priori input $\lambda(c; I)$, a soft-input, soft-output (SISO) ECC decoder computes log-likelihood ratio (LLR) $\lambda(c; O)$ for the coded bits and $\lambda(b; O)$ for the information bits. The latter is used at the final iteration to make a hard decision on the transmitted information bits; whereas the former is interleaved and fed back to the STFC decoder as a priori information. Several SISO algorithms can be applied to compute the ECC decoder output. For the purpose of this study, we consider the use of the Log-MAP algorithm [15].

Recall that the received signal vector is expressed as $\mathbf{y} = \sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G} \mathbf{s} + \mathbf{n}$. The transmitted symbol vector $\mathbf{s}$ can be estimated by a linear MMSE algorithm, i.e.,

$$\mathbf{z} = \mathbf{W}^H \mathbf{y} = \mathbf{W}^H \left(\sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G} \mathbf{s} + \mathbf{n} \right) = \mathbf{U} \mathbf{s} + \mathbf{v}, \quad (11)$$

where the matrix $\mathbf{W}$ is designed to minimize the mean square error $e = E[\|\mathbf{z} - \mathbf{s}\|^2]$, leading to the solution $\mathbf{W} = \mathbf{R}^{-1} \mathbf{P}$, where

$$\mathbf{R} = E[\mathbf{y} \mathbf{y}^H] = E\left[\frac{\rho}{N_t} (\mathbf{H} \mathbf{G} \mathbf{s} + \mathbf{n})(\mathbf{s}^H \mathbf{G}^H \mathbf{H}^H + \mathbf{n}^H)\right] = \frac{\rho}{N_t} \mathbf{H} \mathbf{H}^H + N_0 \mathbf{I}; \quad (12)$$

$$\mathbf{P} = E[\mathbf{y} \mathbf{s}^H] = E\left[\sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G} \mathbf{s} + \mathbf{n} \mathbf{s}^H\right] = \sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G}; \quad \mathbf{U} = \mathbf{W}^H \mathbf{P}. \quad (13)$$

Eq. (12) is derived utilizing the fact that $\mathbf{G}$ is a unitary matrix.

The noise term $\mathbf{v}$ is Gaussian since it is a linear transformation of a Gaussian random vector $\mathbf{n}$ ($\mathbf{v} = \mathbf{W}^H \mathbf{n}$), with zero mean and covariance $\Gamma = E[\mathbf{v} \mathbf{v}^H] = N_0 \mathbf{W}^H \mathbf{W}$. Because the filtered noise $\mathbf{v}$ is no longer white (Γ is not an identity matrix, the elements of $\mathbf{v}$ are correlated), optimum detection involves joint estimation of all the symbols in the vector $\mathbf{s}$, which requires ML or near-ML sphere decoding. However, we have observed from our experiments that the off-diagonal elements of Γ are quite small compared to the diagonal elements. Therefore, we can well approximate Γ as a diagonal matrix. Consequently, each element of $\mathbf{s}$ can be estimated individually, and the receiver design is greatly simplified. The k th element of $\mathbf{z}$, denoted by z_k , can be written as $z_k = u_k s_k + v_k$, where u_k is the k th diagonal element of $\mathbf{U}$, and s_k, v_k are the k -th elements of the vectors $\mathbf{s}, \mathbf{v}$, respectively. The noise term v_k is a Gaussian random

variable with zero mean and variance N_k , which is the k th diagonal element of the matrix Γ . The probability density function (PDF) of the MMSE filter output z_k , conditioned on that the m th PSK/QAM symbol is transmitted, can be expressed as

$$f(z_k|s_m) = \frac{1}{\pi N_k} \exp\left(-\frac{|z_k - u_k s_m|^2}{N_k}\right). \quad (14)$$

In what follows, we derive a general expression for symbol-to-bit LLR mapping scheme based on the PDF function expressed by (14) for different modulation schemes. For a PSK/QAM system, we need to compute LLRs for M coded bits for each symbol s_k , which is one of the $r = 2^M$ possible symbols in the signal constellation. For example, $M = 2$ for QPSK, and $M = 4$ for 16-QAM. Denote the transmitted symbol $s_k = \text{map}\{c_k^{0'}, c_k^{1'}, \dots, c_k^{(M-1)'}\}_{c_k^{m'} \in \{0,1\}}$, the LLR value of the bit $c_k^{m'}$ conditioned on the MMSE filter output z_k can be calculated as

$$\lambda(c_k^{m'}|z_k) = \ln \frac{\Pr(c_k^{m'} = 1|z_k)}{\Pr(c_k^{m'} = 0|z_k)}. \quad (15)$$

