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GRAVITY GRADIENT AND MAGNETORQUING
ATTITUDE CONTROL FOR
LOW-COST LOW EARTH ORBIT SATELLITES
- THE UOSAT EXPERIENCE

Submission for the Ph.D degree
at the University of Surrey

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Summary

An important concern in spacecraft engineering is *attitude determination control and stabilisation* (ADCS) - the combination of applied mathematics, classical physics and modern technology which maintains the pointing direction of one or more axes of an Earth-orbiting satellite. This thesis is a detailed study of a particular type of ADCS which exploits the *gravity-gradient* effect, which is just the weak tendency for an appropriately shaped body to point naturally in preferred directions; reinforced by *magnetorquing*, which is the active interaction of the geomagnetic field with a switched current passing through coils in the spacecraft body. The advantages of this technology is that it is low-cost, non-consumable and has no moving parts - so constituting no limitation to the satellite's life. The thesis is a detailed study of this form of ADCS with specific application to low Earth polar-orbiting (LEO) satellites, for which it is particularly suitable. The work is also a study in attitude determination based solely on a 3-axis *magnetometer measurement* of the geomagnetic field, which is in principle a simple way, in terms of technology, of determining the attitude of the spacecraft, and from this controlling the attitude, if mediated by an on-board computer implementing appropriate algorithms. The results are for the most part practically based on the author's involvement with two satellites over a six year period with the satellites UOSAT-1 and UOSAT-2, which were designed, built, and continue to be controlled from the University of Surrey. A practical innovation in 2-axis attitude control is described: the active 'delibration' by active damping of a gravity-gradient controlled LEO satellite in an attitude-stabilised state, using a *threshold comparison algorithm*. A new theory and algorithms are then developed for 3-axis attitude control, based on a *complementary* use of magnetorquing and gravity gradient. Subject to further development these could alter the general perception of the most effective way of controlling low Earth orbiting satellites.

CONTENTS

[content-0-III]

1 INTRODUCTION

[nt1 -2-III]

- 1.1 Background
- 1.2 Control and Stabilisation
- 1.3 Scope
- 1.4 Results
- 1.5 Acknowledgements

2 DESCRIPTION OF UOSATS

[nt2sa-2-III]

- 2.1 General Systems
 - 2.1.1 Overview of UOSAT-1 systems
 - 2.1.2 Overview of UOSAT-2 systems
 - 2.1.3 Design principles and system functions
 - 2.1.4 Software
- 2.2 ADCS systems [nt2sb-2-III]
 - 2.2.1 Gravity gradient boom
 - 2.2.2 Magnetometers
 - 2.2.3 Magnetorquers
- 2.3 Ground Station
- 2.4 Orbital description
- 2.5 Control facilities and requirements
 - 2.5.1 ADCS provisions
 - 2.5.2 ADCS requirements

3 PASSIVE ATTITUDE STATES & CONTROL

[nt3a-2-I]

- 3.1 Attitude Description and Conventions
 - 3.1.1 1-Axis attitude definition
 - 3.1.2 3-axis attitude definition
 - 3.1.3 Attitude sphere and circle
 - 3.1.4 Mathematical relation and summary
 - 3.1.5 Alternative attitude definition

3.2 Nutation/spin Conventional Equations	[nt3b-2-I]
3.2.1 Kinematics	
3.2.2 Dynamics	
3.3 Natural Torque - Gravity Gradient	[nt3c-2-I]
3.4 G.G Forced Precession	
3.4.1 Geometric interpretation of torque	
3.4.2 Torque equations	
3.4.3 Precession equation	
3.4.4 Locked precession	
3.5 Attitude Capture by Gravity Gradient	[nt2d-2-I]
3.5.1 Pitch differential equation	
3.5.2 Average attitude assuming capture	
3.5.3 Bounds on attitude	
3.6 Modelling of Boom Deployment	[nt3e-2-I]
3.6.1 Numerical calculations	
3.6.2 Results of simulations	
3.7 Simplified Differential Equations	
3.7.1 Magnitude of coupling effects	
3.7.2 Reduction to linear equations	
3.8 Disturbance Torques	[nt3f-0-I]
3.8.1 Solar Radiation torque	
3.8.2 Aerodynamic torque	
3.8.3 Thermo-elastic deformation	
3.8.4 Internal torques	
3.8.5 Engineering context	
3.8.6 Flexible body dynamics	
3.9 List of Symbols	
 4 ATTITUDE DETERMINATION	 [nt4a-2-I]
4.1 The problem of Attitude Determination	
4.2 Spinning/nutating State	

- 4.3 Geomagnetic Field Modelling [nt4b-2-I]
 - 4.3.1 Ideal dipole and polar orbit
 - 4.3.2 Tilted dipole and inclined orbit
 - 4.3.3 Usefulness of models to ADCS design
 - 4.3.4 IGRF field modelling
- 4.4 Libration Measurement
- 4.5 Magnetometer Auto-Calibration
- 4.6 List of Symbols

5 APPLIED TORQUE - MAGNETORQUING [nt5a-0-II]

- 5.1 Spin -Stabilised Control
 - 5.1.1 Z de-spinning
 - 5.1.2 Attitude change by precession
 - 5.1.3 Orbital precession criteria
 - 5.1.4 Denutation [nt5b-0-III]
 - 5.1.5 Denutation + precession
- 5.2 Magnetic Stabilisation
 - 5.2.1 Naive algorithm
 - 5.2.2 Improved algorithm
 - 5.2.3 Quasi-optimal bang-bang control
- 5.3 Delibration [nt5c-0-II]
 - 5.3.1 Threshold comparison algorithm (basic)
 - 5.3.2 Implementation of threshold algorithm - I [nt5d-0-II]
 - 5.3.3 Threshold comparison algorithm (improved)
 - 5.3.4 Implementation of threshold algorithm - II
 - 5.3.5 Implementation in real world IGRF
- 5.4 General Control Laws [nt5e-0-II]
 - 5.4.1 CPL law
 - 5.4.2 Reciprocity Principle
- 5.5 List of Symbols [nt5f-0-II]

6 CONTROL OF UOSATS

[nt6a-0-III]

6.1 Control of UOSAT1

- 6.1.1 Initial appraisal of the system
- 6.1.2 State of UOSAT-1 systems
- 6.1.3 Planned control sequences
- 6.1.4 First succesful de-spin
- 6.1.5 Simple attitude estimation
- 6.1.6 Precession manoeuvres [nt6b-0-III]
- 6.1.7 The boom deployment problem

6.2 Control of UOSAT2 - Part I

[nt6c-0-III]

- 6.2.1 Theoretical preparation
- 6.2.2 Magnetometer and magnetorquer checks
- 6.2.3 Initial acquisition loss and recovery
- 6.2.4 Angular motion after recovery
- 6.2.5 Testing the spin axis magnetorquer
- 6.2.6 Magnetic stabilisation control
- 6.2.7 Boom deployment

6.3 Control of UOSAT2 - part II

[nt6d-0-III]

- 6.3.1 Attitude determination
- 6.3.2 Attitude control

7 ADVANCED MAGNETIC ADCS

[nt7a-0-II]

7.1 General Principles

- 7.1.1 Magnetometer measurements processing
- 7.1.2 Differential equations
- 7.1.3 Analogue spinning estimator
- 7.1.4 Analogue cyclic (pitch) estimator
- 7.1.5 Analogue cyclic with offset (roll) estimator
- 7.1.6 Conversion to sampled data system [nt7b-0-II]
- 7.1.7 Response to control torques

7.2 Derive Innovations

- 7.2.1 The calculations
- 7.2.2 Explanatory theory

7.3 Update Estimators

[nt7c-0-II]

- 7.3.1 Yaw calculations
- 7.3.2 Pitch /roll calculations
- 7.3.3 Explanatory theory

7.4 Simulation Experiment

7.5 The Control Situation - I [nt7d-2-II]

- 7.5.1 Need for spin-plane magnetorquing
- 7.5.2 Commutated damping and spin-control
- 7.5.3 Practical control
- 7.5.4 Effectiveness of torquing
- 7.5.5 Decision control - general damping
- 7.5.6 Results of simulation

7.6 The Control Situation - II [nt7e-2-II]

- 7.6.1 Linearised equations
- 7.6.2 Stability problems
- 7.6.3 Semi-linearised equation
- 7.6.4 3-axis control with magnetorquer
- 7.6.5 Results of simulations

7.7 List of Symbols [nt7f-2-II]

8 REVIEW OF RELATED LITERATURE [nt8a-2-II]

8.1 Control and Determination in General

8.2 Magnetic Control Review [nt8b-2-II]

- 8.2.1 Mobley 1981
- 8.2.2 Schmidt 1981
- 8.2.3 Emsley et.al 1981 [nt8c-2-II]
- 8.2.4 Decanini et.al 1981
- 8.2.5 Dougherty et.al 1981
- 8.2.6 Gran and Proise 1981
- 8.2.7 Junkins et.al 1981

8.3 Miscellaneous Magnetorquing [nt8d-2-II]

- 8.3.1 Stewart et.al 1982
- 8.3.2 Meinzer 1981
- 8.3.3 Goel et al 1979
- 8.3.4 Stickler and Alfrend 1974
- 8.3.5 Teule 1979
- 8.3.6 Takezawa et.al 1980
- 8.3.7 Summary and comparison

8.4 Gravity Gradient Development [nt8e-2-II]

- 8.4.1 Simplified analysis of 3-axis control
- 8.4.2 El-Segundo conference (1969)
- 8.4.3 Anand and Whisnant (1973) [nt8f-0-II]
- 8.4.4 Lerner (1975)
- 8.4.5 Hughes (1986)
- 8.4.6 Siahpush and Sexton (1987)
- 8.4.7 Martel, Pal, and Psiaki (1988)

9 SUMMARY AND CONCLUSIONS [nt9a-0-III]

9.1 Summary

- 9.1.1 Overall mission control
- 9.1.2 Theoretical studies on gravity gradient
- 9.1.3 Magnetorquing manoeuvres
- 9.1.4 Practical 2-axis stabilisation-
- threshold comparison algorithm
- 9.1.5 Stabilisation plus spin control
- 9.1.6 Problem of 3-axis stabilisation
- 9.1.7 3-axis stabilisation of symmetrical
satellite with magnetorquing
- 9.1.8 3-axis stabilisation of asymmetric
satellite with magnetorquing
- 9.1.9 Attitude determination problem
- 9.1.10 1-axis determination in 2-axis
stabilised state
- 9.1.11 Magnetometer auto-calibration
- 9.1.12 3-axis attitude determination

9.2 Future Work [nt9b-0-III]

- 9.2.1 New hardware
- 9.2.2 New software
- 9.2.3 Research aims -accuracy
- 9.2.4 Development plan

9.3 Conclusions

APPENDICES

A Vector Derivation of Nutation	[ntA-0-I]
B Gravity Gradient Induced Precession	[ntB-2-I]
C Libration Bounds Formulae	[ntC-0-I]
D Angular Equations of Motion	[ntD-0-I]
E Magnetic Observations on Nutation	[ntE-2-I]
F Quasi-orthogonal Estimator	[ntF-2-I]
G Magnetic Control Theory	[ntG-0-II]
H Exact precession/nutation formulae	[ntH-0-II]
I Statistically Optimum Estimators	[ntI-0-II]
J Direct Vector Rotation in Orbit	[ntJ-0-II]
K 3-Axis Gravity Gradient Dynamics	[ntk-2-II]

FIGURES	[figs 0 - I]
---------	----------------

ACRONYMS	[acron-0-I]
----------	---------------

REFERENCES	[sources-2-III]
------------	-------------------

BIBLIOGRAPHY

CONFERENCES	[confs-2-III]
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1 INTRODUCTION

This thesis sets out the theoretical basis, supported by practical results, for the attitude control and stabilisation of a particular class of satellite. The satellite is small, it is in a polar low Earth orbit (LEO), and the primary method of attitude stabilisation is by the passive gravity-gradient effect, backed by an essential active control from magnetorquing. The practical results come from my experience with two satellites, UOSAT-1 and UOSAT-2, which were conceived, designed, constructed and operated in orbit from the Department of Electronic and Electrical Engineering of the University of Surrey (NOTE).

UOSAT-1 was launched on 6th October 1981 on a US NASA Delta rocket; while UOSAT-2 was launched on 1st March 1984, again on a Delta rocket from the same launch site (Vandenburg in California). After launch both spacecraft were controlled in orbit - and at the time of writing continue to be controlled - from a ground station designed, built and operated by members of the Department.

The aim was to achieve a stable Earth-pointing attitude for these satellites, within the limitation of a low-cost technology. Based on this experience, a further aim has been to explore in theory the feasibility of extending the performance, for future satellites, using a similar technology. While still limited by a low-cost budget, a continuing theme is the maximisation of signal-processing capability provided by an on-board computer.

1.1 Background

Angular motion of the spacecraft after ejection from the launch vehicle was an uncontrolled tumbling in both cases; the initial question was how to stop it. This meant interpreting the parameters of the motion from

the data transmitted to ground, and getting control, first by magnetorquing alone, and then deploying a 'gravity-gradient' boom to achieve stabilised Earth-pointing of the spacecraft, followed by continued magnetorquing to maintain this stabilisation.

UOSAT-1 was not quite successful, as far as long-term gravity-gradient stabilisation was concerned. Although a variety of attitude changes in an initial *spin-stabilised* state were induced by magnetorquing, and a temporary transition to gravity-gradient control was demonstrated, the final aim of long-term stabilisation failed, due to a mechanical problem with the deployment of the boom (see section 6.1).

I was more fortunate with UOSAT-2 and was able to bring its control and stabilisation problem to a successful solution. Since launch it has been possible to continue the work on improvement of the Earth-pointing attitude state.

As a class, the UOSATs are unique in the following combination of properties: there are no continuously rotating parts (as found with momentum or reaction wheels); no mechanisms for passive damping in any state (NOTE). The satellite is just an orbiting box of electronics surrounded by solar panels; any active control is by 'magnetorquing' - meaning simply the controlled switching of current through coils of wire embedded in the sides of the spacecraft. The resulting torque follows from interaction with the geomagnetic field. It is a practicable proposition only for low Earth orbits.

UOSAT-2 is near axially symmetric (see for example fig.2.1-1) with near-equal moments of inertia about its X and Y axes. The stability at present can be maintained only in respect of the zenith-nadir or Z axis pointing nominally to Earth centre. The literature calls this '2-axis stabilisation'. The satellite is free to spin slowly (yaw motion) around the Z axis.

NOTE Our boom deployment mechanism is a motor. This is however is not an intrinsic feature; booms can be deployed by non-rotating means.

Traditionally, this has always been deemed to be a basic characteristic of an axially symmetric satellite, since there is no yaw-restoring torque generated by the gravity-gradient effect. However, with the inclusion of magnetorquing and an on-board computer there is no reason in principle why a full '3-axis stabilisation' cannot be achieved, for this geometry. It is also well-known that an asymmetric body (having unequal moments of inertia in all axes) under ideal circumstances should exhibit an inherent 3-axis stabilisation. There is good reason to include magnetorquing and computer control in order to *improve* the overall control . Both matters are explored in ~~my~~ *this* thesis, with promising results.

The general aims of the UQSAT satellite program have been described elsewhere (UQSAT-IERE 1982/ UQSAT-IERE 1987). Both spacecraft embody the design goal of relatively inexpensive and cost-effective engineering. For a start they had to be within the resources of a University Department. Even if backed by friendly assistance from industry, there was little funding at the level traditionally associated with commercial and military satellites.

The UQSAT group's intention is to design small satellites which will be able to fulfill a large number of scientific and commercial objectives (Sweeting 1988,1989, Lorenz 1989), which may require only a limited pointing accuracy, and which must use low-cost technology. From the point of view of ADCS (NOTE) this has meant (so far as is possible) the elimination of any dependence on moving, mechanical parts; and to be independent of chemicals and other depletable consumables.

The most suitable technology to meet these requirements, other than exploiting the gravity-gradient effect, would seem to be magnetorquing. The deployment of a single mass on the end of boom to exploit the gravity-gradient effect is not new in space engineering; nor is magnetorquing. What is new in our work is the exploitation of both

NOTE ADCS = 'attitude determination control and stabilisation'

controls simultaneously, for a LEO satellite - in conjunction with digital signal processing, made available by the OBC (on-board computer). With UOSAT-2 we have demonstrated how to use and combine an old principle with a new technology. In the past this may have been seen to be un-interesting but is now in fact highly significant - largely as a consequence of (i) extensive signal-processing capability conferred by on-board computers and (ii) much wider interest in building and flying small, low-cost satellites (conferences AIAA/DARPA 1987, AIAA/USU 1988 , ESRIN 1989) in this 'microsat' mass category.

For both UOSATs, attitude determination was possible only by a 3-axis magnetometer measurement. This might seem self-limiting, and admittedly was to some extent imposed by forces beyond our control; but it was a challenging restriction. By itself, of course, one magnetometer vector measurement cannot determine the attitude of the satellite. The problem was solved in two ways: (i) a method of stabilisation was evolved which works directly from the magnetometer measurement and without seeking to determine the attitude; and (ii) a technique for building up full information using estimating filters.

Very recently there has been published comparable work showing that in the USA space interests are actively exploring ADCS, working within this constraint (Martel et al 1988).

ADCS is an 'overhead' as far as the spacecraft is concerned, because the associated hardware generally does not earn money (or perform experiments), unlike other payloads. The smaller the satellite, the greater the relative volume and power requirements demanded by the ADCS system. The general aim then is to minimise this demand while still seeking the best possible control of attitude.

The pressure for launch opportunities, and the increasing capability of launchers to carry multiple loads, is indicative of new opportunities for small satellites. There will be two new UOSATs (UOSAT-D and E) launched as secondary payload on ARIANE in 1989, each about 2/3 the mass of UOSAT-2, alongside four small US satellites . There is very little

space available to put more elaborate sensors or control mechanisms on these satellites, or 'microsats'. We may expect an increasing interest in flying small satellites; which will consequently fuel the process of developing cost-effective, low power, low mass and low volume ADCS.

1.2 Control and stabilisation

Early in this thesis I have to clarify what seems to me a problem in terminology. In my work I have been attempting both to 'control' a satellite and 'stabilise' it, in respect of its attitude. The meaning of these key words, and the distinction between them, is not as clear-cut as it may seem.

'Stability' in the context of dynamics can be an extremely complicated subject. Hughes (1986) has identified 35 distinguishable meanings ! From elementary physics however one basic meaning is surely that stability is that *passive* characteristic, given a nominally static or equilibrium state, in which a slight disturbance is resisted and the attitude is *restored* to its original state.

For favoured attitudes, a spacecraft body, under the influence of gravity gradient, is commonly held to be 'stable' in this sense (see treatment from section 3.3 and later section 7.6). As is well-known, it is a weak torque effect which tends to align a rigid body in the square-law gravitational field.

But in fact, an attitude controlled by the gravity-gradient effect alone can never be 'stable', using that word in a practical engineering sense, since the 'restoration' is simply by a torque proportional to the amplitude displacement (for small displacements), with no inherent damping. The same practical problem exists for a frictionless ball placed in a concave bowl, under the influence of ordinary gravity.

However the restoration of attitude need not be restricted to the mechanisms of natural physics. A computer and hardware can maintain the attitude, even if the result is not 'stability' in any passive sense.

I have adopted therefore a common-sense, rather than passive, or physical meaning to 'stability'. For a satellite's attitude I will say that stability exists, with respect to one or more parameters of the attitude state, if the parameter(s) always tend(s) to revert to a *required value* - not necessarily zero, when displaced temporarily by some perturbation. The means by which this stability is achieved is deemed to be irrelevant. In the past, and in the gravity-gradient context, 'real' stabilisation was almost always achieved by the addition of passive damping mechanisms (*mechanical or magnetic*). In my work magnetic torquing and computer *control* are needed in order to achieve a real stabilisation.

There is however a difficulty in defining *control*, when gravity-gradient provides at least part of it. I would follow the SOED (NOTE 1) in taking 'control' to require some form of 'apparatus'. 'Control', as applied to the attitude of spacecraft, ideally should mean the directed achievement of some desired attitude state, using this apparatus, governed by some algorithm or law. There is not really a physical 'apparatus' here; but at least there is a simple mathematical law which governs gravity-gradient torque. So in that sense there is 'control'.

But whether or not gravity gradient is *really* a form of 'control' depends on the requirement. The torque constitutes a form of 'control' if and only if the *goal is stabilisation and that stabilisation is directed to a restricted choice of attitudes* (NOTE 2).

NOTE 1 Shorter Oxford English Dictionary

NOTE 2 For 2-axis stabilisation (by gravity-gradient torque alone assuming an axially symmetric satellite) there are two possible attitudes. For 3-axis stabilisation there are notionally 24 possible attitudes although in practice these reduce to 4 (see from section 7.6)

In other contexts gravity gradient is a nuisance; the torque which exists could be seen not as a source of control but a perturbation, tending to drive the system away from some desired state.

Hence, in this work, stabilisation implies control; and control implies stabilisation. I tend to refer loosely to 'attitude stabilisation' or 'attitude control' and mean much the same thing by either. The terminology 'ADCS' embraces the overall aim.

1.3 Scope

The scope of this work then is restricted to cost-effective, attitude determination stabilisation and control (ADCS) techniques, directed towards the needs of Low Earth orbiting satellites (NOTE)

In organisation the thesis is slightly unconventional. In essence, I have put between two covers a coherent and relevant theory, in addition to a practical account, of a large number of matters arising in the ADCS of a small, axially symmetric spacecraft. I draw on past theory and publications known to me, but also develop my own ideas where there seems to be a need.

Chapter 2 provides the setting: a broad description of the UOSAT systems sets up the physical and engineering context which has constrained and guided my efforts; this is followed by a brief account of the controlling ground station; and the orbit characteristics are described, all with a view to clarifying the situation in which the work was done. Finally, the control capabilities and general requirements are identified.

Chapter 3 explains the passive states which can exist without attempting any active (magnetorquing) control. These results can be found in many

NOTE Gravity-gradient stabilisation and control is not useful at geosynchronous altitudes because of the two orders of magnitude reduction in the restoring torque.

texts. However, I have tried to adopt a personal point of view and a unifying approach, which yields a relatively simple body of theory in a straightforward way.

Chapter 4 concerns low-cost methods of attitude determination using the magnetometer measurements only; and involves some novel approaches.

Chapter 5 is a unique account of all the ways that magnetorquing, working from direct measurement of the geomagnetic field, can be used to implement the following: modification of the spin-stabilised state (spin change /denutate/ precession); magnetic stabilisation which forces the spacecraft to follow the geomagnetic field line; active damping (delibration) when in the final stabilised state controlled by gravity gradient. *All these controls are achieved with no attitude determination as such.* This particular aspect of the work is believed to be novel in itself; it has been reported under contract, and accepted by the customer, as 'breaking new ground' (Hodgart 1987).

Chapter 6 reviews the relevant flight history of the two UOSAT satellites to illustrate and demonstrate much of the previous theory.

Chapter 7 consists of several related theoretical steps directed towards a more advanced approach. A 'Kalman-like' estimator is proposed, and it is shown how a heuristic development can lead to a continuous 3-axis, real-time determination of the spacecraft attitude, using the IGRF model of the geomagnetic field as a data base. Given such information, either obtained this way and/or by processing of combinations of other sensors, a more comprehensive control can be attempted. It is shown how a UOSAT-2 type spacecraft can achieve full 3-axis stabilisation, with an interesting azimuth-scanning option. In the past, asymmetric geometries of spacecraft (not UOSAT) were tested for 3-axis gravity-gradient stabilisation, backed up by passive dampers. These attempts were generally not successful. I find, in section 7.6, a theoretical explanation for this (usual) failure. The reason is revealed in a simple mathematical development of the standard linearised equations. It is shown under what circumstances active and additional control should

solve the problem. Magnetorquing is the simplest such addition. On its own however magnetorquing could never achieve the required stabilisation; and gravity gradient is still an inherent part of the control dynamics.

Chapter 8 is a review of past work on magnetorquing and gravity-gradient control and stabilisation on other satellites, selected on the basis of general interest and/or relevance to our work. There is no significant, comparable recent work to review, which is the reason why this matter has been left to the last, where I try to put our UQSAT work in a historical context.

Finally in Chapter 9 I summarise what I have achieved; and consider future directions in the way that work could go in realising fully the aims of the ADCS program for UQSAT.

In support of all this work there is a comprehensive set of appendices.

This thesis has become somewhat extensive. An initial reading by experts can afford to skip some of it. But all should read Ch.2, sections 3.1, 3.5, 3.7; 4.4 and 4.5; Ch.5; section 6.3; Ch.7; and Ch.9. Appendices F, I, and K at least should be of interest.

The ADCS techniques described in this thesis are specifically tailored to the type of spacecraft described and the type of orbit in which they were placed. It should not be expected that they will be necessarily appropriate if there are major changes in dimensions, weights or configuration.

As a cursory inspection will show, I have adopted very little orthodox 'control theory', hardly progressing beyond the classical *position plus rate control*. The real problems have been found to lie elsewhere. The general principles I have applied and found useful owe more to telecommunication, modulation, and general signal processing theory.

1.4 Results

The research led to the successful stabilisation of the UDSAT-2 spacecraft from an uncontrolled tumbling state, achieving a libration control of the order of 10° (see fig.6.3.2-16). It should be pointed out that this was achieved with just the hardware stated, a computer and a few lines of computer code (section 5.3.1). A significant improvement to this can be expected (section 5.3.3), now that we are in a position to utilise a full FORTH operating system which has been installed in the spacecraft. Software already developed allows a mixture of functions: the damping is already combined with a spin-rate control; and the spacecraft's slow residual spin rate is constantly monitored and adjusted within specified limits - all by magnetometer/magnetorquing combination. The potential for development from here is considerable.

Some of the results have been published (IERE 1987) and also presented at the Minneapolis (1988) AIAA/AAS conference by invitation.

Although I summarise more fully the results achieved in chapter 9, a brief list here may assist:

- * Active Earth-pointing state in LEO using previously untried technique
- * rigorous theoretical analysis of gravity-gradient and magnetorquing combinations of control and stabilisation
- * low cost minimal attitude determination techniques
- * identification of basic deficiency in past attempts in continuous 3-axis attitude stabilisation, based solely on passive damping
- * a path created for future research in extending the precision and flexibility of combined gravity-gradient and magnetorquing stabilisation of LEO satellites

1.5 Acknowledgements

I gratefully acknowledge the substantial efforts made over the years by UOSAT team members past and present, in particular Neville Bean, Steve Holder, Jeff Ward and Paul Wright. A much larger group - too long to mention here - were involved in the construction of the two satellites, without whom this work would not have been possible. Little would have happened without the constant stimulus and encouragement (and criticism) due to Martin Sweeting, as leader of the UOSAT team. I would also thank my Head of Department, Professor K.G.Stephens who granted me sabbatical leave at a crucial time, and helped to minimise my administrative and teaching load in order that I could find the time to work on this challenging project. And I should mention the extreme patience shown, and general encouragement given by my wife.

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2 DESCRIPTION OF UOSATS

This chapter looks at the engineering of the satellites in those technical aspects which affect attitude determination and control. The chapter ends with a brief description of the general problems and strategies, and the requirements which the attitude determination and control systems must meet.

This is a personal account of the engineering of the two satellites UOSAT-1 and UOSAT-2. The emphasis is always on the overall relevance to ADCS (attitude determination control and stabilisation). Comprehensive reference information is provided in UOSAT-IERE (1982) and UOSAT-IERE (1987).

2.1 General Systems

The satellites are described in terms of their internal engineering, with emphasis on the telemetry, on-board computer and telecommand systems.

The two UOSAT spacecraft are broadly similar in appearance. They are small, rectangular objects with hardware attached to their outsides - in particular they have antennas. UOSAT-1 weighed in at 52 kgm. UOSAT-2 was heavier at 61 kgm but smaller for the simple reason that the Delta rocket's 'piggy-back' ride offered less space. The attitude of both spacecraft was designed to be stabilised by gravity gradient - a natural effect which makes a long thin body in orbit tend to point up/down. A boom with a tip mass (shown in fig.2.1-1) needs to be deployed to effect this change in geometry.

The satellites are physically small but electronically sophisticated, and able to perform many tasks. For communications the satellites operate within the frequency allocations of the Amateur Satellite Service at v.h.f and u.h.f. UOSAT-2 also has an s.h.f allocation.

The key electronic facility within both spacecraft is an *on-board computer* (OBC), based on an RCA 1802 microprocessor, one of whose roles is to collect data generated internally by the spacecraft systems. Another role is to issue commands to these systems; it is also able to perform general signal processing. The OBC may also generate data of widely varying kinds for transmission over the down-link.

The satellites are designed to accept certain types of up-link or 'ground data'. This divides into two categories: first there are specific encoded commands to the spacecraft. For example a command 'deploy the boom' has a specific code which is recognised as such. The other category of data is highly flexible, being intended for processing by the on-board computer or other 'intelligent' sub-systems. Such data would therefore be part of, or constitute a wholly new program. It could also consist of updating parameters to change the configuration of an existing program. The operating routines of the computer and therefore of the whole spacecraft can be totally re-configured whilst in orbit.

The UOSAT spacecraft have an unusual provision for separate, autonomous signal-creating and processing systems with independent access to the up-links and down-links.

An overview of the two spacecraft follows. A fuller description is provided in sec.2.1.2 where the detail is strictly accurate only for UOSAT-2. Significant differences for UOSAT-1 will be mentioned.

2.1.1 Overview of UOSAT-1 systems

Fig.2.1.1-1 is a way into understanding UOSAT-1. At the top left are depicted the up-link command receivers which accept transmissions in the v.h.f and u.h.f. Command data is input at 1200 b.p.s in a simple form of a.f.s.k coding with frequency modulation (a.f.s.k/fm). Each basic receiver employs a single i.f down-conversion stage, a frequency discriminator, a filter and a decoder. The decoded data's main route is through a *code check module* to the *command decoder* (telecommand).

Sharing the same antennas as the receivers, the down-link transmitters are represented as the *general data beacon* (v.h.f) and *engineering data beacon* (u.h.f). A number of coding/modulation formats are available in principle but in fact the same a.f.s.k/fm scheme as used on the up-links is the one usually adopted in practice (NOTE).

All up-link data and all down-link data is semi-synchronous, meaning that the flow of data on a radio link is transmitted at a constant rate. But in its structure it consists of standard serial words, according to the ASCII convention, with start bit, a parity bit and a stop bit for every 7 bit word. The serial data links within the spacecraft can therefore adopt standard UARTs (Universal Asynchronous Receivers and Transmitters) in order to interface with the internal systems.

The OBC has a dominating role in the satellite but with many important functions sub-contracted to other systems - in particular the majority of measurements are by the *telemetry encoder*. The satellite contains various experiments whose data outputs can be read by the OBC, or measured by the telemetry system. My interest for measurement is in the *magnetometers*, and there is a choice of two. My control interest is in the *gravity gradient boom mechanism* ; and the *X-axis magnetorquer coil*.

2.1.2 Overview of UOSAT-2 systems

The UOSAT-2 systems are designed according to the same general principles - while containing a number of significant improvements and modifications, all of them suggested by flight experience with UOSAT-1.

NOTE In this simple modulation scheme a data '1' encodes one cycle and a data '0' encodes two cycles of a sub-carrier (or the other way round). The sub-carrier then frequency modulates the actual carrier. The technique is not efficient in use of carrier power and it is also expensive in bandwidth. However it has the advantage of supporting very simple receivers and decoders - and is also tolerant to poor phase/amplitude control which may be encountered in the receivers of some UOSAT users.

At the middle top of fig.2.1.2-1 are shown the up-link command receivers - *CMD RX(1)* to *CMD RX(3)* which accept transmissions in the v.h.f, u.h.f and s.h.f bands. Ground data is input at 1200 b.p.s again encoded and modulated according to the a.f.s.k/fm scheme. The decoded data's main route is to the *telecommand system*.

The down-link transmitters are represented at the top right as *down-link(1)* to *down link(3)*, operating in the same range of frequency bands. A number of modulation/coding formats are available in principle but in fact the same afsk/fm as used on the up-links is the one usually adopted in practice, and usually at 1200 bps (with an option to run at other rates, principally at 4800 b.p.s). The data structure is again semi-synchronous.

As in UOSAT-1, the OBC has a dominating role but with many important measurement and control functions 'contracted out'. This satellite also runs various experiments (bottom left) whose outputs can be collected and routed to the transmitters. UOSAT-2 carries additional 'intelligence' and much more independent solid state memory. The *memory experiment* or 'digital store and read out' (DSR) has been useful in ADCS operations. Also of significance is the *digital communications experiment* (DCE at the bottom middle) whose NSC800 computer can double as the stand-by computer in case of failure of the main OBC.

Again ^(C)my interest concerns the ADCS system shown to the middle right: the 3-axis *magnetometer*. The *horizon sensor* and *sun angle sensors* are an experimental package and their measurements would have been useful. Unfortunately they have not functioned properly - hence forcing the concentration in my work on attitude determination purely from magnetic data.

My control interest is the 3-axes *magnetorquer*, and the *gravity-gradient boom drive* (middle right). These are again the sole attitude-controlling sub-systems.

A more detailed description of the non-ADCS systems and their interconnections follows. See also fig.2.1.2-2.

Telecommand system: This is the primary point of access into the spacecraft. It is a hard-wired logical system which accepts three inputs from the receivers plus three 'data present' indications. It also accepts two inputs from the serial network (see also fig.2.1.2-5). Note also the 'by-passes' to the serial network from the receivers. The telecommand's main job is *command verification*. The serial input is converted by an internal UART onto a parallel bus for analysis of 6 consecutive words which embody a direct or 'manual' ground command. This sequence if valid embodies a recognition preamble and then the actual command code. Such a command is passed to 112 bi-state *command latches* which directly connect to the spacecraft sub-systems. A separate *ground data bus* allows an uninterrupted and unverified data sequence to be routed direct to the OBC. A partial list of telecommands which concern ADCS is given in fig.2.1.2-3.

OBC: The heart of the OBC is an RCA 1802 microprocessor which is clocked at the slow rate of approx 1.3 MHz. Its main memory consists of 12 16K * 1 dynamic RAM chips which provide 16 Kbytes storage with Hamming code protection. In addition a further 32K bytes of unprotected RAM is accessible. From the point of view of my work the OBC is a multi-port device with controlled communication in and out to selected peripherals. Details of control lines (from the command decoder) and the practical architecture of the OBC do not concern my account here.

Telemetry: The telemetry has a dedicated measurement function. It has no microprocessor, being 'hard-wired'. It contains an analogue to digital convertor (ADCS) which samples sequentially 60 analogue inputs.

In its normal mode the telemetry generates a standard frame of data for transmission (when selected) over a down-link. The first line of this frame consists of an identifier 'UOSAT2' followed by a useful calendar and clock (resetable by the OBC). The data then follows, identified by

channel numbers (fig.2.1.2-4a). The first 60 channels embody the analogue measurements scaled to the range from 000 to 999 as *characters* using the ASCII convention. These are followed by a further 10 'digital' or status channels (60 to 67 being active), giving a total of 70 channels. The digital channels encode status information in hexadecimal form, giving 12 bits per channel or 96 bits in all. In conformity with the serial signalling convention the numbers '1' through '9' and the letters 'A' through 'F' are encoded as ASCII words. UOSAT-2 also provide automatic error protection on each channel (something which *my* work discovered was essential). Fig.2.1.2-4b shows the telemetry frame with an encoded check sum terminating each channel. The UOSAT-1 telemetry frame is very similar - but having less comprehensive status indications. The time to produce a whole frame of data is about 5 seconds.

Serial network: The left hand side of fig.2.1.2-2 shows the interface of serial lines in and out of what amounts to a general communication system. Note the access to the DCE and DSR experiments. Fig.2.1.2-5 gives the details, showing the possible connections under control of multiplexers. The OBC and the DCE control the multiplexers into their own UARTS. UOSAT-1 had a more rudimentary system of serial connections.

2.1.3 Design principles and system functions

I was not involved directly in the design of either UOSAT-1 or major design aspects of UOSAT-2 (other than specifying what I wanted for the ADCS systems). But *my* operational experiences with the ADCS of UOSAT-1, and the experiences of the UOSAT team, led to significant, technical improvements for UOSAT-2. And it is important to convey the operational constraints and background to the ADCS work. Hence this brief section.

The overall organisation of both UOSATs can be understood by the general principle of not putting all one's eggs in the same basket. For example the OBC may gather data, run algorithms and issue commands; but it can always be over-ridden by ground commands. The separate hard-wired

telemetry system is the principle measurement tool - with its built-in ADCS. Its existence might not be thought necessary given an OBC which could have controlled its own ADCS and performed its own measurements. But being dedicated to this task the telemetry is intrinsically more reliable and, in the event of a computer failure, data would still be gathered and measurements made. It also relieves the OBC of a significant signal-processing load. Also a second computer (the NSC-800 in the DCE experiment) is available as a back-up (NOTE).

The UDSAT team under MNS have designed this spacecraft according to the principle of *redundancy by alternative technologies*. This means that if something fails there should be another way of performing the task but not in the same way and not using the same components.

A consequence of this principle is that everything should be connectable to everything else. The OBC and telemetry are permanently connected to each other - and other systems - by parallel data buses. Of equal engineering significance are the serial communications and various data multiplexers (distributed around the sub-systems) which allow a flexible and variable mix of connections.

Fig.2.1.2-2 presented a simplified overall block diagram. Inputs to the system are at the top right and outputs at the top left. Either ground commands may be verified (1), after reception over the radio link; or alternatively the OBC may generate these commands (2). Whatever the source the commands reach a selected spacecraft system via a *command data bus* and latches (3). The telemetry can read: whatever the computer is programmed to send it (4A); a sub-set of the command latches in a collection of inputs collectively labelled the *command status bus* (4B); other digital 'status' information (4C); and a sequence of analogue measurements, which are converted by its internal ADCS (4D). With all this data it generates the 70 channels of information, which it outputs serially (5).

NOTE UDSAT-1 had an early bipolar F100-L microprocessor as back-up but the on-board VLSI RAM failed after 9 months in orbit.

For UOSAT-2 the telemetry can be completely reconfigured to select a limited number of specified channels, and in any order, by instructions received through a serial port (6). Those instructions emanate from the serial network and can originate therefore from the ground control as 'ground data' or as a request from the computer. This reconfigured mode allows much quicker access to selected data sources, or proportionally faster rate of making a restricted range of measurements.

The all-important magnetometer is accessible in two different ways. Its own ADCS generates privileged access to the OBC (7); but it also is measured routinely by the telemetry in all three channels.

Whatever the telemetry has measured the computer can also know by 'reading' the current telemetry state through a parallel port (8). Therefore the OBC can be programmed to become a telemetry system and generate identical or processed serial data for transmission over selected down-link. For this purpose it has output serial ports (UART1 and UART2) to which the transmitters may be connected rather than the 'real' telemetry system. As an example on UOSAT-1 the OBC is now programmed to compute and add in the check sum , using the parallel port facility, for transmission of a modified telemetry frame.

The OBC may monitor verified ground commands from a *command sense bus* (9). But it also has direct access to an (unverified) ground data through a separate port (10) via a *ground data bus*. This allows it to load software or accept *command packets* - modifications or control blocks to existing software. Exactly the same kind of data can be made available via serial connections (11) from the telecommand system to either UART1 or UART2.

The two UARTS effectively convert the OBC into a 'modem' with respect to the serial network. This feature was brilliantly successful in overcoming an otherwise near-fatal operational failure. A failure of the 'signal-present' circuits from the v.h.f receiver into the telecommand system has disabled that up-link's main point of access (point X). The serial network sets up an indirect alternative route. Any direct ground

command on v.h.f is permanently directed at point (11) into the network and on to the OBC. The latter 'echoes' it back out again into the network and returns to the telecommand system (12), so completing the connection.

2.1.4 Software

The software plays a critical role in all spacecraft operations. The main OBCs of both spacecraft were essentially 'tabula rasa' - blank memory with no prior intelligence. All operational software was designed post-launch and incorporated, with increasing refinements, over the years of operation. Exceptionally the UOSAT-2's OBC had a built-in ROM 'loader' software to enable error-free loading of software from the ground.

Early work was developed by the designer of the computer (Peel from p 137 of UOSAT-IERE 1987) which allowed operations upto and including boom deployment (see section 2.5.1). Later work has benefited from the development of a multi-tasking, concurrent 'Diary' (Holder from p. 143 ibid). This software allows the spacecraft to perform a large number of tasks in addition to ADC or in parallel with it. The Diary monitors up-link commands, controls the collection and distribution of telemetry, generates bulletins, runs ADC operations, collects data for later dump (whole orbit data = WOD) and many other tasks.

Fig.2.1-1 UOSAT-2 perspective

Fig. 2.1.2-1 Block diagram UDSAT-2 systems

Fig.2.1.2-2 UOSAT-2 internal connections

Fig.2.1.2-3 Partial list of telecommands

Telecommand command number	sub-system		status no.
16	g.g caging pyros	safe/arm	13
17	"	hold/fire	14
18	g.g boom deploy	safe/arm	15
19	"	hold/display	16
20	"	extend/retract	17
21	all magnetorquer	safe/arm	18
22	X magnetorquer	off/on	19
23	Y magnetorquer	off/on	20
24	Z magnetorquer	off/on	21
25	all magnetorquer	forward/reverse	22
26	"	low/high	25
27	sun & horizon	on/off	

Fig.2.1.2-4a UOSAT-2 telemetry (no error correction)

```

UOSAT-2      8405174112923
00380 01370 02661 03481 04059 05046 06027 07056 08040 09033
10512 11357 12000 13089 14000 15000 16000 17487 18467 19572
20523 21061 22659 23000 24000 25000 26104 27483 28600 29544
30435 31040 32283 33000 34000 35378 36434 37460 38529 39539
40855 41000 42674 43000 44171 45001 46000 47528 48541 49509
50569 51073 52702 53295 54990 55000 56000 57535 58532 59535
60800 615BC 62800 63024 64000 65000 66C00 67000 68000 69000

```

Fig.2.1.2-4b UOSAT-2 telemetry (with error correction)

```

UOSAT-2      8405174112923
00380B01370502661303481E04059805046706027307056408040C090339
10512711357112000313089314000515000416000717487D18467C195728
20523621061422659A23000124000625000726104127483A28600C29544E
30435131040632283833000034000735378A364346374606385295395395
40855C41000542674343000744171745001046000247528C48541C495091
50569F51073052702253295854990155000056000357535158532959535F
60800E615BC562800C63024364000265000366C00C67000168000E69000F

```

Fig.2.1.2-5 Serial communication net

2.2 ADCS systems

The systems directly concerned with ADCS are the gravity-gradient boom mechanism, the magnetorquer and the magnetometer. Engineering and operational details are given

2.2.1 Gravity gradient boom

At some point in the manoeuvres it was necessary to extend a mass on the end of a boom and thus invoke the stabilising effect of gravity gradient. UOSAT-1 has a 'tip' mass of 2.5 kg for this purpose which can be deployed a maximum distance of 13.7 metres at a rate of 49.8 cm/min. For UOSAT-2 the tip mass is 2.6 kgm and the maximum deployment distance is the same. The measured deployment rate is 46.2 cm/min.

The mechanism consists of a small motor with gear train unit largely filled with a pre-formed beryllium-copper strip wound onto a drum (photograph in fig.2.2.1-1). The tip mass (not shown in the figure) is attached to the free end and was attached also to the top facet of the spacecraft during launch. Many orbits later, when the attitude was correct, two explosive bolt cutters/pin-pullers were fired by ground command , releasing the tip mass from its secure fixing to the spacecraft top facet and leaving it attached just to the strip/tube. On a further command this tip mass was pushed out from the spacecraft body. The strip automatically sprang into a tube as the motor unit started unwinding it from the drum.

The tip mass on UOSAT-1 in fact is also an instrument - a 'scientific' precision 3-axis magnetometer. The reason for mounting it there was obviously to put it at a maximum distance from the spacecraft and therefore experience the least error from stray magnetic fields. It turned out to be an unsound decision, because the cables, which had to trail out after it as the boom deployed, snagged (see chapter 6). Benefitting from this experience, UOSAT-2 had only a simple tip mass - no deployed 3-axis magnetometer and therefore no trailing cables.

2.2.2 Magnetometers

The main attitude-determining sensor is a 3-axis fluxgate magnetometer. UOSAT-1 carries two : the one contained within the deployed tip mass and deemed 'scientific' and the other mounted on one of the 'wings' of the craft which was deemed 'navigational'. UOSAT-2 carries only a navigational magnetometer similarly mounted. Full technical details of UOSAT-1 scientific magnetometer are given by Acuna in UOSAT-IERE (1982).

UOSAT-1 'scientific' The flux-gate magnetometer is inherently an analogue device. In this instrument Acuna created a semi-digital measurement system. A high sensitivity in each channel limits the dynamic range to a nominal ± 8000 nT. This is converted to a 12 bit 'fine code', counting from 0 to 4095 and giving a nominal resolution of 4 nanotesla. Switching in known increments of a locally generated magnetic field component then offsets the real geomagnetic field and allows tracking over a much wider range. Sixteen increments are provided, each of 8000 nT, extending therefore the range of the device over a nominal ± 64000 nT in each channel. Encoding these increments to a 4-bit 'coarse' code and the analogue measurement to a 12-bit 'fine' code then creates 16 bits per channel. This same information is made available on the telemetry but necessarily to a more limited accuracy and resolution.

UOSAT-1 'navigation' The same physical system of measurement provides a single analogue output in each channel which is read just by the telemetry system's ADCS. A gross offset in the Y channel and severe drift of offset in the others limits its usefulness to a qualitative check on the scientific magnetometer. The problem was later attributed to the residual magnetic moment of the VHF/UHF communication antenna strips.

UOSAT-2. This is an improved version of the UOSAT-1 navigation magnetometer, with its analogue outputs available both to the telemetry system and also converted by its own 14-bit ADC direct to the double port on the OBC. The ADC, by Intersil has sufficient dynamic range and

resolution to be able to eliminate the need for the offsetting arrangement used in UOSAT-1. It has a range of ± 50 micro Tesla, and a LSB resolution of approx 8 nanotesla, in each component channel.

This 'navmag' generates measurements of the three field components 10 times a second. This information is passed to the computer as 40 readings per second, cycling between three channels and a temperature reading. The data is in fact digitised to a 16-bit code, with an additional 2 bits identifying which channel is present at any one time.

2.2.3 Magnetorquers

Active attitude control is achieved by 'magnetorquing' - the switching of a current ON and OFF through a coil of wire and creating a magnetic moment which interacts with the geomagnetic field.

UOSAT-1. There are two parallel coils wired into the edges of the +X and -X panels, of 150 turns and dimensions of approximately 425 mm by 425 mm. The resistance in each side of 24 ohms. The two coils were connected in parallel. The nominal power supply voltage of 14V. Defining the Z axis as the natural 'spin' axis of the satellite then this configuration defines a *spin plane magnetorquer*. The windings were contained with the main frame of the spacecraft

UOSAT-2 Based on our experiences with UOSAT-2 it was clear that all directional components of magnetic moment should be available. In addition to the pair of X-axis spin-plane coils it has a matching pair of Y-axis spin-plane coils. These were all wound, unlike UOSAT-1, within a convenient ducts created in the edges of the solar panels. Describing any one of the four spin-plane coils, each consists of 100 turns of Kynar insulated 30 A.W.G wire within a dimension of 537 mm by 337 mm. After installation each coil had a measured resistance of 50 ohms.

Each pair of coils could be connected in either series or parallel. With a power supply voltage of 14V the total current is either 0.56 amps or 0.14 amps.

Additionally a *spin axis magnetorquer* was specified . This consists of two coils wound into the edges of the top and bottom faces (+Z and -Z faces), whose magnetic moment - when a current is passed - is directed along the Z axis. Each of these coils contains 200 turns and have dimensions of approximately 337 mm by 337 mm and a resistance in each of 78 ohms. Only one of these coils is operational, taking a current of 0.18A.

✓ The general configuration of all these magnetorquers is shown in Fig.2.2.3-1. The polarity of all torquers can be set simultaneously and they are all made safe or armed simultaneously. Only one magnetorquer can be switched on at a time.