To simplify (15), we define $\mathbf{I}_{+m}$ and $\mathbf{I}_{-m}$ for $m = 0$ as

$$\mathbf{I}_{-0} = \underbrace{\begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 0 & 1 \end{pmatrix}}_{M \times 2^{M-1}}; \quad \mathbf{I}_{+0} = \underbrace{\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 0 & 1 \end{pmatrix}}_{M \times 2^{M-1}}. \quad (16)$$

Note that for $m = 0$, the first row of matrix $\mathbf{I}_{-0}$ has all elements equal to 0, while the first row of matrix $\mathbf{I}_{+0}$ has all elements equal to 1. The other matrices for $m = 1, 2, \dots, M-1$ can be found by exchanging the 1st row with the corresponding $(m+1)$ th row. Using Bayes' theorem, we can write (15) as

$$\lambda(c_k^{m'}|z_k) = \lambda(c_k^{m'}) + \ln \frac{\sum_{p=0}^{2^M-1} p(z_k|\text{map}\{\mathbf{i}_{+mp}\})e^{\mathbf{L}\mathbf{i}_{mp}}}{\sum_{p=0}^{2^M-1} p(z_k|\text{map}\{\mathbf{i}_{-mp}\})e^{\mathbf{L}\mathbf{i}_{mp}}}, \quad (17)$$

where $\mathbf{i}_{+mp}$ and $\mathbf{i}_{-mp}$ are $(p+1)$ th column vectors of matrices $\mathbf{I}_{+m}$ and $\mathbf{I}_{-m}$. In (17), $\mathbf{i}_{mp}$ is the $(p+1)$ th column vector of matrix $\mathbf{I}_{+m}$ with its m th entry set equal to zero, and $\mathbf{L} = [\lambda(c_k^{0'}) \lambda(c_k^{1'}) \dots \lambda(c_k^{(M-1)'})]$ is a row vector of *a posteriori* LLRs. The second term in (17) is the extrinsic information of bit $c_k^{m'}$. Denoting the extrinsic information of the m th bit by $\lambda_e(c_k^{m'})$, we have

$$\lambda_e(c_k^{m'}) = \ln \frac{\sum_{p=0}^{2^M-1} p(z_k|\text{map}\{\mathbf{i}_{+mp}\})e^{\mathbf{L}\mathbf{i}_{mp}}}{\sum_{p=0}^{2^M-1} p(z_k|\text{map}\{\mathbf{i}_{-mp}\})e^{\mathbf{L}\mathbf{i}_{mp}}}. \quad (18)$$

Substituting (14) into (18) yields

$$\begin{aligned} \lambda_e(c_k^{m'}) &= \max^* \left\{ \frac{-|z_k - u_k \text{map}\{\mathbf{i}_{+m0}\}\mathbf{L}\mathbf{i}_{m0}|^2}{N_k}, \dots, \frac{-|z_k - u_k \text{map}\{\mathbf{i}_{+m(P-1)}\}\mathbf{L}\mathbf{i}_{m(P-1)}|^2}{N_k} \right\} \\ &\quad - \max^* \left\{ \frac{-|z_k - u_k \text{map}\{\mathbf{i}_{-m0}\}\mathbf{L}\mathbf{i}_{m0}|^2}{N_k}, \dots, \frac{-|z_k - u_k \text{map}\{\mathbf{i}_{-m(P-1)}\}\mathbf{L}\mathbf{i}_{m(P-1)}|^2}{N_k} \right\}, \end{aligned} \quad (19)$$

where $\max^*[\cdot]$ is defined as $\max^*[x, y] = \ln(e^x + e^y) = \max[x, y] + \ln(1 + e^{-|x-y|})$, i.e., the max operation compensated with a correction term $\ln(1 + e^{-|x-y|})$. Also $\max^*[x, y, z] = \max^*[\max^*[x, y], z]$, etc..

In the case of QPSK modulation, each QPSK symbol s_k corresponds to two coded bits $c_k^{0'}$ and $c_k^{1'}$. Equation (18) is simplified to [21]

$$\lambda_e(c_k^{0'}) = \ln \frac{P_r(z_k|c_k^{0'} = 1, c_k^{1'} = 0) + P_r(z_k|c_k^{0'} = 1, c_k^{1'} = 1)e^{L_a(c_k^{1'})}}{P_r(z_k|c_k^{0'} = 0, c_k^{1'} = 0) + P_r(z_k|c_k^{0'} = 0, c_k^{1'} = 1)e^{L_a(c_k^{1'})}}, \quad (20)$$

where $L_a(c_k^{1'})$ is the a priori value for the bit $c_k^{1'}$. Substituting (14) into (20) yields

$$\lambda_e(c_k^{0'}) = \max^* \left\{ -\frac{|z_k - u_k s_{10}|^2}{N_k}, -\frac{|z_k - u_k s_{11}|^2}{N_k} + L_a(c_k^{1'}) \right\} - \max^* \left\{ -\frac{|z_k - u_k s_{00}|^2}{N_k}, -\frac{|z_k - u_k s_{01}|^2}{N_k} + L_a(c_k^{1'}) \right\},$$