2.3 Ground Station

The facilities of the ground station are introduced.

The ground station used for mission control is in the Department of Electronic and Electrical Engineering with access to the tracking antennas mounted on the roof almost immediately above.

The station has undergone a number of metamorphoses; its latest form, dating from the summer 1986 is now a highly sophisticated tool. It consists of transmitting and receiving antennas, for operation at v.h.f and u.h.f, which are always steered under computer control in the course of a satellite pass. The station consists of computer controllable receivers and transmitters for these frequencies. As fig.2.3.-1 shows, the control and data-processing is performed by a bank of inexpensive microcomputers (BBC model B and BBC Master) with associated VDUs

(centre). These are linked together by a station data multiplexer co-ordinated by a master microcomputer. A myriad of connections are possible under software control. For routine operation the station can 'run itself' entirely under unattended software control. Orbits are anticipated - receivers set to the correct frequency, the antennas tracked, recorders switched on and off, and many other functions implemented automatically.

The far left of the photograph shows an archiving facility. In clock-wise rotation there are the receivers and transmitters, the 14-screen computer and VDU system; then the tape-recorder, a printer and several other facilities.

Three methods of attitude control of the satellite can be identified (see section 2.5). In the *ground-based control* or *semi-autonomous* spacecraft control, one of the groundstation computers can run a program (mixture of BASIC and machine code). A down-link receiver collects the data from the satellite, routes it to the groundstation computer, which makes the decisions, the results of which are multiplexed to the selected up-link transmitter and directed back to the satellite. A complete round-trip. For *autonomous* control an algorithm is now incorporated in the DBC software and the ground command is limited to setting operating parameters and starting the sequence (see sec.2.1.4).

2.4 Orbital description

A brief summary is given off the orbital conditions common to both UDSATs

The UDSAT orbits are of a particular type: they are low-altitude (500 -700 km), on a circular, near-polar orbit, and sun-synchronous.

The period of UOSAT-1 in March 1986 was 94.2 minutes, implying a mean height of 481.3 km, while UOSAT-2 had a longer orbital period of 98.49 minutes, implying a mean height calculated at 690 km. The satellites do not cross the poles exactly but pass close to them. The orbit configuration is simply described for a circular orbit by the tilt of the orbital plane. Looking down on Earth over the equator and along the edge of this plane the inclination is the angle of this plane to the equator and is about 98° in both cases. This makes the orbits *retrograde*.

The satellites traverse the world between the bounds of maximum and minimum geocentric latitudes of about $\pm 82^\circ$. The orbital plane of each will intersect most of the surface of the Earth every 12 hours as the Earth spins beneath it. Each satellite lies on its own orbit circle orbiting in a time which is unsynchronised to the Earth's spin. Each satellite is visible (to radio) anywhere in the world at some time even at the extremes of the geographic North and South poles. To the user at the latitude of Surrey, the satellites appear irregularly in a cluster of sequential passes. There is alternately a cluster of 2 or 3 passes from North to South and then 12 hours later (measured from the nominal middle) a cluster of passes from South to North. The useful command/data acquisition time within a pass is about 12-15 minutes.

However north of about 76° N (or south of 76° S) the satellite is visible for a time on *every* orbit.

It may be noted that 'visibility' can be considered to be the same at optical frequencies since our choice of frequencies makes communication necessarily line-of-sight at u.h.f and nearly line-of-sight at v.h.f.

The meaning of 'sun-synchronous' in describing an orbit is that the plane of the orbit is not fixed in inertial space but precesses at the same rate as the apparent motion of the *mean sun* in the celestial sphere. Hence whenever the satellite crosses the equator the local time at that instant is nominally a constant. The time on the ascending node of UOSAT-1 was a nominal 3 p.m and for UOSAT-2 was a nominal 9 p.m.

For most latitudes I have stated that the spacecraft are not seen every pass but in clusters every 12 hours. Because of sun synchronicity these visible passes tend to occur at around the same local (Sun) times. For UOSAT-1 they occurred in the vicinity of 3 a.m and 3 p.m while for UOSAT-2 they occur in the vicinity of 9 a.m and 9 p.m. In the years since launch the timing of UOSAT-1 has very considerably drifted due to loss of sun-synchronism, caused by a decay in the orbital height due to atmospheric drag.

2.5 Control Facilities and Requirements

I summarise as briefly as possible the facilities or modes of operation which the combination of satellite and ground station can support. This leads to defining the general ADCS requirements at the various stages after separation from the launch vehicle

After separation from the Delta rocket the satellite in each case ^{was} were thrown free by the apparently simple device of pyrotechnical cutting of a bolt which released a clamp and allowed a spring to eject it out.

It might be expected that the angular motion after launch would be a tumbling one: this is so but complicated in the case of UOSAT-1 by a fairly rapid spinning action, which ~~action~~ was attributed to a torsion in the compressed spring. It was from this point that the satellite comes into the purview of ADCS operations. First however I state what can be done technically in the context provided by the combination of satellites and ground station.

2.5.1 ADCS provisions

As in more orthodox control systems there has to be source of information, decisions made on the basis of this information in the

light of a specified plan or algorithm, and the decisions enacted by the appropriate sub-system.

The most common ADCS requirement reduces to the selection of a magnetorquing algorithm and the running of it. In the course of the manoeuvre a magnetometer is being read and a magnetorquer being controlled. The purpose behind the magnetorquing control varies depending on the stage reached in the mission. Complicated manoeuvres were required during deployment of the boom (described from chapter 6). Also routine commands enacting specific transmitters and receivers, data rates, run times form an essential background to the specific attitude determination and control.

Four distinct modes or ways of 'doing ADCS ' can be identified:

Ground based ADCS: In this mode data is collected, and mixed in with other data as part of a general telemetry package routed onto a down link. At the ground station a computer can be running automatically and with human assistance command decisions are made and a command code is routed onto an up-link. Provided it is supported by the link budget the command is immediately issued to activate the magnetorquer (or whatever). Necessarily this mode is limited to such times when the spacecraft is in view.

Semi-autonomous ADCS: In this mode the OBC is used to mediate between a direct command and its implementation. For example a decision to switch on the magnetorquer for (say) 30 seconds would be transmitted over the up-link and interpreted by a program running on the OBC. A magnetorquer would fire immediately ; however it would then be switched off after 30 seconds without any further intercession from the ground. The important command to deploy the boom was made this way. On careful checking that the time and attitude were right a command was sent to start deploying boom during a pass. After 15 minutes the computer automatically shut it down. After that time of course the spacecraft would be out of sight and a ground command would have been impossible.

Autonomous ADCS: In this mode a ground control command 'starts up' the OBC and thereafter the OBC makes all the decisions itself. Magnetometer and other data are routed directly and processed within the OBC according to the rules of the current algorithm; and the commands issued. The computer can be stopped by direct intercession from the ground during a pass or by reading an internal clock. Normally the routine is embedded within the Diary and is shut down automatically after a passage of hours or days.

Whole Orbit Dump In a pure attitude-determining state the satellite is commanded to start collecting magnetometer data, which it will do for hours or even days. The information is then 'dumped' on command in one or two successive passes and collected in the archive computer for later off-line processing. It is an essential tool for overcoming the limitations of having the satellite in view for only a few minutes on any one pass.

A typical configuration in which the computer is used for *magnetorquing* is shown in fig.2.5.1-1. A *control packet* received on one UART is recognised by previously loaded software and starts a magnetorquing sequence (the packet can contain information such as the number of firings and overall duration of the test etc). As the algorithm runs the OBC monitors the navigation magnetometer, selected telemetry channels (through that parallel port) and, when it wishes, drives the command bus with a code selecting and turning on/off specific magnetorquers. Real-time 'status' messages, designed to be read by the operators at the ground station, are output through UART2 onto a selected down -link. The operation is logged (in memory) and at a later date the result are 'whole-orbit dumped ' out through a UART and again onto a selected down link.

Facilities with UOSAT-1 were more restricted. Difficulties with the command port restricted operations to ground-based ADCS. Up-link commands were direct to the space-craft systems. The OBC software was initially restricted to reading the navigational magnetometer through the special port and directing this data to a down-link. Early

operations were very time-consuming since confirmation that commands had been accepted by the magnetorquer was not provided. At a late stage the programming was developed sufficiently to allow selected reading of the magnetorquer status points via the telemetry port.

Ground-based ADCS naturally requires full DUPLEX communication. UOSAT-1 could only support full duplex by use of the widely separated v.h.f and u.h.f bands. But the u.h.f link provided a poor service - particularly when most needed with the spacecraft tumbling or spinning in non-nominal orientations.

UOSAT-2 performed much better in all respects allowing full DUPLEX operation at v.h.f, if needed, as well as having entirely reliable computer-based semi-autonomous and autonomous control. Even so ground-based control was generally found to be an essential preliminary to testing the state of spacecraft systems and the general feasibility of the various algorithms

2.5.2 ADCS requirements

The polar orbit is vital to ADCS operations. The fact that the spacecraft passes over or close to a geomagnetic pole twice every orbit has been exploited in all manner of ways for both attitude determination and control.

Once in a state of a free angular motion the the satellites were in a state of *nutation* - being a combination of two spins in an (almost) torque-free environment. How were these motions observed? Not very easily is the answer. With only the magnetometer measurement to go by an initial problem was to interpret the motion from the complex waveforms induced as the spacecraft tumbles in the geomagnetic field - which is itself a variable throughout the orbit of course and changes significantly in the space of minutes (see chapter 4).

The general aim was to kill the tumbling motion, re-orientate the spacecraft as necessary and then deploy a mass on the end of a boom to invoke the gravity gradient effect and achieve the desired attitude with the camera and antennas pointing Earthwards. The way to suppress these motions and orientate the spacecraft was to switch on one or more magnetorquers (but remember UOSAT-1 had only one) in the right commutative relationship to the local geomagnetic field. The desired orientation during boom deployment was to have the boom aligned at an angle close to the local vertical. If the angle were too large and/or the angular motion was excessive then boom deployment could not be expected to be successful. Entirely different methods of slowing down and pointing the UOSATs were adopted to get them into this state.

UOSAT-1 had first to be maintained in a spin-stabilised mode with its spin axis pointing in the direction of the orbit normal. Its spin axis (Z-axis) had then to be slewed through practically 90° to get it pointing at a shallow angle to the orbital plane. The manoeuvres required spin rate control, active denutation, and forced precession; all accomplished with the same spin-plane magnetorquer.

With UOSAT-2 life was much easier and it was turned effectively into a kind of compass needle by active magnetorquing with the *spin-axis magnetorquer*, always tracking the geomagnetic field. Boom deployment could be initiated very conveniently on a North to South pass in view from Surrey.

Can this be estimated? ✓

The gravity gradient effect is extremely weak and the boom could only be deployed within a certain attitude window if the gravity lock/capture were to be successful. The difficulties were compounded by the long deployment time of the boom. If the manoeuvre had not been successful then the satellite would have continued to tumble. After deployment there was expected to be a residual oscillation, technically known as a *libration*, which indeed occurred. With UOSAT-1 the boom only went out a short way before a mechanical malfunction, but gravity-gradient capture was observed for a short term.

With UOSAT-2 all went smoothly. Figs.2.5.2-1 are diagrams of UOSAT-2 orientation in the critical stages of boom deployment. The critical feature was to have UOAT2's main Z-axis locked to the geomagnetic field - and known to have been locked under algorithmic control for many orbits previously. A detailed study then showed that the orientation of the field, when command to release the boom was initiated, is about ideal for subsequent capture by gravity gradient (see sec.3.6).

Following capture there followed much work, the theoretical and practical aspects of which fills much of this thesis. One problem was how to determine the attitude from the limited information given by the magnetometer. This has been solved in two different ways (sect.4.4 and chapter 7). The other was how to damp, assist or cause to interact the magnetorquing function with the gravity gradient control in order to damp out this libration. That work is the other main concern of this thesis.

It may be noted that attitude of UOSATs are normally thought of the type which is termed '2-axis stabilised' in the literature. With respect to an orbit-defined co-ordinate system (see section 3.1) control of libration is equivalent to control of rotations around orbit-defined roll and pitch axes. But with their near-symmetry in X and Y it is not normally thought possible to prevent a slow drift in rotation around the Z axis defined by the line of the boom.

That is indeed as far as we have got practically. But as chapter 7 shows there is the capability of imposing 3-axis control by complementary magnetorquing.

Fig.2.2.1-1 Photo of UOSAT-1 boom

Fig.2.2.3-1 Magnetorquer geometry

Magnetorquers spin plane (X and Y) mag moment 10
 spin axis (Z) mag moment 4

Fig.2.5.1-1 Autonomous ADCS

Fig.2.3-1 Ground station

Fig.2.5.2-1 Capture sequence

(a)

(b)

3 PASSIVE ATTITUDE STATES & CONTROL

The attitude states of UOSAT-1 and UOSAT-2 are studied here in the conditions when these satellites are *not* being actively controlled. The topics are

- * attitude description and conventions
 - * nutation - the combinations of two spins in a torque-free environment
 - * nature of gravity-gradient torque
 - * precession induced on a spinning/nutating body induced by gravity gradient
 - * attitude equations and bounds under gravity-gradient capture with simple theory of boom extension
 - * full differential equations for boom extension and libration under capture by gravity gradient
 - * simplified differential equations
-

UOSAT is idealised as a rigid body with a common moment of inertia I_T about its X and Y axes. Thus

$$I_T = I_X = I_Y$$

....3.1

The moment of inertia about its Z axis is symbolised I_Z . The satellite is on an ideal circular orbit over a spherical earth.

The results given in this chapter are a mix of the new and well-known. Nutation is too well-known to need an extended description; but the fundamental facts and equations are summarised here in section 3.2, partly for completeness, but also to establish a notation, for comparison with other attitude states, and as a preliminary to attitude interpretation from magnetometer measurements (in section 4.2).

After summarising the well-known formulae for gravity-gradient torque (section 3.3) the effect of this torque is considered in two different ways: one may seem minor but was important in understanding the motions of the satellites in an uncontrolled state. In such a state, and with the boom not deployed, the gravity- gradient torque cannot capture but will influence the spinning/nutating state. The gravity-gradient torque forces an additional slow *precession*. Section 3.4 gives the essential formulae. The effect is treated only cursorily in Wertz (1978), for a pure spinning body only, and the formula given with its conclusion is in any case wrong. A full treatment is given here which includes the general nutating case, which Wertz does not.

The nutation and precession effect had to be understood in the course of attitude control of UOSAT-1. The problem was by-passed in UOSAT-2 by employing magnetic stabilisation (see from section 5.2 and 6.2.6).

In the second, more important application gravity-gradient torque captures the attitude, provided the ratio I_T/I_z is sufficiently large (by boom deployment), and provided the rotational energy is sufficiently small. What was a residual Z-spin (ordinary spin) is not affected by boom deployment but has an effect on the average pointing of the attitude. And what was the 'T-spin' (transverse spin energy) does not 'disappear' on boom deployment and expresses itself as a *libration*, in which the Z-axis of the satellite experiences an undamped oscillation in both roll and pitch (see section 3.2 for an introduction to these terms).

The aim is to eliminate, or at least reduce the libration by active control (see from chapter 5). In this chapter however the only 'passive' effects are considered. Section 3.5 lists possibly unique equations which establish *bounds* on the librational state. The expected excursions in an idealised case of boom deployment are derived from this analysis. Section 3.6 summarises the differential equations used in a more exact prediction of attitude after boom deployment; and the numerical solution of these equations. This section is kept as brief as possible because it involves no new principles. Finally section 3.7

considers a simplified set of equations, the study of which is important to understanding the effect of active damping control (in chapter 5).

This chapter is supported by a detailed theoretical analysis (from Appendices A through D). The adopted mathematical method was an all-vectorial one in which the simplification, due to the equal moments of inertia about X and Y axes, was systematically exploited.

First however it is necessary to establish a convenient scheme for discussing the ' attitude ' of the spacecraft.

3.1 Attitude Description and Conventions

A reference co-ordinate system is described to which the attitude of UOSAT can be referred. Four angles are defined in order to describe the attitude of the main Z axis of the spacecraft: called here *pitch*, *roll*, *inclination* (or *libration*), and *tilt* angles. If the attitude of the X and Y axes also need to be defined, an Eulerian 1-2-3 system is adopted and a yaw angle established. An *attitude sphere* and *attitude circle* are defined, as means of plotting the attitude of the Z axis only

UOSAT-1 and UOSAT-2 have conventionally defined X,Y and Z 'body' axes. At its simplest the ' attitude of the spacecraft ' means a 1-axis attitude and refers to the orientation of the body Z axis. Sometimes it is necessary to refer to a full '3-axis attitude' definition which specifies also the orientation of the X and Y axes.

3.1.1 1-axis attitude definition

The co-ordinate system is defined by the centre of the Earth and the circular orbit of the spacecraft. A vector which is normal to the orbit plane in a positive (anti-clockwise) sense is the *orbit normal*. A line from the Earth centre passing through the spacecraft defines the local 'up' (i.e zenith). And a third axis in the direction of the velocity component completes a cartesian reference system (fig.3.1.1-1).

The next figure 3.1.1-2 labels the axes as 'pitch', 'yaw' and 'roll' respectively. Unit vectors in these respective directions $\underline{n}$, $\underline{r}$ and $\underline{y}$ are also defined.

None of these axes are invariant: the roll and yaw axes rotate once every orbit (approx 100 minutes) and in the plane of the orbit; while the pitch rotates once every year around a North-South axis because of the induced sun-synchronous orbit (NOTE).

Suppose that the Z axis did in fact lie in the plane of the orbit (fig.3.1.1-3) then the angle of the Z axis to the 'up' $\underline{r}$ vector as a *pitch angle* a_p may be defined, with a positive value as shown if the 'top' of the spacecraft inclines towards the $\underline{y}$ vector. The nominal attitude has a zero pitch angle. A spin-stabilised satellite which is maintaining inertial stability would be constantly decrementing its pitch through the orbit. It is convenient in the latter case to define an *inertial* or *absolute* pitch angle a_{p_i} which is the angle to some invariant radial vector passing through a constant reference latitude - for example over the North pole, or the equator, or the latitude of Guildford (UK) (fig.3.1.1-4). And distinguish the former definition as a 'local pitch'. In either type of pitch definition the range of angle will be defined between $-180^\circ + 180^\circ$. If the radial vector from which the absolute pitch is measured is directed to the ascending node of the orbit then the system is identical to the common 'l b n' system

When the spacecraft's attitude - the attitude of its Z-axis - is not lying in the orbital plane - the pitch angle a_p may be generalised to refer to the angle between the $\underline{r}$ vector and the projection $\underline{Z}'$ of the Z axis onto the orbital plane (fig.3.1.1-5). Then in describing the extent to which the Z-axis is inclined out of the orbit plane it is

NOTE With regret it proved very inconvenient to use the convention defined in Wertz (2.2.2). There the yaw axis is directed towards the nadir, not zenith, and the pitch axis is directed to the orbit anti-normal not normal

convenient to define a *tilt* angle δ - between the orbit normal vector $\underline{n}$ and the Z axis. The range of tilt angle is defined between 0° and $+180^\circ$. It is also useful to define an *inclination* or *libration* angle γ between the $\underline{r}$ vector and the Z-axis. And finally it is convenient to define a *roll* angle a_R which is related to the tilt angle by the simple equation

$$a_R = 90 - \delta$$

...3.1.1.1

and whose range is therefore between -90° and $+90^\circ$.

A nominally aligned spacecraft would have therefore a pitch angle of 0° , a tilt angle of 90° , an inclination of 0° , and a roll of 0° . The attitude is of course over-defined with all these definitions - but they are all useful. For example the inclination angle γ is a general measure of ability to maintain earth pointing, without seeking to distinguish whether its non-zero value is due to a roll or a pitch error. In the literature there seems to be no standardisation of these forms of specification; nor do I see how it is possible; the definitions and the terminology have to be suited to the job in hand (NOTE).

The *roll* definition is rather specialised. It should perhaps be called a 'generalised roll angle' since only for small angles of *pitch* does it correspond to the normal 'aeronautical' sense of a roll. With the roll angle increasing from zero the Z axis progressively heads towards alignment on the orbit normal (the X_0 axis),

No 'yaw' angle is defined here - this only has meaning if one invokes the X and Y axes. A nominally aligned UDSAT with a maintained Z-spin is in a continual state of 'yawing'.

NOTE Where there is no ambiguity or lack of clarity I use symbols 'p' and 'r' rather than a_p and a_R respectively.

3.1.2 3-axis attitude definition

An extension to the above allows a complete 3 axis specification in terms of Eulerian angles (a 1-2-3 rotational set) . It is convenient to define a system of reference axes X_0 , Y_0 , and Z_0 and associated unit vectors (fig.3.1.2-1).

$\underline{x}_0$: a unit vector with the direction of the 'orbit normal' i.e at right angles to the plane of the orbit with a right-handed sense. ($= \underline{n}$)

$\underline{y}_0$: a unit vector in the direction of the backward velocity (assuming a circular orbit) ($= -\underline{v}$)

$\underline{z}_0$: a unit vector directed away the earth centre. ($= \underline{r}$) system)

These unit vectors therefore define the direction of reference axes X_0 , Y_0 and Z_0 respectively.

The attitude of the spacecraft can be defined as ordered series of rotations. Imagining a satellite with its body axes (X , Y and Z) initially aligned on the reference axes X_0 , Y_0 and Z_0 , then the current attitude can be reached by an ordered series of rotations from this null position.

Reference to fig.3.1.2-2 shows first a rotation in *pitch* around the $\underline{x}_0$ vector by an amount α_p .This rotates the vectors (lower case corresponding to upper case axis)

$$\begin{array}{l} \underline{y}_0 \rightarrow \underline{m} \\ \underline{z}_0 \rightarrow \underline{p} \end{array}$$

...3.1.2.1

This is the same as the previous 1-axis definition - if the Z axis should lean towards the forward velocity vector that was counted as a positive pitch angle.

Then a rotation in *roll* around the $\underline{m}$ vector by an amount a_R rotates the axes

$$\begin{array}{l} \underline{x}_0 \rightarrow \underline{l} \\ \underline{e} \rightarrow \underline{z} \end{array}$$

...3.1.2.2

The angle a_R has been called a *roll* angle and is the same as the previous 1-axis definition. To characterise a full three-axis attitude requires a *yaw* rotation completes the description. A rotation in *yaw* around the $\underline{z}$ vector by an amount ϕ rotates the axes

$$\begin{array}{l} \underline{l} \rightarrow \underline{x} \\ \underline{m} \rightarrow \underline{y} \end{array}$$

...3.1.2.3

It should be noted that the rate of change $d\phi/dt$ is not identical with the absolute rate ω_z of the body spinning about its Z axis - the difference is characterised in section 3.7.2. The above Eulerian rotations (1-2-3) is not the standard set.

3.1.3 Attitude sphere and circle

A unit vector $\underline{z}$ pointing in the direction of the body Z axis describes an *attitude sphere* (fig 3.1.3-1). The vector $\underline{z}$ may be resolved into three components using the reference vectors already defined. Thus

$$\underline{z} = z_V \underline{v} + z_N \underline{n} + z_R \underline{r}$$

...3.1.3.1

And looking down on this sphere a projection in the plane defined by the $\underline{v}$ and $\underline{n}$ vectors defines an *attitude circle*, which is convenient for computer-plots (fig.3.1.3-2). The mathematical relation between the vector components and the attitude angles previously defined are shown on this figure.

3.1.4 Mathematical relations and summary

Summarising the various angular definitions and their ranges are

Angle	Symbol	Range °
pitch	p (or a_p)	0 to 360
roll	r (or a_r)	-90 to +90
libration / inclination	y	0 to 180
tilt	δ	0 to 180
yaw	ϕ	0 to 360

From simple spherical trigonometry (or by vector analysis) then

$$\cos(y) = \cos(p) \cos(r) \quad \dots 3.1.4.1a$$

$$\cos(\delta) = \sin(r) \quad \dots 3.1.4.1b$$

$$z_R = \cos(y) \quad \dots 3.1.4.2a$$

$$z_N = \cos(\delta) \quad \dots 3.1.4.2b$$

$$z_V = \sin(\delta) \cos(p) \quad \dots 3.1.4.2c$$

The angular pair (a_p , a_r) is un-ambiguous in specifying the direction of the Z axis) while the angular pair (y , δ) is ambiguous, denoting two possible directions, with the exception when $y = 0^\circ$.

3.1.5 Alternative attitude definition

An entirely different convention is useful for dealing with the spinning or nutating state (see section 3.2). In such a state the Z axis of the spacecraft is well-known to describe a cone in space. The average direction now defines a Z_0 axis and is not necessarily pointing in the local vertical direction. The reference system is further defined by pointing the Y_0 axis in the same plane as the Z_0 axis and the current $\underline{B}$ vector. Axis X_0 completes the orthogonal set. There should be no clash between this magnetic reference system and the orbit-defined reference system, despite using the same symbols (fig.3.1.5-1). The angle between Z_0 and $\underline{B}$ is defined here as χ . Orientation of the body axes X, Y and Z is most easily described by the standard Eulerian 3-1-3 set of rotations (angles ϕ , θ , and ψ).

3.1.1-1 Attitude conventions (i)

3.1.1-2 Attitude conventions (ii)

3.1.1-3 Attitude of pitch only

3.1.1-4 Absolute pitch convention

3.1.1-5 Attitude definitions

3.1.2-1 Related reference systems

3.1.2-2 Rotations in ref system

1-2-3.

3.1.3-1 Attitude Sphere

Point Z describes
locus on attitude
sphere

$$|z| = 1$$

3.1.3-2 Projected attitude circle

$$z_n = \cos(\delta)$$

$$z_v = \sin(\delta) \times \sin(p)$$

$$z_r = \sin(\delta) \times \cos(p)$$

3.1.5-1 Magnetic reference system

$$x_0 \propto \underline{B} \wedge \underline{z}_0$$

3.2 Nutation/spin Conventional Equations

The well-known state of nutation is formalised. The concept of 'Z-spin' and 'T-spin' are introduced in addition to *inertial nutation rate* and *body nutation rate*, finishing with a list of common formulae

Nutation is the well-known generally spinning state of a rigid body not subject to constraining forces or torques. It occurred naturally on the ejection of our two satellites from the launch vehicle and persisted until attitude control was initiated.

The nutation state depends on the ratio of the common moment of inertia I_T to the moment of inertia I_z . For UOSAT-2 the estimated values were $I_T = 2.1$ and $I_z = 1.1$ with the boom NOT deployed (with the boom deployed the value of I_T is greatly increased). For UOSAT-1 the ratio I_T/I_z was about the same and was re-estimated from measurements made during an extended nutational phase.

The angular motion of nutation in this restricted condition of symmetry is characterised by a simultaneous combination of two different spin states.

3.2.1 Kinematics

I have found it helpful to envisage a simple mechanical model of nutation. The model consists (fig.3.2.1-1) of a horizontal turntable which rotates at a constant velocity. The (vertical) rotational axis of the turntable is fixed in space and is termed here the Z_0 axis. Rigidly mounted above this table and directed to its centre is an oblique shaft which defines the Z-axis of the spacecraft.

The spacecraft rotates on this oblique shaft with an angular velocity *relative to the turntable* which is termed the *body nutation rate* ω_B ; while the turntable itself rotates at some *inertial nutation rate* ω_0 .

The angle between the vertical shaft and the oblique shaft is termed the *nutation angle* - notated here as angle θ . As it happens these three parameters are not independent - the dynamics equations permit only two degrees of freedom.

Supplanted

The only defect with this model is that the 'spacecraft' is being translated around with the turntable. This does not happen in practice since the centre of mass remains relatively stationary. In terms of the model, that would require a 'virtual' turntable with the spacecraft half-in and half-out of this level table.

Of course the interpretation of 'horizontal' and 'vertical' in the model is quite arbitrary and has no bearing on the actual orientation of the real space-craft in its orbit.

Thus the spacecraft rotates about its Z-axis; and this Z-axis itself describes a cone in space - maintaining the constant nutation angle to an average direction vector $\underline{z}_0$ (the 'vertical' on the model).

The angular velocity vector describing the complete motion of this body is the sum of these individual rate vectors: the inertial nutation rate vector having a constant magnitude in the $\underline{z}_0$ direction; and a body nutation rate having a constant magnitude but constantly changing in direction. Their vector sum, describing the instantaneous angular velocity of the spacecraft, also has a constant magnitude but also changes direction, while maintaining a constant angle to vector $\underline{z}_0$ (smaller than the nutation angle).

Fig.3.2.1-2 shows these vector spin components and also suggests a different decomposition of the total angular velocity: resolved along the Z axis it may be termed the Z-spin rate or 'body spin rate' ω_z ; and the orthogonal component, normal to the Z axis, as T-spin rate or transverse angular velocity (Wertz p 525) ω_T . The former may be visualised as the absolute spin of the body, additionally modified by a continual slewing due to the T-spin.

Not really, they are just hunting values

There are two important limiting cases : a zero nutation angle defines a case of simple or pure 'Z-spin' (fig.3.2.1-3a) ; and a nutation angle of 90° defines a case of 'flat' or T-spin' (fig.3.2.1-3b). In the former the the decomposition of the total angular velocity into component nutation rates is meaningless. In the latter case the spacecraft is simply tumbling, being whirled round at the T-spin rate, and with a spin axis pointing at some arbitrary direction in the plane defined by the body X and Y axes.

In a torque-free environment the vector $\underline{z}_0$, defined by the centre axis of symmetry of this nutational cone, is invariant, pointing in the same direction inherently as the angular momentum vector $\underline{L}$ of the body. It will not change unless there is an applied torque. In practice it does drift because there are various perturbing torques.

Both UOSATs were ejected in a state very close to a flat spin; because of certain operational problems and the very long time before any control could be implemented on UOSAT-1 this state had changed to one which was very close to a pure Z-spin. And it was from that state that control had to be implemented. By contrast UOSAT-2 was fairly quickly controlled from its initial flat-spin.

Kinematically the relation between the various angular velocities is simply

$$\omega_T = \omega_B \sin(\theta)$$

...3.2.1.1

$$\omega_Z = \omega_A + \omega_B \cos(\theta)$$

...3.2.1.2

3.2.2 Dynamics

Study of the dynamics shows that the $\underline{z}_0$ vector is the direction of the constant angular momentum $\underline{L}$; and there are only 2 degrees of freedom. Given the inertial nutation rate ω_B and the nutation angle θ and knowledge of the moments of inertia I_T and I_Z then it can be shown how the kinematic quantities are related and depend on the mechanical properties of the spacecraft

$\omega_B = (I_T/I_Z - 1) \omega_B \cos(\theta)$	...3.2.2.1
$\omega_Z = (I_T/I_Z) \omega_B \cos(\theta)$	...3.2.2.2
$L = \omega_B I_T$	...3.2.2.3
$L = \omega_Z I_Z / \cos(\theta)$	...3.2.2.4

In the above L is the magnitude of the angular momentum

Alternatively it may be useful to start with assumed values of ω_Z and ω_T plus the value of moments of inertia. Then express all the other parameters from these. By simple algebra

$\omega_B = (1 - I_Z/I_T) \omega_Z$	...3.2.2.5
$L = \sqrt{((\omega_Z I_Z)^2 + (\omega_T I_T)^2)}$	...3.2.2.6
$\omega_B = \sqrt{((\omega_Z I_Z/I_T)^2 + \omega_T^2)}$	...3.2.2.7
$\tan(\theta) = (\omega_T I_T) / (\omega_Z I_Z)$	...3.2.2.8

A consistent convention adopted in all numerical work is to treat ω_T and ω_B as positive quantities. In practice however the body Z spin can have either sign.

To allow both possible states then the range of nutation angle must be defined over the range

$0 < \theta < 90$	<i>forward spin</i>
$90 < \theta < 180$	<i>backward spin</i>

...3.2.2.9

For values of θ exceeding 90 degrees the z spin rate ω_z is negative. For the UOSATS $I_T > I_z$ and therefore ω_A and ω_z are the same sign. For satellites intended to have spin stabilisation (not UOSATS) $I_T < I_z$ and these component spins are of opposite sign.

3.2.1-1 Mechanical model of nutation

3.2.1-2 Angular velocity vectors in nutational state

3.2.1-3a/b Two limiting cases of nutation - Z spin and T spin

3.3 Natural Torque - Gravity Gradient

The standard result for gravity-gradient torque on a symmetrical body is presented in two forms: scalar and vector. Terminology to be used for the rest of this work is also presented

The dominant natural influence on the attitude of UOSAT is gravity-gradient torque. Adopting the simplest possible approach this gradient is simply the non-uniformity of the gravity field - its inverse-square dependence on the distance from the from the centre of the Earth. The force of gravity is weaker on parts of the spacecraft which are furthest away from Earth, and stronger on parts which are closer. For spacecraft with a UOSAT-type symmetry and a circular orbit the net torque is shown by many authors to be (NOTE)

$$N_{gg} = 3 \omega_o^2 (I_T - I_z) \sin(\gamma) \cos(\gamma)$$

...3.3.1

where

γ = inclination angle
 ω_o = $2 \pi / \text{period}$ = orbital rate

Inspection of this equation shows that the torque is zero for three different angles of inclination $\gamma = 0^\circ$, 90° and 180° . Its magnitude is greatest for $\gamma = 45^\circ$ and 135° .

The angle $\gamma = 90^\circ$ is a point of unstable equilibrium. The inclinations of $\gamma = 0^\circ$ and $\gamma = 180^\circ$ are both stable. It is an inconvenient fact of life for UOSATs that there are these two possible stable attitudes.

NOTE This formula can be established simply from a simple 'dumb-bell' model of 4 point masses joined in two orthogonally opposed pairs,

A more fundamental relation than Eq.3.3.1 expresses the g.g torque by a vector relation. In the notation of this work

$$\underline{N}_{gg} = 3 \omega_0^2 (I_T - I_Z) (\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r}) \quad \dots 3.3.2$$

where

$\underline{r}$ = unit vector from the Earth centre to satellite
 $\underline{z}$ = unit vector directed along Z axis

The torque is orthogonal to both $\underline{z}$ and $\underline{r}$.

Almost all texts deal with the general case when moments of inertia I_x and I_y are unequal - with considerably complicating effect on the possible stable states and behaviour. That can all be ignored here; and it may be pointed out that the gravity gradient has no effect - at least in principle for this model - on the state of Z-spin, either in the forced precession case to be considered next, or in the captured librating state.

There is no more which needs to be said about it at this stage other than to point out, as many have discovered in practice, that this torque is extremely *weak*, reflected in the very long period of oscillations of libration. For example, in a pure in-the-orbit-plane with small pitch oscillations it is well-known, for $I_T \gg I_Z$, that the predicted oscillating frequency is only $\sqrt{3}$ times the orbit rate; for example an oscillating period of 58 minutes if the orbit period is 100 minutes.

3.4 6.6 Forced Precession

Equations are presented which show how fast a spinning/nutating body with UDSAT symmetry will precess, at a constant angle to the orbit normal. A final equation is a simple adaption which gives the roll offset when the precession rate is the same as the orbital rate - which is in fact the gravity locked case

The basic text (Wertz 1978) deals with this matter for a purely spinning body; I have extended the analysis to the nutating state. The angular momentum $\underline{L}$ is not invariant because of this imposed torque. It is shown here how the $\underline{L}$ vector maintains a constant angle to the orbit normal but describes a slow cone in inertial space while maintaining this constant angle. In the terminology of section 3.1 and simplifying to the case of a pure Z-spin (no nutation) the tilt angle δ stays constant, while the absolute pitch angle increments or decrements in the course of this precession. Such an effect was corroborated in UOSAT-1. The phenomena was exploited as an inherent part of the attitude changes before deploying the boom on UOSAT-1. *In other words we used magnetorquing to adjust the tilt angle and then waited on this natural precession for the pitch to lie in a suitable attitude window before deployment.*

The problem may be considered in two parts: finding the average torque which actually drives this motion; and then the effect of this torque on the precessional rate. The presented equations are fully supported from Appendix B.

3.4.1 Geometric interpretation of torque

Reference to fig.3.4.1-1 may clarify the effect. The figure shows a UOSAT in three specific points in the orbit. For simplicity the spacecraft is assumed to be simply spinning (no nutation). A positive Z spin is also assumed. Supposing that the attitude were NOT changing because of gravity gradient torque then with our definition of angular change it will maintain a constant tilt angle δ , while the pitch constantly decrements in the course of the orbit. Recalling that 'pitch' is the projection of the Z axis into the orbital plane then the three positions are shown when the pitch angle is zero (i), when it is $\pm 90^\circ$ (ii) and when it is $\pm 180^\circ$ at (iii). In the first and last case there is a g.g torque (into the paper). While in the second case there is NO torque. Therefore there is on average a constant torque in inertial space which is always at right angles to the spin axis $\underline{z}$ and the

is always at right angles to the spin axis $\underline{z}$ and the angular momentum vector $\underline{L}$. Consequently this vector must *precess* around the orbit normal still maintaining a constant tilt angle δ , but forcing a slow but steady change in the inertial pitch angle. For UOSAT parameters and spin rates in the order of minutes this attitude change will be significant in a few days. The only attitude where precession is not induced is with the Z-axis aligned on the orbit normal - the usual direction chosen for spin-stabilised satellites in a low Earth orbit.

3.4.2 Torque equations

As an extension to Eq.3.3.2 for the instantaneous gravity gradient torque it can be shown for the generally nutating case, by a vector expansion, that the effective torque (averaged over one rotation of the nutating cone) is

$$\underline{N}_{gg} = 3 \omega_o^2 (I_T - I_Z) (1 - (3/2) \sin^2 \theta) \cdot \underline{z}_o \cdot \underline{r}$$

...3.4.2.1

Eq.3.4.2.1 is different from the basic Eq.3.3.2 in having a factor involving the nutation angle θ ; and in depending vectorially on $\underline{z}_o$ rather than $\underline{z}$. Here $\underline{z}_o$ is the average Z axis direction NOT the local vertical.

✓ Calculation shows that at a nutation angle = 54.7° will eliminate the g.g torque; a prediction which can be justified if one considers the satellite's density to be 'blurred out' over an extended volume of space by the nutation action.

The derivation assumes that in the course of a nutational cycle that the effect of the g.g torque would be small - so that the direction of the momentum vector - and therefore the average direction $\underline{z}_o$ - does not change significantly (see from Appendix B).

(1178)

The next equation determines a secular averaged torque. It reads

$$\langle N_{gg} \rangle = - (3/2) \omega_0^2 (I_T - I_Z) (1 - (3/2) \sin^2(\theta)) \frac{(\underline{z}_0 \cdot \underline{n})}{(\underline{z}_0 \cdot \underline{n})}$$

...3.4.2.2

where '3/2' substitutes for '3' and the vector $\underline{n}$ substitutes for $\underline{r}$. In scalar terms this reads

$$\langle N_{gg} \rangle = (3/2) \omega_0^2 (I_T - I_Z) (1 - (3/2) \sin^2(\theta)) \sin(\delta) \cos(\delta)$$

...3.4.2.3

Again the validity of this equation depends on an negligible change in the inertial pointing of the $\underline{L}$ vector in the course of one orbit rotation. On comparison with Wertz - and after some substitution of symbols it is clear that his author has erroneously substituted '3' rather than '3/2' in the formula. The editor is also led to the conclusion - or rather the author of this particular section 17.2.1 that 'the rate of precession is a maximum at $\delta = 45^\circ$ or 135° (my change of symbols)' because the value of $|\langle N_{gg} \rangle|$ maximises at these angles. However that statement is not correct.

3.4.3 Precession equation

The above secular torque forces a conical motion at a precession rate ω_N . Exact analysis in Appendix B establishes, in the case of a pure Z spin that the rate of change of momentum

$$d\underline{L}/dt = - I_T \omega_N^2 \cos(\delta) \sin(\delta) + I_Z \omega_N \omega_Z \sin(\delta)$$

...B.2.9

...3.4.3.1

where

ω_N = precession rate

NOTE orbit - averaged

At a sufficiently slow precession rate a good approximation is

$$dL/dt \approx I_z \omega_N \omega_z \sin(\delta)$$

...3.4.3.2a

This incidentally is effectively the form of precession equation, (Goldstein 1950), quoted as if exact in Wertz (Eq 19-6), which can also read

$$| \underline{N_{gg}} | \approx | \underline{\omega_N} \wedge \underline{L} |$$

...3.4.3.2b

which states that the the applied torque is equal to the rate of change of angular momentum, where the latter is expressed by the vector product of the precession rate with the angular momentum. Exact analysis shows that the result is in fact an approximation depending on the condition

$$\omega_N \cos(\delta) \ll (I_z/I_T) \omega_z$$

...3.4.3.3

However the approximate Goldstein form is useful in the generally nutating case. Repeating Eq.3.2.2.4

$$L = \omega_z I_z / \cos(\theta)$$

The effect of nutation is easily encompassed by the 'COS(θ)' term. Substituting this in Eq.3.4.3.2b

$$\langle N_{gg} \rangle = I_z \omega_N \omega_z \sin(\delta) / \cos(\theta)$$

...3.4.3.4

and combining with Eq.3.4.2.3 generates a final expression for the precession rate

$$\omega_N \approx \frac{(3/2) \cos(\theta) \omega_0^2 (I_T - I_Z) (1 - (3/2) \sin(\theta)^2) \cos(\delta)}{I_Z \omega_Z} \quad \dots 3.4.3.5$$

This equation is valid provided the period of the precession is long compared to an orbit. When this is not true the analysis must break down - requiring the exact numerical analysis from section 3.6.

Three conclusions from Eq.3.4.3.5 concern the precession rate: it is inversely proportional to the Z spin rate; it declines with increasing nutation angle; and it falls off as the cosine of the tilt angle. Contrary to what Wertz states it follows that the precession rate maximises when the L is parallel or anti-parallel with the orbit normal (as $\delta > 0$). Only that in the limiting case this precession is not *visible*, since the precession cone has collapsed in this limited case. In principle the precession stops if the tilt angle $\delta = 90^\circ$ i.e putting the L vector into the orbit plane. Plots of typical values are given in fig.3.4.3-1.

3.4.4 Locked precession

Exceptionally it is possible to derive an exact result when the precessional rate is the SAME as the orbital rate. This means that the orientation of the Z vector is fixed in the reference local co-ordinates. The result strictly belongs later to the 'captured state' by gravity gradient, and is discussed in section 3.5 on that basis.

Modifying Eq.3.4.2.3, for the average torque the result is

$$N_{\text{avg}} = 3 \omega_0^2 (I_T - I_Z) \cos(\delta) \sin(\delta) \quad \dots 3.4.4.1$$

Here the factor '3' is correct rather than '3/2' because the torque is actually a *constant*, and does not need to be secularly averaged.

Also the nutation angle is set to zero since otherwise the attitude is not static within the reference frame. Using Eq.3.4.3.1 then, with ω_0 substituted for ω_N , yields this exact formula

$$\frac{N_{gg}}{\omega_0} = -I_T \omega_0^2 \cos(\delta) \sin(\delta) + I_z \omega_0 \omega_z \sin(\delta) \quad \dots 3.4.4.2$$

And simplifying, after combination with Eq.3.4.4.1

$$\cos(\delta) = \frac{\omega_z / \omega_0}{(4 I_T / I_z - 3)} \quad \dots 3.4.4.3$$

This last formula is remarkable for being the one simple exact result in this study. Expressed in terms of a constant roll angle bias r_0 then

$$\sin(r_0) = \frac{\omega_z / \omega_0}{(4 I_T / I_z - 3)} \quad \dots 3.4.4.4$$

Plots of this formula for representative values of I_T/I_z spin periods are given in fig.3.4.4-1,

3.4.1-1 Precession torque geometry

3.4.3-1 Precession rate to spin period UOSAT2

estimated $I_T/I_Z = 1.9$ spin period = 98 mins
 nutation angle $A = 0^\circ$ $B = 10^\circ$ $C = 20^\circ$ $D = 30^\circ$ deg
 PERIOD (DAYS) * COS(TILT)

3.4.4-1 Average roll to Z spin

$$I = \frac{I_T}{I_Z}$$

3.5 Attitude Capture by Gravity Gradient

The standard differential equation for pure pitch libration is first presented and then a further discussion of the average (roll) attitude due to Z-spin. Simple but possibly novel equations determining bounds on roll and pitch are presented, following an extensive derivation in Appendix C. As a useful application the bounds on pitch and roll are given, assuming an ideal case of instantaneous boom deployment. Systematic equations predict a general offset ellipse when plotted in the attitude circle.

This section concerns the intended state of UOSAT-1, and the achieved state of UOSAT-2, when the gravity gradient torque dominates over other disturbance torques or residual angular momentum. In consequence the satellite maintains an Earth-pointing (or near Earth-pointing attitude), at least on average.

Libration is well-known as the state of slow oscillation under gravity gradient control. Here it is expressed in terms of the Z-axis of the satellite describing (ideally) small changes in pitch and roll around the zenith-nadir vector. The exact analysis requires evaluation of the coupled second-order differential equations and their numerical solution; and is considered in the next section 3.6.

3.5.1 Pitch differential equation

Some useful, but not original, engineering conclusions follow from analysing the case of pure pitch libration (the Z-axis always lying in the orbit plane). From Eq.3.3.1 and assuming zero Z-spin then

$$3 \omega_0^2 (I_T - I_Z) \sin(p) \cos(p) = - I_T d^2p/dt^2 \quad \dots 3.5.1.1$$

does approximate to simple harmonic motion for a captured state with small amplitude about an average of zero pitch (or 180° pitch).

For small angles and $I_T \gg I_z$ then the frequency of this pitch oscillation is

$$\omega_p \approx \sqrt{3} \omega_o.$$

...3.5.1.2

It is because this rate is enharmonic with the orbital rate ω_o that the threshold method of libration suppression (section 5.3) is possible.

If both sides in Eq.3.5.1 are multiplied by the rate of change of pitch $d(p)/dt$ and then integrated, a constraint equation emerges

$$\begin{aligned} (3/2)\omega_o^2 (I_T - I_z) (\cos(2 p_1) - \cos(2 p_2)) \\ = I_T (\omega_1^2 - \omega_2^2) \end{aligned}$$

...3.5.1.3

where p_1 and p_2 are the pitch angles at any two points in the motion while ω_1 and ω_2 are the corresponding rates of change in the pitch (Fig.3.5.1-1).

From Eq.3.5.1.3 the amplitude of the pitch displacement and the amplitude of the pitch velocity may be compared. The formula then is

$$(dp/dt)_M \approx \sqrt{3} \omega_o \sin(p_M)$$

...3.5.1.4

where the subscript 'M' implies an amplitude value in either case.

From this there are two useful conclusions:

- * if the rate of change of pitch exceeds ω_p after boom deployment then capture cannot happen.
- * if the pitch velocity at zero pitch angle is the same as the orbital rate then the amplitude of the pitch displacement is given by

$$p_M = \text{ARCSIN}(1/\sqrt{3}) = 35^\circ$$

...3.5.1.5

The last result approximates what happens when the boom actually deploys. If the boom could be deployed from an initial attitude of zero libration angle ($\gamma = 0^\circ$) and instantaneously by an amount which creates a large change in transverse moment of inertia, then this act effectively kills any transverse spin (the tumbling component if it were freely nutating) and brings the satellite to a rotational halt in *inertial* space (it has however no effect on the Z-spin). In the local reference co-ordinates the attitude now has a pitch velocity which is the negative of $-\omega_0$ - the orbital rate within the Earth-referenced system. Hence the minimum libration amplitude expected as a result of this manoeuvre is this figure of 35° .

The same formula Eq.3.5.1.3 can predict the largest value of initial pitch so that the final pitch is 90° - the limit of stability, again assuming a starting pitch rate of $-\omega_0$. The value is 55° in theory, although in practice one would never start the boom deployment at such an extreme value.

3.5.2 Average attitude assuming capture

Section 3.4 derived an exact result for the stable attitude with a finite Z-spin. Repeated here

$$\sin(r_0) = \frac{\omega_z / \omega_0}{4 I_T / I_z - 3}$$

...3.4.4.3

which result is confirmed by the 'bounds' formula of the next section 3.5.3. In this stable state the pitch angle is still zero.

The two possible solutions to bias roll r_0 correctly indicate the two possible roll states, as in Fig.3.5.2-1. The steady-state pitch angle is zero.

A number of conclusions can be drawn:

- * for a zero spin rate then roll angle α_0 is either 0° or 180° ; UOSAT is either exactly the right way up or is exactly upside down. This is consistent with the well-known result that there are two capture states under gravity-gradient.
- * a constant Z spin necessarily causes a roll bias, with the Z axis inclined to the positive side of the orbit plane if the Z-spin is positive, or the negative side if the Z-spin is negative.
- * the deviation of the attitude away from the vertical, whether the right way-up, or upside-down, increases with the magnitude of the Z-spin, but decreases with an increase in the transverse moment of inertia I_T . So the more the boom can be deployed, of a given weight, the more readily a near-vertical attitude can be achieved, and the greater the tolerance towards a residual Z spin.
- * because the magnitude of 'sine' cannot exceed unity the condition is established by inspection for which capture by gravity gradient is at least possible. This condition is simply

$$\omega_z < (4 I_T/I_z - 3) \omega_0$$

...3.5.2.2

This is a necessary but not sufficient condition for capture by gravity gradient.

It may be shown that Eq.3.4.4.3 is still valid at predicting an average roll angle in the presence of substantial libration (the average pitch angle is still zero). For UOSAT-2 the spin period is controllable down to about 10 minutes period at most. This gives a useful insight into the required ratio I_T/I_z for a small pointing error. For UOSAT-2 I made the decision to deploy the boom for 15 minutes. At the measured rate of deployment this would push it out to about 7 metres.

From this and other engineering parameters the calculated ratio $I_T/I_z = 120$. From Fig.3.4.4-1 it seems that an average pointing error is about 1 degree for a residual Z-spin of 10 minute period.

3.5.3 Bounds on attitude

The state of libration can be attributed to a dynamic interchange between kinetic and potential energy. A transverse rotational kinetic energy is associated with a transverse spin - the same 'T-spin' that contributes to nutation in the torque-free spinning case - and slews the Z-axis relative to the local vertical. The libration angle γ is bounded however, since with increasing γ the kinetic energy must be lost by conversion to increased potential energy against the retarding gravity-gradient torque, and eventually cause the attitude to swing back, and γ to decrease.

An analytical solution, expressing the changes of pitch and roll with time does not exist, I believe.

However in common with other problems in mechanics general constraint equations can be found analytically. It is one of the claimed achievements of this work to have extracted what seems to be the equivalent of a kinetic energy and 'dynamic potential' (Hughes 1986) without resort to advanced mechanics (Hamiltonians), by just relying on systematic development of the direct equations of motion using elementary vector theory. First the differential equations of motion are found , in a co-ordinate free representation, and then an energy-dimensioned constraint equation follows (see Appendix C).

It can be shown that

$$\begin{aligned}
 (3/2) \omega_0^2 (I_T - I_Z) \cos(y)^2 & \\
 &= (I_T/2) \omega_{T1}^2 \\
 &+ (I_T/2) \omega_0^2 \cos(\delta)^2 \\
 &- I_Z \omega_Z \omega_0 \cos(\delta) + U
 \end{aligned}$$

y = inclination angle

δ = tilt angle

ω_{T1} = transverse velocity relative to the standard reference system. If zero then the Z axis is not slewing

U = constant energy term

...3.5.3.1

Eq.3.5.3.1 is usefully modified by assuming that there exists a state in which UOSAT is temporarily stationary, with zero relative transverse velocity $\omega_{T1} = 0$. This state is indicated by two angles which will be designated y_0 for the inclination, and δ_0 for the tilt. Simple manipulation then yields

$$\begin{aligned}
 (3/2) \omega_0^2 (I_T - I_Z) (\cos(y)^2 - \cos(y_0)^2) & \\
 &= (1/2) I_T \omega_{T1}^2 \\
 &+ (1/2) I_T \omega_0^2 (\cos(\delta)^2 - \cos(\delta_0)^2) \\
 &- I_Z \omega_0 \omega_Z (\cos(\delta) - \cos(\delta_0))
 \end{aligned}$$

...3.5.3.2

This equation can be used in a number of ways. But if values y , δ and ω_{T1} are taken as 'seed values' and as *constants* then a plot of y_0 against δ_0 or more convenient projections or derivations of these angles, will generate a locus, enclosing a boundary marking the limit of the librational state.

The equations can be simplified by defining some normalising variables:

Ratio of moments of inertia	I	$= I_T/I_Z$
z-spin ratio	n_z	$= \omega_z/\omega_0$
relative-spin ratio	n_{T1}	$= \omega_{T1}/\omega_0$

$$\begin{aligned}
 (3/2) (I - 1) (\cos^2(y) - \cos^2(y_B)) \\
 = (1/2) I n_{T1}^2 \\
 + (1/2) I (\cos^2(\delta) - \cos^2(\delta_B)) \\
 - n_z (\cos(\delta) - \cos(\delta_B))
 \end{aligned}$$

...3.5.3.3

This Eq.3.5.3.3 cannot apply to the case of time-varying ratio I_T/I_Z . However a further useful equation results from assuming that the boom deployed *instantaneously* (see section 3.5.1). In which case the value of $\omega_{T1} = -\omega_{T0}$ which is the component of the constant orbit velocity ω_0 resolved on the normal to the Z axis of the spacecraft . It follows that

$$\omega_{T1} = -\omega_0 \sin(\delta)$$

...3.5.3.4

Substituting into Eq.3.5.3.3 and taking the case of zero Z spin and re-arranging then

$$\begin{aligned}
 (3/2) (I - 1) \cos^2(y_B) + (1/2) I \sin^2(\delta_B) \\
 = (3/2) (I - 1) \cos^2(y)
 \end{aligned}$$

...3.5.3.5

which can be plotted in the attitude circle as a function of starting values of y . The necessary conversions are

$$\sin^2(\delta_B) = 1 - z_N^2$$

...3.5.3.6a

$$\cos^2(y_B) = z_R^2 = 1 - z_N^2 - z_V^2$$

...3.5.3.6b

from which an equation in z_N and z_V is easily constructed.

The plots in fig.3.5.3-1 are seen to be ellipses centred on the origin.

As the more general construction (given below) shows, if there were a finite Z -spin the plot would still be an ellipse but not centred on the origin.

The elliptical shape, compressed in the roll direction (z_R) shows that the 'dynamic potential energy ' is effectively higher than in the pitch direction z_P . This is a consequence of the libration taking place within a rotating frame of reference. It is entirely consistent with later results for the periods of small angle libration (see section 3.7). It also explains the otherwise puzzling sole dependence on the initial inclination angle γ and no other attitude information. For example the elliptical curve B marks the bounds for a boom deployment taking place from an initial attitude anywhere on the $\gamma = 20^\circ$ circle. For varying tilt angles δ , but keeping on a constant γ circle, the variation in kinetic energy due to different initial transverse velocity (Eq. 3.5.3.4) clearly cancels the variation in initial potential energy.

The actual dynamic track depends of course on the initial δ angle as well as the γ angle.

Returning to the general equation Eq. 3.5.3.3 it provides an invaluable cross- check on the accuracy of the numerical calculation based on general differential equations (see section 3.6). The bound on the libration can be computed and then plotted in the attitude circle on which are superimposed the digitally computed time tracks.

GIVEN: a starting roll value r
pitch value p
roll velocity $\dot{r}$
pitch velocity $\dot{p}$

I. Since

$$\cos(\gamma) = \cos(r) \cos(p) \quad \dots 3.5.3.7a$$

$$\cos(\delta) = \sin(r) \quad \dots 3.5.3.7b$$

$$\omega_{T1}^2 = \dot{r}^2 + \dot{p}^2 \cos^2(r) \quad \dots 3.5.3.7c$$

II Then

$$A = (3/2) * (1 - 1/I) \quad \dots 3.5.3.8a$$

$$B = A \cos^2(r) \cos^2(p) - \sin^2(r)/2 - (r^2 + p^2 \cos^2(r))/2 + n_z \sin(r) (1/I) \quad \dots 3.5.3.8b$$

$$C = \frac{n_z^2/I}{2(4I - 3)} \quad \dots 3.5.3.8c$$

$$z_{NO} = n_z / (4I - 3) \quad \dots 3.5.3.9a$$

$$Z_N = \sqrt{((A - B + C)/(A + 1/2))} \quad \dots 3.5.3.9b$$

$$Z_V = \sqrt{((A - B + C)/A)} \quad \dots 3.5.3.9c$$

The general elliptical plot bound in the attitude circle is then

$$\left(\frac{Z_N - z_{NO}}{Z_N} \right)^2 + \left(\frac{Z_V}{Z_V} \right)^2 = 1 \quad \dots 3.5.3.10$$

which is most easily plotted parametrically

$$\begin{aligned} Z_N &= Z_N \cos(\theta) + z_{NO} \\ Z_V &= Z_V \sin(\theta) \end{aligned} \quad \dots 3.5.3.11$$

where

$$0^\circ < \theta < 360^\circ$$

The centre of the ellipse (Eq.3.5.3.9a) co-incides with the average Z-spin induced roll bias of Eq.3.4.4 after appropriate denormalisation of the parameters: a satisfying result since it derives from a completely different analytical route.

3.5.1-1 Pitch and rate of pitch conventions

3.5.2-1 Equilibrium states -earth reference

3.5.3-1 Predicted bounds from instant deployment

Attitude circle 10 degree increment libration circles
Ellipses A,B and C are libration bounds for instantaneous
boom deployment at initial $\gamma = 0, 20$ and 40 degrees

3.6 Modelling of Boom Deployment

The full differential equations in attitude change during and after boom deployment are presented; and the graphical results of numerical solution are given. The theory is not new and confirms only the simple fact that transition to gravity-gradient control from a previously slowly tumbling state is easy provided the prior attitude is 'reasonably' vertical. The results quantify this exactly within the model adopted for UDSAT. Conclusion: there is broad 'window' of attitude from which to deploy the boom, even if that manoeuvre takes as long as 15 minutes. This simulation was further extended to take account the effect of magnetorquing and suppression of libration. (see section 5.3)

The previous section gave a simple analytic formula for predicting the libration after boom deployment if that deployment were *instantaneous*. In this section the full differential equations are given and the results of their solution using a standard Runge-Kutta computation in order to establish what happens when the boom takes a long time (tens of minutes). No particularly new principle is involved, except possibly in using Eulers *modified* equations (see p113 Thomson 1961) which exploits the symmetry of the satellite ($I_x = I_y$), as has been done throughout this text, and which leads to considerable simplification. Detailed derivation of the equations is given in Appendix D.

The equations governing the attitude change are as follows

$$\begin{aligned} \ddot{\rho} \cos(r) I_T = & - \dot{\rho} \omega_z I_z \\ & + 2 (\omega_0 + \dot{\rho}) \sin(r) \dot{\rho} I_T \\ & - (\omega_0 + \dot{\rho}) \cos(r) d(I_T)/dt \\ & - 3 \omega_0^2 \sin(p) \cos(r) \cos(p) (I_T - I_z) \end{aligned} \quad \dots 3.6.1a$$

$$\begin{aligned} \ddot{r} I_T = & + (\omega_0 + \dot{\rho}) \cos(r) \omega_z I_z \\ & - (\omega_0 + \dot{\rho})^2 \sin(r) \cos(r) I_T \\ & - \dot{\rho} d(I_T)/dt \\ & - 3 \omega_0^2 \cos(p)^2 \sin(r) \cos(r) (I_T - I_z) \end{aligned} \quad \dots 3.6.1b$$

The equations solve to the attitude of the Z axis only. A constant Z-spin is allowed and affects to some extent the development of the attitude.

3.6.1 Numerical calculations

The requirement of the usual numerical routines is that equations of second order or higher need to be recast as a set of first order equations, with a normalised time scale. Defining

```
pitch velocity      y[1] =  $\beta/\omega_0$ 
pitch displacement y[2] = p
roll velocity       y[3] =  $\dot{r}/\omega_0$ 
roll displacement  y[4] = r
```

...3.6.1.1

and the first order derivatives

```
pitch acceleration f[1] =
pitch velocity     f[2] = y[1]
roll acceleration  f[3] =
roll velocity      f[4] = y[3]
```

...3.6.1.2

Practical test was only required for zero Z-spin.

After combining Eqs.3.6.1, 3.6.1.1, 3.6.1.2 and re-arranging the equations for zero Z spin (in PASCAL notation) becomes

```
f[1] := -(3/2)*(1-Iz/It)*SIN(2*y[2])
      - (1 + y[1])*It_dot/It
      + 2 (1 + y[1])*y[3] TAN(y[4]);

f[2] := y[1];

f[3] := -(3/2)*(1-Iz/It)*SIN(2*y[4])*SQR(COS(y[2]))
      - y[3]*It_dot/It
      - (1/2)*SIN(2*y[4])* SQR(1+y[1]);

f[4] := y[3];
```

...3.6.1.3

This set was extensively tested prior to boom deployment on UOSAT-2 (in the above $I_t = I_T$, $I_{t_dot} = dI_T/dt$). The above formulation reveals a potential discontinuity for values of $\gamma[4]$ - the roll angle - if it should approach 90° . This is of course a traditional problem associated with the use of Eulerian angles. Fortunately in this application that condition should not happen - the numerical test is restricted in principle to roll angles which are always less than 90° .

A simple model (Fig.3.6.1-1) represents the spacecraft as 2 'boxes' - one the spacecraft proper , the other the boom end. Values used and subsidiary equations are listed in Appendix D.

3.6.2 Results of simulations

Figures 3.6.2-1 through 3.6.2-6 show some typical results. The plots are in the attitude circle with the circles indicating multiples of 10 degrees of inclination angle to the edge at 90 degrees. Figs.3.6.2-1 through 3.6.2-4 show the effect of deploying the boom over 15 minutes from a roll angle of 20 degrees and pitch angles of 0,20,40 and 60 degrees. These represent typical start conditions on release from prior magnetically controlled state. The roll angle is rather worse than values expected from prior stabilisation to the vector magnetic field. The pitch angle more than covers the range expected on a North-South pass over the Surrey ground station.

A 15-minute deploy time was decided as a compromise : it is enough to ensure gravity-gradient capture (the boom goes out by about 7 metres) with the option to extend further at a later date. The total simulated run time is 500 minutes at 1 minute intervals. It can be seen that the best range of initial pitch conditions is roughly within the range from 20 to 40 degrees with the resulting oscillations bound within the 40 degree inclination circle.

Figs.3.6.2-5 and 3.6.2-6 show motion from quite different starting conditions in which the boom is NOT deploying, the purpose being to test

the accuracy of the static bounds computed in the analysis of section 3.5.3. The ellipses additionally bounding the numerical computed plots are analytic results computed directly from equations of that section. There seems to be good agreement.

These particular plots do not check the bounds on the finite Z-spin case; further work - not presented here - has shown these also to be consistent.

3.7 Simplified Differential Equations

The exact differential equations are simplified in two steps: a small signal approximation shows the existence of coupling from roll to pitch state at all amplitudes is shown - which justifies the deliberation methods of chapter 5. A set of linearised equations follow for further use (and compared to Wertz)

As a first step the cosines of attitude angles may be set to unity and the sines of angles set to the angles themselves in Eqs3.6.1. Then

$$\begin{aligned} & \ddot{\theta} I_T + \dot{\theta} \omega_z I_z \\ & - 2 (\omega_0 + \beta) r \dot{\theta} I_T \\ & + 3 \omega_0^2 p (I_T - I_z) = 0 \end{aligned} \quad \dots 3.7.1$$

$$\begin{aligned} & \ddot{\phi} I_T - (\omega_0 + \beta) \omega_z I_z \\ & + (\omega_0 + \beta)^2 r I_T \\ & + 3 \omega_0^2 r (I_T - I_z) = 0 \end{aligned} \quad \dots 3.7.2$$

These equations show clearly that roll velocity couples pitch acceleration and pitch velocity couples roll acceleration.

At infinitesimal amplitudes, for $I_T \gg I_z$ and negligible spin rate ω_z the above reduces to two independent SHM with harmonic rates

$$\begin{aligned} \text{pitch} \quad \omega_p &= \sqrt{3} \omega_0 \\ \text{roll} \quad \omega_R &= 2 \omega_0 \end{aligned} \quad \dots 3.7.3$$

Eq.3.7.2 also predicts a residual roll bias ($\dot{\phi} \rightarrow 0, \ddot{\phi} \rightarrow 0$)

$$r_o = \frac{\omega_z / \omega_o}{(4 I_T / I_z - 3)} \quad \dots 3.7.4$$

which is already known more accurately with the SINE of the roll angle (Eq.3.4.4.4 on the LHS).

3.7.1 Magnitudes of coupling effects

The effectiveness of the simple damping algorithm presented in section 5.3 depends on pitch/roll coupling.

For this section I adopt figures $I_T / I_z = 120$ and a spin rate $\omega_z = 10 \omega_o$ (about the slowest controllable rate on UOSAT-2 corresponding to a spin period of approx 10 minutes)

I argue that multiplying Eq.3.7.1 by $\dot{\phi}$ and Eq.3.7.2 by $\ddot{\phi}$ represents a power flow in the system, from which equal but opposite terms identified in the two equations indicate actual cross -coupling. From which the coupling terms can be identified (NOTE)

	MULTIPLY BY
.... $\ddot{\phi} \omega_z I_z - 2 \omega_o \dot{\phi} \ddot{\phi} I_T$	$\dot{\phi}$
.... $-\ddot{\phi} \omega_z I_z + 2 \omega_o \ddot{\phi} \dot{\phi} I_T$	$\ddot{\phi}$
	...3.7.1.1

NOTE The validity of this approach is justified by comparison with the total energy equation Eq.3.5.3.1. If that equation is differentiated its various terms represent power flow in the oscillating system. If it is simplified with small value approximations made to the cosines and sines then exactly the same equation emerges compared to summing Eqs. 3.7.1. and 3.7.2 after those equations have been multiplied by $\dot{\phi}$ and $\ddot{\phi}$ respectively.

At 0.1 radians amplitude (6°) the linear and square terms in each of Eqs. 3.7.1.1 are comparable. At 0.01 rads (0.6°) of this the second square law term is an order of magnitude less.

Since we are interested only in small amplitude librations then removing squared terms from Eqs. 3.7.1. and 3.7.2 yields

$$\dot{p} I_T + 3 \omega_0^2 p (I_T - I_Z) = - \dot{r} \omega_z I_Z + \underbrace{N_z} \quad \dots 3.7.1.2a$$

$$\dot{r} I_T + \omega_0^2 r (4 I_T - 3 I_Z) = \underbrace{\dot{p} \omega_z I_Z} + \omega_0 \omega_z I_Z \quad \dots 3.7.1.2b$$

These equations show how that roll oscillations couple pitch and vice-versa weakly at all amplitudes (about 5%) provided this finite Z spin is maintained. If the spin rate were increased (to about 5 minute period) then the coupling is doubled (to 10%) but at the expense of an increased roll bias. Contrary to what has sometimes been stated pitch and roll oscillations DO couple even down to negligible amplitudes- provided a significant Z- spin is maintained.

These results are of key significance in explaining the effectiveness of the simple de-libration system (see from section 5.3)

3.7.2 Reduction to linear equations

I propose to ignore the coupling effects for the purpose of simplifying the Kalman estimator approach (see from chapter 7). So linearised the dynamic equations are approximated by the following

$$\dot{p} I_T + (4 I_T - 3 I_Z) \omega_0^2 p = I_Z \omega_0 \omega_z \quad \dots 3.7.2.1a$$

$$\dot{r} I_T + 3 (I_T - I_Z) \omega_0^2 r = 0 \quad \dots 3.7.2.1b$$

$$I_Z \dot{\omega}_z = 0 \quad \dots 3.7.2.1c$$

It is of some interest to compare with the general (unequal I_x and I_y values) linearised equations given in Wertz (P609-610.)

Stripped of modifications due to internal momentum wheels, now setting $I_x = I_y = I_T$ and with change of symbols those equations 18-53 read

$$I_T \ddot{\rho} + 4 \omega_0^2 (I_T - I_Z) \rho = I_Z \omega_0 d\phi/dt \quad \dots 3.7.2.2a$$

$$I_T \ddot{\phi} + 3 \omega_0^2 (I_T - I_Z) \phi = 0 \quad \dots 3.7.2.2b$$

$$I_Z d^2\phi/dt^2 + I_Z \omega_0 \dot{\rho} = 0 \quad \dots 3.7.2.2c$$

The different equations are reconciled when it is realised that the rate of change of yaw $d\phi/dt$ is not identical to the absolute Z spin rate.

Rewriting Eq.3.7.2.2a and comparing Eq.3.7.2.1a then

$$\begin{aligned} I_T \ddot{\rho} + \omega_0^2 (4 I_T - 3 I_Z) \rho &= \omega_0^2 I_Z \rho + I_Z \omega_0 d\phi/dt \\ &= \omega_0 I_Z (\omega_0 \rho + d\phi/dt) \\ &= \omega_0 I_Z \omega_Z \end{aligned} \quad \dots 3.7.2.3$$

Then the identity emerges

$$\omega_Z = \omega_0 \rho + d\phi/dt \quad \dots 3.7.2.4$$

which is easily justified geometrically. It means simply that if there is a finite roll it 'reflects' an advance or retard of the actual rotating frame into the observable Z spin. In practical terms it is proposed to ignore this. For example for ω_Z/ω_0 in the order of 10 and ρ in the order of 0.2 or less the difference is 2% or less.

I note that the equations used throughout this thesis are in a sense more general than the standard linearised equations because they allow unlimited yaw rotation; there is no restriction to some small deviation.

3.6.1-1 Dynamic Model of UOSAT2

3.6.2-1 Boom deployment simulation

Pitch = 0° Roll = 20° Pitch rate = 0.0° Roll rate = 0.0°

Deploy time = 15 mins Delta = 1 min Iterations = 500

3.6.2-2 Boom deployment simulation

Pitch = 20° Roll = 20° Pitch rate = 0.0° Roll rate = 0.0°

Deploy time = 15 mins Delta = 1 min Iterations = 500

3.6.2-3 Boom deployment simulation

Pitch = 40° Roll = 20° Pitch rate = 0.0° Roll rate = 0.0°

Deploy time = 15 mins Delta = 1 min Iterations = 500

3.6.2-4 Boom deployment simulation

Pitch = 60° Roll = 20° Pitch rate = 0.0° Roll rate = 0.0°

Deploy time = 15 mins Delta = 1 min Iterations = 500

3.6.2-5 Libration within static analytic bounds

Pitch = 10° Roll = 30° Pitch rate = 1.0° Roll rate = 0.0°

Deploy time = 0 mins Delta = 1 min Iterations = 2000

3.6.2-6 Libration within static analytic bounds

Pitch = 10° Roll = 20° Pitch rate = 0.5° Roll rate = 0.5°

Deploy time = 0 mins Delta = 1 min Iterations = 2000

3.8 Disturbance Torques

A brief look at known effects which can be described as disturbance torques, on the UOSAT-2 type geometry. Radiation pressure is thought to be a minor problem. Aerodynamic torque can be the dominant cause of pointing inaccuracy in case of maximum air density induced by peak solar activity - even at the 690 km of UOSAT-2 altitude. A known problem of thermal flutter is introduced. Looking ahead, the effect on the eventual pointing accuracy given (1) rate damping (2) position plus rate control from magnetorquing is briefly considered

If gravity gradient were the only cause of environmental torque and the satellite was exactly a rigid body then the full equations would be exact; and the linearised equations would be exact for infinitesimal libration angles. Either full or linearised equations predict that a UOSAT-2 type spacecraft will stabilise to an ideal zero pointing error *on average*, provided there is zero Z-spin. And the linearised equations in this special case predict independent SHM in pitch and roll of constant amplitude.

In this section I consider briefly the extent which these predictions are necessarily untrue. Wertz (1978) and, better, Hughes (1986) summarise the well-established theory which accounts for the best characterised disturbance torques. This theory can be used to assess the ultimate stability of the (nominally) 2-axis stabilised state.

The impact of radiation pressure and air pressure requires a simple geometrical model. I adopt that of Fig.3.6.1-1, to a first approximation, with a physical boom thickness (1/2 " diameter) which must be taken into account.

3.8.1 Solar Radiation Torque

Illumination by the sun of surface causes a radiation pressure W/c on a normal, fully absorptive surface, where W is the solar constant (1358 W/m^2) and c is the velocity ^{of} light. The interaction is slightly more

complicated for interaction on general surfaces. Three coefficients: C_A adsorption coefficient, C_S coefficient of specular reflection, and C_D coefficient of diffuse reflection are bound by the relation

$$C_A + C_S + C_D = 1$$

...3.8.1

Based on the well-known simple equations, using these coefficients, and substituting worst-case values, a torque is trivially estimated whose effect is most easily expressed as a pointing error (libration angle) if it is compensated by gravity-gradient torque alone (NOTE).

The calculation predicts a worst-case steady-state libration angle of $\gamma \approx 0.25^\circ$ due to sun-light. The boom is the dominating source of torque. This worst case is simplest to calculate with the sun assumed to be constantly illuminating the satellite at an angle normal to the nominal Z axis. The result might appear to indicate that sun-light is not a significant problem in the UOSAT-2 context. However the sun-synchronous orbit UOSAT-2 does fly into night zone briefly on every orbit. So in principle sun-light can 'pump' the librating state.

3.8.2 Aerodynamic torque

The aerodynamic torque is as easily estimated, provided one makes a major assumption that any surface exposed to the slip stream completely cancels the energy of any incident particle. Hughes re-derives the well-known result that the pressure is proportional to the air density and the square of the relative air-velocity, subject to this and three other related assumptions.

Hughes also derives the very much more complicated formulae if these assumptions are revoked.

NOTE Assuming that the responding torque is = $3 \omega_o^2 (I_T - I_Z) \gamma$

Again for UOSAT-2 a worst-case aerodynamic torque is easily calculable, if one has a figure for the air density, expressible as a pointing error in γ which the gravity gradient has to compensate.

According to both Wertz and Hughes there is a real problem in establishing a meaningful figure for air density. According to Wertz's Appendix L, at an orbit height at 500 km (slightly higher than) UOSAT1's orbit), the air density could be anywhere between $1.3E-14$ to $3E-11$ with a mean figure of $7E-13$. No minimum or maximum or figures are offered at higher altitudes for UOSAT-2.

Wertz does remark (section 4.4) the ' 11 year cycle of solar activity', as causing the greatest fluctuations, and a ' 11 year variation in density is the largest, amounting to order-of-magnitude fluctuations at 350 km'.

Hughes (1986) has gone back to the original NASA (1971) data and plots air densities for day and night time maxima in conditions of high solar activity and low solar activity(p. 259). This data indicates that aerodynamic torque varies between being dominant and negligible depending on the time within the 11 year cycle. At 690 km (UOSAT-2 height) and from his graph the densities for high activity lie between $7 E-13$ to $1.4E-12$ (kgm/m^3), between day and night time. At this level the aerodynamic torque would seem to be the worst source of pointing error causing (at least) a systematic bias in pitch. On the other hand the potential range of densities is very large - at low activities it is more than 2 orders of magnitude less - which makes the errors negligible from this cause.

For UOSAT-2 parameters the worst-case deflection estimates to an equivalent pitch bias error between ≈ 1.4 and 2.8° . Since UOSAT-2 constantly flies briefly (about 1/3) of the orbit into night one may expect a parametric excitation to the pitch state from variations in the air density between these worst-case levels. Again, the main cause of this torque is the deployed boom. There is, in principle, a cyclical component of air velocity, due to rotation of the earth. As UOSAT-2

crosses the equator it must experience a peak 'cross wind' amounting to about 7% of the full forward velocity. Again in principle this can amount to a pumping effect, tending to build up the libration (see Hughes p.259). A peak of solar activity is next expected in late 1990. So it will be interesting to test the proposed ADCS algorithms in this worst case, either on UOSAT-2 if still functioning, or on the next series of UOSATs.

3.8.3 Thermo elastic deformation

A notorious problem with the type of boom which UOSAT-2 employs is a thermally induced deformation. I report more fully from the literature in section 8.4. Broadly I understand it to be a mechanism in which sun-light causes thermal expansion on the sun-illuminated side of the boom, and a twisting effect which brings a previously cool part under illumination. The previously hot side starts to cool and an oscillation builds up of large amplitude. As Hughes says (p.268) this is not so much an additional (and undesirable) environmental torque but an existing (i.e gravity- gradient) torque which must be recalculated. If the entire boom is bent over in an arc then the tip mass is not where it ought to be ! Numerous technical fixes have been proposed (see Hughes also p.337). Since on UOSAT-2 we have not experienced any catastrophic libration effects I tentatively would claim that our maintenance of a continual Z-spin has prevented it happening.

On the other hand we are vulnerable to any persistent geometric error or distortion in the deployment of the tape boom. Evidence for a consistent 'bend' away from the nominal Z axis is shown in latest figures 6.3.2-17 and 6.3.2-18, which is discussed at the end of chapter 6

3.8.4 Internal torques

As a matter of observation we see a tendency in the stabilised UOSAT-2 to change its spin state. This is not consistent. But typically from a spin period of 5 - minutes it could take a week - if it were not dealt with - to accelerate steadily to a period of 1 minute or less. Sometimes it has been observed to go the other way. Irrespective of the cause we have to deal with it by magnetorquing.

3.8.5 Engineering context

Sections 3.8.1 through 3.8.4 list a substantial number of effects which potentially de-stabilise UOSAT-2. And as a matter of empirical fact if UOSAT-2 is left alone - as operationally we had to do on occasions - then it has destabilised.

Phenomena from 3.8.1 through 3.8.3 would seem to indicate that all these effects exist largely because of the interactions on the boom used to deploy the tip mass. Why not deploy it less distance (than the estimated 7 metres) or why deploy the tip mass at all ?

The argument for deploying the tip mass on UOSAT-2 to about 7 metres was in order to create a two orders of magnitude ratio in I_T/I_Z . And the reason for doing that was to seek a reasonable insensitivity to 'tumbling'. Essentially this ratio of I_T/I_Z was chosen to keep a small roll bias while maintaining an easily measurable and controllable yaw drift state (see Eq.3.4.4.4).

The satellite configuration can therefore tolerate a spin period, in the range of 10 minutes to 1 minute, which is both easily measurable relative to the local B vector and controllable without recourse to absolute attitude determination (see section 5.1). In addition a large I_T/I_Z is necessary in order to tolerate changes in the spin, caused by necessary spin-rate manoeuvres (3.8.4).

If however the Z spin rate can be reduced by an order of magnitude - to a period in the order of one hour - then in principle a much lower ratio of I_x/I_z can be admitted - and gravity gradient stabilisation achieved with a far shorter boom deployment, thus hopefully reducing the effects 3.8.1 through 3.8.3. Thus reducing to about 1.5 metres deployment, and yielding an $I_x/I_z \approx 12$ and calculates equivalent sun pointing errors of 0.12° , and between 1.8° 0.9° aerodynamic errors, as a directly comparable figures - a modest improvement (NOTE).

However this was not thought to be a sensible option for UOSAT-2, partly because its thermal design expects a 'barbecue' mode for a minimum thermal gradient and a significant Z spin needs to be maintained.

The final pointing error given a control loop cannot, it seems to me, be easily established a-priori. For one thing it depends crucially on the proposed mode of control. If this control is a simple *damping*, then a proportional rate control torque is being added to the basic libration equations. Such an approach can clearly do nothing about a systematic bias (d.c level) from any disturbance torque.

Simple damping has been the method used historically for gravity-gradient stabilisation. The active magnetorquing described and implemented in chapters 5 and 6 is still effectively a damping technique, being sensitive only to attitude velocity (and achieved without attitude determination).

NOTE Another option for reducing inherent error, on future missions, is to increase the tip mass: the increase in gravity-gradient torque greatly dominates over the minor increase in cross-section. The obvious penalty is the loss of volume and mass to an otherwise useless lump of metal. The same calculations show a useful reduction in pointing error if for example a 4 kgm tip-mass had been employed (rather than 2.6 kgm). Comparable figures again at 1.5 metres deployment of radiation 0.1° , 0.6° to 0.4° aerodynamic. The even more obvious option of attempting some form of *geometric compensation* (to cancel out sun and aerodynamic torques - for example by deploying two booms up and down - is probably not a cost-effective option

The situation is in principle quite different if one can provide additional *position* control in addition to rate control.

I have started to consider this matter, assuming magnetorquing as the additional control element in sections 7.5 and 7.6). Real-time full 3-axis attitude determination is now required. In principle the pointing error can be reduced inside the 'limits' set by the bias terms roughly established above, since the system knows what the pointing error is at any time and can assist the gravity gradient effect to overcome it.

Section 7.5 and 7.6 shows that position plus rate control by complementary magnetorquing will naturally achieve a full 3-axis stabilisation or greatly improve the inherent 3-axis stabilisation (NOTE) .

If full 3-axis stabilisation is achieved then by definition there is no residual Z spin; and in this context the boom length might be substantially contracted, working at a much lower I_T/I_Z ratio. In which case one might hope to reduce the perturbations encountered from a long boom length. On the other hand, even with this advanced use of magnetorquing, one cannot expect to dispense with the gravity-gradient effect entirely. To some extent a 'dumb-bell' object must be created in the satellite. This is because magnetorquing can only generate a torque vector $\underline{N}$ which is orthogonal to $\underline{B}$ (see discussion in section 5.4). Passive gravity-gradient torque is needed 'in the background', in order to complement the magnetorquing, however comprehensive the attitude determination and the control algorithms.

NOTE Section 7.5 continues with the UOSAT-2 type geometry which has no inherent yaw stiffness. Section 7.6 deals with an arbitrary asymmetric satellite which nominally achieves full 3-axis stabilisation by gravity gradient effect alone.

3.8.6 Flexible body dynamics

All dynamic equations used in this thesis assume a rigid body. This is fundamentally an approximation.

Complete equations, in which 'everything is put in', are useful only for numerical computation - they do not provide conceptual insight. I am guided by Hughes to understand that one can adopt rigid-body assumptions as long as one remains aware of the limitations. For one thing the flexing in a structure dissipates energy; and that the effect of energy dissipation can be assessed analytically. The rigid-body equations cannot comprehend this effect. The main benefit of dissipation analyses is to confirm whether or not notionally stable states are in fact. Hughes' analysis confirms the 'common-sense' intuition that the rigid 'dumb-bell' satellite is genuinely stable in the expected two vertical configurations but not horizontal (see p.302).

In section 7.6 I look further at the role of magnetorquing in combination with an asymmetric satellite which (nominally) can achieve 3-axis stabilisation if controlled only by gravity gradient alone. It is well-known that a consequence of unequal moments of inertia $I_x \neq I_y$ is a form of yaw stabilisation. Hughes' very useful analysis shows that there are strong restrictions, because of internal flexing, on the choice of different moments of inertia, and effectively only four stable attitudes for a given geometry (see section 7.6).

The other consequence of internal structural flexing he considers is an increase in the number of internal degrees of freedom. This apparently has a bearing on the inherent stability of spin-stabilised and dual spin stabilised spacecraft - but is not within our province

3.9 List of Symbols

X, Y, Z	Body axes of spacecraft
I_x	Moment inertia about X axis
I_y	Moment inertia about Y axis
I_T	Common transverse moment inertia
I_z	Moment inertia about Z axis
X_o, Y_o, Z_o	Axes in orbit reference co-ordinate system
$\underline{n} \quad \underline{x}_o$	unit vector orbit normal
$\underline{v}$	unit vector forward velocity
$\underline{y}_o$	unit vector backward velocity
$\underline{r} \quad \underline{z}_o$	unit vector to zenith
X_o, Y_o, Z_o	<i>Alternative reference axes magnetic definition</i>
x_o	unit vector normal to momentum $\underline{L}$ and $\underline{B}$
y_o	unit vector in plane of $\underline{L}$ and $\underline{B}$
z_o	unit vector in direction of $\underline{L}$ vector
γ	inclination (libration) angle
δ	tilt angle
ϕ	yaw angle
$p \quad (a_p)$	pitch angle
a_{pi}	inertial pitch angle
$r \quad (a_R)$	roll angle
$\underline{l}, \underline{m}, \underline{p}$	unit vectors of intermediate rotations
z_v	component of $\underline{z} \cdot \underline{v}$
z_R	component of $\underline{z} \cdot \underline{r}$
z_N	component of $\underline{z} \cdot \underline{n}$
$\underline{L}$	Angular momentum
ω_A	body nutation rate
ω_B	inertial nutation rate
θ	nutation angle
ω_z	Z-spin rate
ω_T	T-spin rate
ω_o	orbit rate
ω_N	Precession rate

N_{gg}	Gravity gradient torque
$\langle N_{gg} \rangle$	Secular averaged torque
r_0	constant bias (dc) of roll angle
r_1	roll without offset (dc) value
ω_p	pitch oscillation rate
ω_R	roll oscillation rate
ω_1	first value of pitch rate
ω_2	second value of pitch rate
p_1, p_2	two values of pitch angle
p_m	pitch amplitude
ω_{T1}	transverse T-spin relative to orbit co-ordinate system
γ_s	stationary libration angle
δ_s	stationary tilt angle
U	arbitrary energy constant
n_z	normalised Z-spin
n_{T1}	normalised T-spin relative to co-ordinate system
I	normalised moment of inertia
$\dot{\alpha}$	roll rate
$\dot{\beta}$	pitch rate
A, B, C	calculated intermediate steps in bound calculation
z_{NO}	bias or offset in projected roll on $\underline{n}$ vector
z_N	axis of elliptical plot in $\underline{n}$ vector
z_v	axis of elliptical plot in $\underline{v}$ vector
$\ddot{\alpha}$	roll acceleration
$\ddot{\beta}$	pitch acceleration
$y[J]$	computer variables in Runge Kutta solution
$f[J]$	first order derivatives in Runge Kutta solution
I_{t_dot}	rate of change of transverse M of I for boom deployment
I_t	transverse moment of inertia

4 ATTITUDE DETERMINATION

This chapter shows how the attitude of UOSATs may be inferred from magnetic vector measurement, as obtained from a 3-axis magnetometer. The topics discussed are

- * (4.1) the general problem of attitude determination solely from a magnetometer measurement
 - * (4.2) Spinning/nutating state observed in the short term , where the angle and magnitude of the magnetic field to the spinning body is assumed to be constant
 - * (4.3) qualitative description of magnetic field over a polar orbit, the dipole approximation and the 'law of cycles'
 - * (4.4) libration measurement in the gravity-gradient captured state
 - * (4.5) in-flight magnetometer recalibration in the gravity-gradient captured state by non-linear statistical processing
-

The spinning case is considered here in simulation (but there are some actual records for UOSAT-1 described in chapter 6). The point of this work was important historically for UOSAT-1. For other, future applications the results given here will be useful in interpreting attitude at early stages in mission deployment. Even with more sophisticated spacecraft carrying a variety of other sensors, it is frequently the case that immediately after spacecraft deployment the commissioning of the various sub-systems can be a prolonged process; in which case the information available from sun-sensors, and horizon sensors, let alone gyroscopes and star sensors might not be immediately available. The great advantage of the magnetometer reading is of course its ability to function in all possible attitudes and independently of any illumination conditions. The ability then to make deductions about the attitude within minutes of switch-on under any conditions whatsoever is extremely useful. I am not aware of other systematic work which relates the spinning/nutating state directly to magnetometer measurements. A much simpler analysis was still a topic deemed worthy of presentation at a recent conference (Wilkinson 1988).

The second case, of libration measurement under gravity-gradient control, being the most important, is illustrated directly from records. What is presented is believed to be a unique 'method of bounds' which exploits the polar orbit of UOSAT-2 and the fact that the magnetic field orientation lies close to the orbit plane.

Finally in-flight recalibration is an important matter in order to overcome the problems of offset/scaling error accumulating after years of operation. The method is successful and specialised to this special case of gravity gradient control with a simultaneous yaw rotation.

4.1 The problem of Attitude Determination

A 3-axis magnetometer cannot be used to measure instantaneously the attitude of any one axis of a satellite except when one of these axes is aligned parallel or anti-parallel to the current magnetic vector.

This work aspires to be a compendium of useful information in addition to presenting absolutely new results. A good starting point is one which has to be explained to all engineers who are not directly concerned with attitude determination work. Without exception the intuitive belief is that if there exists a 3-axis magnetometer, which is ideally accurate, and knowledge of the field external to the spacecraft related to some known co-ordinate system, then somehow the spacecraft's attitude is properly and uniquely determined. This is of course quite untrue; (NOTE) and it may be of interest to consider a simple way of refuting this mistaken belief. Reference to fig.4.1-1 clarifies the point.

NOTE In principle full 3-axis attitude determination is possible if a state estimator can be described which updates the attitude of the spacecraft according to a known model. Results achieved by this approach are given in chapter 7.

Let there be a known $\underline{B}$ vector and an attitude of the spacecraft such that equal component quantities are measured in all three channels i.e. $B_x = B_y = B_z$. It is then clear that the craft could be in a state of rotation 'around' the $\underline{B}$ vector and the component field values would be exactly the same. And this arbitrary and undetermined rotational state is true for any component values. There is however an important and useful special case. If B_x and B_y values are relatively small to zero the Z-axis is then close to, or aligned on, the $\underline{B}$ vector (or its inverse). In this condition the Z axis is then known to lie somewhere on the surface of an 'uncertainty cone' (fig.4.1-2) of a known small angle; and the smaller this angle the better. The angle of the Z-axis is also well-determined relative to the vertical if and when the $\underline{B}$ field is known to be closely aligned to the vertical. The combination of these two simple ideas has been exploited in a powerful way in recent UOSAT2 operations (see from section 4.4). Of course by 'attitude' being closely determined one can only refer to that particular Z axis.

4.2 Spinning/nutating State

All the kinematic parameters of the nutational/spinning state may be measured from simultaneous plot of three components of the magnetic field resolved on the body axes of the spacecraft. The method is best for small nutation angles. The oscillation rate in B_z gives the inertial nutation rate f_B . The nutation angle and the angle of the magnetic vector to the angular momentum vector can be computed from minimum and maximum excursions in this same B_z waveform. The mean oscillating frequency (from zero crossing count) in both B_x and B_y gives the sum of f_B and f_A the body nutation rate. From these measurements all the parameters can be calculated including the estimated ratio I_T/I_z

The parameters of nutation can be interpreted from records of the magnetometer data B_x , B_y , and B_z . These waveforms depend not only on the nutational state but also on the angle χ defined as the angle between the $\underline{B}$ vector and the $\underline{z}_0$ vector (direction of the $\underline{L}$ angular momentum), adopting the magnetic inertial reference system (sec.3.1.5).

All results given here assume that the spin rates are fast compared to the rotation of the magnetic field - which slowly changes in the course of the orbit. So the measurements are inherently approximate (but could be improved in some cases). It ^{should} be emphasised that the discussion is restricted to the case of $I_T > I_z$; which configuration is true of both our satellites.

In section 3.2 the component angular velocities and equations involving them - ω_A , ω_B , ω_z and ω_T - for the general nutation state were presented; these velocities are used here but in the Hertzian scaling i.e f_A , f_B , f_z and f_T .

The set of figures 4.2-1 illustrate typical plots of the magnetic field components to be seen with different combinations of the angle X and the nutation angle θ . The underlying equations are well-known but are repeated for convenience in Appendix E.

In fig.4.2-1a $X = 60^\circ$ while the nutation angle $\theta = 30^\circ$. The ratio $I_T/I_z = 2.8$ corresponds roughly to UOSAT-1 geometry with the boom not extended. The B_z component has an exact sinusoidal appearance (plus a d.c offset) : while the B_x and B_y components are approximately sinusoidal, being subject to modulation. Although modulated these components maintain a quadrature phase relationship, with B_x lagging B_y in this case. The frequency of the a.c component in B_z is the inertial nutation rate f_B . Analysis shows (see Appendix E) that the waveforms B_x and B_y are the result of the summation of three different frequencies, comprising

body nutation rate

$$f_A$$

difference

$$f_D = f_B - f_A$$

...4.2.1

sum

$$f_S = f_B + f_A.$$

...4.2.2

In this plot the sum frequency appears to capture the other two because its amplitude dominates the others. The mean zero crossing rate will

estimate this sum frequency directly. This sum frequency is similar but not identical to the Z spin rate of the satellite. Taking Eq.3.2.1.2 and changing all angular frequencies ω to Hertzian frequencies f then

$$f_z = f_A + f_B \cos(\theta)$$

...4.2.3

which is the same as f_B only for zero nutation angle. If this particular pattern is termed 'type I' then a quite different pattern - type II - can be generated (NOTE).

In fig.4.2-1b the inertial nutation rate f_B is kept the same as in fig.4.2-1a but the value for nutation angle and $\underline{B}$ angle have been interchanged ($\theta = 60^\circ$ and a $\underline{B}$ angle $\chi = 30^\circ$). Appendix E shows why the B_z plot is unchanged. Since the nutation angle is larger the body nutation rate f_A must be slower for the same rate f_B , according to Eq.3.2.2.1 here repeated (after conversion to Hertzian)

$$f_A = (I_T/I_z - 1) f_B \cos(\theta)$$

...4.2.4

but in any case the the pattern of B_x and B_y waveforms is quite different. The key geometrical difference is that in type I case the $\underline{B}$ vector points *outside* the nutational cone while in the type II situation it is *inside* (see Fig.4.2-2). While the three frequencies still exist as components of the B_x and B_y waveforms it may be shown that their relative amplitudes are quite different. Generally in this type-II case the body nutation rate f_A dominates the oscillation but only provides a qualitative basis for measurement unless special precautions are taken. A hint of the problem is already conveyed in fig.4.2-1b where one can see at point A the waveform starting to 'double back' on itself in an extra local oscillation. When this local wriggle occurs near B_x or $B_y = 0$ then a higher rate of zero crossings is greater than would be expected from the rate f_A alone. It is shown below that this problem is worsened with increasing nutation angle.

NOTE this designation type I and type II is the author's.

The distinction between type I and II case should be quite clear. Given an orbital plot and a low nutation angle it is best to check the record only in the type I case. Clearly the method gives the greatest opportunity for observation the smaller the nutation angle.

From section 3.2 the inertial nutation rate f_n is *defined* to be a positive quantity. In the previous figures the Z-spin rate f_z is positive. But it could of course be negative in practice. This is equivalent to setting a nutation angle greater than 90° . Thus in Fig.4.2-1c the nutation angle $\theta = 150^\circ$ while the angle of the B vector has been reset at $X = 120^\circ$. It is still a type-I case with similar-looking waveforms to fig.4.2-1a but the difference is that the B_x now leads on B_y instead of lagging. Therefore it is easy to decide if the satellite is forward or back spinning in type I case. As yet another variation let the nutation angle be 30° again (satellite forward spinning) but this time set the X angle to 120° instead of 60° (fig.4.2-1d). This is still type I - with the B vector outside the nutational cone - but the effect is to shift the d.c or average level of B_z .

The excursions in B_z allow rapid assessment of both the nutational angle θ and the B-angle X . It is readily shown that

$$\begin{aligned} B_{z\max} &= B_M \cos(X - \theta) \\ B_{z\min} &= B_M \cos(X + \theta) \end{aligned}$$

...4.2.5

where B_M is the magnitude of the total field.

From $B_{z\max}$ and $B_{z\min}$ then the angles X and θ are determinate but with a 4-fold ambiguity. That ambiguity is resolved from further information. Assuming an ample supply of whole-orbit magnetometer data it is best to wait for a region in which the configuration is type I, as can be identified by inspection of the record. And again by inspection it can be seen if the spacecraft is forward or back spinning.

For a type I plot *only*:

FORWARD SPINNING	BACK SPINNING
$0 < \theta < 90$	$90 < \theta < 180$
$\theta < \chi < 180 - \theta$	$180 - \theta < \chi < \theta$

...4.2.6

which restrictions are sufficient to resolve the values of θ and χ .

The observation of B_z gives the inertial nutation rate f_B , while the observation of B_x and B_y over a sufficient number of cycles gives the rate $f_A + f_B$. The accuracy of this obviously improves for smaller nutation angles. Having independent measurements then substitution in Eq.4.2.4 allows an in-flight check of the ratio I_T/I_Z .