where s_{mn} denotes the symbol corresponding to the bits $c_k^{0'} = m$, and $c_k^{1'} = n$. Similarly,

$$\lambda_e(c_k^{1'}) \approx \max^* \left\{ -\frac{|z_k - u_k s_{01}|^2}{N_k}, -\frac{|z_k - u_k s_{11}|^2}{N_k} + L_a(c_k^{0'}) \right\} - \max^* \left\{ -\frac{|z_k - u_k s_{00}|^2}{N_k}, -\frac{|z_k - u_k s_{10}|^2}{N_k} + L_a(c_k^{1'}) \right\}.$$

Two bit-to-symbol mapping schemes, namely, Gray and anti-Gray are considered in this work, for QPSK and 16-QAM systems, respectively. The results will be shown in Section V.

For a multi-stream system, the received signal can be written as

$$\mathbf{r} = \sum_{i=1}^{N_t} \sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G}_i \mathbf{s}_i + \mathbf{n}, \quad (21)$$

where $\mathbf{G}_i$ is the encoding matrix for the i th stream and $\mathbf{s}_i$ is the i th source symbol vector. In order to facilitate MMSE decoding, we reformulate (21) as

$$\mathbf{r} = \sum_{i=1}^{N_t} \sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{G}_i \mathbf{s}_i + \mathbf{n} = \sqrt{\frac{\rho}{N_t}} \sum_{i=1}^{N_t} \mathbf{H}_i \mathbf{s}_i + \mathbf{n} = \sqrt{\frac{\rho}{N_t}} \underbrace{\begin{bmatrix} \mathbf{H}_1 & \dots & \mathbf{H}_{N_t} \end{bmatrix}}_{\mathbf{H}_{eq}} \underbrace{\begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_{N_t} \end{bmatrix}}_{\mathbf{s}_{eq}} + \mathbf{n}, \quad (22)$$

where $\mathbf{H}_i = \mathbf{H} \mathbf{G}_i$. The rest of the derivation follows similarly to (11) – (18), with $\mathbf{H} \mathbf{G}$ replaced by $\mathbf{H}_{eq}$, and $\mathbf{s}$ replaced by $\mathbf{s}_{eq}$.

V. NUMERICAL RESULTS

Simulation settings are summarized as follows:

- 1) A convolutional code (with block size of 512 coded bits, code rate $R_c = 1/2$, constraint length 3, and generator polynomials (5, 7) in octal form) is used in Figs. 5 – 7; a Reed Solomon code (each RS codeword includes 6 RS information symbols and 2 redundancy symbols, and each RS symbol corresponds to 4 bits) is used in Fig. 8.
- 2) MIMO frequency selective channel has channel order $L = 1$ (2-path except in Fig. 6(b) where 7-path channel is assumed), and each path experiences independent Rayleigh fading. Channel power delay profile is assumed to be uniform.
- 3) $N_t = N_r = 2$, $N_F = 4$, $T = 2$, and $N_C = 32$,
- 4) the number of STFCs for joint CDC and ECC is K , and the number of STFBs within one STFC for one block of 3-D CDC and ECC is N_a .

Performance of different decoding algorithms for the joint CDC-ECC system with QPSK modulation are demonstrated in Fig. 5. Comparing the two non-iterative schemes, soft sphere decoding (SD) [11] shows 1 dB gain at $\text{BER}=10^{-4}$ compared to the MMSE scheme with gray mapping. However, we observed that the performance of the MMSE decoding can be much improved by using anti-gray mapping and iterative decoding, which is slightly better than or comparable to the non-iterative soft SD decoding over a wide range of SNRs.

Fig. 6 shows the performance of the MMSE decoding for 16-QAM modulated CDC-ECC STFC system with gray and anti-gray mapping for 2-path and 7-path channels, respectively. For the anti-gray system, there is a significant performance improvement by applying an iterative process if we compare the topmost curve representing the first-iteration of CDC based STFC MMSE decoding and Log-MAP ECC decoding with the bottom curve representing the performance of iterative decoding upon convergence. The most significant gain is obtained at the second iteration. Note that no gain can be obtained by performing the iterative process for the systems with gray mapping, in which the bits are mapped to I and Q channels independently [21]. The iterative MMSE decoding with anti-gray mapping outperforms the one with gray mapping at the 4th iteration when $E_b/N_0 > 26.1$ dB and $E_b/N_0 > 22.2$ dB for 2-path and 7-path channels, respectively. This suggests that if the 16-QAM system operates at low SNR, gray mapping can be applied. Otherwise, anti-gray mapping and iterative decoding would be preferred. By comparing Fig. 6(a) with Fig. 6(b), it is evident that the proposed system can exploit the diversity gain provided by multipath propagation.

Fig. 7 shows the performance comparison between ECC-only STFCs and iterative CDC-ECC STFCs as well as the impact of the parameter N_a on the STFC system performance. To maintain the same data rates among ECC-only STFCs and iterative CDC-ECC STFCs, we construct ECC-only STFCs by using identity matrices for $\mathbf{G}_{STFB}$ in CDC based STFCs. Clearly, iterative CDC-ECC STFCs outperform ECC-only STFCs, especially at higher SNRs and when the iterative scheme converges. Consistent with the analysis in Section III, in Fig. 7, the system using full diversity 3-D CDC based STFC with $N_a = 1$ outperforms that with $N_a = 4$.

Fig. 8 shows the results of joint full diversity 3-D CDC and ECC with Reed-Solomon (RS) codes. Each RS coded stream is across N_a 3-D CDC STFCs, and one RS codeword is only across one STFB within each STFC. The number of RS symbols of one RS codeword within one STFB is N_g . In the simulations, hard sphere decoding for 3-D CDC STFCs and hard decisions for RS codes are chosen. From Fig. 8, one can see that with the same configurations of RS codewords, STFC using symbol-interleaved coded complex diversity coding (SICCDC), i.e. $N_g = 1$ significantly outperforms STFC without SICCDC, i.e. $N_g = 4$. Considering the case when a pair of RS codewords have the minimum distance, i.e., 2 RS symbols, the probability of two different RS symbols being encoded into two different STFBs over time is i) 1 in the SICCDC case; ii) 4/7 in the case without using SICCDC. As shown by (10), for the SICCDC case, $\min \{rank(\Lambda_{(a,b)})\} = 2 \min \{N_t N_r T(L+1), N_r T N_C\}$; whereas for the case without SICCDC, $\min \{rank(\Lambda_{(a,b)})\} = \min \{N_t N_r T(L+1), N_r T N_C\}$. Note that they are the lower bounds, and when a pair of RS codewords differ in more than 2 RS symbols, $rank(\Lambda_{(a,b)})$ may be much higher than $\min \{rank(\Lambda_{(a,b)})\}$ in the SICCDC case. It is observed from Fig. 8 that the SICCDC with full diversity, proved in Section III, yields superior performance due to its better diversity properties.

VI. CONCLUSIONS

Joint 3-D space-time-frequency complex diversity coding and channel coding has been investigated in this paper. Our theoretical analysis reveals that by exploiting diversities over all three physical dimensions (spatial, time, and frequency), the joint code design has the potential to achieve a diversity order of $\min \{K, d_{\min}\} \min \{N_t N_r T(L+1), N_r T N_C\}$, where N_t is the number of transmit antenna, N_r is the number of receive antennas, N_c is the number of subcarriers per antennas, L is the frequency selective channel order between any pair of transmit and receive antennas, $d_{\min}$ is the minimum distance of the employed channel code, and K is the number of 3-D CDC over time. This paper proposes and proves full diversity construction with 3-D CDC and channel coding, bit-interleaved coded complex diversity coding and symbol-interleaved coded complex diversity coding.

The iterative decoding of ECC and CDC has been investigated in order to exploit the diversity potential inherent in the joint CDC-ECC STFC system. In particular, a low-complexity MMSE iterative decoding scheme with anti-gray mapping is proposed, and is shown to achieve the performance of soft sphere decoding, and at the same time, reduce the complexity from exponential to polynomial. A multi-stream CDC-ECC architecture is also introduced and is shown to have comparable performance to a single-stream system with reduced complexity and decoding latency due to its parallel structure.

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Fig. 1. Block diagram of the transmitter for the STFC system.

Fig. 2. Structure of Multi-stream joint CDC and ECC STFC

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Fig. 3. The structure of the STFC code.

Fig. 4. Structure of the iterative STFC decoding.

Fig. 5. Performance comparison of MMSE and sphere decoding in ECC-STFC system with QPSK modulation. MMSE with gray mapping and soft SD decoding schemes are non-iterative. The curve for MMSE with anti-gray mapping is plotted at the 4th iteration.

(a) 2-path channel.

(b) 7-path channel.

Fig. 6. Performance of iterative MMSE decoding. The employed modulation scheme is 16-QAM. For the systems with anti-gray mapping, the top curve represents the first iteration of the CDC-ECC decoding, and the bottom curve represents the 5th iteration of the CDC-ECC decoding.

Fig. 7. STFC performance comparisons between iterative CDC-ECC and ECC-only. The top curve represents the first iteration, the bottom curve represents the 4th iteration.

Fig. 8. The effect of using SICCDC on the performance of joint 3-D CDC and RS codes