Important special cases are for a motion which is either a true Z-spin or the nutation angle is very small. A type I graph will nearly always be seen in the data because the cone is so narrow that the $\underline{B}$ vector will generally lie outside it. Fig.4.2-1e plots such a situation for $\theta = 5^\circ$ and $\chi = 45^\circ$. The oscillations in B_x and B_y are nearly perfect and the residual nutation is betrayed sensitively by the modulation in the B_z waveform.

The other important special case is with the spacecraft in a near flat-spin. In nearly all cases a type II graph will be produced because the cone is so broad that the $\underline{B}$ vector lies inside it. Fig.4.2-1f is an example with $\theta = 89^\circ$ and $\chi = 45^\circ$. Some little extra interpretation is needed here. What is happening is that spacecraft is tumbling head over heels ('head' = + Z, 'heels' = - Z). If it were exactly a flat spin then Eq.3.2.2.1 says that there would be no body nutation. In which case all THREE axes would detect the body nutational frequency. Whereas with almost a flat spin there exists a relatively small body nutation rate (the spacecraft is still twisting around its Z axis). In this particular graph the Y axis happens to start off aligned very close to the Z_0 axis and it initially detects a *near-constant* value B_y . At this time the X-axis is being swept round and 'picking up' the same

inertial nutation rate as the B_z waveform. But as time elapses the B_y waveform starts to pick up oscillations in f_B while B_x start to lose it, because of the additional rotation around the Z axis. After quarter of such a rotation of the body the response in X and Y axes is inter-changed. After one cycle at the *body* nutation rate the situation returns (approximately) to this waveform. It follows that a zero-crossing count in the type II configuration is not a reliable measure of anything. On the other hand if one roughly measures the time interval between peaks in B_x or B_y (which is not so easy) then this would still be a correct measure of the body nutation rate.

4.1-1 Ambiguity in attitude determination

4.1-2 Attitude uncertainty cone

For given measurements B_x , B_y and B_z the Z-axis lies somewhere on a cone of constant angle α to the B vector

4.2-1a Plot of B field components in nutating spacecraft

4.2-1b Plot of B field components in nutating spacecraft

4.2-1c Plot of B field components in nutating spacecraft

4.2-1d Plot of B field components in nutating spacecraft

4.2-1e Plot of B field components in nutating spacecraft

4.2-1f Plot of B field components in nutating spacecraft

4.2-2 The Nutational Cone

4.3 Geomagnetic Field Modelling

The adoption of an ideal dipole model of the geomagnetic field and the assumption that in a polar orbit the magnetic field rotates once an orbit in the plane of the orbit may be very rough; but is in fact a dominating consideration in both attitude control and attitude determination. An IDPO, a TDIO, and the full IGRF models are compared. The useful 'cycle' rule is introduced

Detailed accounts of the geomagnetic field have been summarised in Wertz and elsewhere. Only matters relevant to UOSAT control are summarised here.

4.3.1 Ideal dipole and polar orbit

It is well-known that the geomagnetic field in order to be reliably characterised in precision work requires an 8th multipole model and evaluation of the field by a Legendre series expansion. A particular set of multi-pole coefficients constitutes the IGRF or International Geomagnetic Reference Field. A particular problem is the continual drift in the coefficient values. The most recent reference apparently is (Baraclough 1981)

Nevertheless a completely idealised model of a magnetic dipole is useful; and the idealisation can be taken a step further in relation to the polar orbit by supposing that the magnetic field lies entirely in the orbit plane, with the satellite passing directly over the poles. Defining an orbital 'advance' λ as an angular equivalent of a mean anomaly - but measured from an intercept over the South pole then

$$\begin{aligned} B_{x0} &\approx 0 \\ B_{y0} &\approx -B \sin(\lambda) \\ B_{z0} &\approx 2B \cos(\lambda) \end{aligned}$$

...4.3.1.1

as measured in the standard reference co-ordinate system. I call this the IDPO model. Symbol B is a reference magnitude - say 20 μT

The equations predict the well-known variation in magnitude of 2:1 from poles to equator, and showing a single rotation of the magnetic vector in the course of one orbit. In terms of this model the B vector rotates one cycle per orbit, but not a constant speed, varying between a rate $\omega_0/2$ when the satellite is over the poles to a speed $2\omega_0$ when over the equator. These are speeds relative to the local reference co-ordinate system.

In this model the B vector always lies in the orbit plane. To an imaginary observer travelling with the spacecraft the field vector is pointing downwards over the North magnetic pole, upwards over the South magnetic pole, and the field vector is horizontal over the equators. Some tests using a full IGRF model and running over a 24 hour simulation gave a figure for an inclination of the B vector out of the orbit plane as a maximum of 26° - but for most of the time it is much less.

The predicted rotation leads to a useful 'law of cycles' which is valid even for the realistic IGRF model:

1 if the B_z field describes ONE cycle per orbit then gravity gradient lock has been acquired (however rough)

2 if the B_z field describes TWO cycles per orbit then a state approximating inertial stability is being observed (Z axis pointing to a fixed point in space) (NOTE)

The 'law of cycles' is an effective and immediate check if capture by gravity-gradient has occurred - or has been lost.

NOTE Within an inertial frame of reference the local co-ordinate system rotates 1 cycle and the B vector field rotates 2 cycles. For case 2 it is always possible that the spacecraft is tumbling forwards by FOUR cycles inertially (or THREE relative to the local co-ordinate system) to give the same effect. In practice however this is an unlikely co-incidence and has never been observed by us.

4.3.2 Tilted dipole and inclined orbit

A step towards greater realism is to incline the ideal magnetic dipole by an amount which is indicated by the lower degree dipole components of the IGRF model and also to take account of the effect of the polar orbit being inclined typically by 98° orbit inclination

From IGRF list and following p.782 (Wertz Appendix H) the components of the main dipole are

$$\begin{aligned} g_2^1 &= -30109 \\ g_1^1 &= -2006 \\ h_1^1 &= 5704 \end{aligned}$$

...4.3.2.1

which implies a single dipole of strength 30710 and a tilt of the dipole away from the vertical of 11° (actually 169°)- to the nearest degree.

It is well known that the magnetic field due to a vector dipole $\underline{m}$ is given by

$$\underline{B} \propto \left\{ 3(\underline{m} \cdot \underline{r}) \underline{r} - \underline{r} \right\}$$

...4.3.2.2

where $\underline{r}$ is the unit vector from Earth centre. Computer calculation is simplest if instead of calculating B in terms of geocentric orbits and then transforming to local co-ordinates the components of $\underline{m}$ are transformed first. If m_{x0}, m_{y0}, m_{z0} are components of $\underline{m}$ which have been transformed from 4.3.2.1 to the reference co-ordinate system at a given point in the orbit it follows from Eq.4.3.2.2 that

$$\begin{aligned} B_{x0} &\propto -m_{x0} \\ B_{y0} &\propto -m_{y0} \\ B_{z0} &\propto 2 m_{z0} \end{aligned}$$

...4.3.2.3

since $\underline{r} = (0,0,1)$. This TDIO model (tilted dipole and inclined orbit) was adopted in testing the threshold deliberation package of section 5.3.

4.3.3 Usefulness of models to ADCS design

Rough though the above models are it is a fact that they act as a major motivation in designing both attitude control (using magnetorquers) and attitude determination (using magnetometers). The IDPO model (ideal dipole and polar orbit) is often implicitly or explicitly assumed - see for example a special session on magnetic attitude control (Guidance and Control Conference 1981). It was adopted as simple assumption for UOSAT-1 in the initial stages of manoeuvres, with the boom yet to be deployed (see flight experience section 6.1).

It is a useful principle to think in terms of the IDPO model for attitude DETERMINATION provided it is used only as a qualitative guide, but always basing the actual calculations on the IGRF model. For the determination technique to be described next the IDPO model suggests that narrow bounds can be calculated for the measurement of the attitude, and is achieved without actually involving any dynamic model of the attitude change.

4.3.4 IGRF field modelling

I have worked on some inherited software and I am also indebted to the assistance of Dr. P. Sweeney of this Department. In operation the user enters known orbital parameters: the date, time of start of run; and time of finish: and the FORTRAN program uses the full multipole model to calculate the reference B field in X_0 , Y_0 and Z_0 co-ordinates, having first calculated the orbital position from data provided.

4.4 Libration Measurement

A method of estimating the crucial inclination or libration angle γ by 'method of bounds' is presented with examples taken from real spacecraft data

The method constructs continuous upper and lower bounds to γ , based only on the magnetometer data and the IGRF predictions. A characteristic attitude 'signature' is generated and intermittent instants when the bounds lie close to each other and the actual γ angle then closely estimated.

The fundamental data is a list of $\underline{B}$ vector measurements compiled by the on-board computer, stored in memory over the course of a few hours ('whole orbit data') and then down-linked in the course of a pass. The start time of this data must be known (usually standardised at 00h00m by the spacecraft clock); also the data interval (4.84 seconds per address increment). The off-line processing on the ground runs as follows:

For each data sample vector $\underline{B} = (B_x, B_y, B_z)$ compute the transverse

$$B_T = \sqrt{ B_x^2 + B_y^2 } \quad \dots 4.4.1a$$

Within the IGRF model and from the vector $\underline{B}_0 = (B_{x0}, B_{y0}, B_{z0})$ compute the transverse

$$B_{T0} = \sqrt{ B_{x0}^2 + B_{y0}^2 } \quad \dots 4.4.1b$$

There are then three angles to consider

* the angle α between the $\underline{B}$ vector and the local vertical $\underline{r}$

$$\alpha = 2\text{ARCTAN}(B_{z0}, B_{T0}) \quad \dots 4.4.2a$$

* The angle β between the $\underline{B}$ vector and the Z-axis of the spacecraft

$$\beta = 2\text{ARCTAN}(B_z, B_T) \quad \dots 4.4.2b$$

* the angle γ between the local vertical (ZO axis) and the satellite body Z axis

All angles lie between the range 0 to 180°. The relation between the three angles is given in fig.4.4-1.

Now if the attitude were perfect ($\gamma = 0$) the plots of α and β should be identical - at all times $\alpha = \beta$. If they differ then the libration angle $\gamma \neq 0$. More usefully bounds γ_1 and γ_2 may be calculated

$$\gamma_1 = \text{ABS}(\alpha - \beta) \quad \dots 4.4.3a$$

$$\gamma_2 = \text{ABS}(\alpha + \beta + 180) \text{ MOD } 360 - 180 \quad \text{NOTE} \quad \dots 4.4.3b$$

Consideration of the geometry checks that

$$\gamma_1 < \gamma < \gamma_2 \quad \dots 4.4.4$$

The difference between the bounds contracts to zero, and actually measures γ if

1	$\alpha = 0^\circ$
2	$\alpha = 180^\circ$
3	$\beta = 0^\circ$
4	$\beta = 180^\circ$

...4.4.5

The aim is to measure the librational state for which a sufficient number of independent samples of γ is needed. Conditions 1 and 2 require the B vector to point up or down at some time - which condition is never likely to be maintained exactly; but it is at least likely to be close on a polar orbit.

NOTE This means simply that either $\gamma_2 = \alpha + \beta$ or $\gamma_2 = 360 - \alpha - \beta$ whichever is smaller. One solution will be less than 180°.

Conditions 3 and 4 are more subtle. They say that an attitude fix is obtained if and when the B vector is parallel or antiparallel to the Z axis (a condition looked-for in section 4.1). Such a condition is likely to be impartial to the librational state, tending to sample it randomly, and just as likely to pick up large excursions (at or near peak libration) as at small angles, again provided B comes at some times close to the vertical.

From practical results typical values of B_z and B_r in June 1986, and plotted in Fig.4.4-2a show the expected cyclical behavior in B_z of one cycle per orbit. This value and the calculated B_r are much rougher than the simple dipole models would lead one to expect, even allowing for incomplete data records.

The measured and theoretical magnitudes B and B_o according to the sum of squares can be calculated

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \dots 4.4.6a$$

$$B_o = \sqrt{B_{xo}^2 + B_{yo}^2 + B_{zo}^2} \quad \dots 4.4.6b$$

should agree (in a perfect world) since B should be independent of the actual attitude. As Fig.4.4-2b clearly shows they do not agree well, although there is rough qualitative agreement. The difference is attributed to a drift in the magnetometer calibration - how this is cured is explained in section 4.5.

The plots for a rather badly librating UOSAT-2 is given then in fig.4.4-3 for α and β and the bounds plot in fig.4.4-4 has picked up four quite clear indications of attitude:

- * Point 'A' samples an inclination angle of 22-23°.

- * Point 'B' observes a sweep of inclination from about 25° to 15°
(the plotting needs improvement here)

* Point 'C' observes a dramatic sweep IN from about 33° coming into about 9°

* Point 'D' observes a sweep OUT from about 12° to about 17°

These attitude measurements were made before the delibration trials described from section 5.3 and 6.3

4.5 Magnetometer Auto-calibration

An auto-calibration technique is demonstrated which can correct both offset and scaling on 'raw' 3-channel magnetometer data. The technique is demonstrated on real data. It is valid only for a gravity gradient controlled spacecraft with a persistent slow drift/spin in the vertical (Z) axis

Some considerable improvement was made in dealing with the magnetometer data, by an automatic new calibration.

The improvement is shown in Fig.4.5-1 (better) and then Fig.4.5-2 (best). The formulae used are given in these figures. It is conveniently seen as a two-step process. The first step depends on the existence of one cycle per orbit oscillations in B_z and a record persisting over several orbits. Referring to the formula on the figure the first step assumes a common scaling error t_1 for both B_x and B_y ; and an offset error c_0 and a scaling error c_1 in B_z . These are all estimated iteratively.

The second step depends on the existence of Z-spin in the satellite and a period to this spin of at least one order of magnitude less than the orbit period. Here (fig.4.5-2) different offset errors a_0 and b_0 are assumed in B_x and B_y respectively; and a common but *differential* scaling error d_1 in these measurements. The combination of sum and

differences in t_1 and d_1 allows independent re-scaling of B_x and B_y .

A full account is given in Appendix F. The fundamental source of information is the disagreement E between the squared magnitudes of the measured and the computed field strengths. This error is correlated in a number of different ways with various functions of the measured fields B_x , B_y and B_z to calculate t_1 , c_0 , c_1 , a_0 , b_0 , and d_1 , by simple formulae. Having estimated these the measured fields are re-calculated and iterated again; it is found that the mathematical calculation rapidly converges in a few iterations. I believe this theory to represent a novel contribution - although not of course the principle of least mean square estimation. It might be called a 'quasi-orthogonal' mean-square estimator - the form of which is specially dictated by the nature of waveforms B_x , B_y and B_z . It is quite specific to the waveforms produced by a satellite locked by gravity gradient and slowly spinning - it could not work on any other set of waveforms that did not correspond to this situation.

As a practical example the telemetry system yields a number N in the range 0 to 1000. For example, back in June 1986 the re-calibration compared to the original yielded

Channel	latest (June 1986)	original (onlaunch)
1	$B_x = (0.1468 N - 68.3)$	$(0.1485 N - 68)$
2	$B_z = (0.1435 N - 64.9)$	$(0.1523 N - 69.3)$
3	$B_y = (0.1500 N - 69.8)$	$(0.1507 N - 69)$

Note the major change is in the Z axis - both in offset and scaling.

The method automatically checks its own success; and an important point is that the good fit was achieved only by time shift in the data. This quantifies as 95 seconds (± 5). The source of the error was traced to timing problem in the software clock on-board the satellite, so that the data was not starting exactly at 00h00m as planned.

The analysis also enabled a prediction to be made of the potential accuracy in measuring the attitude. Subject to further work and a re-think my current assesement is 0.7 μ T r.m.s in any one channel. This still substantial error is almost certainly due to the known non-linearity in the magnetometer measurements made through the telemetry system, at the most negative end of the measurement. This effect can be clearly seen in the poor fit at points A and B in the repeated fig.4.5-2 coinciding with the minimal Z measurement seen in Fig.4.5-3.

The measurement error then determines a worst case pointing error of 2.8° r.m.s over the equator or 1.4° over the poles. The method would achieved much better results if it were not for a known navigation magnetometer non-linearity.

The new calibration then creates more convincing measurements of attitude. Figures 4.5-4 and 4.5-5 are directly comparable to 4.4-3 and 4.4-4 respectively.

At point A the γ angle is somewhere between 20 and 22°. From B to C there is tight bound showing the angle coming in from 20° as far possibly as 15° and then exiting out further than 30°. From D to E is seen a violent swing across from 32°, in as far possibly as 6° and then out again to at least 36°. It almost certainly went no further than 38° (point F) before swinging back in. This does not mean that the Z- axis attitude reversed its path - simply that the axis swung up to the vertical and then carried straight on *through* the vertical (or close to it). In the region from G to H it comes as close possibly as 13° before going back out again - point I.

The fit of data is still not as good as one would like: and efforts are being to improve it. In particular the computer can be used to read the parallel port which is known to have a more linear characteristic.

4.6 List of Symbols

save nt4c side2 disc 1 linespacing 45 216

B_x, B_y, B_z	Components of magnetic field in body axes
f_A	Body nutation rate (Hz)
f_B	Inertial nutation rate (Hz)
f_z	Z-spin rate (Hz)
f_T	T-spin rate (Hz)
f_D	Difference frequency (Hz)
f_S	Sum frequency (Hz)
χ	Angle between $\underline{B}$ vector and $\underline{L}$ vector
θ	Nutation angle
λ	Orbital advance
B_{x0}, B_{y0}, B_{z0}	Components of magnetic field in ref system
B	Scaling value of magnetic field OR general magnitude
m_{x0}, m_{y0}, m_{z0}	Components of magnetic moment in ref system
α	Angle between $\underline{B}$ vector and $\underline{r}$ vector (vertical)
β	Angle between $\underline{B}$ vector and $\underline{z}$ vector
γ	Angle between $\underline{z}$ vector and $\underline{r}$ vector
y_1	lower bound on y
y_2	upper bound on y
B_m	Magnitude of calculated field

4.4-1 Attitude measurements II

Attitude angles

4.4-2a B_z and B_T plots

4.4-2b Magnitude plots

— measured
 model

4.4-3 Plot of $\underline{B}$ to vertical α and $\underline{B}$ to Z axis β

4.4-4 Bounds on attitude $\underline{Z}$ to local vertical y_1, y_2

4.5-1 First stage of flight calibration

THEORY: Treat X, Y, and Z imperfect data which is modified in this first stage

$$\hat{B}^2 = (X^2(1 + t_1)^2 + Y^2(1 + t_1)^2 + (Z + o_0 - c_1 Z)^2)$$

and it is proposed to minimise the mean square error

$$E^2 = (B^2 - \hat{B}^2)^2$$

where B is assumed to be the true field. Simple differentiation establishes

that orthogonality conditions are $\langle EZ \rangle = 0$ $\langle EZ^2 \rangle = 0$ $\langle ET^2 \rangle = 0$

where $T = \int (X^2 + Y^2)$

and these orthogonality conditions are easily found by iteration

Assuming formulae $X = (n_x - o_x) s_x$, $Y = (n_y - o_y) s_y$, $Z = (n_z - o_z) s_z$

then the iterative corrections are

$$s_x = s_x' / (1 + t_1) \quad s_y = s_y' / (1 + t_1) \quad s_z = s_z' / (1 + c_1) \quad o_z = o_z + c_0 / s_z$$

found by solving the equations

$$\begin{aligned} \langle EZ \rangle &= 2 c_0 \langle Z^2 \rangle \\ \langle EZ^2 \rangle &= 2 c_1 \langle Z^4 \rangle + 2 t_1 \langle Z^2 T^2 \rangle \\ \langle ET^2 \rangle &= 2 c_1 \langle Z^2 T^2 \rangle + 2 t_1 \langle T^4 \rangle \end{aligned}$$

Convergence very rapid on sample data given.

4.5-2

Second stage of flight calibration

THEORY In addition to the model given with Fig 6-1 Treat X Y and Z as imperfect data which is to be modified

$$\hat{B}^2 = (X - a_0 - d_1 X)^2 + (Y - b_0 + d_1 Y)^2 + Z^2$$

Again minimising the mean square error dictates the orthogonality condition

$$\langle EX \rangle = 0 \quad \langle EY \rangle = 0 \quad \text{and} \quad \langle E(X^2 - Y^2) \rangle = 0$$

The iterative corrections are then

$$s_X = s_X / (1 + d_1) \quad s_Y = s_Y / (1 - d_1) \quad o_X = o_X + a_0 / s_X \quad o_Y = o_Y + b_0 / s_Y$$

found by solving the equations

$$\begin{aligned} \langle EX \rangle &= 2 a_0 \langle X^2 \rangle \\ \langle EY \rangle &= 2 b_0 \langle Y^2 \rangle \\ \langle ED^2 \rangle &= 2 d_1 \langle D^4 \rangle \end{aligned}$$

$$\text{where } D = \sqrt{X^2 - Y^2}$$

This is combined simultaneously with all the other work of Fig 6-1 (see program B.RMproc2 - the earlier B.RMproc1 carries only the first stage.) The method relies on a spin 2- 6 minute period of the spacecraft to generate sufficient orthogonality in the data. $X^2 - Y^2$ is then the second harmonic of the spin.

4.5-3a B_z and B_T plot

4.5-3b Magnitude plots

4.5-4 Plot of β to vertical α and β to Z axis β

4.5-5 Bounds on attitude Z to local vertical y_1, y_2

5 APPLIED TORQUE - MAGNETORQUING

This chapter describes all the various magnetorquing techniques used on the two UOSATs, in three distinct modes: spin-stabilised, magnetically stabilised and gravity-gradient stabilised. In the first and last modes, magnetorquing modifies the stabilised state: in the second it is the sole influence. A mix of old techniques (in a new format) and claimed new techniques are itemised. Supporting diagrams emphasise a geometric interpretation. The chapter finishes with a critical review of the CPL law (Cross Product Law) and suggests a possibly new concept linking magnetorquing to magnetometer measurements - the Reciprocity Principle

Although a compact, detailed treatment of magnetorquing is given in various papers (reviewed in chapter 8) and also from p.644 of (Wertz 1978), the treatment here is believed to embody a unique unified approach; with some but not all techniques being original (NOTE).

A completely self-contained account is given here. In section 2.5.2 the control aims with UOSAT-1 were described. From a spin-stabilised state the requirement was to slow the spin rate, change the attitude of the spin-stabilised spacecraft before boom deployment, and damp out nutation in the course of these manoeuvres. The control aims with UOSAT-2 were different. Initially a direct magnetic stabilisation was attempted, with the Z-axis tracking the geomagnetic field. On conversion to stabilisation by (passive) gravity-gradient control, after extending the boom, the aim was to implement active libration damping. Since UOSAT-2 has a tendency to increase naturally its Z-spin in this state, in addition to building up libration it is also necessary to maintain Z spin state. Magnetorquing has a fundamental role at all stages in the control.

NOTE Wertz' treatment introduces the concept of energy minimisation and a theorem based on such a criterion. I am doubtful that this is a useful approach - at least not on UOSAT type spacecraft- because the limited duty cycle and occasional use of the magnetorquers does not make it a major consumer of the power budget

I have not heard of any other satellite attempting gravity-gradient control with active magnetic damping - other than Mobley (1981).

The magnetorquing manoeuvres fall therefore into three classes

- 1 modification to the spin-stabilised state (5.1)
- 2 magnetic stabilisation (tracking) (5.2)
- 3 damping (delibration) in the gravity gradient stabilised state (5.3)

Although all 'magnetorquers' are mechanically the same - coils of wire wound on the edges of each face for UOSAT-2 - there is an important distinction in classification, depending on the orientation of the magnetic moment. Referring back to section 2.2.3 the two geometries were distinguished - spin-axis and spin-plane. The former creates a magnetic moment M_z in the direction of the controlled Z-axis while the latter creates a moment which is orthogonal to it. UOSAT-1 has only a spin-plane magnetorquer. There are two spin-plane magnetorquers on UOSAT-2 with orientation in X and Y, creating moments M_x and M_y . All torquers are switchable on or off with either polarity; so these moments can be effectively assigned values +1, 0 or -1 in arbitrary units.

While some of the algorithms given here are well-known, those stated for precession and nutation control in the spin stabilised state using the *spin-plane* magnetorquer are believed to be novel. A new algorithmic principle for slewing the spin-axis into the orbit plane is given ; and the well-known QOMAC principle compared to it. The magnetic tracking concept may be new ; and the simple delibration control is also believed to be new. In order to provide a coherent theme and provide a basis for comparison it is necessary to include the well-known algorithms - although they are formulated differently. As will be apparent they are almost always related to direct measurements from an associated 3-axis magnetometer, so requiring a minimum of signal processing and computer

support. The practical reason was that, at least until recently, the software needed for the OBC (On Board Computer) on UDSAT-2 had not been developed sufficiently to support a high-level language. It is suggested that the need of the same computer to do other tasks (support experiments, data collection and overall management) will often favour simple ADCS solutions such as these.

The chapter finishes with a comparative assessment of the so-called Cross Product Law (in two versions) and finishes with what is believed to be a more useful observation which I call the Reciprocity Principle. It aims to show the intimate connection between attitude control and attitude determination when both are magnetic.

Mathematical proof for the various algorithms is provided in APPENDIX G.

5.1 Spin-stabilised Control

All combinations of techniques are presented for spin rate control; precession; denutation; and combined precession with denutation control, using both spin-axis and spin-plane torquers.

5.1.1 Z despinning

ASSUMED STATE: Z-spin
 ASSUMED CONTROL: spin-plane magnetorquer
 CONTROL AIM: reduce the spin rate

Choosing just one torquer then arbitrarily the X axis is chosen. With a magnetic moment M_x then the torque components created are

$$N_z = M_x B_y \quad \dots 5.1.1.1a$$

and

$$N_y = - M_x B_z \quad \dots 5.1.1.1b$$

For a satellite which is simply spinning (about its Z axis) then a sinewave is generated in the components B_x and B_y as detected in the 3-axis magnetometer. The de-spinning N_z torque requires then the polarity of M_x to commutate on the sign of B_y . The rule

$\{ M_x \} = s \{ dB_x/dt \}$	$B_x \approx 0$, $B_z \approx 0$	NOTE 1
$s = +1$ increase		...5.1.1.2
$s = -1$ decrease		

will increase the *magnitude* of the spin ($s = +1$) or decrease its magnitude ($s = -1$) irrespective of the *sign* of this spin. If it is rapidly spinning (tens of seconds periods to 1 minute) then the B field rotation during the course of an orbit can be ignored and as far as the user is concerned there is an absolute change in the spin rate.

The maximum torque occurs on a maximum or minimum in B_y , coinciding with a zero-crossing in B_x . Rather than timing the magnetometer to come on for very small fraction of the Z spin period the most efficient use requires a near 100% duty cycle and effectively commutation, or at last controlled on/off operation, approaching 50% duty cycle. Fig.5.1.1-1 illustrates this. The additional restriction $B_z \approx 0$ is usually desirable but not always essential to keep the torque $N_y \approx 0$ and to eliminate unwanted attitude change or nutation.

On a polar orbit, for the spin stabilised state; and for any attitude, this opportunity occurs at least four times per orbit (NOTE 2)

NOTE1 For greater clarity I use the symbolism $\{ \}$ as a function to indicate the *sign* of the interior quantity. Thus $\{ M_x \}$ has a value only of +1 or -1

NOTE2 The Z despinning algorithm may be used without alteration in the gravity gradient controlled state. In which case the opportunity occurs TWICE.

It is important to get some feel for the rates of possible acceleration or de-acceleration. Without significant magnetic material the magnetic moment can be calculated simply by multiplying the number of turns by the area by the current. i.e

$$M_x = n S I$$

...5.1.1.3

where

n = number of turns
S = coil area
I = current

An orthogonal orientation to the local B field gives a maximum torque

$$N = M_x B$$

...5.1.1.4

where B is the magnitude of the magnetic field

Ideally with the spinning satellite and commutating every half cycle (a full 100 % duty ratio) then the average torque is

$$N_{av} = \frac{2 M_x B}{\pi}$$

...5.1.1.5

So the rate of de-acceleration is at best (NOTE)

$$d\omega_z/dt = \frac{2 M_x B}{\pi I_z}$$

...5.1.1.6

where I_z is the moment of inertia on the Z-axis

NOTE Of course both the X and Y spin plane torquers could be used in alternation, commutating every quarter cycle. The average torque would improve by simple calculation from $2/\pi = 0.64$ of peak to $2 \sqrt{2}/\pi = 0.9$ of peak - a useful increase.

A physical interpretation is best given in the time taken to bring to a dead halt in space from a given spin rate. Analysis gives

$$T_{\text{DESPIN}} = \frac{\pi^2 I_z}{T_z M_x B} \quad \dots 5.1.1.7$$

where T_{DESPIN} is the time to despin and T_z is the initial spin period.

Using the figures of sec.2.1.2 the value of M_x or M_y calculates to 10.1 A metre² (say 10 A metre²). A suggested figure for I_z was 1.1. This was estimated from consideration of the various masses distributed in the satellite. Since the purpose is only order of magnitude calculations let the magnetic moment be 10, the moment of inertia $I_z = 1$, and the value of the ideal B field is assumed to vary from 20,000 over the equator to 40,000 over the poles. Two cases are considered with and without possible sign reversal of the magnetorquer.

Continuing the example and supposing that the initial Z spin period $T_z = 60$ seconds then T_{DESPIN} can be tabulated (in secs)

TIME TO DESPIN

duty factor	polar	equatorial
100%	411	822
50%	822	1644

These figures of course assume most favourable B vector orientation

5.1.2 Attitude change by precession

ASSUMED STATE: pure Z-spin
 ASSUMED CONTROL: spin-axis OR spin-plane magnetorquer
 CONTROL AIM: slew the spacecraft without changing Z-spin

Using a *spin-axis* torquer is simple : with a M_z magnetic moment firing the Z torquer has no effect on the Z spin.

Then set

$$\{ M_z \} = - s \{ \omega_z \}$$

...5.1.2.1

s = sign of precession

As is well-known the Z -axis starts to describe a cone around the direction of $\underline{B}$, attempting to maintain the initial angle β between vectors $\underline{z}$ and $\underline{B}$. Following usual convention a positive s value implies $\underline{z}$ rotating in a right-handed corkscrew around $\underline{B}$. Of course in the polar orbit the angle of $\underline{B}$ is constantly altering - so that the cone of precession is not maintained. The angular velocity of the precession due to magnetorquing, derived in Wertz p.637 and adapted to my symbols, is

$$\Omega_N = \frac{M_z B}{L}$$

...5.1.2.2

This quantity is not very informative in my view since the cone of precession is never completed in normal operations. More useful is the idea of a slew rate Ω_B which describes the rate of change of the momentum vector.

This slew rate is just

$$\Omega_B = \frac{dL/dt}{L} = \frac{M_z B \sin(\beta)}{\omega_z I_z}$$

...5.1.2.3

The smaller the precessional cone the less actual coverage there can be of the attitude sphere. The slew rate equals the precession rate only for $\beta = 90^\circ$.

I define a figure of merit μ in which the slew velocity is multiplied by the radius of the *small circle* traced out in precessing state (if the cone were ever to complete). This radius is just $\sin(\beta)$ - expressing the idea that as β approaches 90° so the coverage of the attitude sphere is maximised (and at $\beta = 90^\circ$ it describes a great circle).

Figure of merit

$$\mu = \frac{M_z B \sin^2(\beta)}{\omega_z I_z}$$

...5.1.2.4

This figure of merit quantifies the point that the most effective manoeuvring occurs with $\beta = 90^\circ$.

Using a *spin-plane* torquer is more complicated. The idea is to keep the satellite steadily spinning, and fire torquer pulses with alternating signs symmetrically on zero-crossings in B_y - or if not changing the sign of the magnetorquer fire on a zero-crossing of B_y in one sense (Fig.5.1.2-1). The choice of polarity depends on the way one wishes to slew the satellite.

The rule is

$$\text{or } \{ M_x \} = -s \operatorname{SGN}(dB_y/dt) \operatorname{SGN}(B_z) \quad \dots 5.1.2.5a$$

$$\{ M_x \} = s \operatorname{SGN}(B_x) \operatorname{SGN}(\omega_z) \operatorname{SGN}(B_z) \quad \dots 5.1.2.5b$$

$$s = \text{sign of precession}$$

Reference to figures 5.1.2-2 show how this algorithm works. The figures use the magnetically defined reference system of sec.3.1.5. To repeat, the direction in which the angular momentum $\underline{L}$ points defines a reference $\underline{z}_0$ vector (just as was assumed in the nutational analysis earlier). The cross-product of $\underline{B} \wedge \underline{L}$ defines a reference $\underline{x}_0$ vector. And a final reference vector $\underline{y}_0$ completes the set (NOTE).

NOTE the same symbolism is used for the Earth-referenced co-ordinate system . Since they are never used simultaneously, there should be no confusion.

In this convention the component B_{Y_0} is always positive, but the component B_{Z_0} can be either sign. In order to precess the satellite without causing a spin change a torque is directed along the $\underline{x}_0$ direction (as in all these figures) - or the inverse of this. The former state will be defined as a positive precession while the latter defines a negative precession. In either case the $\underline{L}$ vector will start to perform a cone in space, maintaining the current angle to the magnetic field $\underline{B}$.

As the satellite spins, and using an X-axis spin-axis magnetorquer the optimum condition occurs with the X-axis aligned on $+Y_0$ or $-Y_0$ (the figures only give the former case). The four diagrams give the four possible conditions for positive/negative B_{Z_0} and positive/negative spin. For a negative spin the $\underline{z}$ vector opposes the $\underline{z}_0$ vector. The Eqs.5.1.2.5 give the correct sense of precession for any arbitrary geometry and sign of spin. An exact justification is given in Appendix B

The effectiveness of precession by spin-plane magnetorquing is not the same as by spin-axis magnetorquing; for if $\underline{B}$ were pointing in the direction of $\underline{L}$ ($\beta = 0^\circ$) there would be maximum of torque but it would get nowhere, with the precessional cone having zero radius; yet if it were pointing in the Y_0 direction there would be no torque. The 'figure of merit' idea expresses this point simply:

The slew rate is now

$$\Omega_B = \frac{dL/dt}{L} = \left\{ \frac{2}{\pi} \right\} \frac{M_x B \cos(\beta)}{\omega_z I_z} \quad \dots 5.1.2.6$$

The factor $2/\pi$ is the same spatial averaging effect as applied in the spin change of sec.5.1.1. In principle it could be improved to $2\sqrt{2}/\pi$ using X and Y commutation. Figure of merit is now

$$\mu = \frac{2 M_x B \sin(\beta) \cos(\beta)}{\pi L} = \frac{M_x B \sin(2\beta)}{\pi \omega_z I_z} \quad \dots 5.1.2.7$$

This result quantifies the intuitive idea that the best starting attitude is to have the Z axis vector pointing in the region of about $\beta = 45^\circ$ (or $\beta = 135^\circ$) to the B vector.

Of academic interest is the fact that the formula for precession - attributed to Goldstein - is not exact - as has been pointed out already in sec.3.4.3 in connection with gravity-gradient precessional tracking. An exact derivation (appendix H) shows a neat convergence on the ordinary nutational state.

5.1.3 Orbital precession criteria

A special requirement with UOSAT-1 was to slew the spin axis (Z axis) away from its natural orbit-normal direction and direct it into the orbit plane, requiring a slew in the order of 90° . Using the attitude definition of this work the aim was to change the *tilt* angle from near 0° (or 180°) towards 90° . The method relies on the fact that the magnetic field is rotating and that a detectable change in B_z can be seen in a few minutes. The rule works in an ideal IDPO case (sec.4.3.1).

TO INCREASE TILT ANGLE	$s = -$	$\text{SGN}(dB_z/dt)$
TO DECREASE TILT ANGLE	$s = +$	$\text{SGN}(dB_z/dt)$

...5.1.3.1

and will work better if normalised $B_z \rightarrow b_z$ so that the dependence is on purely an angular change - not on the changing magnitude of the B vector .

This is justified from reference to Fig.5.1.3-1. The spacecraft is depicted as having its Z axis in the plane of the paper which is also the plane of the orbit. In other words the tilt angle near to 90° is considered as having been achieved; and the aim is to increase or decrease it. The orbit normal is directed into the paper.

The vector $\underline{B}$ therefore rotates as shown. The two diagrams correspond to a positive and negative spin sense. In the latter case the $\underline{L}$ vector is directed anti-parallel to the $\underline{z}$ vector. The direction $\underline{B}_0$ gives the strongest precessional effect using the spin-axis magnetorquer. The vicinity of directions $\underline{B}_1$ and $\underline{B}_2$ give the strongest effect using a spin-plane magnetorquer. In the geometry shown the sign of dB_z/dt is negative. A positive precession s will slew the Z-axis such as to increase the tilt angle; while a negative value to s will decrease the tilt. If the direction of $\underline{B}$ were reversed then opposite is the case but the rules above are still valid because the sign of dB_z/dt is now positive. The method works best when the Z axis is closest to the orbit plane.

On a computer-controlled system this would be relatively simple to program - requiring a change every quarter orbit (relative to the spin-stabilised state the field rotates twice per orbit).

The manoeuvre can also induce a shift in the absolute pitch (see section 3.1.1 for the definition); however this is not so significant since this is steadily incrementing (or decrementing) anyway under the effect of gravity gradient torque (see section 3.4.3 for theoretical explanation). In other words the manoeuvre consists of changing the tilt angle and then waiting on passive gravity gradient effect to move the absolute pitch as required..

Despite the similarity of a commutation four times per orbit the manoeuvre is not the same as the the well-known QOMAC principle(Quarter Orbit Magnetic Attitude Control). QOMAC operates in a different attitude regime - with the spin axis always close to the orbit normal (or antinormal) which in my convention corresponds to a tilt angle near 0° or 180° . The aim with QOMAC is to achieve fine control in this region - which attitude changes are best referred to as changes in *right ascension* or *declination*.

In this regime changes in dB_z/dt are not observable, or at least not reliable, and the QOMAC principle must be imposed as a *pre-calculated* phase, deduced from other orbital measurements, rather than in response to this observed local field.

5.1.1-1 Optimal magnetorquer firing for Z despin

5.1.2-1 Precession control

5.1.2-2a Optimal attitudes for precession

5.1.2-2b Optimal attitudes for precession

5.1.2-2c Optimal attitudes for precession

5.1.2-2d Optimal attitudes for precession

5.1.3-1 Polarities for required precession sign

Orbit normal $\underline{n}$ into paper. As shown a positive precession of $\underline{L}$ around $\underline{B}$ vector increases tilt angle δ (angle between $\underline{z}$ and $\underline{n}$). In both cases $dB_z/dt < 0$. From this and other geometries - to increase tilt angle precession sign

$$S = -\text{SGN}(dB_z/dt) \quad \text{INCREASE tilt}$$

$$S = +\text{SGN}(dB_z/dt) \quad \text{DECREASE tilt}$$

5.1.4 Denutation

ASSUMED STATE: Z- spin and T-spin (general nutation)
 ASSUMED CONTROL: spin-axis magnetorquer or spin-plane magnetorquer
 CONTROL AIM: remove the nutation without changing the Z-spin rate

Using a *spin-axis* torquer the rule is simple (and well-known). It is

$$\{ M_z \} = - \{ dB_z / dt \}$$

...5.1.4.1

With a M_z magnetic moment, firing the Z torquer has no effect on the Z spin.

How to use the *spin-plane* torquer required some thought. The rule is in fact

$$\{ M_x \} = \text{SGN}(B_z) \text{SGN}(B_x) \text{SGN}(dB_z/dt) \quad B_y \approx 0$$

...5.1.4.2

Fig.5.1.4-1 shows the optimal instant in every half cycle of spin when $B_y = 0$ and the body axes X and Z can be shown to be coplanar with the magnetic vector B. If dB_z/dt has a discernable value at this instant it can only be the result of a angular motion of the Z- axis, a significant fraction of which lies in the plane of the diagram. Turning on the magnetorquer with the correct sign would create a torque into or out of the paper to resist this motion. In the diagram it can be seen that M_x should be of the same sign as dB_z/dt where both B_x and B_z are positive - agreeing with the above. Arbitrarily different signs to these last two values will still produce the correct sense of torque providing this equation is obeyed.

The technique may make obsolete any requirement for passive nutation dampers on intended spin-stabilised spacecraft. A full mathematical justification is given in Appendix G.

5.1.5 Denutation + Precession

ASSUMED STATE: Z-spin and T-spin (general nutation)
 ASSUMED CONTROL: spin-axis or spin plane magnetorquer
 CONTROL AIM: remove the nutation AND precess the attitude

For the *spin-axis* torquer it is possible to combine precession control with denutation. Comparing 5.1.2.1 and 5.1.4.1 one can see to monitor the B_z component field and switch on the magnetorquer when the same value of M_z is required by both equations. This is a well-known manoeuvre

For the *spin-plane* torquer it is also possible to combine precession control with denutation. Comparing Eq.5.1.2.5 with Eq.5.1.4.2 one can again wait for the same M_x value to be required by both equations. For that desirable state it can be seen that *cycles in B_z must be enharmonic with respect to the cycles in B_x and B_y* . And from Eq.3.2.2.2 it can be seen that the ratio of spin rate to inertial nutation rate f_z/f_n is in the order of I_T/I_z for a small nutation angle. Now f_z is approximately the rate of oscillations as seen in B_x and B_y and f_n is the rate as seen in B_z . Therefore the ratio of moments of inertia for the spacecraft must not be equal or be close to an integer.

Fig.5.1.5-1 is a simulation very similar to waveforms seen with the spacecraft UQSAT-1. The polarity of the magnetorquer pulse is one which would consistently denutate. In each instant the sense of the simultaneously induced precession is indicated. Therefore all that is required is to fire the magnetorquer only when the precession is pushed in the direction one wants - reducing the duty factor to 50% or less if both signs are allowed, or 25% or less if only one sign is employed. It is believed that this simple formulation is new.

5.2 Magnetic stabilisation

Three algorithms in order of increasing complexity permit magnetic tracking; in which the +Z and -Z ends of the satellite become effective 'poles' and the satellite points in the direction established by the local magnetic field vector

ASSUMED STATE: T-spin (flat-spinning case)
 ASSUMED CONTROL: spin-axis magnetorquer
 CONTROL AIM: eliminate flat-spin and lock to the B field

The following algorithms keep the Z axis aligned on the B field. As explained in sec.2.3.2 this was deemed an essential preliminary before deploying the boom. After ejection from the Delta launcher UOSAT-2 was tumbling with no detectable Z-spin (see section 6.2). If there *had* been a significant Z-spin then the simple algorithm of 5.1.1 could have been used to eliminate it.

The procedure here turns the spacecraft effectively into a compass needle. However a continuous d.c current to the spin axis magnetorquer cannot be expected to accomplish anything, unless there exists some significant electromagnetic damping within the satellite (we tried it on UOSAT-2 - it didn't work). The need is to interrupt the current at best times in such a way that one effectively gets active and *directed* damping which would cause the Z axis to end up aligned on the B vector, having started from some arbitrary state. The first step is to decide which alignment is wanted. Specifically for UOSAT-2 the spacecraft was to be pointing correctly over the North Pole; therefore the z vector had to point in the opposite sense to B. This state I call $s = -1$; the opposite of course being $s = +1$.

5.2.1 Naive algorithm

The simplest possible algorithm is

$$\begin{aligned} \{ M_z \} &= s \text{ (i.e ON) if } \text{SGN}(dB_z/dt) = -s \\ \{ M_z \} &= 0 \text{ (i.e OFF) if } \text{SGN}(dB_z/dt) = +s \end{aligned}$$

....5.2.1.1

The general justification is as follows. The angle β between the $\underline{z}$ vector and the $\underline{B}$ vector should ideally be either 0° or 180° . If the attitude is improving then $\beta \rightarrow 0^\circ$ or 180° , depending on choice of sign s . The algorithm simply states that if the attitude is improving then leave the torquer OFF. But if the attitude is worsening then switch the torquer ON to oppose the direction. The rule given above will be seen to express this requirement. For UOSAT-2 and requiring $s = -1$ and the magnetorquer only comes on if the sign of dB_z/dt is positive (Fig.5.2.1-1).

The assumption here is that the angular motion is fast compared to the rate of change of the *magnitude* of the geomagnetic field. As is shown in section this rate of change becomes comparable with the libration rate on gravity-gradient capture. It was shown empirically that this very simple algorithm is only sufficient for slowing down a tumbling motion and achieving a rough capture. The defects are:

- * dB_z/dt does not effectively measure the relative change of the Z axis when β is near 0° , or - as in this case - near 180° . A particular problem is that the magnitude of the actual field is changing at a considerable rate in the course of the manoeuvre.

- * no provision for control concepts which would maximize speed of convergence on the required alignment, and minimise residual oscillations.

The ideal solution to the first problem obviously requires measurement of the actual angle of $\underline{B}$ to $\underline{z}$. This requires trigonometric calculations. Performed in fixed point operations this problem and others similar has been solved in an elegant way by Meinzer (1981) but really requires a higher level language for easy implementation. In early days this facility did not exist on UOSAT-2.

5.2.2 Improved algorithm

This improvement requires a program which can calculate a 'transverse' field.

$$B_T = \sqrt{B_x^2 + B_y^2}$$

...5.2.2.1

This can be written easily in assembler code and there is a simple, well-known iterative routine for finding the square root in fixed point arithmetic. This computation is insensitive to any residual Z-spin.

Restricting to the case of reverse alignment $s = -1$ then the rule is

```

IF
    EITHER
        Bz > - BT AND SGN(dBz/dt) > 0    condition ( 1 )
    OR
        Bz < - BT AND SGN(dBT/dt) > 0    condition ( 2 )
THEN
    Mz = -1    (turn on )
ELSE
    Mz = 0    (turn off)

```

...5.2.2.2

Condition (1) is essentially the same as previous simple algorithm but only applies when the attitude is more than 45° out during tumbling or wide oscillations. Condition (2) comes into play when vector $\underline{z}$ points somewhere within an arc of +/-45° of the required direction (fig.5.2.2-1). In this region and if dB_T/dt is positive then the attitude

is judged to be diverging. This is increasingly sensitive to real attitude change and decreasingly sensitive to magnitude change in the magnetic field as the attitude improves. A slightly cruder version of this idea was used on UOSAT-2 stabilisation and was found to be effective.

5.2.3 Quasi-optimal bang-bang control

The following has been tested in simulation but not in practice. Assuming that either we work with B_z field or better with a computation of the angle β directly then the aim is to simulate the damping characteristics of a standard 2nd order loop, although restricted to on/off states with the control Z-magnetorquer. To simplify the argument I assume that sign $s = +1$; and that the desirable attitude is therefore $\beta = 0^\circ$. In pseudo-code the algorithm reads

```

    initialise  $\beta_0 = 0$ 
    Measure  $\beta$ 
    Torquer OFF
100:  $\beta_{PREV} = \beta$ 
    Measure  $\beta$ 
    if ON : if  $\beta > \beta_0$  : reset  $\beta_0 = \beta$ 
    if ON : if  $\beta < \eta \beta_0$  : torquer OFF; goto 100
    if OFF: if  $\beta > \beta_{PREV}$  : torquer ON : reset  $\beta_0 = \beta_{PREV}$ 
    goto 100

```

...5.2.3.1

In this algorithm η is the fixed threshold (typically = 0.9).

It was tested in the IDPO model in which β is attributed to a pitch difference, between the pitch angle a_B of the B vector in the plane of the orbit and the pitch angle of the Z vector a_P , also in the plane of the orbit (both measured with respect to the local co-ordinate system).

For this simple simulation it is necessary to define a *signed* angle

$$X = a_B - a_P$$

where $|X| = \beta$

...5.2.3.2

In the algorithm if $|X|$ is increasing then the torquer should be ON and providing de-acceleration. Eventually the torquer will succeed in containing this swing and it will come back. The maximum error - the amplitude of the oscillation - is monitored and when the error has fallen to some fraction of the previous maximum the current is switched OFF. Now the angular error will (usually) continue to decrease of its own accord. When the magnitude is detected as showing signs of increase again the decision is made to switch on again

The test assumed the differential equation

$$d^2a_p/dt^2 + \frac{M_z(t) B}{I_T} (a_p - a_B) = 0 \quad \dots 5.2.3.3a$$

or

$$d^2X/dt^2 + \Omega_0(t)^2 X = d^2a_B/dt^2 \quad \text{NOTE} \quad \dots 5.2.3.3b$$

where

$$\Omega_0(t) = \sqrt{(M_z(t) B / I_T)} \quad \dots 5.2.3.4$$

and $M_z(t)$ with $\Omega_0(t)$ are on/off time-varying functions under control of the algorithm. With M_z left ON and no acceleration d^2a_B/dt^2 then the motion is an undamped oscillation with frequency Ω_0 .

How strong should the magnetorquer be? There is at least one perturbing torque and that is the created by the geomagnetic field B itself; relative to either an inertial system or the uniformly rotating local system the B field does not rotate uniformly: there is a significant angular acceleration and de-acceleration.

NOTE In Eq.5.2.3.3b the variable X is needed here in this differential equation. The algorithm of course can only measure $\beta = |X|$.

The effect of acceleration is to cause an offset with the error angular difference β having some finite value on average. Even if the oscillation is damped out the offset remains. This is an unavoidable consequence of the fact that the angle a_m and a_p must differ if some torque is to be generated; and this torque is required to maintain an angular acceleration to match the acceleration of the vector $\underline{B}$.

$$\text{Offset error} = \frac{\ddot{\underline{B}}}{\Omega_0^2}$$

...5.2.3.5

For an orbit rate ω_0 it is easy to show, for the IDPO model (see section 4.3) a peak acceleration $\approx 2.1 \omega_0^2$. So even if there were no oscillations there would be an average tracking error given by

$$\begin{aligned} \text{error} &= 2.1 (\omega_0/\Omega_0)^2 && \text{radians} \\ &= 120 (\omega_0/\Omega_0)^2 && \text{degrees} \end{aligned}$$

...5.2.3.6

So that in an orbit of 100 minute period, a magnetorquer-induced oscillation of 10 minute period would control the offset to 1.2 degrees. The oscillation period may sound very long but of course the magnetorquing is a weak phenomenon. The ideal theory also predicts that this maximum acceleration is near to the equator - where the field is weakest.

Some results of simulation are given. Fig 5.2.3-1 is a case of zero acceleration so that convergence on zero error is possible. A 'threshold fraction' $\eta = 0.90$ was satisfactory. The occasions when the torquer comes ON and OFF are clearly marked. It can be seen that the duty factor of the torquer is about 50 percent, in this initial acquisition phase.

The behaviour is quite different when there is significant angular acceleration in the B field. From a state of being OFF the acceleration in B rapidly drives the error up again and causes it come ON without waiting for the angular error to cross zero. The overall effect is a smooth damping towards the fixed offset (fig.5.2.3-2). The convergence is never total and there will be a residual oscillation which is however no more than about 10 percent off the offset in all cases - as can be seen in the blow-up (fig.5.2.3-3). With any detectable angular acceleration it settles to a state with the torquer ON for a duty cycle approaching unity.

5.1.4-1 Optimal instant for denutation

5.1.5-1 Denutation with spin-plane torquer + precession

5.2.1-1 Simplest algorithm for magnetic stabilisation

5.2.2-1 Improved simple algorithm

5.2.3-1 Angular magnetic stabilisation with active damping

5.2.3-2 Angular magnetic stabilisation with active damping

5.2.3-3 Angular magnetic stabilisation with active damping

on

off

5.3 De-libration

The method of active delibration or ' active damping ' of the satellite using a magnetorquer is described. In its simplest implementation this *threshold algorithm* has been tested in practice (see flight experience Chapter 7). Further consideration has been given to its refinement in order to reduce libration to a smaller angular error. These latest ideas are still in the process of testing.

The attitude control required may be summarised as

ASSUMED STATE: passive undamped gravity gradient control
 ASSUMED CONTROL: spin-axis magnetorquer
 CONTROL AIM: reduce amplitude of the libration

5.3.1 Threshold comparison algorithm (basic)

The algorithm is

$$\begin{aligned} \{ M_z \} &= - \{ dB_z/dt \} \\ \text{iff } | dB_z/dt | &> | dB_{oz}/dt | \end{aligned}$$

...5.3.1.1

dB_{oz}/dt is a fixed threshold.

The general principle is more sophisticated than it might appear. The aim is to decide on some maximum rate which $| dB_z/dt |$ would have globally, assuming that the spacecraft was pointing in the nominal up/down direction and had *no* libration. Setting dB_{oz}/dt to some margin greater than this and the algorithm will not ever trigger. Now let there be *some* libration and with sufficient amplitude the condition of 5.3.1.1 will occasionally happen.

Fig.5.3.1-1 shows the configuration when the algorithm is likely to operate. With the satellite passing over the equator with and pointing nominally (not librating) the $\underline{B}$ vector is orthogonal to the Z axis and the rate of change of dB_z/dt is greatest. Let there now be some actual libration and assume at this point in the orbit the attitude is 'back-pitching' from the near vertical attitude - as seen in the diagram. Then the algorithm would be triggered and rotational energy removed from the system. Of course the satellite could be forward pitching - which case nothing would happen: or it could be mostly in a state of roll motion; in which case it is again unlikely that anything would happen.

The advantage of the scheme is the *elimination of any attitude determination*. Effectively it is a semi-passive damping method - or 'assisted' damping.

The effectiveness of such a scheme therefore depends on certain basic properties of the librational state

- * in a pure pitch oscillation the periodicity is $\sqrt{3} \omega_0$. this means that from any initial phase of oscillation there must be enharmonic possibilities when something approaching this near ideal back-pitching attitude will occur eventually.

- * in a general mixed oscillation of pitch and roll the two states couple; so that even if the energy in the librational state is only extracted directly from a temporary mostly pitching state ALL the energy will be extracted indirectly.

- * an adequate stability of the static dB_z/dt (i.e in the absence of libration) is needed. The threshold dB_{0z}/dt presumably has to be set higher than the largest static value in order that there should be no 'false triggering'. Looking ahead, the actual rate of change shows a considerable variation (see sec.5.3.5) so that the setting of dB_{0z}/dt has to be a compromise between failure to respond to residual libration (too high) and a certain fraction of false firings (too low)

The discussion and numerical analysis of section 3.7.1 showed the coupling effect between pitch and roll - a second order effect at large amplitudes and a residual linear interaction which is valid down to arbitrarily small values. The latter depends on the maintenance of the Z-spin rate (see Eqs. 3.7.1.1).

5.3.2 Implementation of threshold algorithm -I

It has been found from experience that it is not enough to formulate an algorithm and hope that it will somehow 'work'. Many other points have to be taken into consideration, in order to get efficient functioning software. In this particular case the limited computing power of the 1802, the many other tasks the computer has to do, and the need in early UOSAT years to program in Assembler required the greatest simplicity as possible (see UOSAT-IERE 1987 pp.143 -150)

Since UOSAT must have a slow spin, in this basic implementation the spin was exploited. A slow spin in the Z axis realises sine-wave like oscillations in B_x and B_y (see idealised Fig.5.1.1-1). To simplify the programming, and for various other reasons, a positive zero-crossing in B_x was used to time a measurement of B_z .

Such a decision means that the slope measurement would be computed from just two consecutive B_z measurements separated by the spin period (typically 3- 5 minutes). Another factor to take into account is that firing of the torquer and magnetometer measurements must not be simultaneous (because of distortion to the local field). Logically then the firing of the magnetorquer would be timed to lie between the measurement samples. In the initial study - again for the purpose of simplifying programming - the fire time was selected to be a constant.

Having established these ground rules, there are now seen to be two distinct modes of operation:

* Anticipation mode (fig.5.3.2-1): the first and second of a pair of measurements determine the firing of a magnetorquer pulse in the next interval, even if there is a pulse already fired *between* these measurements.

* Delay mode (fig.5.3.2-2) : the first of the pair of measurement vectors is allowed only *after* the firing of a pulse; therefore there must be a delay of at least one spin period before acquiring a pair of vectors and deciding to fire again.

There is no difference between the modes as long as the torquer is *not* fired. Whether the torquer fires or not a measurement of B_z is regularly acquired every Z-spin period. The anticipation mode allows a duty factor to approach 100% ; while the delay mode cannot let the duty factor exceed 50% . The anticipation mode is not correct 'academically' since it applies a corrective pulse before checking that it is still needed.

Measurement of the dB_z/dt had to cope with a varying spin period in the practical work described in chapter 6. The solution was to compare the difference ΔB_z between a current sample of B_z and previous measurement B_z' (one spin period earlier) against a variable computed threshold :

$$\Delta B_{oz} = T_z dB_{oz}/dt$$

...5.3.3.1

where dB_{oz}/dt has been explained and T_z is the period of the Z spin.

In order to test the effectiveness of the algorithm the programs used to describe libration and the effect of boom deployment - as discussed in section 3.6 - were extended to allow for magnetorquing (appendix D gives full details). The full set of differential equations were adopted (Eqs.3.6.1) and solved on the standard Runge-Kutta- Merson method provided by the PRIME system. In the course of development first an IDPO model (section 4.3.1) was tried.

This did not work well. Whereas the TDIO model (section 4.3.2) delivered good results. The TDIO model embodies the effect of rotation

of the Earth in 24 hours and the fact that on some orbits the satellite passes near-directly over the pole and the magnetic field lies in the orbit plane ; but in other orbits there can be a significant pitch component of magnetic field lying outside the orbit plane- and helpful in suppressing roll motion.

The results suggested the following parameters

- * the Z-spin period is optimally 5 minutes
- * the fire time should be set to 3 minutes
- * the threshold value (dB_{oz}/dt) needed by the algorithm must be optimised
- * the algorithm is run in an *anticipation mode* (see below)

Fig.5.3.2-3 through fig.5.3.2-9 are attitude plots. Each plot is a map of an attitude circle (c.f sec.3.1.3). A correctly pointing satellite is point in the middle. A satellite lying on it side is a point somewhere on the outer circle. The inner circle marks a constant 10° inclination to the vertical.

Each plot simultaneously prints relevant parameters. At the top right is printed an assumed initial attitude expressed as an initial PITCH angle and an initial ROLL angle. Each figure then records *part* of the attitude track between times START PLOT TIME and FINISH PLOT TIME. For clarity these times usually record only part of the total operational time; and several figures are needed in order to record the progress of the deliberation.

Other parameters are the SPIN PERIOD, and the FIRE time. This last parameter is the fixed time which the magnetorquer fires following a decision to fire. The ACCUMULATED FIRE TIME is then a multiple of this, accumulating the total time that the torquer has been left on.

Additional parameter Bz_dot_lim is the same as dB_{oz}/dt . At one time it was thought to be a good idea to limit the decision to fire on an

additional constraint that the value of B_z should be less than some B_{z_lim} . But this idea was abandoned in practice (to be resurrected for the improved algorithm)

Other relevant parameters not plotted in the figures are (i) an assumed orbit period of 100 mins (ii) and assumption that the boom had deployed to nearly 7 metres in the assigned 15 mins.

It is convenient to envisage the algorithms being switched on at midnight 00.00 am. By mid-day of the first day then 12 hours have elapsed, by mid-night of the first day 24 hours and so on.

The plots assume the ANTICIPATION mode. This is because in all cases it gave a marked decrease in deliberation time. Fig.5.3.2-3 and fig.5.3.2-4 with a fire time of 2 minutes show that the algorithm must run for at least 10 days (14400 mins) before approaching the same order of small residual libration, for a spin period of 5 mins and a fire time of 2 mins. In an endeavour to speed things up a longer fire time of 4 mins has been tried, for the same spin period; but instability is generated - after an apparent immediate success (fig.5.3.2-5 through fig.5.3.2-7). A choice of 3 mins fire-time for a 5 min spin time gives a good compromise: with the libration substantially damped by mid-day of the 3rd day - see fig.5.3.2-8 and very successfully killed by mid-day of the 7th - see fig.5.3.2-9.

All the above assumes of course an arbitrary initial state of libration of 60° pitch and 30° roll. From an actual librational state with greater or less amplitude the convergence time will be correspondingly be greater or less.

The limitation of all these predictions is in the simple TDIO model of the geomagnetic field. Even so I judged that the method ought to work in rough acquisition of the satellite from a wide librating - or even tumbling state. This proved to be correct - as seen in the experimental results of section 6.3.

5.3.1-1 Favoured orientation for effective delibration

5.3.2-1 Example sequence in anticipation mode

5.3.2-2 Example sequence in delay mode

5.3.2-3/ 5.3.2-4 Threshold algorithm test

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 2.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS) 0
FINISH PLOT TIME (MINS) 720
ACCUM FIRE TIME (MINS) 18

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 2.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS) 14400
FINISH PLOT TIME (MINS) 15840
ACCUM FIRE TIME (MINS) 224

5.3.2-5/ 5.3.2-6 Threshold algorithm test

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 4.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS)
4320
FINISH PLOT TIME (MINS)
5040
ACCUM FIRE TIME (MINS)
268

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 4.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS)
5040
FINISH PLOT TIME (MINS)
5760
ACCUM FIRE TIME (MINS)
308

5.3.2-7/ 5.3.2-8 Threshold algorithm test

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 4.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS) 6480
FINISH PLOT TIME (MINS) 7920
ACCUM FIRE TIME (MINS) 376

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 3.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS) 3600
FINISH PLOT TIME (MINS) 5040
ACCUM FIRE TIME (MINS) 162

5.3.2-9

Threshold algorithm test

START PITCH 60.0
START ROLL 30.0

SPIN PERIOD (MINS) 5.0
FIRE TIME (MINS) 3.0

BZ_LIM 0.5
BZ_DOT_LIM 2.0

START PLOT TIME (MINS)
9360
FINISH PLOT TIME (MINS)
10800
ACCUM FIRE TIME (MINS)
270

5.3.3 Threshold comparison algorithm (improved)

With the development of an operating system for the QBC, and provision of a high-level language facility, more elaborate algorithms can be studied. Some results are presented in simulation here, resulting from a more advanced approach. The low resulting libration (less than a degree if the roll bias is discounted in this simulation) was achieved by tightening up the various requirements:

- 1) determining the rate of rotation $d\beta/dt$ of the B vector relative to the Z axis and comparing that statistic against a threshold $d\beta_0/dt$ (rather than comparing dB_z/dt against a dB_{0z}/dt threshold)
- 2) aiming for a technique which is able to make as accurate an estimate as possible of the true *differential* rate of change (true slope) $d\beta/dt$ from samples of the geomagnetic field which are separated by a significant interval in time
- 3) setting and comparing against a threshold $d\beta_0/dt$ to an accuracy better than 1%
- 4) restricting the firings to a specific 'latitude' region in the orbit defined by a ratio $g_0 = |B_z|/B_T$
- 5) making the duration of magnetorquer fire time proportional to the margin over threshold
- 6) making the intervals ΔT between measurements independent of the spin rate. We expect to operate in the band of 5 to 10 minute spin period, which is too long an interval for accurate work, if this were to govern the sample interval.

On (1) it is the angular motion of the satellite which we are really trying to detect and strictly it is the rate of rotation of the B vector against which it should be compared. The measure dB_z/dt is strictly only a measure of the rate of rotation (i) when $B_z \approx 0$ and (ii) the magnitude of the field is a constant.

On (2) a practical magnetometer measurement requires a reasonable time separation or record of samples measurements in order to make a sufficiently accurate measure of the change in field direction. Given the magnetometer's sensitivity of 8 nanoT/ bit (see section 2.2.2) a minimum time ΔT between measurements is in the order of a minute, in order to accumulate a change in B_z in the order of 1 μT . In such a time the instantaneous rate of change may have altered; so the estimation ideally should estimate a true slope referred to some instant in time within this interval of measurement.

On (3) the point is that either too high or too low a level will leave a libration or *induce* a libration. The simulation revealed that this order of precision is required.

On (4) the firings aim to be restricted if at all possible to a 'belt' of approximately the same latitude surrounding the Earth. The reason for this is explained in section 5.3.5 as desirable on measurement grounds. The simulation here tests if such a restriction limits the algorithm's effectiveness - and confirms that it does not.

On (5) the aim is to impose roughly constant damping - to remove a constant fraction of the detected velocity independent of the amplitude - in the expectation of optimising the convergence.

On (6) the point is that we want to approximate a rate measurement or slope in the vicinity of a critical ratio of B_T to B_z

Summarising then

$$\{ M_z \} = \{ d\beta/dt \}$$

$$\text{iff } | d\beta/dt | > d\beta_0/dt \text{ and } | B_z | \approx g_0 B_T \quad \dots 5.3.3.1a$$

$$\text{fire time } T_F = k_0 (| d\beta/dt | - d\beta_0/dt) \quad \dots 5.3.3.1b$$

$d\beta_0/dt$ is a fixed threshold g_0 = latitude ratio k_0 = torquer fire gain

5.3.4 Implementation of threshold algorithm - II

As discussed in section 3.7 and summarised in Eqs.3.7.1.2 the libration at low amplitude can be modelled by two weakly coupled oscillating systems. The degree of coupling is proportional to the Z- spin rate (ω_z). If this spin rate does not change the equations are linear.

Since there is no reason to change the Z-spin and the use of spin-axis magnetorquer will not alter it, a much simpler mathematical modelling is possible than the Runge-Kutta-Merson calculation. As is used extensively in chapter 7 a simple *one-step* computation is sufficient to reproduce the behaviour. The details will be omitted here except to say that a simple Z -transform set of equations will adequately model the oscillation.

Figs.5.3.4-1 through 5.3.4-4 give some typical results. The plots are: SINE YAW which remains unchanged; TORQUER which indicates when the magnetorquer fires; and plots of ROLL and PITCH. For compactness the parallel plots are allowed to overlap. The initial conditions are 5° pitch amplitude, 5° roll amplitude and a normalised yaw rate of 20 (in an orbit of about 100 minutes corresponding to a 5 minute spin period). The assumed satellite parameters are $I_z = 1$; $I_T = 125$ and $M_z = 4$ (magnetic moment of the spin axis magnetorquer). The number of samples per orbit was arbitrarily set at 80, corresponding to about 70 secs interval for UOSAT-2.

The orbit parameters have been set at a slightly higher altitude than UOSAT-2. This is in order that there should be an integer number of orbits in 24 hours - here 14 . Each plot runs then for exactly half a simulated day.

In line with operational limitations the firings of the magnetorquer are restricted to one polarity. The first three figures restrict firings to zero-crossing in B_z .

Fig.5.3.4-1 shows the characteristic effects. The torquer fires only once (point 'A') in the first 7 orbits so that the motion is almost unconstrained. The simulation confirms the unequal oscillation rate in roll and pitch ; the offset in roll due to the roll bias phenomenon ; and the coupling effect which causes a ' beating ' in amplitude and alternating exchange of energy between the two states.

Fig.5.3.4-2 shows what has happens up to 28 orbits (2 days). A consecutive burst of firings has 'picked' up on the negative-going pitch state. The torquer fire gain k_0 has been adjusted empirically for most effective results. Which happens when the torquer fires is a reversal of the direction of pitch velocity (the energies are so small at these small pitch amplitudes that there is no physical problem in achieving this effect from the actual magnetorquers). The gain k_0 is limited so that the amplitude after this reversal is not greater than the amplitude before firing (see point 'B' for example). It can be seen that the scheme effectively removes energy from the roll oscillation in consecutive firings.

A similar effect is shown after 70 orbits (5 days) in fig.5.3.4-3, when the libration has been removed. it shows again the effect of pitch reversal as an effective method of damping the roll state. The conclusion is that this 'single point' firing is a highly effective method of removing energy from the system. Further points:

(1) the TDIO model brings the 'static' value of $d\beta/dt$ upto the threshold value $d\beta_0/dt$ only once every 24 hours for a few orbits. This happens when the orbit plane is most closely aligned with the plane of the tilted dipole . The simulation realistically demonstrate the effect then of a 'sparse' opportunities for delibration

(2) roll-pitch coupling and the extraction of energy from the pitch state forces the pitch oscillation to a rate which is nearly the natural roll rate $2 \omega_0$, rather than its own natural $\sqrt{3} \omega_0$ value. It would be unfortunate if this were exactly true since the resulting synchronisation would

leave the damping state 'stuck' with no chance generally of being damped by the rotating $\underline{B}$ vector. Fortunately the residual natural pitch state forces a phasing effect by which the oscillation always 'drifts' into the best point for being damped -eventually.

(3) it can be shown in general that there is an optimum rate for extracting energy from one element of a coupled oscillating system if both systems are to be drained of energy. If the controllable element is damped too rapidly then the coupling disappears and the uncontrolled element is left 'hanging'. The damping must be applied at no faster rate than the coupling allows interaction between them so that BOTH can be brought down together. It follows that the limited occasions for firings are not inconveniently infrequent.

The torquer fire gain k_0 to optimise this rate was found to be $90000 \text{ sec}^2 \mu\text{T}^{-1}$ (for substitution in Eq.5.3.3.1b)

Fig.5.3.4-4 is a demonstration of a modified technique firing on a simple point transition $|B_z| = B_T$. The fire gain constant has to be raised in order to deliver the same effective de-acceleration. A value $k_0 = 100000 \text{ sec}^2 \mu\text{T}^{-1}$ is appropriate. The point of this last plot is to show that it is possible to work the algorithm effectively from a non-zero magnetic latitude

The simulation emphasised the importance of making an accurate rate estimate. Here it is produced by a simple calculation

$$dB/dt = k \frac{B_z - B_z'}{B_T + B_T'}$$

...5.3.4.1

which can be shown on an exact dipole orbit model (IDPO) to produce an exact and constant value at all points in the orbit for arbitrary constant time separation ΔT between readings.

The simple proof is as follows. From Eqs.4.3.1.1

$$\begin{aligned} B_z' &= 2 B \cos(\lambda) \\ B_z &= 2 B \cos(\lambda + \omega_0 \Delta T) \end{aligned} \quad \dots 5.3.4.2$$

$$\begin{aligned} B_T' &= B \sin(\lambda) \\ B_T &= B \sin(\lambda + \omega_0 \Delta T) \end{aligned} \quad \dots 5.3.4.3$$

where ΔT is the time interval between measurements. Then substitution in Eq.5.3.4.1 yields

$$dB/dt = 2 k \tan(\omega_0 \Delta T/2) \quad \dots 5.3.4.4$$

For a true rate estimate the constant k should be adjusted for whatever latitude of the orbit the measurement is being made. For the purposes of the algorithm however setting $k = 1$ is adequate since only relative measurements are being made. On the TDIO model the estimate shows then a smooth accurate characteristic over a wide range of relative latitudes for coarse time separation ΔT .

Although the results of the simulation are strictly valid only for TDIO model they can be used to anticipate a real-life performance on the IGRF model (see next section)

5.3.5 Implementation in real world IGRF

The simulation shows that the improved algorithm requires a reasonable fraction of orbits in 24 hours which have a stable, static dB_0/dt . Does this condition exist in the real geomagnetic field? The answer seems to be yes. Fig.5.3.5-1 is a typical print-out for a 24 hour run with UOSAT orbit parameters entered into the standard IGRF model. The final column is an estimate of the static dB/dt in crossing through $B_z = 0$. The estimate is calculated using Eq.5.3.4.1, based on $\Delta T = 60$ second intervals. It can be seen that 8 out of the 27 crossing the peak values are within 1.5 % of each other. The contrast with the ΔBZ readings (last but one column) is marked. It would seem then that the estimated rate of rotation of the B vector, as determined by this simple formula, is a reasonably stable statistic even for the real geomagnetic

(multipole) field - at least in the restricted zone defined by $B_z \approx 0$. More extensive investigation may reveal specific latitudes (found by selecting a non-zero value to the latitude ratio g_0) and/or modified estimation procedures which will yield even better stability.

Finally, since the simulation shows that the damping can be confined to a specific latitude 'belt' and since the computer program can be told the time by the on-board real-time clock, it follows that the a threshold 'signature' can be built up - a list of $d\beta_0/dt$ values stored by longitude and to be selected by the algorithm in the course of the orbit. It is also distinctly feasible that the elements of this array of thresholds can be updated *adaptively* in the course of time - so liberating the system from drift problems in the magnetometer measurements (see section 4.5) and uncertainty about correct values from the old IGRF model.

The significant understanding gained from the estimation here are on the point of being tested on the real satellite (Feb 1989)

5.3.4-1/ 5.3.4-2 Refined libration control $B_z \approx 0$

5.3.4-3 Refined libration control $B_z \approx 0$ 5.3.4-4 Refined libration control $B_z \approx B_T$

5.3.5-1 Table of orbit parameters IGRF model

RANGE OF TIME-TS(hhmm),TF(hhmm),TD(secs) 0000,2359,60

UOSAT ORBIT NORMALISED MAGNETIC FIELD COMPONENTS

Orbit Number:12000

Altitude: 700.6 Kms

Period: 98.93 mins

Inclination: 98.22 degs

Equatorial Crossing Time(minutes after mid-day): 1255.

" " " (hhmmss): 0.00000000

(N.B.Codip is defined as Broll/Byaw and is in degs)

TIME	LAT	LONG	BABS	BPITCH	BROLL	BYAW	DBYAW	Dbeta
1459.0	3.2	-37.0	0.2063	-0.0359	0.2016	0.0248	0.0266	0.132
1499.0	8.5	133.2	0.2743	0.0345	-0.2722	-0.0016	-0.0377	-0.138
1548.0	-9.4	-58.9	0.1904	-0.0076	0.1900	0.0110	0.0228	0.122
1598.0	6.7	108.2	0.2883	0.0435	-0.2840	-0.0236	-0.0428	-0.149
1646.0	-11.2	-83.1	0.2044	0.0458	0.1991	0.0061	0.0238	0.118
1696.0	8.5	84.0	0.2819	0.0548	-0.2765	-0.0014	-0.0424	-0.150 *
1746.0	-5.8	-108.9	0.2230	0.0677	0.2122	0.0120	0.0268	0.121
1795.0	6.7	59.0	0.2581	0.0482	-0.2534	-0.0093	-0.0389	-0.150 *
1846.0	-0.3	-134.7	0.2325	0.0721	0.2196	0.0252	0.0286	0.124
1893.0	8.4	34.7	0.2442	0.0365	-0.2414	-0.0053	-0.0385	-0.156 *
1945.0	1.5	-159.7	0.2390	0.0755	0.2261	0.0163	0.0303	0.126 *
1992.0	6.6	9.7	0.2291	0.0502	-0.2209	-0.0343	-0.0348	-0.151 *
2045.0	7.0	-185.5	0.2452	0.0762	0.2320	0.0218	0.0296	0.120
2091.0	4.8	-15.3	0.2118	0.0769	-0.1956	-0.0264	-0.0278	-0.130
2144.0	8.8	-210.5	0.2603	0.0567	0.2540	0.0047	0.0339	0.130
2192.0	-4.3	-41.9	0.1926	0.0869	-0.1718	-0.0058	-0.0195	-0.100
2243.0	10.6	-235.5	0.2811	0.0406	0.2773	0.0222	0.0400	0.143
2293.0	-13.3	-58.5	0.1906	0.0390	-0.1865	-0.0020	-0.0226	-0.117
2342.0	12.4	-260.6	0.2912	0.0366	0.2857	0.0429	0.0432	0.150 *
2391.0	-11.5	-92.7	0.2108	0.0019	-0.2106	-0.0086	-0.0277	-0.131
2440.0	10.6	-284.8	0.2775	0.0261	0.2748	0.0287	0.0420	0.153 *
2488.0	-6.1	-116.1	0.2249	-0.0066	-0.2248	-0.0001	-0.0295	-0.131
2538.0	8.9	-309.0	0.2546	0.0323	0.2523	0.0118	0.0371	0.147
2586.0	-4.3	-140.4	0.2340	-0.0083	-0.2334	-0.0137	-0.0307	-0.132
2637.0	10.7	-334.1	0.2436	0.0320	0.2412	0.0110	0.0364	0.151 *
2684.0	-2.6	-164.6	0.2450	-0.0108	-0.2434	-0.0251	-0.0327	-0.135
2736.0	12.5	-359.1	0.2333	0.0109	0.2318	0.0236	0.0348	0.151 *
2781.0	2.9	-188.1	0.2509	-0.0074	-0.2501	-0.0185	-0.0330	-0.133
2833.0	7.1	-382.5	0.2136	-0.0251	0.2119	0.0081	0.0293	0.140
2878.0	8.3	-211.5	0.2615	0.0192	-0.2608	-0.0008	-0.0345	-0.133

5.4 General Control Laws

In contrast to the specific algorithms for dealing with specific spinning, nutating, or librating states, the CPL or Cross Product Law is a known alternative with general application. It is critically reviewed and found not to be so useful as might be thought. To finish: a different general observation - the Reciprocity Principle - is proposed, which establishes the complementarity between magnetometer measurements and magnetorquing

5.4.1 CPL law

The CPL law states that if $\Delta \underline{L}$ is the required change in momentum $\underline{L}$ then point $\underline{M}$ in the direction indicated by the vector cross product of $\underline{B}$ and $\Delta \underline{L}$ i.e

$$\underline{M} = k \underline{B} \wedge \Delta \underline{L}$$

...5.4.1.1

where k is some scalar factor

The law is apparently 27 years old (White 1961). It aims to maximise the scalar product

$$p = \underline{N} \cdot \Delta \underline{L}$$

...5.4.1.2

The proof is very simple - just depending on vector identities (see Appendix G).

If the above may be called the exact CPL law then a variant has been used by Meinzer on Oscar 10 and 13. It applies when there is not complete freedom to point the magnetic moment $\underline{M}$ in any desired direction. In his system this magnetic moment vector is restricted to the spin-plane of his spinning satellites. A '3-phase' commutating system switches between three equi-angularly spaced magnetic rods, and maintains an effective constant pointing $\underline{M}$ in the spin plane.

His software calculates the vector product

$$\underline{v} = \underline{B} \wedge \underline{\Delta L}$$

...5.4.1.3

and, not generally being able to point $\underline{M}$ in the same direction as $\underline{v}$, does the next best thing and aligns $\underline{M}$ (which can only be rotated in the spin plane) until it coincides with the *projection* of $\underline{v}$ on the spin plane. It can be shown that this again maximises the scalar product $\underline{N} \cdot \underline{\Delta L}$ within the limitations of the constraint on the pointing of $\underline{M}$ (proof in Appendix G - as supplied by me to Meinzer).

I suggest that this may be called a *projected cross-product law* PCPL which essentially states that if $\underline{M}$ cannot be pointed directly in the direction $\underline{v} = \underline{B} \wedge \underline{\Delta L}$ then the next best thing is to point $\underline{M}$ in a direction as close an angle to $\underline{v}$ as is allowed by whatever constrains $\underline{M}$. In some cases this might amount to only a choice of sign - no other degree of freedom being offered.

An examination of all cases considered : on spin change, precession, and denutation , shows that this law is always compatible with the algorithms given. They are all cases of either the direct or projected CPL law.

		CPL	$\underline{N} \parallel \underline{\Delta L} \quad ?$
5.1.1	Z despin	exact	Yes if $B_z = 0$
5.1.2	precess-axis	projected	Yes
	precess-plane	projected	Yes
5.1.4	denutate-axis	projected	Yes
	denutate-plane	projected	Yes
5.2	mag stabilise	projected	Yes
5.3.1	libration control	projected	Yes

However that does not mean that the CPL law as a universal principle replaces them; because it is easy to show that only with this choice of

states and desired changes - - as in sections 5.1.1, 5.1.2, and 5.1.3 - does the generated torque $\underline{N}$ actually point in the desired direction.

To change an angular momentum by $\Delta \underline{L}$ the laws of classical physics state that the torque

$$\underline{N} \propto \Delta \underline{L} \quad \dots 5.4.1.4$$

But the torquing law

$$\underline{N} = \underline{M} \wedge \underline{B}$$

forces $\underline{N}$ to be always orthogonal to $\underline{B}$. The CPL law does not bring about Eq.5.4.1.4, unless constrained in the special situations studied in this chapter. Simply optimising the product in Eq.5.4.1.2 does not ensure that the torque is actually directed along $\Delta \underline{L}$; and in general it is not.

As a particular example suppose that the aim is a change in the direction of $\underline{L}$ without changing its magnitude. Therefore $\Delta \underline{L}$ is always orthogonal to $\underline{L}$. The aim is to direct $\Delta \underline{L}$ in some chosen direction within the spin plane, which is defined by the magnetic reference axes X_0 and Y_0 , when the nutation is zero. Two limiting cases can be identified, and explained using the magnetic reference system of sec.3.1.5.

1 $\Delta \underline{L}$ points in direction $+X_0$ (orthogonal to $\underline{L}$ and $\underline{B}$ - (figs 5.4.1-1a). No problem here. A direct CPL law requires $\underline{M}$ to point in the plane of $\underline{L}$ and $\underline{B}$ (fig.5.4.1-1b); while the projected CPL law allows either version of the precession laws as presented in section 5.1.2 to do the job

2 $\Delta \underline{L}$ points in direction $+Y_0$ (in plane with $\underline{L}$ and $\underline{B}$ as in fig.5.4.1-2a). This requires setting $\underline{M}$ in the $-X_0$ direction (figs.5.4.1-2b); which is in fact the configuration normally used to change the SPIN state of the satellite. Only a component of $\underline{N}$ is directed along the required $\Delta \underline{L}$. So there is generally

no way of avoiding an unwanted spin change: with the exception of special and degenerate condition if the B vector should happen to point parallel (or antiparallel) to the L vector.

From which I conclude that the generality of the CPL law is more apparent than real.

Even if there were not these cautionary points to be expressed about the CPL law, its explicit use will usually require more information than can be supplied in the limited data supplied by only the magnetometer data.

Conclusion: application of CPL law outside the special cases and restrictions stated always causes unwanted attitude changes; and can demand more data than is simply provided under the assumptions of this work (NOTE).

5.4.2 Reciprocity Principle

A more interesting observation than the CPL law can be made: and establishes the strong link between measurement and control when both are entirely magnetic. In the absence of claims to the contrary I call this the Reciprocity Principle.

Qualitatively it states that only the angular motion which is measurable by a magnetometer is controllable by a magnetorquer, in the sense of changing the rotational energy of the spacecraft. The principle affects therefore the spinning/de-spinning cases; and denutating; and after applying a threshold condition, to deliberating case also.

NOTE I believe that Meinzer gets away with it by setting up logical constraints and thresholds. If for example the aim is an attitude change then the spin rate is monitored; and if this is going too fast or too slow then the torquing routine is interrupted until such time that a change orientation in B will be favourable to both attitude AND spin rate change.

More precisely it states that a magnetic moment $\underline{M}$ which is directed to be parallel or anti-parallel to the detected normalised rate of change $[\underline{db}/dt]$ will cause the maximum rate of increase or decrease in the rotational energy of the system. In other words if

$$\underline{B} = B_x \underline{i} + B_y \underline{j} + B_z \underline{k} \quad \dots 5.4.2.1$$

is normalised by dividing by

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \dots 5.4.2.2$$

so that

$$\underline{b} = \underline{B}/B = b_x \underline{i} + b_y \underline{j} + b_z \underline{k} \quad \dots 5.4.2.3$$

then the rate of change in the body axes defines a vector

$$[\underline{db}/dt] = [db_x/dt] \underline{i} + [db_y/dt] \underline{j} + [db_z/dt] \underline{k} \quad \dots 5.4.2.4$$

and if $\underline{M}$ is pointed in the same direction as $\underline{db}/dt$ then it will achieve the maximum rate of change in energy which is possible magnetically

A proof is given in Appendix G which also shows that the rate of change $\underline{db}/dt$ is a measure of only a vector *component* of the total angular velocity of the body: the proof shows that it is just this measurable component which is accelerated or de-accelerated instantaneously by application of the magnetorquer.

Conversely, given the need to impose the maximum rate of change on the angular kinetic energy, and having only a magnetorquer with which to do it, the Reciprocity Principle states that a magnetometer provides exactly the right information to determine the magnetic moment $\underline{M}$.

This seems to me to be an important principle: it shows the essential complementarity between all magnetic determination and control. Also it shows that the sole magnetometer measurement is not so limited as might be thought.

Fig. 5.4.1-1 Momentum change CPL law Required $\Delta \underline{L} \propto \underline{x}_\theta$

5.4.1-2

Required $\underline{L} \propto \Delta \underline{y}_\theta$

Fig. 5.4.1-2 Momentum change CPL law

(a) Achieved $\underline{N}$ not $\propto \Delta \underline{L}$

(b)

5.5 List of Symbols

$\{ \}$	function returning sign of variable (+1,0, or -1)
B_x, B_y, B_z	components of geomagnetic field along body axes
M_x, M_y, M_z	components of magnetorquer mag moment
$\underline{N}$	torque vector
N_x, N_y, N_z	components of torque
L	angular momentum
s	sign = +/-1
n	no of turns in magnetorquer coil
S	area of coil
I	current in coil
B	geomagnetic field magnitude
I_z, I_T	Moments of inertia
T_{DESPN}	time to despin to stationary
T_z	Z spin period
X_0, Y_0, Z_0	magnetically defined reference system
B_{Y0}, B_{Z0}	components in mag reference system ($B_{X0} = 0$)
Ω_N	precession rate
Ω_B	slew rate
θ	angle between $\underline{B}$ vector and $\underline{Z}$ axis
μ	figure of merit
B_T	transverse component of geomagnetic field
$\underline{B}_0, \underline{B}_1, \underline{B}_2$	special cases geomagnetic vectors
θ_{PREV}, θ_0	algorithmic values of field vector angle
η	threshold level in algorithm
χ	signed angle between $\underline{B}$ vector and $\underline{Z}$
α_B	pitch angle of B field in plane of orbit
α_P	pitch angle of Z axis in plane of orbit
Ω_0	resonant frequency under magnetorquing
dB_{0z}/dt	threshold set = $B_{z_dot_lim}$
ΔB	variable increment in B_z
ΔB_{0z}	threshold increment in B_z

g_0	pre- set ratio of $ B_z $ to B_T
ΔT	sampling time between measurements
$d\beta_0/dt$	threshold rate of change in β
T_F	fire time for magnetorquer
k_0	torquer fire gain
k	arbitrary constant
λ	orbital advance (ideal rotation from North Pole)
$\Delta \underline{L}$	vector change in angular momentum
$\rho, \underline{v}$	constants in calculation
b_x, b_y, b_z	normalised geomagnetic field components

6 CONTROL OF UOSATS

This chapter describes the principle events in the attitude control of UOSAT-1 and UOSAT-2. For UOSAT-1 it is shown that the precession and denutation methods using a spin-plane magnetorquer did in fact work, although stabilisation by gravity gradient was not achieved. For UOSAT-2 the achievement of magnetic stabilisation, capture by gravity gradient and then delibration suppression was demonstrated.

The main problem with UOSAT-1 was the impossibility of effecting any autonomous, reliable control using the on-board computer, due to a malfunction in the command bus. This malfunction made attitude (and other) control possible only when the spacecraft was in view and accepting commands from mission control. The tumbling state of the satellite forced arbitrary drop-outs in both up and down-links - so control algorithms had to be designed to cope with frequent poor communications.

Apart from this a large number of technical problems had to be overcome, in development of software and setting up the ground station. Ultimately in March 1983 UOSAT-1 was orientated favourably, under magnetic control, and there followed a dramatic session in attempting to switch over from active control to passive control under gravity gradient. The control requirements and overall aim is explained in sec.2.3.2. Unfortunately there was a mechanical failure and the boom 'stuck' after deploying only a short distance. While Earth pointing was achieved for brief interval of days, this could not be maintained. Subsequent efforts to deal with this problem were not successful. Still, much was learnt. The flight experience recorded here emphasises the successful aspects, particularly claimed novel method of using spin plane magnetorquer for general spin change, denutation, and precessional change.

The ADCS of UOSAT-2 can be judged a success. After launch there was an initial hiatus when communication was lost but ultimately restored. The control history divides into two phases. Some months immediately after launch the spacecraft was successfully put into a

gravity gradient controlled state. This was achieved from an intermediate stage of magnetic stabilisation, using a new spin-axis magnetorquer. Further attitude refinement has to wait until the winter 1986/87, when de-librating tests were successfully carried out.

6.1 Control of UOSAT-1

Down-linked data confirmed a wide-angle nutational state immediately after launch. After a passage of months following loss and then recovery of control, the nutation had suppressed itself and the attitude was pointing near normal to the orbit plane. Successful de-spinning was followed by precessional change forcing the attitude back into the orbit plane. After various problems were solved, this was accomplished along with novel combined active denutation procedures. The attitude could be forced into the orbit plane with the low nutation, while waiting on natural gravity-gradient precession to bring this attitude to a suitably low pitch angle. Everything worked until the final phase of boom deployment.

6.1.1 Initial appraisal of the system

In the early days of UOSAT-1 orbital operations, data handling and presentation were primitive. Fig. 6.1.1-1 is an example. Later on more sophisticated plots were capable of being generated. However, crude though it is, information can be derived from it. Uncertainties in scaling and offsets made it unwise to depend on magnitudes in any of the channels. However the characteristic periodicities are clearly seen (see discussion in section 4.2). In the Z channel the inertial nutation rate is seen; the modulation effect in this channel is nothing to do with the changes in angular motion but is due to changes in the direction of the $\underline{B}$ vector through a pass. A slower near-sinusoidal effect in the X and Y channels effectively measures the body nutation rate. The condition is type II as discussed in section 4.2 with a wide nutation angle and the angular momentum direction lying close to the orbit plane.

The reason for the near disappearance of the B_z modulation towards the end of the record is due to the $\underline{B}$ vector nearly aligning with the angular momentum, confirming that the spacecraft's momentum was closely aligned to the orbit plane (see IDPO model in section 4.3.1).

The time interval between samples is precisely the time taken to transmit one telemetry frame. This was 4.8 secs.

The periodicities estimated 'by hand' off these graphs gives

Inertial nutation period	= 25.5 seconds
Body nutation period	= 48.8 seconds

The data available for the spacecraft before launch gave figures for the moments of inertia: $I_x = 2.6$, $I_y = 2.6$ and $I_z = 0.72$ in an 'all stowed' condition, meaning that the gravity-gradient boom and various other antennae had not been deployed. The calculated ratio $I_y/I_z = 3.60$. If this can be believed then the nutation angle can be estimated from equation 4.2.4 as 78° . There were strong grounds for suspicion that this data referred to UQSAT-1 in an early unfinished state. Later work with an autocalibration procedure, and from studying the motion more accurately, established a more likely ratio of $I_y/I_z = 2.8$ and therefore a nutation angle of 73° . The formulae relating all these quantities are given in section 3.2.

The plot of fig.6.1.1-1 also shows that the rate of information provided by the telemetry is simply not fast enough for any controlled operation. What the plot does not show is that there are occasional gross errors in the readings - this is due to an 'aliasing' effect (see next section)

6.1.2 The State of the UOSAT-1 systems

A detailed check of the satellite's state after launch established certain problems in the sub-systems

- * insufficient isolation between the TX and the RX systems in the spacecraft prevented simultaneous operation of up-link and down-link at VHF (2 metres), and simultaneous operation of up-link and down-link at UHF (70 cm).

- * the up-link sensitivity at 70 cm was poor and needed development of a more powerful transmitter in the ground station, needing ultimately 200 watts and an antenna gain of 22 dB (the onboard receivers are not very sensitive).

- * the spacecraft's down-link power at 70 cm was low and needed a far more sophisticated ground station receiver for reliable reception.

The communications on the down-link was originally possible only at 2m. also it was the only good up-link (but not at the same time!). So full duplex mode was not possible - only half duplex. Ultimately a sensitive 70 cm receiver/decoder for the down-link was developed. To add to the problems

- * of the two magnetometers the 'navmag' was defective; it suffered excessive drift in the X and Y component channels; and the Y channel was down.

- * the other magnetometer, the 'scientific' magnetometer, designed and built by Acuna always worked well. But the unusual data format (as discussed in section 2.1.4) where each magnetic component measurement was split into separate 'coarse' and 'fine' measurements caused problems. In the standard telemetry format these were so far separated as to constitute a serious 'aliasing' error.

- * the telemetry data was slow. It provided access to the magnetometer channels only once every 4.8 seconds which was slow compared to the tumbling/spinning rate which UOSAT-1 initially had on launch and was far too slow compared to the spin rate which the satellite was found to have after recovery (in the order of 12 seconds).

- * the rapid tumbling motion caused the communication channels to suffer from drop-outs (in both directions), and corruption of data.

- * the on-board 1802 computer had a command problem; an error in circuit design prevented reliable commanding of the s/c sub-systems - including the all important magnetorquer. Fortunately it could read sub-systems - including the scientific magnetometer.

From launch, great efforts were made to solve some of these problems by redesign and rebuilding of ground-station antennae, transmitter and receivers.

UOSAT-1 went down from April '82 due to accidental switching on both beacons at once. Contact was resumed in 1st September 1982, courtesy of Stanford Research Institute, who used massive EIRP to break into our 'de-sensed' receivers

6.1.3 Planned control sequences

After all this time without being in contact the initial examination of the magnetometer data showed that UOSAT's attitude had entirely changed. The nutation angle had largely collapsed - to 16 degrees - and a Z spin rate had very substantially speeded up to a period of 12 secs. The angle of the B field seemed to be always in the order of a right angle to the Z axis, implying that the spin axis had re-orientated itself all the way from lying in the spin-plane to lying close to the orbit normal (or anti-normal)

The contraction of the nutation angle was surprising since it contradicts the rules for spinning satellites. Having a greater I_x (and I_y) than I_z is supposed in standard theory to drive a spinning body into a flat spin. However the slewing effect directs the spin axis to the state of Thomson equilibrium (see Hughes 1986 p 357-362) and clearly dissipative mechanisms driving to that state were also enforcing spin stability.

This very drastic change in orientation was bound to cause a re-think in the intended attitude control sequence. However it did not alter the basic requirement to achieve SOME sort of control; and the first step - of attempting to reduce the Z-spin rate - was still a plausible and simplest manoeuvre to attempt. OBC Software had now been developed (R.Peel) which could read the scientific magnetometer directly, at sufficient speed, and by-passing the problem with the telemetry, relaying the data to the down-link.

By November '82 it was possible to report that an overall command sequence had been implemented and might have reduced the spin rate by a few percent. Various other problems had been discovered and solved in order to get this far.

A sensitive receiver/decoder for detecting with low-error the weak down-link transmissions at 70 cm. had been developed. This at last made possible a two-way control link (full duplex) and made possible a more robust approach. This was to

- * wait for UOSAT on a pass and check that its v.h.f beacon was off

- * send a command via the v.h.f up-link to an on-board (previously loaded) program which switched on the 70 cm (u.h.f) beacon and then read, processed, and down-linked the magnetometer data at 70 cm. The fast reading facility, complete with error-checking, now overcame the limitations of slow data rate by the standard telemetry

* on the ground use a computer /operator to analyse this data and with operator over-ride send on/off commands to the magnetorquer over the v.h.f up-link (having first armed the magnetorquer via this link).

* at the end of the pass send a string of OFF commands to the torquer and instructions to switch OFF the u.h.f beacon.

* if it was not clear that the the u.h.f beacon was really off then do nothing until the next pass

* but if the u.h.f. beacon was clearly off one could send commands to the satellite to switch on the v.h.f beacon under an internal timer control (another program) and (usually) transmit telemetry with the aim at some later time of expecting the v.h.f beacon to have switched itself off and allow a repeat on a later pass.

6.1.4 First successful De-spin

The fact that only minor alteration to the spin rate had been achieved was traced to incorrect labelling of magnetometer axes. After a necessary re-interpretation of the channels the space-craft was slowed down for the first time using the simple algorithm of section 5.1.1 and as detailed in fig.6.1.4-1.

Dividing the timing figures by 60 it is seen that the spin period was increased from $954/60 = 15.9$ seconds to $1800/60 = 30$ seconds by the end of the sequence. The data was analysed more fully on this last pass and then compared with an early one. Summarising in fig.6.1.4-2 this showed a reduction of the Z spin rate to 40% of its start value and also a reduction in the transverse T spin to 60 % of its start value.

6.1.5 Simple attitude estimation

Ground-station software was at last available to process and plot graphically selected data available from the telemetry

Fig.6.1.5-1 was about the first available, and confirms the attitude because of the consistent small average value in the normalised b_z component. It is seen to be a type-I waveform (see section 4.2) with the Z spin identifiable at once from the normalised b_x and b_y waveforms. Assuming the IDPO model then the observed

$$b_z = \sin(\delta) \cos(a_p - a_B) \quad \dots 6.1.5.1$$

where

δ is the current tilt angle
 a_p is the current pitch angle (in the orbit plane)
 a_B is the current angle of the $\underline{B}$ vector

The angle δ lies between limits

$$\delta_1 = \delta_0 - \theta < \delta < \delta_2 = \delta_0 + \theta \quad \dots 6.1.5.2$$

where δ_0 is an average value to the tilt and θ has been introduced as the nutation angle.

The cycles seen in b_z are due then to the nutation action - so the period of this can be roughly inferred. While the average also goes through a cyclical action - one harmonic in the course of an orbit. The interval here is of 200 seconds only. If either the maximum and minimum of the average is also seen then the $\cos(a_p - a_B)$ term in Eq.6.1.5.1 can be assumed to be +1 or -1, and the average tilt value deduced.

Very roughly if that was the case at the beginning of the plot then the two limits $\delta_1 = -30^\circ$ and $\delta_2 = +8^\circ$ imply $\delta_0 = -11^\circ$ suggesting that the Z axis lies close to the orbit normal. The suggested nutation angle is clearly $(30 + 8)/2 = 19^\circ$.

Actually the estimation is ambiguous: the answer could equally well be $180 - 11 = 169^\circ$ - and later evidence suggested that this was the case (see below)

The estimation improved when the plots became available over whole orbit - not just over a brief pass as here

6.1.1-1 ASCII MAGPLOT

```

Program to read magnetometer data from file and print calculated values
Filename to read input data : 1

```

INUT36

INPUT

-19
-17
-16
-15
-15.1
-14
-13
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
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44
45
46
47
48
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55
56
57
58
59
60
61
62
63

70°N

$\lambda = \frac{v}{f} = \frac{300}{\frac{1}{25.5}} = 790$

$$ZS^{\circ} \text{ (N)}$$

6.1.4-1 First ever successful de-spin

ORBIT	Fire time s	Subsequent observed periods
6064	7.5	954, 966, 975, 993, 1014, 1022, 1036, 1063, 1079, 1088 1112, 1139, 1152, 1175, 1201, 1217
6065	10	1280, 1325, 1356, 1385, 1374
6070	10	1474, 1527, 1463, 1582, 1636, 1628, 1721, 1734, 1756, 1800

6.1.4-2 Initial and final states from first de-spin sequence

ORBIT	Inertperiod	Bodyperiod	Z period	T-period	nutangle
5837	33.55	19.8	12.67	110.09	17.7
6070	77.2	47.5	30	190	24

6.1.5-1 Magnetometer plot for orbit 6071M

6.1.6 Precession Maneuvres

To change the attitude prior to boom deployment we needed to maintain the Z-spin, and with this spin defining an angular momentum, force a magnetic precession of this momentum. While Wertz (1978) may explain in principle how to do this we needed something far simpler to cope with the rough and ready operating conditions.

As has been explained in section 5.1.2 the general principle is to commute every ON pulse of the X-axis magnetorquer symmetrically within a half-cycle of a component B_x i.e about a zero-crossing in B_y . That way there is no net torque tending to change the z-spin rate. The algorithm was given in Eq.5.1.2.5

The required sign of the precession is determined by the second algorithm of Eq.5.1.3.1. Repeating,

TO INCREASE THE TILT ANGLE $S = - \text{SGN}(dB_z/dt)$
 TO DECREASE THE TILT ANGLE $S = + \text{SGN}(dB_z/dt)$

where the meaning of a tilt angle was defined in section 3.1.1 as the angle between the Z axis and the orbit normal vector.

The choice depends on which side of the orbit plane the satellite was pointing. If it was pointing close to the orbit normal then the tilt angle needed increasing (from near 0°); if pointing close to the orbit anti-normal then the tilt needed decreasing (from near 180°). There was no way of resolving this ambiguity from the geomagnetic measurements alone.

All evidence pointed to the latter situation, based on the temperatures observed on the various faces. It also became clear that the satellite was back-spinning (negative ω_z) so that in effect the angular momentum $\underline{L}$ was pointing in the orbit normal direction. By definition, in a sun-synchronous orbit the orbit plane maintains (roughly) the same orientation to the sun. On an orbit defined by 3.p.m South to North

equatorial crossing the sun must lie on the side of the plane defined by the normal direction (Fig.6.1.6-1). If the spacecraft is pointing towards the orbit anti-normal then its base would be consistently illuminated by the sun (except for a brief interval in the earth's shadow), keeping its base illuminated and warm, and the top in shadow and cold. All this seemed to be confirmed by temperature and illumination sensors which the UOSAT-1 possessed.

The tilt angle therefore was near 180° (not 0°) and needed to be decreased in order to swing it into the orbit plane.

Early trials at precession change showed signs of success. For example fig.6.1.6-2 shows a tilt angle backed off from 180° by $(160 + 128)/2 = 144^\circ$, again assuming that the beginning of the plot was a stationary value in the orbit. Then came realisation that the we were about to lose the spacecraft. Fig.6.1.6-3 is a plot of battery temperature, starting from orbit 6360; the scale is in 'telemetry units' which invert the true temperature scale. Equivalent temperatures have been marked. During manoeuvres the battery temperature had been steadily declining. According to the book, a temperature of -20 degrees Celsius represents a point of no return. Fig.6.1.6-4 shows how the attitude geometry could bring about this effect.

There was only one thing to do - and that was to spin the spacecraft right down : this would hopefully make the gravity-gradient induced precession speed up (see sec.3.4.3), so that the base would rapidly come back into the sunlight.

Comparison of figs.6.1.6-5 and 6.1.6-6 between passes on orbit 6466 and orbit 6519 show the drastic change effected on the spin state, which the temperature plot clearly showed had the right effect (magnetic plots are now all of normalised values)

Returning after a seasonal break it became clear that at low spin rates the energy-dissipating mechanism causing the spin axis to realign on the orbit normal had been in evidence . Fig.6.1.6-7 which showed a tilt

increase back to 166 degrees. This was the first full orbit plot which was obtained. Also the Z-spin seemed to have increased.

There seemed to be no recourse but to continue the manoeuvres. But in order to get the required tilt more quickly set a slower Z spin. The consequent reduction in angular momentum should make it easier to slew the satellite and take a correspondingly shorter time to get the attitude right again. It should also get us out of the previous problem by making the gravity gradient precession move faster and prevent the low temperature problem by not exposing the back panel for extended periods. On the other hand we would have to force the magnetic precession more frequently; to compete with the 'natural collapse' effect we would have to run just to stand still.

From now on it was intended just to keep 'pushing' the tilt attitude until the rest of the team had got together required control software. It was necessary however to develop the special denutation procedure as outlined in section 5.1.4 and combine the two (as in sec.5.1.5).

Fig.6.1.6-8 taken on a whole orbit plot of 7734 shows that the satellite was at last effectively beginning to be slewed into the orbit plane (estimated tilt angle 110°) and with low nutation. So, after a very long haul the operational effectiveness of the attitude changing system was demonstrated. *It may be the first time that a direct control from a ground station has simultaneously precessed and denutated a spinning spacecraft using a spin-plane magnetorquer.* I found no reference later in the literature.

It is not intended to describe here in detail all the subsequent work. The general principle was to keep pushing the tilt angle towards 90° and deploy the boom on a North to South pass having checked at the right time (at the beginning of the pass) that the pitch angle could also be judged small. Effectively all we had to do was to wait for the natural precessional effect (see section 3.4) under gravity gradient to bring the pitch attitude to a reasonable degree of verticality before deploying the boom.

A complicated set of manoeuvres then followed, starting from pass 7741 and continued to the final pass, with the aim (i) keeping down the roll (ii) preventing build-up of nutation (iii) trying to anticipate /control a convenient pass in advance when the attitude would be in bounds. This turned out to be a hit-or-miss affair; after some cogitation I decided that for the final pass the only measurement I would consider was the B_z component at the beginning of the pass. Better software for example fig.6.1.6-9 now converts the B_z field to an equivalent angular deflection (as Eq.6.1.5.1). Notice the tilt at $\delta = 198^\circ$ - within 18° of the vertical on that part of the orbit. And therefore satisfactory for boom deployment.

6.1.7 The boom deployment problem

The following table was drawn up

Observation	Decision
$-1000 < B_z < -670$	OK to deploy
$-670 < B_z < -300$	maybe (OK if increasing)
$-300 < B_z < 0$	wait and try again

On pass 7826 the measured B_z values were taken down at the beginning by hand. These values (1000 is the normalised value) were

-641 -645 -648 -651 -654 -658 -661 -664 -668 -674 -677 -680 -683 -685

Since they were in bounds and going the right way I judged that our manoeuvrings had been successful. The rest of the team then went into action (Sweeting, Peel and Bean) The consequences are shown dramatically in figs.6.1.7-1 and 6.1.7-2.

It can be seen that the B_z continued negative; but an ominous cyclical rotation was detected in B_x and B_y . Then approximately 2 minutes after the boom deployment began it collapsed. The most likely source of the problem was the set of magnetometer feed cables. These had been wrapped around inside the central core of the spacecraft and were supposed to pull out as the magnetometer ascended. Clearly they stuck.

Nevertheless gravity gradient lock WAS achieved for a short time: a whole orbit dump number 7826 taken immediately after (fig.6.1.7-3) shows that there is only one cycle of B_z effectively in the course of one orbit - consistent only with this interpretation.

Clearly the boom had only gone out a very short distance - perhaps 1 metre - and the gravity gradient torques were overcome by other disturbing torques. So the capture was lost - as Fig.6.1.7-4 clearly shows.

6.1.6-1 Projection of sun and UOSAT1 ascending node

6.1.6-2 Magnetometer plot for orbit 6353M

6.1.6-3 The cooling of UOSAT1's base

6.1.6-4 Illumination conditions

6.1.6-5 Magnetometer plot for orbit 6466M

6.1.6-6 Magnetometer plot for orbit 6519M

6.1.6-7 Magnetometer plot for orbit 6891

6.1.6-8 Magnetometer plot for orbit 7734/5

6.1.6-9 Magnetometer plot for orbit 7802

6.1.7-1 Bz mag reading as boom deployed

6.1.7-2a B_x mag reading as boom deployed 6.1.7-2b B_y mag reading as boom deployed

6.1.7-3 Plot of nav magnetometer for orbit 7826

6.1.7-4 Plot of nav magnetometer for orbit 7848

6.2 THE CONTROL OF UOSAT-2 -part I

In this first phase magnetic stabilisation was achieved and the transition to gravity-gradient ADCS worked smoothly, using effective attitude determination from a magnetically locked state

A second satellite UOSAT-B was planned, and construction started on it in earnest in October 83. After launch in March 1984 of the spacecraft, with the designation 'UOSAT-2', control and switch-on were achieved immediately but then communications were lost over the period from March 1 to May 14 1984. The problem proved traceable and was able to be circumvented due to the multiple redundancy feature within the command system. The nature of this problem and its solution has been outlined in chapter 2 (see particularly sec 2.1.3).

The strategy outlined in section 2.5.2 was followed, made possible by a more elaborate magnetorquing system, and the magnetically stabilised s/c was switched over to gravity-gradient stabilisation with success.

As from February 1987 successful OBC software has been able to control libration using on-board algorithm, principles of which have been discussed in section 5.3.

6.2.1 Theoretical preparation

Section 2.5.2 spelt out the ideal course of events in making a transition from magnetically stabilised UOSAT-2 to gravity- gradient controlled state. The two figures 2.5.2-1a and 2.5.2-1b indicate an idealised state of transition from a state of pure pitch.

The aim was to find criteria for safe deployment of the the gravity - gradient boom. From what initial attitude, would it be safe to switch off the magnetic stabilisation and deploy the boom? Also, given the limited attitude determining capability of the magnetometer measurement, how would one *know* what this attitude was in the first place?

The answer to the first part of the question has been presented in sections 3.5 and 3.6 showing a broad tolerance in prior attitude provided the Z axis starts from a forward-pitched state from the vertical. Specific cases were analysed and solved by the numerical solution by Runge-Kutta-Merson algorithm of the appropriate coupled differential equations (section 3.6), and checked by analytical work on bounds of libration (section 3.5).

6.2.2 Magnetometer and magnetorquer Checks

Prior to launch, the UOSAT team took UOSAT-B to the Goddard Flight Centre in Washington D.C. Fig.6.2.2-1 shows a sketch of the magnetometer geometry, with the labelling of axes of the spacecraft, the location of the magnetometer on one of the wings, and its relation to local North .

A table of measurements of the magnetometer output is given in fig.6.2.2-2. It will be recalled (see chapter 2) that this output was accessible by two different routes: by ADC conversion to 14-bit precision and feeding to the parallel input port of the 1802 microprocessor; or the three channels were accessible through the telemetry system, whose ADC looked at it in sequence with all the other analogue channels. Only the direct parallel port inputs are identified here. The 3 channels are arbitrarily numbered. The contents of each channel are in 'hex' code. The temperature is a decimal number read from one of the telemetry channels. The same magnetic channels were measured in the telemetry.

During the calibration, starting from a forced zero field the magnitude increments were in multiples of 15 microtesla, and in the directions North, South, East, West, 'up' and then 'down' respectively. Bringing the field back to zero values at intervals serves to test the stability of the ADC conversion.

The tables show that that channel 1 responds inversely to the B_x component, channel 2 responds inversely to the B_z component, while

channel 3 responds inversely to the B_y component. This non-systematic order is characteristic of what can be problems in large-scale system integration and emphasise the need for final checks and tests. The plots all established that the basic resolution in any channel is approximately 8 nanotesla, and effectively linear for our purposes.

Although the results are not shown here there was a last minute check done at Vandenberg AFB on the magnetorquers. The fields were checked using a hand-held magnetometer. All settings in the nominal 'forward' and 'reverse' settings, in both high power and low power mode, were checked

6.2.3 Initial Acquisition, Loss and Recovery

After launch on March 1 1984 the telemetry was switched on at v.h.f and three orbits of data were obtained. A rough plot of the motion on orbit 1 showed a flat spin state with a period of 21 secs. The satellite was evidently in a flat spin because the other two channels all showed the same frequencies (see section 4.2).

The telemetry then shut off (prematurely due to an error in the software coding) after 1.25 hours; subsequent efforts to switch it back on were in vain. UOSAT-2 was heard by Stanford Research Institute (outpost in Greenland) listening to the 1.2 GHz local oscillator. On May 11th this was confirmed, as was the orbit position. This test confirmed that UOSAT-2 was still basically operational.

On Monday 14th May, on orbit number 1076, UOSAT-2 was successfully turned back on, with the 145 MHz transmitting telemetry. The up-link turn-on command was successful at 70 cm (u.h.f) having tried repeatedly over the previous weeks at both this and at 2 metres. Credit for sheer persistence goes to N.Bean who kept trying contact over a period of 10 weeks, until a response was obtained.

Study of the telemetry data confirmed that the spacecraft was in perfect condition apart from its reluctance to accept up-link commands. From then on an engineering rescue mission was supervised by M.N.Sweeting - doing much of the work himself - to put it right.

Telemetry was successfully gathered in the course of the week. The command problem was then seen to lie in the decoding stage within the satellite. The problem was ultimately traced to a failure in the Data Presence Detector circuitry. Work started on 'patching' a connection withing the spacecraft - exploiting the multiple independent paths available (outlined in sec.2.1.3).

6.2.4 Angular Motion after Recovery

The angular motion of UOSAT- 2 was then estimated from the magnetometer data extracted from the pass over Guildford on orbit 1076 (fig.6.2.4-1). A similar plot was obtained from orbit 1090 (fig 6.2.4-2). This revealed that there was still this 'flat spin' or 'tumbling' state, defined by a rotation of the z-axis about a spin axis in the plane of the X and Y axes of the spacecraft. This arbitrary spin axis was roughly parallel to the plane of the orbit. The period estimated off the graph was 42 seconds. The attitude configuration was much like UOSAT-1 immediately after launch.

But these graphs showed some intriguing features. I came to the conclusion that here was not a standard near-flat spin. While there was a consistent T-spin component there was not the expected residual constant Z-spin but a Z axis *oscillation*. From the graph one would estimate this to have a period of 165 secs and an amplitude of approximately $\pm 45^\circ$.

This effect is not predicted from the standard rigid body nutation in a torque-free environment. Fascinating though it was there was no opportunity to study it further.

A temperature graph (not shown) compiled from channel 57 over the range of orbits from 1076 up to 1237 clearly revealed the cycling effect with orbital precession under gravity gradient, the nature of which was studied in section 3.4. The temperature evidence suggested that the spin axis was slowly precessing over approximately a 9 day cycle.

6.2.5 Testing the spin-axis magnetorquer

At this time it was not possible to test out commutated magnetorquer control. It was not possible from the ground since the patching of reliable two-way communication was still being worked on; and it was not possible autonomously since the requisite software had yet to be developed. There seemed however little harm in testing if the magnetorquers would come on, and see what effect - if any - followed from permanently leaving on the Z-magnetorquer (i.e spin-axis magnetorquer). It was always possible that there would exist sufficient magnetic damping in the spacecraft that the Z-axis would start to align on the magnetic field - this being the essential preliminary of the control manoeuvres. This plan was finalised and agreed by 29th May .

On May 29th (orbit 1301) the Z magnetorquer was switched on. This was a 3-stage process. First a continuous burst of commands were sent. Success in this was observed in the status points of telemetry transmission. Since the torquers were already in the state forward it remained then to switch on. This registered immediately in a status point ; and was confirmed by the battery current jumping to 590 -600 mA .

There was no immediate change in spin rate from this pass to the next (1302) roughly measured at 51 secs. It was noted however that the spin had already slowed down since recovery - presumably due to magnetic interaction with the activated circuits in the S/C. By June 6 the period of the Z -spin may have shows a slight increase but it was not significant.

Meanwhile continued testing by MNS and Bean using round-link tests showed that with some 'tweaking' to the antennae on favourable passes and attitudes that commanding was tolerable at u.h.f for significant groups of commands. More round-link tests were done at v.h.f using a remote facility with receivers sufficiently far from the transmitter. This showed perfectly good quality in data demodulation and decoding at v.h.f and isolated the problem of the Data Presence Detector which failed to toggle high on data received.

Seeing that their success was imminent, I started to plan for some ground control and initiate active damping. We now had the benefit of installation of a number of BBC model B microcomputers which represented an order of magnitude advance over our old Commodores in their superior graphics, serial port facilities and procedural BASIC.

The work of the team had by now confirmed the detection problem in the v.h.f receiver. It proved possible to power up the second computer normally dedicated to DCE (data communications experiment) and with this to by-pass the detector problem with the appropriate software.

On the 7th June the magnetorquer was switched off because of concern over the power consumption and because it seemed to be having no particular effect, confirming that without active control there was no way of achieving magnetic stabilisation (see section 5.2).

6.2.6 Magnetic Stabilisation Control

By the 15th June there existed an effective and reliable two-way communication system existing between spacecraft and ground. Up-link and down-link could function simultaneously at 2 metres, which was always more reliable than the 70 cm links.

A series of programs had been developed on the BBC micro which accepted a frame of 'dwell' telemetry of B_x , B_y and B_z data from the magnetometer along with status information. I decided to fire the Z

torquer only - because there was known to be no Z-spin and it was undesirable to induce any. The simple procedure for slowing down and capturing a tumbling satellite runs as follows (in pseudo-code). This implements the simplest algorithm of section 5.2.1 firing the magnetorquer (reversed) on a detected positive slope dB_z/dt .

```

load(timeout,timefire,threshold)

1:  getmagdata(Bznow,timenow)
2:  Bzprev = Bznow ,tprev = timenow
3:  getmagdata(Bznow,timenow)
    if tnow > tprev + timeout goto 1
    if abs(Bznow - Bzprev) < threshold goto 3
    if Bznow < Bzprev goto 2
    firemagnetorquer(tfire )
    goto 1

```

The BBC computer allowed a mix of fast BASIC for the signal processing and machine code for handling the input and output routines. This was tried out on passes 1535, 1536, 1544, and 1549. Comparative measurements on earlier passes 1521 and 1529 showed spin periods of 64.6 ± 1.2 secs and 65.1 ± 1.2 secs. respectively. While measurements on 1557 and 1558 showed an increase to 81.6 ± 0.4 and 82 ± 0.13 secs. So there was no doubt that the torquing was taking effect.

Then ground station torquing was continued on subsequent passes (typical fire time about 50 secs) until it was judged that the tumbling had stopped and an oscillating capture on the geomagnetic field had been achieved (orbit 1617). This was confirmed by whole-orbit data collection using a program which had been developed by R.Peel. Analysis of 1630 showed about $\pm 75^\circ$ degree swing. Leaving on the magnetorquer permanently was shown to maintain lock but a progressive deterioration of this acquisition was observed with increasing amplitude (orbits 1637, 1639, 1660, 1661). Active control was always needed.

Subsequent passes proved the futility of trying to damp-out this motion within the limited period of a pass because of the very slow periods (typically 10 minutes) which gave little opportunity within a working

pass of 10 - 12 minutes to find favourable phases. From this time it was obviously necessary to develop and test out on-board software for automatic control.

Pass 1676 with improved whole-orbit data collection showed strong oscillations with max deviation of $\pm 154^\circ$. On pass 1681 the first on-board 1802 program was loaded and run; it was turned off on the next pass (1682) to allow data collection. This showed an immediate effect in reducing the amplitude to $\pm 102^\circ$. This algorithm was then left running to test its ultimate effectiveness; which proved that it could reduce oscillations to around 60° amplitude with an occasional wider excursion to 85° (orbit 1697). This algorithm still implemented the same simple idea as used in the ground control .

I was looking for magnetic stabilisation with a residual amplitude of the Z axis oscillation relative to the B vector no greater than $10-20^\circ$. was that it took not account of the change in *magnitude* of the geomagnetic field during the orbit. This would clearly influence the rate of change dB_z/dt . An improved program, much on the lines of section 5.2.2 was tried. As discussed , the algorithm effectively measures the angle between the Z-axis and the B vector with almost no signal processing. The program was developed and tried out, testing purely in observation to begin with, and then developed with full control of the magnetorquer for the first time on orbit 1739 and its effect observed by 1740.

Program development continued - and by about orbit 1813 it seemed to be working well. Analysis showed that oscillation were reduced to $\pm 30^\circ$ in amplitude at worst and with clear evidence of a 'rocking' motion passing through the B vector while magnetorquing; then with the torquer left on this converts to a gyration around the B vector with uneven amplitude - which gradually increase in amplitude.

6.2.7 Boom Deployment

The critical problem remained how to make the actual attitude measurement during the boom deployment. The plan was to have the satellite approaching on a North to South pass and notionally locked to the local geomagnetic field. If the stabilisation were perfect then attitude of Z axis would equal the attitude of B vector. On a known orbit and for a known geomagnetic field the attitude is therefore determinate at every moment during a pass. In the course of a pass then at any time, the consequences of starting to deploy the boom could be predicted. Knowing the direction of the IGRF field in the course of a pass and using the results of section 3.6.2 imply, in general it would always be safe at the beginning and becoming more problematic towards the end of the pass - because the pitch was too large before the start of the boom deployment.

So the attitude of the Z-axis was *deemed* to be identical with the known pitch and roll of the B vector within the reference frame, where this value was precalculated and known at any instant, from reading a time-synchronised file. This file contained predicted values from knowledge of the orbit and the IGRF model of the geomagnetic field - the IGRF mode, not the idealised IDPO model (see sec.4.3.4). This file was created on a main-frame computer (PRIME) and then transferred to a floppy disc in advance of the critical pass, to be read by a BBC computer synchronised in advance for the correct time.

As the spacecraft came into view, a master program was running on the BBC. This accepted inputs on the down-link of 'dwell' telemetry making measurements of components B_x , B_y and B_z . Comparison with IGRF data established the *error* in the nominal attitude. At any instant - for all these values need to be updated every few tens of seconds - worst-case pitch and roll values could be calculated. Combined with the input from the floppy disc allowed the calculation and display on the screen of an uncertainty rectangle (fig 6.2.7-1). The centre of this rectangle is the computed orientation of the B vector (taken from the floppy disc and updated as time elapses). Each edge of the rectangle marks a

possible current attitude. Strictly the display should be an ellipse but the rectangle bounds is easier to compute and plot; and is therefore more pessimistic.

It was thought important to make an estimate of rates of change in pitch and roll just in case these were excessive. To do this a key was pressed at the start of the pass and the window in possible pitch and roll options created. Then about 2 minutes later another key was pressed. This initiated calculations within the program which checked how much the three worst case edges of the window had moved and calculated worst-case velocities. By looking up another reference data file (created by the numerical integration work) all worst case edges on the attitude window could be checked in order to decide if it was safe or not to deploy.

If all possible combinations were safe then the screen display validated the decision to deploy and the team could enable commands to start boom deployment. So that on the pressing of a key at the start of the pass and the pressing of a key one or two minutes later the distillation of all these sources of information would converge on the decision to deploy or not.

Despite the complexity of the system planning everything went smoothly. All this came together on pass 2112 in July when UOSAT-2 was primed ready for deployment on pass 2113 (tape count 1301). On pass 2113 the data was acquired and the decision made to deploy based on the display on the BBC after about 2 minutes. The display read out worst case possibilities in pitch and roll and included worst-case change in velocity.

UOSAT ATTITUDE LIMITS (3 sides)

p°	r°	$dp/dt *$	$dr/dt *$
22.8	6.6	0.6	0.0
5.4	6.6	0.1	0.0
14.1	15.3	0.0	0.2

* normalised to orbital rate

Compared with a previous look-up of permitted values then this was judged favourable for boom deployment. The boom was deployed for a time of 15 minutes with automatic shut-down by the on-board program. On pass 2114 the whole orbit dump confirmed that there was only one cycle of Z data observable in one whole orbit - indicating a lock onto gravity gradient. This pattern was observed subsequently on further orbits. A collection of dumps 2185, 2208, 2221, 2231, and 2245 was combined with an improved IGRF-generating program was used to test the attitude over North and South poles, when possible. At that time the signal processing, software computing and graphics facilities were simply not adequate for making reasonable measurements of attitude. Figs 6.2.7-2 show the one cycle effect per orbit - confirming gravity gradient was achieved.

6.2.4-1 Mag data on orbit 1076

6.2.4-2 Mag data on orbit 1090

6.2.7-1 Signal processing in boom deployment phase

6.2.7-2a/d B_z data on orbits after deployment

6.3 The Control of UOSAT-2 - Part II

In this second phase the technique of deliberation by application of the threshold algorithms was demonstrated and a best result showed capture from a tumbling initial state to gravity gradient lock within a 10° libration angle

The attitude determination control of UOSAT-2 was effectively interrupted in the late summer of 1984. It did not resume until April/June 1986. During that time attitude control development went into 'hold' with no effort available to make further improvements. Operations were limited to two activities:

- * monitoring the spin rate - which built up from a negligible value and had at intervals to be suppressed.

- * needing to allow increasing libration and recovery loss of capture by occasional re-application of the magnetic stabilisation algorithm NOTE

6.3.1 Attitude determination

Better software (C. Underwood) and improved ground data-handling facilities made it possible to develop the bounded attitude estimation technique, even though we had few control facilities. The method has been explained in section 4.4 with examples given there

NOTE There is a strong resemblance between the stabilisation routine and the deliberation routine since both fire on the rule $\{ M_z \} = \{ dB_z/dt \}$. MNS used this routine 'by hand' not expecting the stabilisation routine to recover on magnetic stabilisation (it did not) but simply to remove transverse energy - which it seemed to.

6.3.2 Attitude control

Renewed efforts started in November 1986, and positive results were obtained from February 1987. Prior to that some misguided 'manual' efforts at stabilising the spacecraft had only caused it go into a systematic tumble with a period of 17.5 mins (the boom extended and the situation is as bad as it could possibly be). Fig.6.3.2-1 is a plot of B_z - (first and last are frame counts which converts to time by multiplying by 4.84 secs). In this period manual control was being attempted - on passes only - because of uncertainties about the state of the spacecraft software. Details are not given here but it involved the spin-plane torquer. Despite many attempts the tumble rate period had been lengthened only to 23.7 mins by 13th February (fig. 6.3.2 -2).

Finally the on-board software was ready and run. The software was intended to implement the threshold algorithm of sec.5.3.1 firing on the rate of change dB_z/dt compared against a threshold. There was an immediate, dramatic difference see fig.6.3.2-3 taken from the 16th February. The loaded threshold was 5 magnetometer units per sec (equivalent approximately to 40 nT/sec or 480 nT/min).

This turned out to be set too low - causing the satellite to go into 'semi-tracking' of the magnetic field judging by fig 6.3.2-4.

By then it became possible to get up-to-date routine α and β plots. Fig.6.3.2-5 through 6.3.2-7 show very clearly the transition to a rough gravity gradient lock. This was achieved by backing off the threshold to 7 units per sec or 56 nT/sec. However this got worse (fig.6..3.2-8) and was finally lost again fig.6.3.2-9. Fig 6.3.2-10 taken one day later shows the first γ plots indicating the attitude in poor state. Investigation showed that the magnetorquer was simply not coming on at a reasonable frequency.

The solution turned out to be a forgotten condition embedded in the software. Referring to fig 6.3.2-11 it can be seen the bottom graph of B_x has two horizontal lines above and below which the magnetorquer was programmed not to fire. While the decision to fire was governed always by the zero-crossings of B_x . For too slow a spin rate and too narrow a window all opportunities are lost as can be clearly seen. This B_x restriction was removed and recovery achieved (fig.6.3.2-12). In any case, as predicted in the simulation of sec.5.3.4, the magnetorquers fire only rarely. The next fig 6.3.2-13 is interesting in showing the pulses appearing in B_x when this happens..

Fig. 6.3.2-14 is a plot of theoretical against 'measured' magnitude and shows a great discrepancy. This was greatly improved as of fig.6.3.2-15 using the autocalibration method, the full details of which are given in section 4.5.

Not detailed here is the fact that we have constantly to monitor and adjust the spin rate. If done from the ground this is bound to perturb the librating state. The remedy is to run spin -changes also from the spacecraft since one can control the times when minimum damage is done. Referring to section 5.1 this condition is for $B_z \approx 0$. Following that treatment the attitude greatly improves. Fig. 6.3.2-16 is the best yet showing excursions in γ confined within a band less than 10° .

All the work of those weeks had been with threshold set at 7 units per sec. There is evidence that this^{is} still too low and we are still stimulating libration rather than suppressing it. There must exist an optimum. If set too low it stimulates libration; if too high it lets libration build up before suppressing it. Experiments are continuing.

The threshold de-libration algorithm of section 5.3.1 can be improved on in many important ways. The immediate plan is refine the scheme on the lines outlined in sec.5.3.3 and after. More interesting would be to achieve a full 3 axis attitude determination - using a Kalman-like estimator working on the magnetometer data. Given this information far more comprehensive active torquing can be envisaged. These matters are

studied in the next chapter 7. These will be investigated just so far as we can develop the software to run on the OBC.

A limitation in all this work previously had been the need for the programmer (S.Holder) to do all his work in 1802 assembler code. This did not allow for complex decisions nor for any higher mathematics.

Only since January 1989 after a long program of development by the UOSAT team is there now installed an on-board multitasking combined operating system and language (POLYFORTH). Fig.6.3.2-17 (α and β plots) and fig.6.3.2-18 (y bounds) shown here are a first fruits of this system, providing the remarkable detail of measurements every 5 seconds over 4096 data points. Collected on-board by the computer it is then down-linked from memory in a packet communication system which ensures perfect data transmission. Fig.6.3.2-17 show a residual amplitude of fast oscillation in the order of $\pm 6^\circ$ which seems to be a feature associated with the Z spin. We can only attribute this to an actual bending of the boom out of the centre line by $\sim 6^\circ$.

Fig.6.3.2-18 shows a residual libration $\sim 15^\circ$. The noteworthy feature of this result is that it is a steady-state condition obtained after about 1 year of operation with the threshold delibration program switched OFF. But all this time a spin-control algorithm has been in place. In other words an autonomous program monitors the Z-spin rate and if it is too fast or too slow then a compensating small spin-plane torquing pulse is applied. The conclusion is that the control of the Z spin energy indirectly applies a crude delibration control. I have no theory to explain this effect, which I can assume only arises from a dissipative interaction between the spin state and the librating state, and enabled by the flexible tape boom. This matter clearly needs to be investigated further.

I am very grateful to P. Wright for the able assistance which made the rapid production of these graphics from fig.6.3.2-1 through 6.3.2-16 possible.

Fig. 6.3.2-1 Tumbling state at start of manoeuvres - Bz values

Fig. 6.3.2-2 Slower tumbling from 'manual' attempts Bz values

Fig. 6.3.2-3 Drifting state after first application of algorithm

Fig. 6.3.2-4 Near magnetic capture

Fig 6.3.2-5 Nearly captured state α and β plots

Fig. 6.3.2-6 Roughly captured state α and β plots

Fig. 6.3.2-7 Reducing libration

Fig. 6.3.2-8 Increasing libration

Fig.6.3.2-9 Attitude now lost

Fig.6.3.2-10 Lost attitude first y_1 and y_2 plots

Fig. 6.3.2-11 Cause from insufficient firing Bx and By plots

Fig. 6.3.2-12 Recovery achieved - α and β plots

Fig. 6.3.2-13 Plots showing mag firing on ascending B_z slope

Fig. 6.3.2-14 Check on poor calibration - field magnitude

Fig.6.3.2-15 Improvement after autocalibration routine

Fig. 6.3.2-16 Best plot obtained on y_1 and y_2 bounds

Fig.6.3.2-17 α and β plots - fine detail plots (4.8 secs int)

Fig. 6.3.2-18 γ_1 and γ_2 bounds with de-spin only

7 ADVANCED MAGNETIC ADCS

This chapter in part investigates the use of magnetometer data to make a complete 3-axis attitude measurement. Detailed simulation shows that it can be done with *estimators*. From a dynamic model of libration, individual estimators can be recursively calculated, while being fed intermittently *innovations* which express the error between the estimated state and the evidence of the magnetometer data. The key concept of *double innovation* is introduced to make it work properly on a polar orbit for the librating state (sections 7.1 through 7.4). Section 7.5 effectively inverts the concepts discovered, and investigates full 3-axis attitude *control* using complementary magnetorquing and gravity gradient on the symmetric UOSAT-2 type geometry. Section 7.6 is an analysis on 3-axis stabilisation by gravity gradient effect alone on a general body (not current UOSATs). It is shown how this complementary technique should overcome past stabilisation problems.

Previous chapters have introduced the theory, and demonstrated the effectiveness in simulation and practice, of ADCS based on incomplete attitude determination (chapters 4,5,6).

Despite the enhancements to the basic threshold algorithm (discussed from sec.5.3.3) it is obvious that best control requires proper attitude determination - whether the control aim is just to maintain 2-axis stabilisation and an uncontrolled yaw - or a full 3-axis stabilisation. The obvious first step to improved control is full attitude determination, on all 3 axes, in real time . The question is how this can be done, if only one 3-axis magnetometer is available as the sole attitude-determining sensor.

A combination of magnetometer measurement and a *state estimator* should yield the required information. And a polar orbit is particular favourable, because of the significant changes in the direction of the magnetic vector in the course of the orbit.

My approach to a best estimator has been basically ad-hoc, eventually favouring something which is a 'filter', and which could be called a

'Kalman filter' if it were linked to a stochastic model and known system and measurement noise. Although it becomes ultimately computer code, executed on sampled data, the 'filter' is most simply visualised as a real machine, acting on a continuous flow of data, almost a 'ghost' satellite, which librates in pitch and roll, and yaws in obedience to the defining differential equations.

Such a 'machine' is self-maintaining, being driven by a mix of internal feed-back and external data. As it runs on a regular sampling basis, the current estimate of an attitude state, at each sampling instant, is used to read a geomagnetic vector and calculate an estimated body vector $\underline{B}^{\wedge}$. The estimate each time can be compared with an actual measured $\underline{B}$ and the vector difference fed back into the 'machine' in such a way, that over a period of time its state comes into line with the satellite's actual attitude.

Although my solution is a set of *deterministic* estimators, these same estimators can be optimised in a *statistical* sense, with optimum solutions calculable in terms of system noise and measurement noise - see Appendix I.

This chapter shows one consistent approach which seems to work well (at least in simulation). By going back to first principles, and trying to keep the maths simple, and testing the work in progressive stages, it has been possible to overcome a number of problems in what is essentially a non-linear system estimation (if one includes all the magnetic vector calculations).

It can be shown that the underlying maths for 3-axis *control* is much the same. Broadly, if an estimator can be driven by magnetic data to follow the real, live librating satellite then much the same maths can be made to drive the real satellite to a desired state using the magnetorquer - another useful illustration of the Reciprocity Principle (sec. 5.4.2)

Finally I have attempted a systematic attack on the problem of trying to understand the basic problem with (intended) full 3-axis stabilisation

by gravity gradient, which is the potential of any general asymmetric body, in the gravity field. Complementary magnetorquing would seem to overcome basic difficulties and lead to virtually a new control principle suitable for low Earth orbiting satellites.

7.1 General Principles

The magnetometer data can in principle yield roll, pitch and yaw states. From the quasi-linear differential equations analogue 'machines' can be constructed which track these motions continuously, although fed data intermittently. One is a spinning estimator, the other two are cyclic or harmonic estimators. The conversion of these machines to sampled data calculations is discussed

7.1.1 Magnetometer measurements processing

Using the figures 3.1.2-1 and 3.1.2-2 a simple set of equations relate the reference geomagnetic vector $\underline{B}_0$ to the local vector $\underline{B}$ as seen in the body co-ordinates. From vector $\underline{B}_0$ a first transformation yields a vector $\underline{B}_1$

$$B_{x1} = B_{x0} - B_{z0} r \quad \dots 7.1.1.1a$$

$$B_{y1} = B_{y0} + B_{z0} p \quad \dots 7.1.1.1b$$

$$B_{z1} = B_{x0} r - B_{y0} p + B_{z0} \quad \dots 7.1.1.1c$$

These equations are valid only for small pitch and roll angles. A second transformation to the measurable $\underline{B}$ vector (in the body axes of the spacecraft) is

$$B_x = B_{x1} \cos(\phi) + B_{y1} \sin(\phi) \quad \dots 7.1.1.2a$$

$$B_y = -B_{x1} \sin(\phi) + B_{y1} \cos(\phi) \quad \dots 7.1.1.2b$$

$$B_z = B_{z1} \quad \dots 7.1.1.2c$$

The yaw angle ϕ can have any value.

On our near-polar orbit and on two occasions per orbit there is a known time when the B_{z0} field component is actually zero (true even with the full IGRF model).

Eqs.7.1.1.1 shows that if $B_{z0} = 0$ then $B_{x1} = B_{x0}$ and $B_{y1} = B_{y0}$. And a reasonable time window exists in the vicinity of the equator for which $B_{x1} \approx B_{x0}$ and $B_{y1} \approx B_{y0}$. From Eq.7.1.1.2, and at that time, and measuring B_x and B_y , the yaw state ϕ is known.

The yaw rate can be expected to be constant for at least a quarter orbit - certainly justified for UOSAT2 - which means that the yaw angle is predictable when the satellite reaches a position near magnetic pole one 1/4 orbit later.

At a point of closest proximity to the pole the direction of the B_0 field will be the nearest to the vertical (typically within $5 - 10^\circ$). The equations can then be interpreted in a different way. Vector B_1 can be back-calculated from B using the inverse of Eq.7.1.1.2 and an estimate of ϕ , in order to measure the current roll and pitch values via Eq.7.1.1.1: and because the components B_{x0} and B_{y0} will be relatively small the roll and pitch values can be estimated with the best precision, by multiplication through the scaling B_{z0} ; with least dependance on offsets B_{x0} and B_{y0} .

With this information available at the poles and a suitable time-varying model the pitch and roll values can be extrapolated to the next equatorial crossing. A formal calculation using Eqs.7.1.1 and 7.1.1.2 can be used to improve the next yaw state estimation, over an extended window in which B_{z0} does not have to be zero (NOTE)

NOTE In principle a pitch estimate can be attempted at the equator. On the IDPO model at a suitable point the measurement B_z is most sensitive to a pitch measurement because of the dominating component B_{y0} (Eq.7.1.1.1). This additional information may be adopted in later work.

Conclusion: the magnetometer data can be made to measure all three attitude angles but only intermittently: pitch and roll data in a time window near the poles; yaw data in a time window near the equator. In combination with some suitable model all three values can be determined continuously.

7.1.2 Differential equations

A dynamic model is needed for the librating state. In the absence of applied torques and for small angles Eqs.3.7.2.1 may be modified to read

$$\ddot{r} + \omega_R^2 r = k_{R0} \omega_R^2 d\phi/dt \quad \dots 7.1.2.1a$$

$$\ddot{p} + \omega_p^2 p = 0 \quad \dots 7.1.2.1b$$

$$d^2\phi/dt^2 = 0 \quad \dots 7.1.2.1c$$

where ω_R and ω_p are the fixed oscillating rates respectively in roll and pitch; and k_{R0} is a known constant which can be traced back to the relative moments of inertia of the satellite. Repeating Eqs.3.7.3 then $\omega_p = \sqrt{3} \omega_0$ and $\omega_R = 2 \omega_0$.

The parameter ϕ , the yaw rotation, according to this equation does not accelerate. As explained back in sec.3.7.2 this is an approximation but not exceeding a few percent in error.

The motions in pitch and roll can be viewed as simple uncoupled s.h.m of known period. The yaw motion can be viewed as simple and constant Z spin.

So the solutions are ideally

$$r = r_0 + R_1 \cos(\omega_R t + \theta_R) \quad \dots 7.1.2.2a$$

$$p = P_1 \cos(\omega_p t + \theta_p) \quad \dots 7.1.2.2b$$

$$\phi = \phi_0 + \omega_z t \quad \dots 7.1.2.2c$$

The constant roll bias

$$r_0 = \omega_R^2 k_{R0} \omega_z$$

...7.1.2.2d

Roll and pitch, amplitude and phase, and the yaw phase and spin rate constitute six constant parameters: R_1 , θ_R , P_1 , θ_P , Φ_0 , ω_z . In principle the attitude is entirely determined for all time once these six parameters are known, if this model were exact. In reality they can never be known with total accuracy ; and in any case the state is bound to changing (and the model itself has limitations). Some means is needed in which all these parameters are not determined permanently but are averaged over a period of time and also updated regularly. The analogue estimators described next do this without explicitly solving for these fixed parameters

7.1.3 Analogue spinning estimator

Eq.7.1.2.1c modified by a possible control acceleration reads

$$d^2\phi/dt^2 = N_z/I_z$$

....7.1.3.1

To implement this equation two integrators connected in series will emulate a spinning system with the output ϕ identified as the angular rotation of a shaft (fig.7.1.3-1). Such a system provides a model for the yaw state of UDSAT. The effect of a control acceleration can be accommodated at the input (i) of the first integrator. When no control is applied (most of the time) then the yaw state is an incrementing or decrementing ramp (NOTE).

The aim is to measure the state of rotation. But ϕ is not continuously accessible; as information it is available only intermittently.

NOTE The inconvenient property of true integrators running into top and bottom limits is ignored here

The solution is to build an estimator, or 'copy system', consisting again of two integrators, whose output is an estimate $\hat{\phi}$ which can be read at all times, and which tries to track ϕ .

The system requires the estimate $\hat{\phi}$ - at point (ii) - to be subtracted from the true ϕ making an *innovation* v - an expression of the disagreement between them, following Kailath's terminology (Kailath 1971).

Assuming that this innovation is available it is then applied, plus a summation of its first derivative, through a switch S and the appropriate gains, as an equivalent acceleration into the estimator's first integrator (point iii). In practice an indirect calculation generates a scaled innovation directly from the magnetometer data and $\hat{\phi}$, without first calculating ϕ (details from section 7.3).

The governing equations are

SYSTEM

$$d^2\phi/dt^2 = N_z/I_z \quad \dots 7.1.3.1$$

INNOVATION

$$v = \phi - \hat{\phi} \quad \dots 7.1.3.2$$

ESTIMATOR

$$d^2\hat{\phi}/dt^2 - 2a dv/dt - c^2 v = N_z/I_z \quad \dots 7.1.3.3$$

where a and c are gain constants feeding the estimating filter.

From Eqs 7.1.3.1 through 7.1.3.3

$$d^2v/dt^2 + 2a dv/dt + c^2 v = 0 \quad \dots 7.1.3.4$$

The above is a standard differential equation.

Convergence is controlled by setting a damping factor

$$\zeta = a/c = 1/\sqrt{2}$$

...7.1.3.5

for a good combination of fast acquisition and minimum overshoot. The system has an effective bandwidth depending on the value of c . And the settling properties are identical to a second order phase-lock-loop (PLL).

It is the estimate $\hat{\phi}$ which feeds a controller - box at bottom of fig.7.1.3-1. And if this controller makes a decision, to accelerate or deaccelerate the spinning spacecraft, that acceleration is fed to both the 'real' system and the estimator, as seen in the diagram. The mathematics then show that the estimator is not affected by any control decision.

Given continuous information the innovation will eventually stabilise on zero and is interruptable; the switch S may be opened and the estimator will carry on ramping up/down with $\hat{\phi}$ regardless (although with time increasingly running into error) until such time that the information is back on stream. When the switch S is closed the system gets back into line. Thus it can 'run' on intermittently provided data —?

7.1.4 Analogue cyclic (pitch) estimator

The following description applies to the *pitch* only - a modification is needed for roll estimation. Eq.7.1.2.1b, as modified by a control acceleration, reads

$$d^2p/dt^2 + \omega_p^2 p = N_{x0}/I_T$$

...7.1.4.1

where N_{x0} is a torque component resolved on the X_0 axis.

To implement this equation the system consists of two successive integrations - see the top of fig.7.1.4-1.

Unlike the yaw model there is also feed-back. The system therefore oscillates (although unlike a true oscillator there is no mechanism to initiate the process of limiting the amplitude). The response to a torque - applied at point (i) - is a calculated acceleration due to magnetorquing. This acceleration is in response to whatever decision a controller may make (bottom of the figure).

The aim is to measure the (pitch) state p of the oscillator. But p is not continuously accessible. The solution is to build an estimator, or 'copy system', consisting again of two integrators , whose output $\hat{p}$ can be seen at all times, and which tracks p .

The estimate $\hat{p}$ - at point (ii) - is subtracted from the true p making an *innovation* v_p . In practice an indirect calculation generates an innovation directly from the magnetometer data and $\hat{p}$, without explicitly involving p .

This innovation is applied, plus also its first derivative, through a gain ω_e , and series switches S_A and S_B respectively, and fed into the estimator at the two different points shown.

The governing equations are

SYSTEM

$$d^2p/dt^2 + \omega_p^2 p = N_{x0}/I_T \quad \dots 7.1.4.1$$

INNOVATION

$$v_p = p - \hat{p} \quad \dots 7.1.4.2$$

ESTIMATOR

$$d^2\hat{p}/dt^2 - 2\omega_e dv_p/dt + \omega_p^2 \hat{p} = N_{x0}/I_T \quad \dots 7.1.4.3$$

where ω_e = a gain applied to the estimator.

From Eqs 7.1.4.1 through 7.1.4.3

$$d^2 v_p / dt^2 + 2 \omega_c dv_p / dt + \omega_p^2 v_p = 0 \quad \dots 7.1.4.4$$

Estimate $\hat{p}$ feeds a controller - box at bottom of the figure. And if this controller makes a decision, to suppress the oscillation, then an acceleration is fed to both the 'real' system and the estimator, as seen in the diagram. The mathematics show that the estimator is not affected by any control decision.

The innovation v_p obeys the standard form of a second-order differential equation. Unlike the PLL yaw case there is no specific damping factor. The resonant frequency ω_p is fixed; and for sensible values of the variable ω_c in Eq.7.1.4.4 that equation will be substantially underdamped, with possibly many cycles of oscillation before it finally settles down. Despite this, a plot of the estimate $\hat{p}$ against the driving waveform p will appear to converge monotonically. This is because pitch p is itself oscillating at the same frequency.

The innovation v_p will eventually stabilise on zero, in the absence of perturbing torques. The system is interruptable; the two switches S_A and S_B may be opened and the estimator will carry on oscillating regardless (although in time increasingly running into error), until the innovation is switched back on.

Double innovation, having two switches feeding both the innovation AND its derivative, is important. It is an essential feature of the A.D.C estimator.

The reason is as follows. It may be supposed that the switches are 'mostly off' but with occasional small windows (much shorter than the oscillating period) when they may be ON.

With the switches S off, the model $\hat{\phi}^P$ and the estimator $\hat{\phi}^{\hat{P}}$ will oscillate independently of each other at the same frequency ω_p , but having arbitrarily different phases and amplitudes. Therefore the innovation v_p will also be a sinewave (of the same frequency), but having some arbitrary amplitude and phase. There will be no damping because the switches are not closed. Assume that only ONE switch can be ^{closed} opened briefly just once per cycle, on a particular phase of the innovation. Through switch S_A a sample of the 'straight' innovation v_p would fail to correct the estimating loop, on one sample whenever $v_p = 0$. Through switch S_B a sample of the derivative of dv_p/dt would fail to correct the estimating loop, on one sample, whenever $dv_p/dt = 0$.

However v_p and dv_p/dt are in *phase quadrature* because each is a (co)sinusoid. Therefore the two worst-case situations outlined above cannot happen simultaneously. So that, on any timing or synchronisation of the switch opening a corrective innovation is always available. The estimator can 'run' on intermittently provided data, whatever its timing/phase.

7.1.5 Analogue cyclic with offset (roll) estimator

The following applies the small modification needed for roll estimation. Eq.7.1.2-1a modified by a possible control acceleration reads

$$d^2r/dt^2 + \omega_R^2 r = k_{RO} \omega_R^2 d\phi/dt + N_{VO}/I_T.$$

...7.1.5.1

Fig.7.1.5-1 is similar to fig.7.1.4-1. The 'model' of the oscillating system again consists of two successive integrations. Unlike the yaw model there is also feed-back. The estimator inherently oscillates at frequency ω_R (which is not the same value as ω_p). Unlike the pitch model a compensating term is applied at the point (iv) shown to account for the offset. This offset is derived from the yaw estimator,

at the point where it generates $d\phi^{\wedge}/dt$. Again the response to a torque - applied at point (i) in the model - is a calculated acceleration due to magnetorquing. This acceleration is in response to whatever decision a controller may make (bottom of the figure).

The differential equations are necessarily more complex because of the second input from the yaw estimator.

SYSTEM

$$d^2r/dt^2 + \omega_R^2 (r - k_{RO} d\phi/dt) = N_{YO}/I_T \quad \dots 7.1.5.1$$

INNOVATION

$$v_R = r - r^{\wedge} \quad \dots 7.1.5.2$$

ESTIMATOR

$$d^2r^{\wedge}/dt^2 - 2 \omega_E dv_R/dt + \omega_R^2 (r^{\wedge} - k_{RO} d\phi^{\wedge}/dt) = N_{YO}/I_T \quad \dots 7.1.5.3$$

where ω_E is the same gain constant as used in the pitch estimator

From Eqs.7.1.5.1 through 7.1.5.3

$$d^2v_R/dt^2 + 2 \omega_E dv_R/dt + \omega_R^2 v = \omega_R^2 k_{RO} (d\phi/dt - d\phi^{\wedge}/dt) \quad \dots 7.1.5.4$$

The provision of 'double innovation' is particularly important for the roll estimator. Because the frequency of the roll oscillation $\omega_R = 2 \omega_D$ this means that the sampling windows in which innovations are available (over the poles) will be synchronised to a roll oscillation. Without ' double innovation ' there would always be critical phases in the roll innovation for which the correction simply does not work (confirmed in simulation runs).

Exact: $\frac{d^2 \phi}{dt^2} = T(t) \Rightarrow \frac{\phi(s)}{s^2} = \frac{1}{s^2}$

Discrete:

$$G(z) = (1-z^{-1}) \mathcal{Z} \left(\frac{1}{s^3} \right)$$

$$= (1-z^{-1}) \frac{\Delta^2}{2} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} = \frac{\Delta^2}{2} \frac{z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}} = \frac{\phi(z)}{T(z)}$$

Matrix Form: $\underline{x}(k+1) = A \underline{x}(k) + B T(k) \quad \text{and} \quad \phi(k) = C \underline{x}(k)$

Estimator: $\hat{\underline{x}}(k+1) = A \hat{\underline{x}}(k) + B T(k) + G [\phi(k) - \hat{\phi}(k)] \quad G = [g_1, g_2]$

$\Rightarrow \underline{e}(k+1) = \underline{[A + GC]} \underline{e}(k) \quad \text{For} \quad \underline{e}(k) = \underline{x}(k) - \hat{\underline{x}}(k)$

Determine dynamics of estimator (place estimator roots by choice of G)

$$\frac{d^2 \hat{\phi}}{dt^2} - 2a \frac{d\hat{\phi}}{dt} - c^2 \hat{\phi} = \frac{N_2}{I_2} = 0$$

$$\approx \frac{1}{\Delta^2} (1-z^{-1})^2 \phi(z) - 2a \frac{1}{\Delta} (1-z^{-1}) N(z) - c^2 N(z) = 0$$

$$(1-z^{-1}) \phi(z) = 2a\Delta N(z) + \frac{\Delta^2 c^2}{(1-z^{-1})} N(z)$$

7.1.6 Conversion to sampled data system

A first step to a realistic system replaces every analogue estimator by a discrete sampling estimator, essentially replacing the integrators by sample and hold elements (figs.7.1.6-1 through 7.1.6-3); and the differentiators by sample and hold elements with feed-forward connection. The synthesis of response to applied control torques is a more complicated question. This question is side-stepped here by setting all torques to zero (see sec.7.1.7).

Mathematically the conversion consists in each case

- * ignore torques
- * integrate both sides of the main estimating equation
- * replace a derivative with a multiplying factor $(z - 1) \frac{1}{\Delta} (1 - z^{-1})$
- * replace an integration with a dividing factor $(1 - z^{-1}) \frac{1}{\Delta}$
- * re-scale gain constants
- * sketch block diagram of the system

Significantly the integration is not just the inverse of the differentiation. An explicit 'time advance' or multiplication by 'z' operator is required so that the pitch and roll estimators can resonate correctly at a real frequency matching the analogue system

Yaw estimator

Following rules from Eq.7.1.3.3

$$\frac{(1 - z^{-1})}{(z - 1)} \Phi = 2 \alpha N + \frac{y z}{(1 - z^{-1})} N$$

....7.1.6.1

where

- $\alpha = a \Delta$
- $y = c \Delta$
- $\Phi = z$ -transform of yaw estimate
- $N = z$ -transform of innovation

Fig 7.1.6-1 follows. The sampled data system has approximately the same tracking behaviour as an analogue system (improving as α and $\gamma \rightarrow 0$) . With the innovation switched off the system will ramp up or down on a sampled basis, at a constant increment or decrement.

Pitch Equation Following rules from Eq.7.1.4.3

$$P (z - 1) + \frac{k_p^2 P}{(1 - z^{-1})} = 2 k_e N_p \quad \dots 7.1.6.2$$

where

$$\begin{aligned} \omega_e &\rightarrow k_e \\ \omega_p &\rightarrow k_p \\ P &= z\text{-transform of the pitch estimate} \\ N_p &= z\text{-transform of pitch innovation} \end{aligned}$$

Fig 7.1.6-2 follows. The feature of making both the innovation and its differential input at two different points is preserved - for reasons explained in sec.7.1.4 and sec.7.1.5.

An explicit value for k_p is found by solving Eq.7.1.6.2 for transformed innovation $N_p = 0$. Then

$$P (z - 1) (1 - z^{-1}) + k_p^2 P = 0$$

or

$$P (z^{1/2} - z^{-1/2})^2 + k_p^2 P = 0 \quad \dots 7.1.6.3$$

Resonance occurs at frequency

$$\omega = \omega_p$$

or equivalent

$$z = e^{j\omega_p \Delta} \quad \dots 7.1.6.4$$

where Δ is the sampling interval

Eq.7.1.6.3 with Eq.7.1.6.4 then solve

$$k_p = 2 \sin(\omega_p \Delta/2)$$

...7.1.6.5

An exact copy of the oscillating waveform can be achieved (but sampled at intervals Δ). But the system cannot match exactly the tracking response to the innovation input. That however does not matter; and setting

$$k_e = \omega_e \Delta$$

...7.1.6.6

will give effectively the same response time to the innovation as the analogue system.

Roll Equation Following rules from Eq.7.1.5.3

$$R(z-1) + \frac{k_R^2 (R - R_0)}{(1-z^{-1})} = 2 k_e N$$

...7.1.6.7

where

$$\begin{aligned} \omega_e &\rightarrow k_e \\ \omega_R &\rightarrow k_R \\ R &= z \text{ transform of the roll estimate} \\ R_0 &= z \text{ transform of the roll bias} \\ N_R &= z\text{-transform of the roll innovation} \end{aligned}$$

Fig 7.1.6-3 follows. The feature of making both the innovation and its differential input at two different points is preserved - for reasons explained in sec.7.1.4 and sec.7.1.5.

For the roll estimator the feed back gain

$$k_R = 2 \sin(\omega_R \Delta/2)$$

...7.1.6.8

and an offset term is again introduced to account for the bias due to offset.

7.1.7 Response to Control Torques

It can be shown that restricting the control regime to correctly timed torque pulses allows exact replication in the digital sampled system of the response in the real (analogue) system - so ensuring the cancellation required in the estimator of the effects of magnetorquing

With care it is possible to match response to torques exactly. By keeping to the regime in which a symmetrical magnetorquing pulse is centred on a sampling instant, the response of the actual (analogue) satellite can be matched exactly by a dirac-delta input at the appropriate points shown. This is still true even with the effect of cross coupling from the yaw to the roll estimator, by some careful re-configuration of the block diagram.

7-1 Attitude determination IDPD model small attitude angles

Fig.7.1.3-1 Analogue spinning estimator

Fig.7.1.5-1 Analogue cyclic with offset (roll) estimator

FIG 7.1.6-1 SAMPLED DATA CONVERSION OF YAW ESTIMATOR

FIG 7.1.6-2 SAMPLED DATA CONVERSION OF PITCH ESTIMATOR

FIG 7.1.6-3 SAMPLED DATA CONVERSION OF ROLL ESTIMATOR

7.2 Derive Innovations

There is no direct access to the value of p, r and ϕ as assumed in previous, simplified discussion. All that the real-world measurement provides are the components in $\underline{B}$, which may be compared with calculated reference fields $\underline{B}_0$. These measurements are compared and combined with current estimates $p^\wedge$, $r^\wedge$ and $\phi^\wedge$ to generate scaled innovations

7.2.1 The calculations

The following calculations are repeated after every physical sampling of field values B_x , B_y and B_z and the computation of B_{x0} , B_{y0} and B_{z0} from the IGRF model and orbital position (details for these calculations not given here)

Variables set, stored and carried over from previous iteration:

VARIABLE	MEANING
yaw _x	COS($\phi^\wedge$)
yaw _y	SIN($\phi^\wedge$)
$p^\wedge, r^\wedge$	pitch and roll estimates
bv _{ϕ_x}	current scaled cosine yaw innovation
bv _{ϕ_y}	current scaled sine yaw innovation
bv _p	current scaled pitch innovation
bv _r	current scaled roll innnovation

In the following Bz_norm and Bt_norm are empirically determined scaling values used to normalise the magnetic measurements.

From the $\underline{B}$ vector back transform using COS and SIN yaw estimates

$$B_{xz} = B_x \text{ yaw}_x - B_y \text{ yaw}_y$$

$$B_{yz} = B_x \text{ yaw}_y + B_y \text{ yaw}_x$$

...7.2.1.1

From B_0 forward transform using pitch and roll estimates

$$B_{x1} = B_{x0} - r^{\wedge} B_{z0} \quad \dots 7.2.1.2a$$

$$B_{y1} = B_{y0} + p^{\wedge} B_{z0} \quad \dots 7.2.1.2b$$

$$B_{z1} = r^{\wedge} B_{x0} - p^{\wedge} B_{y0} + B_{z0} \quad \dots 7.2.1.2c$$

Normalise:

$$\begin{aligned} b_x &= B_x / B_{t_norm} \\ b_z &= B_z / B_{t_norm} \end{aligned} \quad \dots 7.2.1.3$$

Shift current scaled yaw innovation:

$$\begin{aligned} bv_{\phi_x}' &= bv_{\phi_x} \\ bv_{\phi_y}' &= bv_{\phi_y} \end{aligned} \quad \dots 7.2.1.4$$

Calculate new scaled yaw innovation

$$\begin{aligned} bv_{\phi_x} &= b_{x1} b_{x2} + b_{y1} b_{y2} \\ bv_{\phi_y} &= -b_{x1} b_{y2} + b_{y1} b_{x2} \end{aligned} \quad \dots 7.2.1.5$$

Shift current scaled pitch and roll innovations

$$\begin{aligned} bv_p' &= bv_p \\ bv_r' &= bv_r \end{aligned} \quad \dots 7.2.1.6$$

Calculate new pitch and roll scaled innovations

$$bv_p = \text{SGN}(B_{z0}) (-B_{y0} + B_{y2} - B_{z0} p^{\wedge}) / B_{z_norm} \quad \dots 7.2.1.7a$$

$$bv_r = \text{SGN}(B_{z0}) (+B_{x0} - B_{x2} - B_{z0} r^{\wedge}) / B_{z_norm} \quad \dots 7.2.1.7b$$

Calculate reference transverse field

$$B_{t0} = \sqrt{(B_{x0}^2 + B_{y0}^2)} \quad \dots 7.2.1.8$$

Calculate valid windows:

$$\text{IF } \text{ABS}(B_{z0}/B_{T0}) < \text{yaw_T} \text{ THEN update yaw_estimator} \quad \dots 7.2.1.9a$$

$$\text{IF } \text{ABS}(B_{z0}/B_{T0}) > \text{pr_T} \text{ THEN update pr_estimator} \quad \dots 7.2.1.9b$$

The values yaw_T and pr_T are pre-calculated thresholds whose significance is explained in the next section.

7.2.2 Explanatory theory

Eqs.7.2.1.1 and Eqs.7.2.1.2 take the components in $\underline{B}$ and the components in $\underline{B}_0$ respectively and rotate to intermediate axes $\underline{B}_1$ and $\underline{B}_2$ respectively .

If the current estimates $\hat{r}$, $\hat{p}$ and $\hat{\phi}$ were exactly correct then vector $\underline{B}_1$ would agree with $\underline{B}_2$.

The remainder of the mathematics extracts a scaled yaw innovation assuming that the roll and pitch estimates are correct; or scaled pitch and roll innovations assuming that the yaw estimate is correct. There is no logical or mathematical conflict here because, as explained in sect.7.1.1 the innovations are never switched into the estimators at the same time.

Yaw innovation

Suppose that angles r and p are correct. Define ϕ_B as the angle of the transverse component of the $\underline{B}$ vector in the plane of axes X_1 and Y_1 . Then

$$\begin{aligned} b_{x1} &\propto \cos(\phi_B) \\ b_{y1} &\propto \sin(\phi_B) \end{aligned}$$

...7.2.2.1

Then ϕ is the true (but unknown) rotation of X, Y body axes from the X_1, Y_1 . So

$$\begin{aligned} B_x &\propto \cos(\phi_B - \phi) \\ B_y &\propto \sin(\phi_B - \phi) \end{aligned}$$

...7.2.2.2

The back rotation of $\underline{B}$ by the current $\phi^{\wedge}$, as required by Eqs.7.2.1.1 is equivalent to

$$\begin{aligned} b_{x2} &\propto \cos(\phi^{\wedge} - v_{\phi}) \\ b_{y2} &\propto \sin(\phi^{\wedge} - v_{\phi}) \end{aligned} \quad \dots 7.2.2.3$$

where

$$v_{\phi} = \phi - \phi^{\wedge} \quad \dots 7.2.2.4$$

is the actual (but not calculated) innovation in yaw. The cross multiplication required then by Eqs.7.2.1.5 yields

$$\begin{aligned} bv_{\phi x} &\propto \cos(v_{\phi}) \\ bv_{\phi y} &\propto \sin(v_{\phi}) \end{aligned} \quad \dots 7.2.2.5$$

Innovations $bv_{\phi x}$ and $bv_{\phi y}$ are therefore the COSINE and SINE of the yaw error estimates but scaled by field magnitudes.

These values are used directly in the yaw estimator, rather than attempting to calculate v_{ϕ} and use that.

Pitch innovation

The pitch estimator works as follows. Repeating Eq.7.2.1.2b

$$B_{y1} = B_{y0} + p^{\wedge} B_{z0}$$

Suppose that the yaw value is correct. Then

$$B_{y2} = B_{y0} + p B_{z0} \quad \dots 7.2.2.6$$

then

$$\begin{aligned} -B_{y0} + B_{y2} - B_{z0} p^{\wedge} &= B_{y2} - B_{y1} \\ &= (p - p^{\wedge}) B_{z0} \\ &= v_p B_{z0} \end{aligned} \quad \dots 7.2.2.7$$

Therefore bv_p from 7.2.1.7a is a scaled pitch innovation

Roll innovation

The roll estimator works as follows. Repeating Eq.7.2.1.2a

$$B_{x1} = B_{x0} - \hat{r} B_{z0}$$

Suppose that the yaw value is correct. Then

$$B_{x2} = B_{x0} - r B_{z0}$$

...7.2.2.8

then

$$\begin{aligned} B_{x0} - B_{x2} - B_{z0} r^{\wedge} &= B_{x1} - B_{x2} \\ &= (r - r^{\wedge}) B_{z0} \\ &= v_r B_{z0} \end{aligned}$$

...7.2.2.9

Therefore bv_r is a scaled roll innovation.

These innovations are conveniently calculated for every sample instant, on all orbital positions. *But they are never used simultaneously*. The simple criterion of Eq.7.2.1.9a restrict the use of the yaw innovation to windows near the equator; and the other simple criterion of Eq.7.2.1.9b restricts the use of the pitch and roll innovations to windows near the poles.

Typical values which seem to work well in simulation (for realistic multipole model of the geomagnetic field) are

$$\begin{aligned} yaw_T &= 0.5 \\ pr_T &= 3.0 \end{aligned}$$

The lower the value of yaw_T the closer must be operations to the equator; and the higher the value of pr_T the closer to the poles; these values are seemingly a good compromise between not providing sufficiently wide windows for the estimators to work properly and making them so wide as to induce computational instability. A region of the orbit has to be reserved for doing nothing ; where

$$0.5 < B_{z0}/B_{T0} < 3$$

...7.2.2.10

7.3 Update Estimators

The updating of the yaw estimate imitates a linear 2nd order PLL at low deviations with a non-linear facility for automatic capture at arbitrarily low loop bandwidth. The pitch and roll updating is routine.

7.3.1 Yaw calculations

Current yaw estimate

$$\phi = 4 \text{ATN}(\text{yaw}_x, \text{yaw}_y) \quad \dots 7.3.1.1$$

Possibly update new yaw rate:

IF yaw_update THEN

$$\Delta\phi = \Delta\phi' + k_1 (bv_{\phi_y} bv_{\phi_x'} - bv_{\phi_x} bv_{\phi_y'}) + k_2 bv_{\phi_y} \quad \dots 7.3.1.2$$

where k_1 and k_2 are re-scaled values from α and γ .

Update new yaw value

$$\phi = \phi + \Delta\phi \quad \dots 7.3.1.3$$

Recalculate COSINE AND SINE yaw estimates

$$\begin{aligned} \text{yaw}_x &= \text{COS}(\phi) \\ \text{yaw}_y &= \text{SIN}(\phi) \end{aligned} \quad \dots 7.3.1.4$$

7.3.2 Pitch/Roll calculations

Calculate new roll bias

$$r_0 = k_{ROT} \Delta\phi \quad \dots 7.3.2.1a$$

Constant k_{ROT} is a proportionality constant calculated (once) from

$$k_{ROT} = k_{RO} \Delta \omega R^2 \quad \dots 7.3.2.1b$$

Update increments

$$\begin{aligned} \Delta p &= \Delta p^I - k_p^2 p \\ \Delta r &= \Delta r^I - k_r^2 (r - r_0) \end{aligned} \quad \dots 7.3.2.2$$

Possibly further update with innovation differential

$$\begin{aligned} \text{IF } pr_update \text{ THEN} \\ \Delta p &= \Delta p^I + k_e (bv_p - bv_p') \\ \Delta r &= \Delta r^I + k_e (bv_r - bv_r') \end{aligned} \quad \dots 7.3.2.3$$

where k_e constant has already been explained

Update

$$\begin{aligned} p &= p + \Delta p \\ r &= r + \Delta r \end{aligned} \quad \dots 7.3.2.4$$

Possibly further update with innovation

$$\begin{aligned} \text{IF } pr_update \text{ THEN} \\ p &= p + k_e bv_p \\ r &= r + k_e bv_r \end{aligned} \quad \dots 7.3.2.5$$

7.3.3 Explanatory theory

A sampled data modification, developed from the 'ideal' analogue system, were sketched in figs.7.1.6-2 and 7.1.6-3. When it comes to actual implementations there are some further necessary modifications described here.

The estimator stores the cosine and sine (yaw_x , yaw_y) of the current yaw estimate $\phi^{\wedge}$ rather than the actual value. However the value of $\phi^{\wedge}$ is wanted on every calculation. Therefore as a first step Eq.7.3.1.1 is needed, where the notation '4ATN' conveys the idea of a four-quadrant arctangent conversion.

If v_{ϕ} and $v_{\phi'}$ are current and previous yaw innovations (never actually calculated) then the term in Eq.7.3.1.2 with Eqs.7.2.2.5 yield

$$bv_{\phi_y} bv_{\phi_x'} - bv_{\phi_x} bv_{\phi_y'} \propto \text{SIN}(v_{\phi} - v_{\phi'})$$

And after multiplication by k_1 this term represents therefore a frequency error, sensing continually if the rate of change of the yaw estimate is too fast or slow.

Provided the magnitude of this rate of change does not exceed π between samples (which anyway constitutes an aliasing limit) this frequency error signal has always the correct sign. The other term

$$bv_{\phi_y} \propto \text{SIN}(v_{\phi})$$

is proportional to the sine of the phase error. And after multiplication by k_2 this term serves to lock the estimate to the true value. When the error is small the calculations act quasi-linearly and confers a standard 2nd order PLL tracking characteristic (provided k_1 and k_2 , and also the preliminary scaling Bt_{norm} , Bz_{norm} are properly chosen). But on initial acquisition the loop acts non-linearly and serves to acquire lock of the estimator on the spinning satellite however small the effective bandwidth determined by k_1 and k_2 .

The combination of Eqs.7.3.2.2 and Eqs.7.3.2.4 by themselves serve to maintain an oscillation automatically - with a pitch frequency controlled by the parameter k_p , and roll frequency by k_R .

The condition 'pr_update' is the equivalent of opening switches in the system block diagrams. While Eqs.7.3.2.3 and Eqs.7.3.2.5 serve to introduce the scaled innovation at the two required points in each estimator.

7.4 Simulation Experiment

Top-down versus bottom-up computer simulation. Development from IDPO to TDIO to IGRF model. Broad outline of simulation is given. The method works and is tolerant to rough field models. There is a need for an initial acquisition phase in yaw before switching on roll and pitch estimators. Slightly different results depending on initial roll phase are presented.

There are at least two opposing philosophies to computer simulation. One approach is start from a 'top-down' structural design; and then to fill out the details arriving at a sufficiently expanded program, which (hopefully) (i) will actually run, and (ii) will produce believable answers. In my experience this only works if one has complete understanding of the problem and knows the answer almost completely in advance. The alternative is a 'bottom -up' approach: to treat the simulation as a form of *experiment*: develop the simulation in easy stages, checking each step for consistency and believability, creating (and storing) a dozen or so programs of increasing complexity. I adopted this latter approach. The intrinsic advantage is that one can be alert for surprises and difficulties; having built up the 'world' in stages, and having reasonable confidence in earlier programs, one is aware at what point the development has produced strange or bad results, and is best placed for finding the solution.

This second approach also pays off because it will usually suggest new ideas, in addition to correcting conceptual errors. In this particular work simulation identified the need for *double innovation* - described first in sec.7.1.4 - the need to route innovations both directly and in differential form at two separate points into the roll and pitch estimators. This discovery was forced by poor results which improved only when this technique was adopted.

The gradual stages developed here: first a perfect polar orbit over an ideal dipole geomagnetic field (the IDPO model as sec.4.3.1); then an inclined sun-synchronous orbit over an offset dipole Earth (TDIO).

In order to prove the concept properly however it was necessary to generate a realistic IGRF type field based on the standard model of spherical harmonics, following the standard treatment of Wertz.

The strategem was adopted of assuming a sun-synchronous orbit where the orbit period is an exact sub-multiple of 24 simulated hours. Also it was assumed that the estimator updates an integer number of times per orbit. In consequence there are an integer number of estimates made in 24 hours AND the sub-satellite point returns to exactly the same point over the Earth. Therefore a generated set of magnetic data - which takes a long time to synthesise on my desk-top computer- can be used repeatedly if the estimates are viewed over more than 24 simulated hours.

It is not proposed to describe the program in detail - a full listing is available on request. In principle a model is created in which the satellite describes an orbit while it spins and librates. At regular sampled times the latitude and longitude of the subsatellite point is computed and the reference field B_0 at that point is computed (or if already computed, then looked up). The standard calculations yield a vector components referred to spherical co-ordinates. Only the radial component is correct - the other two need a further rotation to yield the local reference for that particular orbit at that particular position. Rather than implement a complicated series of forward and back rotations a one-step mathematical transform was analysed and implemented (Appendix J).

From the current attitude, and knowing this reference field, the local field B is computed (in body axes). Of course this part of the program is NOT the estimator - it is merely creating the environment in which the rest of the program (the actual estimator) can operate. No synthesised 'data ' is fed to that part of the program which, being an 'estimator', could not know this information in the real world.

The first step within the actual estimator is a calculation of a vector, consisting of all the innovations discussed in section 7.2. Then, if flag 'yaw_update' is set a procedure called 'yaw estimator' runs.

While a flag 'pr_update' allows another procedure to update both pitch and roll. As was explained they never occur together with the former limited to operations over the simulated equator and the latter to operations over the simulated poles. Empirically it was also found necessary to forbid any 'pr_updating' on initial acquisition until the yaw estimator has sufficient time to 'settle down' and achieve some sort of lock by itself. There are in principle many other methods of defining these estimating windows. Here it seems that 'simplest is best' - the method described in section 7.2 was the only one that survived up-grading the simulations to the complex IGRF field model.

Fig.7.4-1 is a plot showing a typical acquisition sequence based on a full IGRF data set (up to the 8 th harmonic) and shows that the acquisition works perfectly. The relevant parameters are shown on the graphics and an explanation of the various terms and concepts given in subsidiary tables. The first graph is an overlay of actual (sparse dots) and estimated sine of the current yaw *SINE YAW*. The estimate is plotted as either fine dots - when it is simply 'coasting' or as full line - when the yaw estimator is in actual operation.

The second graph yields two *ERRORS* which monitor the success of this yaw estimator. One of them the estimator cannot 'know' and is the difference between true yaw and its estimate - the yaw innovation. This is the quasi-triangular plot. Its appearance is due to the intermittent operation of the yaw estimator; between updates there is no error correction; so the innovation must diverge at some fixed rate. The second more irregular superimposed plot is what the estimator does know and is its 'belief' as to what is the true error. This is $bv_{\phi y}$ - the scaled *SINE* innovation described in section 7.2 and Eqs.7.2.1.5 - which differs from the true innovation by being weighted by the current value of the transverse magnetic field (B_T), amongst other things.

Fig.7.4-1 and some that follow have been selected to try and obtain the fastest possible acquisition. Because I start the yaw estimator from rest the error is seen to be highest at the start - and then wanders

erratically positive and negative until the roll and pitch estimators are switched on.

The final two graphs show *ROLL* and *PITCH*. Again the intermittent dots mark the true values. The full lines are the estimated values when the estimators are 'on'. The fine dotted lines mark the estimates when the estimators are 'off'. The yaw update can be seen to gradually 'settle in' and then the roll and pitch estimators are allowed on. "frac delay before r and p update = 3 " means that these second estimators are not allowed on until 3 orbits have passed. After some initial overshoots the entire estimator settles down. The parameters were all chosen by trial and error to give the fastest acquisition - damping and the two ratios of B_t/B_z need to be optimised.

The item " roll phase (relative to orbit) = 0.0 " links the roll oscillations to the orbit. As was implied in sec.7.1.2 the roll phase is locked to the orbit oscillating twice per orbit. Fig.7.4-1 was chosen as a worst case with a roll phase of 0° . This means that the roll amplitude is a maximum through the polar window AND the equatorial window. Rather different tracking characteristics were found depending on this initial phase. Fig.7.4-2 with a roll phase of 90° shows that the initial acquisition is now much faster in yaw tracking. For a good explanation one looks at Eqs.7.1.1.1. With 90° phase shift on the roll oscillation the amplitude of the roll is zero when passing through the equatorial window. In the initial orbits, when roll and yaw estimators are yet to be switched on, the adverse effect on the yaw estimate is therefore minimised. Going back to fig.7.4-1 it can then be explained why there is the zig-zag error in the yaw estimate which does not start to damp down until the pitch and roll estimators start to come on.

Figs.7.4-3 revert to the zero roll phase and show clearly the effect of reducing the gain of the roll and pitch estimators - the overall attitude acquisition is then starting to come right after only 12 or so orbits. The advantage of a lower gain in practice is expected to be a greater tolerance to measurement and IGRF model errors (see below).

Fig.7.4-4 is a check on the supposed reference and model fields. The TOTAL plot of the magnitude of the mag field shows the characteristic variation of roughly 2:1 between passing over the poles to passing over the equator. One notes the curious step effect early on in B_y is presumably associated with the Brazilian anomaly (roughly gauging where the simulated orbit passes over at this time).

It is of course important to assess the 'robustness' of this process. A computer simulation can be very deceptive if a good result is a consequence of the high precision 'data' which the simulations generated and which in no way correspond to conditions found in real measurement.

One major problem in practice may be the IGRF field model which now in the late 80's is living off borrowed time. As a simple way of testing tolerance to a poor model the estimator was deliberately limited to low coefficient values of the IGRF model. Thus figures 7.4-5 operates under exactly the same conditions as figs .7.4-3 except that the model used by the 'estimator' only works up to the 4 th harmonic (the computation of the local field in the outer part of the program uses the full 8 harmonics but the rest of the program is only 'told' about the first four). It can be seen that sensible operation is still possible although inaccuracies are appearing. The yaw error estimate never settles down (providing an excellent self test in real trials) and the inaccuracies in pitch and roll approach 0.5 degree peak.

Figures 7.4-6 simulate even worse modelling with a 3rd harmonic model. Although the parameters have probably not been optimised it can be seen that the worst case errors now approach 1 degree peak. The plots of fig.7.4-7 show how much the fields assumed by model and for 'real' differ from each other in this case.

Fig. 7.4-1

**Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimate5= 1988 June 18**

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of model = 8.0 harmonic number of reference = 8.0

pitch amplitude (deg)	=	3.0	roll amplitude (deg)	=	3.0
roll phase (relative to orbit)	=	0.0	yaw rate (revolutions per orbit)	=	6.0
norm convergence rate yaw	=	10.0	damp control in yaw	=	1.0
norm convergence rate pitch/roll	=	10.0	frac_delay before r and p update	=	3.0
Bz0/Bt0 threshold yaw	=	0.5	Bz0/Bt0 threshold pitch/roll	=	3.0
no of up_dates per orbit	=	80.0	update interval (mins)	=	1.3

Fig.7.4-2

Acquisition sequence by State Estimator on IGRF field
adcm.kalman.estimate5e 1988 June 18

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
 number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of aodel = 8.0 harmonic number of reference = 8.0

pitch amplitude (deg)	=	3.0	roll amplitude (deg)	=	3.0
roll phase (relative to orbit)	=	90.0	yaw rate (revolutions per orbit)	=	6.0
norm convergence rate yaw	=	10.0	damp control in yaw	=	1.0
norm convergence rate pitch/roll	=	10.0	frac_delay before r and p update	=	3.0
Bz0/Bt0 threshold yaw	=	0.3	Bz0/Bt0 threshold pitch/roll	=	3.0
no of up_dates per orbit	=	80.0	update interval (mins)	=	1.3

Fig.7.4-3a

Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimate5e 1988 June 18

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
 number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of model = 8.0 harmonic number of reference = 8.0

Fig. 7.4-3b

pitch amplitude (deg)	=	3.0	roll amplitude (deg)	=	3.0
roll phase (relative to orbit)	=	0.0	yaw rate (revolutions per orbit)	=	6.0
norm convergence rate yaw	=	10.0	damp control in yaw	=	1.0
norm convergence rate pitch/roll	=	2.0	frac_delay before r and p update	=	3.0
Bz0/Bt0 threshold yaw	=	0.5	Bz0/Bt0 threshold pitch/roll	=	3.0
no of up_dates per orbit	=	80.0	update interval (mins)	=	1.3

Fig.7.4-4

**Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimate5e 1988 June 18**

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

magnetic plots

harmonic number of model = 8.0 harmonic number of reference = 8.0

pitch amplitude (deg)	=	3.0	roll amplitude (deg)	=	3.0
roll phase (relative to orbit)	=	0.0	yaw rate (revolutions per orbit)	=	6.0
norm convergence rate yaw	=	10.0	damp control in yaw	=	1.0
norm convergence rate pitch/roll	=	10.0	frac_delay before r and p update	=	3.0
Bz0/Bt0 threshold yaw	=	0.5	Bz0/Bt0 threshold pitch/roll	=	3.0
no of up_dates per orbit	=	80.0	update interval (mins)	=	1.3

Fig. 7.4-5a

Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimate5e 1988 June 18

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of model = 4.0 harmonic number of reference = 8.0

Fig.7.4-5b

Fig.7.4-6a

Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimateSF 1988 June 18

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of model = 3.0 harmonic number of reference = 8.0

Fig. 7.4-6b

pitch amplitude (deg)	=	3.0	roll amplitude (deg)	=	3.0
roll phase (relative to orbit)	=	0.0	yaw rate (revolutions per orbit)	=	6.0
norm convergence rate yaw	=	10.0	damp control in yaw	=	1.0
norm convergence rate pitch/roll	=	2.0	frac delay before r and p update	=	3.0
Bz0/Bt0 threshold yaw	=	0.5	Bz0/Bt0 threshold pitch/roll	=	3.0
no of up_dates per orbit	=	80.0	update interval (mins)	=	1.3

Fig.7.4-7

Acquisition sequence by State Estimator on IGRF field
adcs.kalman.estimate5F 1988 June 18

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

harmonic number of model = 3.0 harmonic number of reference = 8.0

7.5 The Control Situation - I

The possibility of achieving 3-axis control on UOSAT-2 by a combination of gravity-gradient stabilisation and active magnetorquing is investigated theoretically. Among the conclusions are (1) spin-plane magnetorquers should be used instead of the spin-axis magnetorquer; (2) a commutation argument shows that independent control of pitch, roll and yaw state is achievable in a polar orbit on average; (3) a dynamic control law can be formulated which will achieve a stable convergence in all three axes. Final pointing error is not assessed, but the ability to monitor and control stochastic disturbances in yaw is demonstrated

In this section the assumption is that the attitude is at all times completely determined; and a way of achieving full 3-axis control, based on this information, but just using magnetorquing control and passive gravity-gradient stabilisation, is now considered.

A potential advantage of 3-axis control realised by this combination of techniques, on the UOSAT-2 type of satellite, is the yaw controllability. The stable state is a zero pitch and roll angle; but the yaw angle is arbitrary. Because the yaw angle is controllable the satellite can be rotated around the vertical axis and centred on any azimuth angle within a scan of 360° , under command.

Section 7.1 showed that the attitude estimator using magnetic data is independent of any attitude controller; that whatever rules govern the controller do not influence the estimating process, provided that any real torque generated by magnetorquing is matched by a computation applied within the estimating algorithm. Consequently there are no constraints on the nature of the attitude control from the point of view of that attitude estimator.

Conversely the controller is independent of the method of attitude determination. The attitude can be determined using any other combination of sensors. So that, although intended for UOSAT-2, the control method can be applied to future UOSATs using other methods of

attitude determination, provided they maintain the same axial symmetry and a boom deployed on the zenith/nadir axis.

However it may be noted that the control method needs to measure the local geomagnetic field, using a magnetometer, before making any decision to fire the magnetorquer, even if this measurement is not used to determine the attitude (and the geomagnetic field is not modelled)

It is further assumed here that, before start-up of the controller, gravity-gradient stabilisation has been achieved in the normal simple manner, as described in chapter 3, and that the libration angle has already been limited to a small value by the other simpler techniques.

7.5.1 Need for spin-plane magnetorquing

The practically demonstrated de-libration technique uses the spin-axis torquer and is based on the threshold algorithm (sec.5.3.1). Now the spin-axis torquer simply accelerates (angularly) the Z axis towards (or away from) the $\underline{B}$ vector. It was argued that the threshold algorithm operates only in the vicinity of the equator, when the $\underline{B}$ vector is roughly orthogonal to the Z axis; and when the satellite is back-pitching against the current rotation of the $\underline{B}$ vector.

A geometrical argument shows that the spin axis torquer can never remove roll energy adequately, whatever the algorithm, even with full attitude knowledge. This is simply because the 'attitude' of the $\underline{B}$ vector is mostly a pitch deviation, converting to a roll deviation for only a small fraction of the orbit when the satellite makes a nearest approach to the magnetic poles. One can see, for an ideal IDPO model (orbit in plane of dipole field) with infinitesimal roll amplitude, when the Z axis and the magnetic $\underline{B}$ vector both lie in the orbit, that the $\underline{B}$ vector is always in a 'pitch' state. Consequently there is never any roll-altering torque using the spin-axis torquer - in this limiting special approximation to the real geomagnetic field.

This means that the pitch libration can be suppressed directly: but not the roll libration, which is suppressed only by indirect roll/pitch coupling associated with a sufficiently fast Z-spin (as described in sec.3.7.1). With the requirement for full 3-axis control, and therefore a nominal zero Z -spin when that control is achieved, such a coupling disappears and the need exists to remove roll energy directly.

It follows that the spin-plane magnetorquers must be used. This conclusion is demonstrable by simple algebra. Repeating Eqs.4.3.1.1 for the magnetic field in reference co-ordinates on the IDPO model (ignoring a minus sign on B_{y0}) then

$$\begin{aligned} B_{x0} &= 0 \\ B_{y0} &= B \sin(\lambda) \\ B_{z0} &= 2 B \cos(\lambda) \end{aligned} \quad \dots 7.5.1.1$$

where

B is a reference value.
 λ is the advance in orbit

Measuring a time advance since a polar crossing at $t = 0$

$$\begin{aligned} B_{y0} &= B \sin(\omega_0 t) \\ B_{z0} &= 2 B \cos(\omega_0 t) \end{aligned} \quad \dots 7.5.1.2$$

Magnetic torques are created (in general)

	CHANGE	TORQUE
pitch	$N_{x0} =$	$M_{y0} B_{z0} - M_{z0} B_{y0}$
roll	$N_{y0} =$	$- M_{x0} B_{z0}$
yaw	$N_{z0} =$	$M_{x0} B_{y0}$

...7.5.1.3

where ($M_{x0} M_{y0} M_{z0}$) are current resolutions of the magnetic moment (from whatever source) in the reference co-ordinate system. The actual moments M_x and M_y determine

$$\begin{aligned} M_{x0} &= M_x \cos(\phi) - M_y \sin(\phi) \\ M_{y0} &= M_x \sin(\phi) + M_y \cos(\phi) \\ M_{z0} &= M_z \end{aligned} \quad \dots 7.5.1.4$$

The algebra confirms that

- * M_z creates a pitch torque only
- * roll suppression is due to M_{x0}
- * M_{x0} depends on a mix of M_x and M_y depending on state of rotation Φ .
- * component M_{x0} will in general induce a spin change (because of field B_{y0}) at the same time

It is necessary then to look at the *spin-plane* torquing as a means of direct roll suppression, in addition to pitch ; while needing to consider how to deal with possible problems of induced changes to the Z-spin (NOTE)

7.5.2 Commutated Damping and spin control

The assumed (unconstrained) equations of motion at small librations and neglecting coupling between roll and pitch, were Eqs.3.7.2.1. These have solutions (repeating Eqs.7.1.2.2)

$$r = r_0 + R_1 \cos(\omega_r t + \theta_r) \quad \dots 7.1.2.2a$$

$$p = P_1 \cos(\omega_p t + \theta_p) \quad \dots 7.1.2.2b$$

$$\Phi = \Phi_0 + \omega_z t \quad \dots 7.1.2.2c$$

where time t can be arbitrary.

The first point to establish is if it is possible to control the pitch, roll or yaw states independently of each other.

NOTE The IDPO model is useful in providing a semi-quantitative backing to the general arguments. I am careful not to assume it in the algorithms to be described, which all depend on actual measurements in the real geomagnetic field

Although Eqs. 7.5.1.3 indicate that roll and yaw changes are apparently tied together there is apparent hope in that roll changes are most easily induced over the poles (high $|B_{z0}|$) while yaw changes are most easily induced over the equator (high $|B_{y0}|$). The possibility exists then of achieving independence in an average sense when considered over a period of one orbit.

A necessary condition is to show that at least on average active damping can be imposed on the libration, during the orbit. The 'effectiveness' of the libration damping can be expressed as the product of the rate of change of pitch with the pitch torque; and the rate of change of roll and the roll torque, both of which should average to a negative value. And assuming some initial Z spin the need is to show that on average, a consistent N_z torque can be maintained. In tabular form

	$\langle dp/dt N_{x0} \rangle$	$\langle dr/dt N_{y0} \rangle$	$\langle N_{z0} \rangle$
PITCH	< 0	$= 0$	$= 0$
ROLL	$= 0$	< 0	$= 0$
YAW	$= 0$	$= 0$	$= s N$

In this table the quantity N is some positive quantity and $s = +/- 1$; while the $\langle \rangle$ notation means 'on average over an orbit' (or fraction of an orbit).

Analysis shows that the above requirements can indeed be met if the magnetic moments can be made *variable* and *functions of time*, according to certain *commutation laws*. Adopting a notation $m_{x0}(t)$ and $m_{y0}(t)$ these laws are as follows

pitch delib

$$m_{y0}(t) = m \cos(\omega_p t + \theta_p) \cos(\omega_0 t) \quad \dots 7.5.2.2a$$

roll delib

$$m_{x0}(t) = m \cos(3 \omega_0 t + \theta_R) \quad \dots 7.5.2.2b$$

yaw change

$$\begin{aligned} m_{x0}(t) &= s m (\sin(\omega_0 t) - \sin(3 \omega_0 t)) \\ &= -2 s m \sin(\omega_0 t) \cos(2 \omega_0 t) \end{aligned} \quad \dots 7.5.2.2c$$

In these equations

$$s = +/ - 1$$

m = maximum achieved magnetic moment

$$\omega_R = 2 \omega_0$$

Creating a magnetic moment $m_{y0}(t)$ controls pitch without affecting roll and yaw at any time. Creating a magnetic moment $m_{x0}(t)$, according to the above equations, will achieve independent control of roll and yaw on average; which point is readily shown by a harmonic analysis.

It can also be shown that the effectiveness of damping in roll is equal to the damping in pitch (using just the spin-plane magnetorquer). Therefore, on grounds of being impartial to the nature of the libration, it is proposed not to use the spin-axis magnetorquer at any time.

7.5.3 Practical control

The commutation argument says that full 3-axis control is available only if continuously variable $m_{x0}(t)$ and $m_{y0}(t)$ can be realised. The practical realities (for UOSAT-2)

1 the magnetic moments (M_x M_y), permanently aligned on the body axes, are controllable on UOSAT-2 with a choice of only three discrete states: being either ON (positive or negative); or OFF. For reliability in the electronics it is highly desirable that this restriction be maintained

2 the yaw angle Φ during acquisition can have any value

3 the magnetorquing must be interrupted for measurement of the B vector

4 the commutation must be embodied within the operation of the discrete Kalman filter if we use this magnetic data estimator.

Taking the points (3) and (4) first, I currently envisage a repeated sampling rate of 50- 100 times per orbit (the estimator simulations had 80). The analysis is not incorporated in this thesis but the effect of magnetorquing on the estimator - is easily calculated as an impulsive acceleration, provided the actual magnetorquer current pulse is symmetrically positioned around the regular sample points in time when the estimator updates. This leads to the proposed scheme of Fig.7.5.3.1, where application of magnetorquing pulses alternate at a constant sampling interval with the innovations applied to the estimators. Such an 'alternating estimator' still updates every sample interval Δ but alternately between the control effect and the innovation effect

A regular and repeated sampling-and-firing scheme seems necessary even if the attitude determination is derived from other systems entirely (NOT as UOSAT-2). The reason lies in the need to create a magnetic moment in the spin plane, of effectively *variable* magnitude and direction decided by a control law. Given the limitations of points (1) and (2), the problem is solved in principle by the combination of *pulse width modulation* firing for variable times T_{Fx} and T_{Fy} , alternation between M_x and M_y torquers, and a constant up-date interval Δ . For fire times T_{Fx} and T_{Fy} respectively then *modulated and directed* magnetorquer moments averaged over a time 2Δ are

$$m_{xo} = \frac{(s_x T_{Fx} \cos(\phi) - s_y T_{Fy} \sin(\phi)) M}{2 \Delta} \quad \dots 7.5.3.1a$$

$$m_{yo} = \frac{(s_x T_{Fx} \sin(\phi) + s_y T_{Fy} \cos(\phi)) M}{2 \Delta} \quad \dots 7.5.3.1b$$

where

Δ is the sampling interval
 $s_x s_y$ are assigned polarities (+1 or -1)

The discrete values of magnetorquer moments imply that

$$\begin{aligned} M_x &= s_x M \\ M_y &= s_y M \end{aligned}$$

...7.5.4

These equations merely confirm that a selection of the correct polarities for M_x and M_y , and a calculation of fire times T_{Fx} and T_{Fy} will create a magnetic moment of desired strength and directed wherever required in the 4 quadrants of the spin plane. To work consistently however the sample interval Δ must be a constant: the response of the system must extend over many multiples of these intervals; and the required fire times T_{Fx} and T_{Fy} must be less than Δ in most cases. The last two points are valid provided Δ is in the order of 1-2 minutes while the fire times are usually much less

7.5.4 Effectiveness of torquing

To check the last point it is simple to demonstrate that a sufficient 'clout' can be delivered in required short fire times. There would seem to be no problem here, at least for the current UOSAT-2 parameters. Assuming just a pitch libration and from Eq.7.1.2.2b, differentiation yields

$$dp/dt = -\omega_p P_1 \sin(\omega_p t + \theta)$$

...7.5.4.1

and the magnitude of the peak angular momentum

$$\begin{aligned} L &= I_T (dp/dt)_{\max} \\ &= I_T P_1 \omega_p \end{aligned}$$

...7.5.4.2

A torque pulse of anything up to a few minutes is effectively an impulse at the slow librational (oscillating) rate. Therefore, to kill the motion with one pulse requires a fire time

$$T_F = \frac{I_T P_1 \omega_p}{M B}$$

...7.5.4.3

here M is the achievable magnetic moment. Taking $M = 10$ (and $I_T = 120$) for UOSAT-2, and assuming $B = 20 \mu T$ (as a minimum over the equator) then then the fire time needed to stop the motion for a residual libration of 1° is 20 seconds (very nearly). This figure makes the practical commutation scenario quite credible.

7.5.5 Decision Control- general damping

The commutation argument merely showed that individual control of all three motions is possible on average. It is now proposed to abandon Eqs.7.5.2, which can be considered to be purely motivational, and look for rules which are effective instantaneously in achieving full 3- axis control.

It is convenient to visualise a vector libration 'angle'

$$\underline{y} = \underline{v} p + \underline{n} r$$

...7.5.5.1

The two components have been resolved along the defining unit vectors $\underline{n}$ and $\underline{v}$ of the attitude circle (c.f sec.3.1.3). These vectors have been chosen here because a small positive roll and pitch angle, by the conventions of this thesis, shift the attitude point in forward directions along these axes. The magnitude of this vector $|\underline{y}|$ is identical to inclination or libration angle γ used throughout this thesis.

The angular velocity

$$d\underline{y}/dt = \underline{v} dp/dt + \underline{n} dr/dt$$

...7.5.5.2

and angular acceleration due to torques N_{x0} and N_{y0}

$$d^2\underline{y}/dt^2 = (\underline{v} N_{x0} + \underline{n} N_{y0}) (1/I_T)$$

...7.5.5.3a

$$= (\underline{v} M_{y0} - \underline{n} M_{x0}) \left(\frac{B_{x0}}{I_T} \right)$$

...7.5.5.3b

The simplest aim is to create an acceleration which is opposed to the velocity and and proportional to its magnitude - a simple 2-D vector extension of an active damping principle

After some considerable analysis and simulation the following rules seem to be effective

Pitch/Roll controller

On every sampled instant in the orbit calculate

$$\begin{aligned} dy_1/dt &= \dot{\rho} \cos(\phi) + \dot{\rho} \sin(\phi) \\ dy_2/dt &= -\dot{\rho} \sin(\phi) + \dot{\rho} \cos(\phi) \end{aligned} \quad \dots 7.5.5.4$$

which is simply a calculation of $\dot{y}$ in components which are rotated by angle ϕ from the original $\underline{y}$ and $\underline{n}$ axes. Then set

$$\begin{aligned} \{ M_x \} &= \{ dy_2/dt \} \\ T_{Fx} &= k_{PR} |dy_2/dt| B_z \end{aligned} \quad \dots 7.5.5.5a$$

and

$$\begin{aligned} \{ M_y \} &= - \{ dy_1/dt \} \\ T_{Fy} &= k_{PR} |dy_1/dt| B_z \quad (\text{NOTE}) \end{aligned} \quad \dots 7.5.5.5b$$

which establishes the polarities and fire times required to drive an acceleration opposing the velocity. The constant k_{PR} controls the general rate of the vector damping.

Yaw controller

For spin (yaw) control the near-simultaneous use of both M_x and M_y and the use of pulse-width modulation brings about a basic improvement on the simple de-spin method of sec.5.1.1. It can now be arranged for the effective magnetic moment in the spin plane to be always orthogonal to the $\underline{B}$ field vector component which lies in the spin plane. In other

NOTE The notation $\{ \quad \}$ always means 'sign of ' and returns a value +1, 0 or -1

words although the spacecraft may be spinning about its Z axis the created magnetic moment is contra-rotating and keeping the same angle to the B field in the short term. The required torque is therefore realised always most efficiently.

The aim is not only to slow down the spin but bring the yaw angle to some desired angle Φ_0 . To this end a control error may be constructed

$$e_z = k_{z1} \, d\Phi/dt + k_{z2} \, \text{SIN}(\Phi - \Phi_0) \quad \dots 7.5.5.6$$

Here parameters k_{z1} and k_{z2} are constants.

Then setting

$$\begin{aligned} \{ M_x \} &= - \{ e_z B_y \} \\ T_{Fx} &= | e_z B_y | \end{aligned} \quad \dots 7.5.5.7a$$

$$\begin{aligned} \{ M_y \} &= \{ e_z B_x \} \\ T_{Fy} &= | e_z B_x | \end{aligned} \quad \dots 7.5.5.7b$$

can be shown to create the required orthogonal M vector. The simulation to follow shows that the yaw controller slows any initial z-spin and ultimately causes the yaw angle Φ to converge on the desired Φ_0 , according to the basic behaviour of a 2nd order loop. Setting of the parameters k_{z1} and k_{z2} determines the rate of convergence and overshoot.

Since control sub-algorithms I and II cannot be formulated simultaneously the same criterion as used in the magnetic data estimator is followed - of reserving different sectors of the orbit to the two different requirements. The pitch and roll control are confined to a polar sector. The yaw control is restricted to equatorial sectors. The reasons are exactly the same, just reflecting the relative magnitudes of B_z field and transverse field B_T . A separate and essential

requirement is therefore to set two threshold ratios yaw_TC and pr_TC and permit only this restricted operation

pitch roll controller	$ B_z > pr_TC B_T$
yaw controller	$ B_z < yaw_TC B_T$

...7.5.5.8

7.5.6 Results of simulation

The model adopts an integer number of orbits per 24 hours. The magnetic field is calculated according to the TDIO model (in order to speed up calculations). The evolution of attitude is solved as in the estimator study again using the simple Z- transform, with one-step updates per sample time on the orbit. The only difference from the estimator study is that the code which previously implemented the estimated attitude state now simulates the 'real' attitude state. The effect of firing a specific magnetorquer is very easily modelled on the assumption of a torque applied at each sampled instant converting to an impulse - realising a sudden change in angular momentum.

Fig 7.5.6-1 shows a typical result. In this simulation the start pitch and roll were set at 5° and the yaw rate at 10 per orbit (roughly 10 minute period). The number of updates was set at 40 per orbit. The top plot of YAW shows the slope of the yaw angle reducing and then yaw state being captured. The full range of yaw angle is plotted from -180° to $+180^\circ$. The second plot of TORQUER indicates the 'business' of the magnetorquing. The positive bars show the strength of response in the yaw control which comes only during the equatorial crossings (I). The negative bars show the strength of response in the pitch/roll control which comes on only during the polar crossings (II). As can be seen all three states converges on zero in a well-behaved manner.

The yaw control damping and gain listed in the table, 'norm damp pitch/roll and 'norm yaw disp' (0.4/0.02), are set sufficiently low so that convergence only occurs over several orbits - effectively using Earth as a 'super-sampled' data control system, updating every

equatorial crossing. Similarly the parameter 'norm damp pitch/roll' is set low so that libration is damped over several orbits. These three parameters, as listed in the table at the bottom of the figure are related to the constants k_{z1} , k_{z2} and k_{PR} respectively by simple formulae.

The second figure 7.5.6-2 adopts different values to the parameters, in particular the yaw gains have been increased by an order of magnitude (3.00/0.7); instead of values (0.4/0.02): the yaw window has been opened by increasing 'Bz/Bt threshold yaw' (yaw_TC in Eq.7.5.5.8) and the frequency of samples increased to 80 per orbit. And in this program a white noise in yaw is introduced so that the yaw is pushed into error when the yaw state is open-loop between equatorial sectors. As a test of the system's ability, non-zero values are set to the attitude state (a 2° roll and pitch error are introduced and a yaw rate of 10). It can be seen that yaw state is captured after 2-3 orbits and in a stochastic 'steady state' after about 9-12 orbits. The random drift in yaw error between equatorial passes is clearly shown (YAW plot), which is consistently forced back to zero within each equatorial pass. It should be noted that the range of yaw being plotted is from -15° to +15°. The figure shows that there is no significant feed back to the roll and pitch states despite the large increase in yaw control gain. There would seem then to be no difficult stability problems,

The success - in simulation - of this 'intra-sector' yaw correction prompts the consideration that the same strategy can and should be adopted for roll and pitch correction: that within one pass over the pole any error accumulated from the previous pole ought to be correctable within one polar pass. This directed displacement control would use further information provided by the the vector inclination angle γ in addition to its rate of change dy/dt . Such a formulation could work to overcome any systematic bias from perturbations to the pitch and roll state. A precise formulation of the required control law needs further work.

Fig.7.5.3-1 Alternating Estimator

Fig.7.5.6-1 3-Axis Controller (low yaw gain)

Control Acquisition by G.G and Spin-Plane Magnetorquer
adcs.libration.control9 1989 Feb 10

orbit radius (km) = 7271.9 orbit period (min) = 102.9 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

Tilted Dipole Inclined Orbit model

UOSAT-2 parameters IT = 125 IZ = 1 HT = 10 HZ = 4

pitch amplitude (deg)	=	5.0	roll amplitude (deg)	=	5.0
roll phase (relative to orbit)	=	0.0	roll phase (relative to orbit)	=	0.0
yaw rate (revolutions per orbit)	=	10.0	norm damp pitch/roll (secs)	=	5.0
norm yaw damp (secs)	=	0.400	norm yaw disp (secs)	=	0.020
Bz /Bt threshold yaw	=	0.5	Bz /Bt threshold pitch/roll	=	3.0
no of up_dates per orbit	=	40.0	update interval (mins)	=	2.6
pr noise level simulation	=	0.0	yaw noise level simulation	=	0.0

Fig.7.5.6-2 3-Axis Controller (high yaw gain)

Control Acquisition by G.G and Spin-Plane Magnetorquer
adcs.libration.control10 1989 Feb 10

orbit radius (km) = 7271.9 orbit period (min) = 6171.4 orbit inclination (deg) = 99.0
number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

Tilted Dipole Inclined Orbit model UDSAT-2 parameters IT = 125 IZ = 1 MT = 10 NZ = 4

pitch amplitude (deg)	=	2.0	roll amplitude (deg)	=	2.0
roll phase (relative to orbit)	=	0.0	roll phase (relative to orbit)	=	0.0
yaw rate (revolutions per orbit)	=	10.0	no of up_dates per orbit	=	80.0
norm pitch/roll damp (secs)	=	8.0	norm pitch/roll disp (secs)	=	0.0
norm yaw damp (secs)	=	3.0	norm yaw disp (secs)	=	0.7
Bz0/Bt0 threshold yaw	=	1.0	Bz0/Bt0 threshold pitch/roll	=	3.0
pr noise level simulation	=	0.000	yaw noise level simulation	=	0.050

7.6 Control Situation - II

This final section considers asymmetric satellites (not UOSATs so far). It is argued that the fundamentals of 3-axis passive attitude stabilisation by gravity gradient, exploiting the unequal moments of inertia on all three axes, may not have been sufficiently understood. The concept of 'tumbling sensitivity' is introduced which can be understood from (non-linear) differential equations valid for all yaw angles. It is easily demonstrated that the cost of passive 3-axis stabilisation over simpler 2-axis stabilisation is a magnified tumbling sensitivity at large yaw angles. The implication is that a proportional rate control (damping), which has always been acknowledged as necessary, is not generally sufficient; there is also needed a position-plus-rate control which can operate at least intermittently in order to constrain the yaw angle (and possibly the other angles). In this respect magnetorquing seems ideally suited.

The goal of inherent, passive 3-axis stabilisation by gravity gradient has had a long theoretical history and been the subject of much experimentation in the past. I review briefly the background in section 8.4. A thorough theoretical treatment, involving much advanced mathematics, with a survey of experimental work, is found in a recent text book (Hughes 1986) on whose results I draw in part.

7.6.1 Linearised Equations

As is well-known, the method simply exploits the unequal moments of inertia on the three principal axes of a general body in orbit, which can be realised by exploiting just the inherent shape of the spacecraft or by extending one or more booms with a tip mass.

The well-known linearised equations, valid only for infinitesimal deflection angles in roll, pitch and yaw, are given here, in my notation and according to my axis convention.

The equations are

$$I_x \frac{d^2 p}{dt^2} + 3 \omega_0^2 (I_y - I_z) p = N_x \quad \dots 7.6.1.1a$$

$$I_y \frac{d^2 r}{dt^2} + \omega_0 (I_x - I_y - I_z) \frac{d\phi}{dt} + 4 \omega_0^2 (I_x - I_z) r = N_y \quad \dots 7.6.1.1b$$

$$I_z \frac{d^2 \phi}{dt^2} - \omega_0 (I_x - I_y - I_z) \frac{dr}{dt} + \omega_0^2 (I_x - I_y) \phi = N_z \quad \dots 7.6.1.1c$$

Here torques N_x , N_y , N_z represent the net effect of any applied control plus perturbation torques on each axis.

These libration equations indicate a simple harmonic motion in pitch and a coupled resonance in roll and yaw - all undamped. The roll/yaw is a linear mixture of oscillations at two frequencies. Since the oscillations are undamped, according to these equations, a persistent libration in all angles is implied, even if the attitude is stable. Not all values of I_x , I_y and I_z are permitted, there being two regions of stability in roll/yaw (mapped in two co-ordinates of dimensionless ratios derived from the moments of inertia). According to Hughes (p.297), of the two stable regions, the Lagrange and the DeBra-Delp, only the former is realistic, if any dissipation is present. He finds that the latter is an 'interesting artefact' i.e of no engineering use. Mathematically the sensible Lagrange region is defined, after taking into account the need for pitch stability, by the inequalities

$$I_x > I_y > I_z \quad \dots 7.6.1.2$$

in the notation consistently used in this thesis (Hughes and Wertz interchange the meaning of X and Y axes). In sec.8.4.1 I give a simple non-mathematical justification of this inequality.

The coupling terms in the roll and yaw equations both contain the factor $(I_x - I_y - I_z)$, which is generally non-zero since

$$I_x \leq I_y + I_z \quad \dots 7.6.1.3a$$

A special case is a 'planar' satellite (PLANARSAT ?), effectively a 'two-dimensional' body, flattened into a sheet parallel to the orbit plane. Necessarily for this geometry

$$I_x = I_y + I_z$$

...7.6.1.3b

The linearised equations then simplify to

$$d^2p/dt^2 (I_y + I_z) + 3 \omega_o^2 (I_y - I_z) p = N_x$$

...7.6.1.4a

$$d^2r/dt^2 I_y + 4 \omega_o^2 I_y r = N_y$$

...7.6.1.4b

$$d^2\phi/dt^2 I_z + \omega_o^2 I_z \phi = N_z$$

...7.6.1.4c

There are now just three, independent simple harmonic motions around the three axes. The roll-yaw interactions have disappeared. Although it is not physically realisable exactly, this 'spacecraft' can be closely approximated; and PLANARSAT shows a strong resemblance in its dynamics to the UOSAT-2 geometry, in that the pitch rate $\omega_p \approx \sqrt{3} \omega_o$, and the roll rate $\omega_r = 2 \omega_o$. What is new is a yaw restoring oscillation. There is of course no actual gravity-gradient restoring torque in the yaw axis. Its physical origin is simply the stability conferred by a 'major axis spin' - rotating at the orbit rate. It may be noted that the roll bias effect, which was such a characteristic feature in the axially symmetric case, has apparently disappeared. In other words the yaw rate does not induce any roll coupling - at least according to the linearised equations.

7.6.2 Stability problems

Hughes cites past work, and also analyses for himself, sources of instability found for finite librations, for some ratios of moments of inertia, even within the supposedly stable Lagrange region.

These sources are (1) non-linear 'internal resonance', (2) the effect of mechanical damping (3) a pitch resonance excited by elliptical

perturbations in the orbit. None of these effects of course are derivable from the linearised equations.

To avoid the effect of 'internal resonance' one must avoid ratios of I_x , I_y and I_z values which would solve, through Eqs.7.6.1.1, to a pitch resonant frequency which is close to either (1) twice the lowest roll/yaw frequency or (2) the sum of the two roll/yaw frequencies or (3) the difference between the two roll/yaw frequencies. Then Hughes uses also his own analytical techniques to find that mechanical i.e. internal damping leads to certain restrictions on usable parts of the Lagrange region (NOTE 1). He also analyses the well-known condition in which ellipticity in a nominally circular orbit can induce a pitch resonance.

It is of interest to take a UOSAT-2 model with an initial $I_T = 125$ and $I_z = 1$ and then consider its 'conversion' to a nominally 3-axis stable satellite, created perhaps by deployment of a second boom (along the roll axis). Such a deployment (with a tip mass of a few kgm) could create an increment ΔI such that

$$\begin{aligned} I_x &= I_T + \Delta I \\ I_y &= I_T \\ I_z &= I_{z0} + \Delta I \end{aligned}$$

...7.6.2.1

where I_{z0} is the initial value of the Z moment of inertia. It is easily demonstrated that very small increments in ΔI fail his damping test and/or the internal resonance tests. While very large extensions fail the internal resonance test. But in the range (ill-defined) of roughly $2 < \Delta I < 40$ all of Hughes' tests are passed (and in this range the geometry closely resembles PLANARSAT) (NOTE 2).

NOTE 1 Hughes shows that the effect of mechanical damping is far more complicated than just placing linear damping terms in the linearised equations.

NOTE 2 There is also a regime available $50 < \Delta I < 124$ bounded at the high end by failure of pitch suppression.

The passing of these theoretical tests does not guarantee that a satellite design will perform well in practice.

Much ingenuity was expended in the past on passive 'dampers'; since it seems clear that a damping mechanism is needed to remove librational energy, in aiming for a static as well as stable state.

Ignoring the complications and distinction between external and internal damping, Hughes asserts, and it seems reasonable, that 'even the best transient performance and response times cannot overcome the the fact that the characteristic time in the dynamics is the orbital period'. Further, there generally has to be a 'compromise between transient performance and steady state response to disturbance torques' (p 331).

In many practical cases there were problems which damping would seem not to overcome. Apart from the general imprecision of pointing, which was always worst in the yaw angle, a typical experience would be of an ever-widening yaw libration, leading to a general tumbling. Or the satellite might appear to stabilise at first and then mysteriously fall apart later. Hughes mentions the librational consequences of (1) aerodynamic drag (2) meteoroidal disturbances (3) the consequence of non-inertial precession of the orbit (4) unintended magnetic torques (5) structural flexibility. In fact he rounds off his chapter by roundly declaring (p. 345) that 'three axis stabilisation almost never seems to function as intended'.

The nub of the problem is what happens to the dynamics of the spacecraft, with the ever-widening libration caused by perturbation torques. The linearised equations seem to provide no information. Although there is obviously the procedure of formulating the exact differential equations and solving them numerically such an approach notoriously does not provide insight, and is usually specialised to the the particular spacecraft in question.

The linearised equations by definition do not state the range of librational amplitudes within which they are even approximately true.

There is therefore a fundamental problem in analysing the acquisition of the desired attitude from some arbitrary state; conversely, should there be a particularly bad 'glitch' or disturbance torque in the system there is no information on what drives the attitude to recovery (if at all). From reported experience, the worst case is the libration in yaw, as might be expected from its having the weakest correction torque. What happens at wide yaw angles ?

7.6.3 Semi-linearised equations

All the equations used in this and previous chapters ^{except the linearised equations.} have necessarily been valid for any yaw rotation, because the yaw angle is not constrained for the nominal 2-axis controlled symmetrical satellite, with $I_x = I_y$.

A set of differential equations is readily derivable for the general case (having unequal I_x and I_y), valid for any yaw angle, by a natural extension of the simple mathematics I have used to analyse the axially symmetric UOSAT-2. The derivation appears in Appendix K.

These approximate equations for the asymmetric satellite are valid for arbitrary values of I_x , I_y and I_z , but assume small (ideally infinitesimal) pitch and roll angles. Appendix K shows that such equations are easily derived, provided the meaning of roll angle, and pitch is redefined. It is necessary to define the 3-axis attitude first by an unconstrained yaw rotation (about Z_0), then followed by small rotations in pitch and roll (small angles) about the body axes X and Y (NOTE).

This is in the reverse order to the standard definition and makes the roll and pitch angles *space-craft fixed* rather than *orbit-defined*. The new 'pitch' and 'roll' correspond only to the old pitch and roll in the nominal yaw state of $\phi = 0^\circ$. Otherwise they are mixed.

NOTE Equivalent to an Eulerian 3-2-1 rotation or a 3-1-2 rotation.

For example at $\phi = 90^\circ$ they are swapped; with 'pitch' = roll, and 'roll' = - pitch. To avoid confusion I shall adopt this 'pitch' and 'roll' notation.

Such equations should therefore provide (1) a continuity with the UOSAT-2 case and (2) shed light on the problem of instability associated with excessive yaw librations. To examine the second point I look at the special set of PLANARSAT equations because they are again simplest. These are

'PITCH'

$$(I_Y + I_Z) \frac{d^2 p}{dt^2} - \frac{2 \omega_0 \sin(\phi) I_Z}{\sin(2\phi)} \frac{d\phi}{dt} \\ + \omega_0^2 \sin(\phi) \cos(\phi) (I_Y - I_Z) r \\ + (3 + \sin^2(\phi)) \omega_0^2 (I_Y - I_Z) p = N_x$$

...7.6.3.1a
...K.2.3

'ROLL'

$$I_Y \frac{d^2 r}{dt^2} \\ + \omega_0^2 \sin(\phi) \cos(\phi) I_Y p \\ + \omega_0^2 (3 + \cos^2(\phi)) I_Y r = N_y$$

...7.6.3.1b
...K.3.3

YAW

$$\frac{d^2 \phi}{dt^2} I_Z + \frac{2 \omega_0 \sin(\phi) I_Z}{\sin(2\phi)} \frac{dp}{dt} \\ + (1/2) \omega_0^2 I_Z \sin(2\phi) = N_z$$

...7.6.3.1c
...K.3.4

In these equations the underlined terms are a 'pitch'- yaw interaction, and are new. Their $\sin(\phi)$ dependence explains why. The yaw-restoring dependence on $\sin(2\phi)$ in Eq.7.6.3.1c confirms that not only $\phi = 0^\circ$ but also $\phi = 180^\circ$ are attitudes of nominal stability.

Eq. 7.6.3.1c also confirms that excessive yaw libration will pump the 'pitch' state. An elementary simulation easily confirms this. The interaction will be greatest at $\phi = 90^\circ$.

This 'pitch' to yaw interaction is the missing factor which the linearised equations apparently ignore. This interaction is always present in the general asymmetric case (see Appendix K) and is worst at $\phi \rightarrow \pm 90^\circ$

At the extreme yaw angle $\phi = \pm 90^\circ$ the semi-linearised equations can be checked from the standard linearised equations *simply by re-labelling the axes* (just interchange subscripts X and Y).

In particular a revealing relationship is found from the *standard* Eq.7.6.1.1b, after swapping I_x and I_y . A roll angle (ordinary definition) can be calculated, after setting $d^2r/dt^2 = 0$

$$r_0 = - \frac{(d\phi/dt) (I_y - I_x - I_z)}{4 \omega_0 (I_y - I_z)}$$

...7.6.3.2

The physical meaning is that the satellite has skewed round by $\pm 90^\circ$ yaw and is still Z-spinning at a rate $d\phi/dt$. The value r_0 is a *roll bias* which would need to exist at that instant in time to balance exactly the roll acceleration created by this yaw (Z-spin). It is a simple measure of the propensity of the satellite to tumble. It should be compared with the standard roll bias formula Eq.3.7.4 for the symmetric UQSAT. From which follows a normalised (and positive) factor, which I interpret as a

$$\text{tumble sensitivity} = \frac{I_x - I_y + I_z}{I_y - I_z}$$

...7.6.3.3a

For comparison, a normalised 'restoring spring stiffness' taken from Eq.7.6.1.1c

$$\text{yaw stiffness} = \frac{I_x - I_y}{I_z}$$

...7.6.3.3b

To see what this means one can visualise a symmetrical satellite with equal moments of inertia in all axes I_0 . From this satellite along X and Y axes two unit tip masses are deployed distances d_r and d_z respectively. The two previous equations become

$$\text{tumble sensitivity} = \frac{I_0 + 2 d_r^2}{d_z^2 - d_r^2} \quad \dots 7.6.3.4a$$

$$\text{yaw stiffness} = \frac{d_r^2}{I_0 + d_r^2} \quad \dots 7.6.3.4b$$

I take this to mean that the pursuit of yaw stiffness is incompatible with stability. For a given vertical extension d_z the greater the extension d_r - needed to create yaw stiffness - the more prone the spacecraft is to tumble over - if and when the yaw angle starts to approach $\pm 90^\circ$. Of course for a given d_r the vertical boom can be deployed an increased d_z to reduce tumbling sensitivity - but that practically risks uncharacterised torques (see from sec.3.8).

7.6.4 3-axis control with magnetorquer

The preceeding argument strongly suggests that, in order to acquire and maintain 3-axis stability, it is *not enough just to damp* the libration. There should also be *control*. In other words apply a standard (or near standard) *position plus rate control law*. The semi-linearised equations suggest that this control should be applied *routinely*.

A prime contender for such treatment is the yaw state since this is subjected naturally to the weakest restoring torque. And it is now shown how lack of control over the yaw angle is the cause of satellite tending to tumble. The argument acquires greater force when all the known real-world disturbance torques are taken into account.

An objection to such a proposal is that it destroys the apparent advantages of gravity-gradient stabilisation, since there is now a need for (1) restoring torque(s) provided by by some other mechanism, and (2) actual attitude determination.

To see what this means one can visualise a cuboid mass of uniform density and dimensions d_x , d_y and d_z . Using the simple relations for moments of inertia then the two previous equations become

$$\text{tumble sensitivity} = \frac{2 d_y^2}{d_z^2 - d_y^2} \quad \dots 7.6.3.4a$$

$$\text{yaw stiffness} = \frac{d_y^2 - d_x^2}{d_y^2 + d_x^2} \quad \dots 7.6.3.4b$$

I take this to mean that the pursuit of yaw stiffness is incompatible with stability. For a given vertical extension d_z the greater the margin of d_y over d_x - in order to improve the yaw stiffness - the more prone the spacecraft is to tumble over - if and when the yaw angle starts to approach $\pm 90^\circ$. For a second boom extension in the extended UOSAT example, a value (say) of $\Delta I = 12$ makes the satellite nearly 30 times more likely to topple over.

7.6.4 3-axis control with magnetorquer

The preceeding argument strongly suggests that, in order to acquire and maintain 3-axis stability, it is *not enough just to damp* the libration. There should also be *control*. In other words apply a standard (or near standard) *position plus rate control law*. The semi-linearised equations suggest that this control should be applied *routinely*.

A prime contender for such treatment is the yaw state since this is subjected naturally to the weakest restoring torque. And it is now shown how lack of control over the yaw angle is the cause of satellite tending to tumble. The argument acquires greater force when all the known real-world disturbance torques are taken into account.

An objection to such a proposal is that it destroys the apparent advantages of gravity-gradient stabilisation, since there is now a need for (1) restoring torque(s) provided by by some other mechanism, and (2) actual attitude determination

The counter argument admits this but points out that such an additional control and determination need not be applied *all the time*. These additional resources may be partial and intermittent. In a sense, gravity gradient provides a mechanism of 'inertial navigator' -- providing the attitude continuity between occasional correction.

The natural complementary control mechanism is *magnetorquing*. As demonstrated in the previous section 7.5 magnetorquing can provide occasional correction, by alternating between roll/pitch and yaw control during an orbit.

Section 7.5 explores the possibility of additional spin-plane magnetorquing in trying to force 3-axis control on UOSAT-2. Sec.7.5.6 showed how well the imposed algorithm matches the requirement, by simultaneously damping out roll and pitch motions while *controlling* the yaw state.

7.6.5 Results of simulations

Now virtually the same algorithm is applied, in which it now 'backs-up' the yaw-restoring torque, taking a simple PLANARSAT as a model ($I_y = 125$, $I_z = 12$). The choice of values is arbitrary (NOTE).

The model in this simple simulation goes back to the linearised Eqs.7.6.1, which can be taken to be valid because all angles are small and the yaw angle is being *held* to small values.

As in the previous fig.7.5.6-2, noise has been introduced in yaw. As can be seen, after an initial transient the system is brought to stability with the effect of any yaw disturbance not being just damped but controlled back to the nominal zero. The algorithm can be seen to be

NOTE One minor modification is a algorithmic change to prevent lock, with the satellite pointing back to front (with $\phi = 180^\circ$).

systematically 'clamping' the yaw every equatorial pass. The operation of the magnetorquer is essentially intermittent and the gravity-gradient torques in all three axes have an essential function in maintaining stability when the magnetorquer cannot assist.

The parameters used in the figure are not intended to be representative of any particular model satellite, merely to illustrate the validity of this mixed control method.

However comparing fig 7.6.5-1 with fig 7.5.6-2 it is seen that, for exactly the same injected noise acceleration, the perturbations are now roughly halved. This can be attributed to the assistance of passive yaw restoration. But the real improvement is surely better, since no account has been taken of a factor of 12 increase in I_z . If the actual torque disturbances in yaw were purely internal (NOTE) then the yaw error would drop by a further factor of 12.

A further comparison can be made with fig.7.6.5-2, which is essentially the same model operating with exactly the same parameters as in fig.7.6.5-1, except that the 'position' control response is switched off ('norm yaw disp = 0'), leaving just the 'rate'. The effect of the remaining damping is to 'freeze' the yaw motion on an equatorial pass. This can be taken as a crude representation of what might happen on average using the best possible mechanical damper, over a period of time, under white noise perturbation. As is easily seen the general response is significantly worse (and this is in the most idealised and favourable modelling of a damping system).

Finally in Fig.7.6.5-3 it is shown that noise introduced in all three axes can be constantly damped and controlled, and individually. The sparse nature of the roll/pitch torquing pulses is due to an asymmetric damping idea (not firing if the attitude position is improving).

NOTE For example interaction between currents delivered by the power supply to all the on-board electronics interacting with the geomagnetic field.

Complete knowledge of the attitude during the orbit is assumed, possibly provided by a magnetometer and the Kalman-like estimator introduced at the beginning of this chapter. However the control concept can obviously survive intermittent attitude information as well. This feature could be very important in our continued investigations of low-cost options in attitude-determining sensors (NOTE).

More study is needed in order to establish the relative effects of real disturbances. For a real asymmetric satellite, operating with this assisted magnetorquing, it is not clear what increment in ΔI is desirable (as obtained possibly by deploying a second boom). The greater the increase in ΔI the smaller the resulting accelerations from internal torques. But the longer the boom the greater the exposure to external torques. It also weakens the pitch restoring torque from the increasing I_z (Eq.7.6.1.1a). Much more drastically is the way it increases the tumbling propensity of the satellite. The similar consideration extends to the main boom and tip mass (on the Z axis). The arguments involving mixed gravity-gradient stabilisation and magnetorquing are complex and need much more detailed modelling. The general principle however does seem sound.

Although one should be cautious in making excessive claims, it may be that the history of attempted 3-axis gravity-gradient stabilisation has been dominated by the thinking "libration is undamped $\rightarrow$ must damp $\rightarrow$ damping is enough". Hughes makes no mention of the simple tumbling sensitivity argument. If this point has been missed, as also the 'semi-linearised' equations are not well-known, then the need and justification for additional, if intermittent, proportional plus rate control about the yaw axis (at least) may have been missed.

NOTE For example a low-cost option of sun-sensors and optical/near infra-red horizon sensors cannot maintain continuous attitude information on a typical sun-synchronous orbit.

Fig.7.6.5-1 3 axis Controller (high gain)

Control Acquisition by G.G and Spin-Plane Magnetorquer
 adcs.libration.control12A 3-axis basic planar
 1989 Feb 20

orbit radius (km) = 7271.9 orbit period (min) = 6171.4 orbit inclination (deg) = 99.0
 number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

Tilted Dipole Inclined Orbit model planar parameters IX = 137 IY = 125 IZ = 12 MT = 10 MZ = 4

pitch amplitude (deg)	=	2.0	roll amplitude (deg)	=	2.0
roll phase (relative to orbit)	=	0.0	roll phase (relative to orbit)	=	0.0
yaw rate (revolutions per orbit)	=	2.0	no of up_dates per orbit	=	80.0
norm pitch/roll damp (secs)	=	15.0	norm pitch/roll disp (secs)	=	0.0
norm yaw damp (secs)	=	24.0	norm yaw disp (secs)	=	8.0
Bz0/Bt0 threshold yaw	=	1.0	Bz0/Bt0 threshold pitch/roll	=	3.0
pr noise level simulation	=	0.000	yaw noise level simulation	=	0.050

Fig.7.6.5-2 3-axis Controller (damping only)

Control Acquisition by G.G and Spin-Plane Magnetorquer
 adcs.libration.control12A 3-axis basic planar
 1989 Feb 20

orbit radius (km) = 7271.9 orbit period (min) = 6171.4 orbit inclination (deg) = 99.0
 number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

Tilted Dipole Inclined Orbit model planar parameters IX = 137 IY = 125 IZ = 12 NT = 10 NZ = 4

pitch amplitude (deg)	=	2.0	roll amplitude (deg)	=	2.0
roll phase (relative to orbit)	=	0.0	roll phase (relative to orbit)	=	0.0
yaw rate (revolutions per orbit)	=	0.0	no of up_dates per orbit	=	80.0
norm pitch/roll damp (secs)	=	15.0	norm pitch/roll disp (secs)	=	0.0
norm yaw damp (secs)	=	24.0	norm yaw disp (secs)	=	0.0
Bz0/Bt0 threshold yaw	=	1.0	Bz0/Bt0 threshold pitch/roll	=	3.0
pr noise level simulation	=	0.000	yaw noise level simulation	=	0.050

Fig.7.6.5-3 3-axis Controller (noise all axes)

Control Acquisition by G.B and Spin-Plane Magnetorquer
 adcs.libration.control12A 3-axis basic planar
 1989 Feb 20

orbit radius (km) = 7271.9 orbit period (min) = 6171.4 orbit inclination (deg) = 99.0
 number of orbits = 14.0 start longitude (deg E) of a.node = 0.0 start at equator/pole (e/p) = e

Attitude plots

Tilted Dipole Inclined Orbit model planar parameters IX = 137 IV = 125 IZ = 12 MY = 10 MZ = 4

pitch amplitude (deg)	=	2.0	roll amplitude (deg)	=	2.0
roll phase (relative to orbit)	=	0.0	roll phase (relative to orbit)	=	0.0
yaw rate (revolutions per orbit)	=	2.0	no of up_dates per orbit	=	80.0
norm pitch/roll damp (secs)	=	25.0	norm pitch/roll disp (secs)	=	0.0
norm yaw damp (secs)	=	24.0	norm yaw disp (secs)	=	8.0
Bz0/Bt0 threshold yaw	=	1.0	Bz0/Bt0 threshold pitch/roll	=	3.0
pr noise level simulation	=	0.020	yaw noise level simulation	=	0.050

7.7 List of Symbols

special line spacing 45 216

$B_{x0} B_{y0} B_{z0}$	B vector in reference co-ordinates
$B_{x1} B_{y1} B_{z1}$	B vector in intermediate co-ordinates
$B_{x2} B_{y2} B_{z2}$	B vector in intermediate co-ordinates
$B_x B_y B_z$	B vector in body co-ordinates
r	roll (estimate in some formulae)
p	pitch (estimate in some formulae)
ϕ	yaw (estimate in some formulae)
R_1	roll amplitude
θ_R	roll phase
P_1	pitch amplitude
θ_P	pitch phase
ϕ_0	yaw start value
ω_z	yaw rate (approximation)
N_z	torque on body axis
I_z	Z moment of inertia
v	innovation
$\phi^{\wedge}$	yaw estimate
a	yaw analogue estimator gain
c	yaw analogue estimator gain
δ	damping factor analogue estimator
ω_P	pitch oscillating rate
N_{x0}	pitch torque
$p^{\wedge}$	pitch estimate
v_P	pitch innovation
ω_R	roll oscillating rate
N_{y0}	roll torque
$r^{\wedge}$	roll estimate
v_R	roll innovation
k_{R0}	yaw rate to roll bias proportionality constant
ω_0	estimator gain for roll and pitch estimators
I_T	transverse moment of inertia
Δ	sampling time interval
z	the z operator

P	z-transform of pitch estimate
R	z-transform of roll estimate
N	z-transform of innovation
R_o	z-transform of roll bias estimate
α	conversion of gain a for sampled yaw estimator
γ	conversion of gain c for sampled yaw estimator
k_p	conversion of pitch oscillating rate
k_R	conversion of roll oscillating rate
$r_o^{\wedge}$	roll bias estimate
k_c	conversion of estimator gain for roll and pitch estimators
$yaw_x \ yaw_y$	cosine and sine of yaw estimate
bv_{ϕ_x}	scaled cosine yaw innovation (' suffix = previous)
bv_{ϕ_y}	scaled sine yaw innovation (' suffix = previous)
bv_p	scaled pitch innovation (' suffix = previous)
bv_r	scaled roll innovation (' suffix = previous)
$\underline{B}_1$	transformation from $\underline{B}_o$ using roll and pitch estimates
$\underline{B}_2$	back-transformation from $\underline{B}$ using yaw estimate
Bz_norm	scaling change in Z value
Bt_norm	scaling change in T value
$\underline{b}_1$	normalised from $\underline{B}_1$
$\underline{b}_2$	normalised from $\underline{B}_2$
yaw_T	threshold yaw estimator window ratio
pr_T	threshold pitch/roll estimator window ratio
Φ_B	notional angle of $\underline{B}$ vector in plane of X and Y axes
v_{ϕ}	notional actual innovation in yaw
$\Delta\phi$	yaw increment (rate) estimate
k_1, k_2	rescaled gains for sampled yaw estimator
Δp	pitch increment (rate) estimate
Δr	roll increment (rate) estimate
k_{ROT}	yaw increment to roll bias proportionality constant
λ	orbital advance (rotation in orbit plane)
ω_o	orbital rate

M_x M_y M_z	body magnetic moments
M_{x0} M_{y0} M_{z0}	transformed magnetic moments
$m_{x0}(t)$ $m_{y0}(t)$	equivalent modulated moments
m	max value of modulated moment
T_{Fx} T_{Fy}	specific fire times on X and Y spin plane torques
T_F	fire time on unspecified magnetorquer
B	reference field magnitude
M	reference spin plane magnetic moment magnitude
s_x s_y	assigned polarities to spin-plane moments
$\underline{v}$	unit vector in forward velocity direction
$\underline{n}$	unit vector in orbit normal direction
k_{PR}	pitch roll controller gain
k_{z1} k_{z2}	yaw controller gains
e_z	yaw control error
B_T	transverse component of vector field from B_x and B_y
B_z	Z field component in body axis
pr_TC	threshold yaw controller window ratio
yaw_TC	threshold pitch/roll controller window ratio
ΔI	Increment in moment of inertia assymmetric test

8 REVIEW OF RELATED LITERATURE

The purpose of this chapter is to try and put the University of Surrey satellites in perspective, from the point of view of attitude determination and control. The use of gravity gradient and magnetorquing on other spacecraft, as cited in the literatures, is reviewed, where the techniques are similar or relevant to UOSAT practice. The chapter starts with a rapid survey of all other standard methods of attitude determination and control

The selection of literature here has concentrated on magnetic control (firing of magnetorquers) and gravity gradient control. In order to give a focus and a theme to the work I am fortunate to obtain an entry via two special conferences. Magnetic control was given special treatment in the Guidance and Control Conference (1981) under the auspices of the American Astronautical Society. This set of unified papers presented are summarised here as a whole rather than breaking into separate topics. Other papers not connected with this conference are summarised subsequently. I traced a Symposium on Gravity Gradient Attitude Stabilisation (1969) under the auspices of the Aerospace Corps and Airforce System Command. The set of paper presented give a very interesting review of the problems then current. And very recently Hughes's excellent book (1986) also gives a detailed review of gravity- gradient work, in theory and practice/

Magnetorquing has always found applications in many classes of satellites but the use of gravity gradient in recent years may have entered a backwater, because of the many problems and difficulties encountered, despite early excitement about its possibilities. But a recent revival of interest in small spacecraft may be a sign that gravity- gradient control is due for rehabilitation. There is for example a recent study for Globesat by Siahpush and Sexton (1986) specifically on this form of control for small satellites (reviewed briefly in sec.8.4.6). More interesting studies by Martel et.al (1988) are reviewed in sec.8.4.7.

The summary of the literature given here is therefore inherently selective, seeking to identify useful concepts in (i) magnetic control (ii) passive gravity-gradient control (and in combination).

8.1 Control and Determination in general

In all standard methods of ADCS, magnetorquing is always found in combination with other control techniques. Gravity-gradient control is also found in combination with other techniques - from passive damping to assisted stability from additional momentum, due to spinning wheels.

In this brief round-up I avoid any attempt to summarise advanced or specialised control functions - as needed for interplanetary probes or any function involving special attitude change. I am concerned solely with Earth-referenced satellites where the aim is to acquire and stabilise on a fixed attitude.

The simplest class of controlled satellite is well-known to be a 'spinner', in which the body of the spacecraft spins around an axis. This confers angular momentum on the body and therefore some resistance to disturbance torques. The natural attitude has the spin axis aligned on the orbit normal. It is also a theoretical, natural equilibrium point into which a dissipating spinning body may settle (according to Hughes 1986). At any other orientation the spin axis is subjected to a precessional torques due to gravity gradient (see chapter 3). In practical engineering terms, the supposedly spin-stabilised state, because of disturbance torques, or mission requirements, will generally require active maintenance. For low Earth orbits, magnetorquing is very suitable, particularly on a polar orbit, because the geomagnetic field strength is adequate for effective control (much less so at geo-synchronous altitudes)

An early development was from a spin-stabilised spacecraft to *dual-spin*. Here a spinning part maintains stability while another part - typically

a directed antenna - is held 3-axis stable within the Earth-referenced frame.

For 3-axis control of a whole spacecraft development has followed different directions. The dual-spin concept has evolved into *momentum bias* design in which one or more spinning flywheels are buried within the spacecraft as a control sub-system. These provide an inherent stability to the entire structure because of their maintained angular momentum. In a single-wheel design, if the wheel spin axis is aligned along the pitch axis then changes to its speed must alter the pitch of the satellite body. By making this speed variable and part of a control loop then an external disturbing torque can be compensated and control provided in pitch. This wheel (or wheels) must be backed-up by control methods which can re-orientate the basic spin direction, correct for attitude drifts under disturbing torques, and correct in roll and yaw.

An alternative is an all *reaction wheel* design. In such a design there are three or more flywheels aligned along different axes capable of spinning in either sense (having a nominal speed of zero). Such a spacecraft has no inherent stability and pointing accuracy has to be maintained constantly from attitude measurements and by changes to the speed in one of three or more reaction wheels. Magnetorquing has no direct role in attitude change (but see below).

The relative advantages of the two approaches would seem to be finely balanced and need to be examined in each application. A good introductory treatment is given in Wertz (1978) (NOTE)

A control system obviously depends on attitude measurements, and for an Earth-referenced spacecraft the primary sensor must be a horizon sensor. Although its function is indirect, next on the list comes magnetometer

NOTE For fast slewing to arbitrary pointing directions there is a revival of interest in CMG (control moment gyros)

measurement. More complex schemes contain alternatively (or additionally) gyroscopes, sun sensors and star sensors.

Using an Earth horizon-sensor it is possible to measure pitch and roll errors but not yaw. With the spinning satellite - or the momentum bias design of 3-axis control - any pointing error in this spin axis alternates every quarter orbit between a roll and a yaw error.

With the former measured by a horizon-sensor the expectation is that controlling the roll error will automatically control the yaw, relying on the momentum bias to maintain this automatic conversion every quarter orbit.

Using an all-reaction wheel design for 3-axis control a yaw error must be estimated instantaneously, in addition to the pitch and roll. Since this cannot be done with a horizon sensor there is generally a need for inertial package based on gyros, this being the only attitude - determining source which can be relied on for continuous information.

Three-axis control using either momentum or reaction wheels needs to be backed up by the technique of *momentum dumping* in which the momentum accumulating in the wheels from continued disturbance torques is discharged. Otherwise the speeds of the momentum or reaction wheels will eventually exceed their design range and control will fail. For LEO satellites momentum dumping is conveniently achieved by magnetorquing; but for geosynchronous satellites this is not a popular option - the field is relatively weak and unstable (NOTE).

The other commonly-found application of magnetorquing is the suppression of nutation. Although passive nutation dampers are also common, an active method, based on the schemes outlined in section 5.1, is generally convenient to implement.

NOTE The use of hydrazine (reaction jets) are common for station-keeping : these can also be used for attitude control

An entirely different method of controlling attitude is by gravity-gradient effect. Simple 2-axis stabilisation - revived for UOSATs - was tried first; and passive 3-axis stability has also been attempted. In the notation of this work the requirement with 2-axis stability is to align the Z axis on the reference Z_0 (or its inverse). All the flight experience cited here refers to the '60s and early 70's when this kind of work was done.

Although the effect exists in principle for any geometry (except the equivalent of a sphere) it was very common practice to deploy at least a mass at the end of a boom along the nominal zenith-nadir (+ Z axis or - Z axis). This creates a dominant I_x and I_y over the I_z moments of inertia (see section 3.3) and achieves 2-axis stability in principle. The spacecraft may continue to spin about the Z axis, which yawing motion is not influenced.

If the moments of inertia I_x and I_y are unequal and various other conditions are met (see sec.7.6), then a weak stability in principle can be conferred in YAW, giving full 3-axis stability. As Hughes (1986) has remarked these (intended) three-axis stabilised systems generally have not worked well in practice.

Whether intended 2-axis or 3-axis stabilisation there is no inherent damping for a rigid body. A number of methods of passive damping have been tried out or suggested.

Hybrid schemes are also possible in which gravity gradient control is augmented by spinning flywheels either to promote greater roll/yaw stability or, if described as gyrotorquers or 'control moment gyros', to assist in damping.

NOTE The literature always refers to ' 2-axis control ' meaning control of roll and yaw angles about orbit-defined co-ordinates. I would prefer the terminology ' 1-axis', meaning the control of the *pointing of an axis*. This is then consistent with the literature on attitude determination.

8.2 Magnetic Control Review

The 1981 Guidance and Control conference of the American Astronautical Society held a special survey on the use of magnetorquing in spacecraft control. The conference performed the useful task of getting the leading engineers from the various corporations and agencies to review their general approach over more almost 2 decades of effort. As might be expected there was considerable independence of approach between the various groups. These group of papers effectively survey the American space effort in 'magnetics' i.e magnetorquing at that time.

The aim here is to describe magnetorquing practice rather than attempt to summarise detailed theory. The main physical principles invoked in these papers are simply

- 1 Angular momentum is conserved within a torque-free environment. So a change of speed of an incorporated flywheel (if there is one) must induce an attitude rotation on the same axis of the body containing it.
- 2 An applied torque must induce a vectorial change in the vector angular momentum;
- 3 The torque induced by a magnetorquer is equal to the vector cross-product of the magnetic moment and the geomagnetic field.

8.2.1 Mobley :1981.

The first paper describes the work of APL (Applied Physics Laboratory John Hopkins University). This paper also described their early work on gravity-gradient control. They were the first to get a successful 2-axis stabilised small satellite into space.

Back in the 1960's this group used an electromagnet for initial acquisition of the satellite's attitude, and then gravity-gradient stabilisation for permanent alignment (TRANSIT 5A-3) after switching the magnet off.

However it was quite insufficient just to switch on an electromagnet and hope that the satellite would lock to the geomagnetic field. They had previously used magnetic hysteresis rods (TRANSIT 2A) and found them effective. Further information was referenced from Fischell (1964).

Early work on de-spinning relied on direct coupling between the magnetometer measurements in X and Y and then amplifying or switching the magnetorquer coils (DME-A).

The early gravity-gradient controlled satellites employed passive damping using a magnetically anchored damping - (more fully described in Wertz 1978).

Their work on a Small Astronomy Satellite required a passive nutation damper for this spacecraft which was intended either to be single spin or dual spin (constantly rotating internal momentum wheel). They developed a passive device; which would appear to be about the same size as UOSAT-2.

Their work on magnetics was applied to a near-synchronous, gravity gradient controlled satellite (DODGE). The application of magnetics here is limited by the very weak geomagnetic field. The historically earlier approach of using magnetic hysteresis rods was ingeniously extended here by a magnetometer-magnetorquer combination which amplified the local B field by 3-4 orders of magnitude while keeping the SAME vectorial direction. While the satellite was arbitrarily tumbling this locally induced field was static. Impacting this on a suitable material then provided the mechanism for energy dissipation. The spacecraft was intended for gravity-gradient stabilisation and needed passive damping. They deployed a boom on the end of a wire - quite independently of the spacecraft axes - which oscillated independently of the librating spacecraft and whose relative oscillations they damped out using various pieces of passive hardware. This satellite is discussed further in sec.8.4.3.

The major part of this paper concerned the scientific satellite MAGSAT which was designed to measure the geomagnetic field. It employed an on-board microprocessor the RCA 1802 chip (which was used on both UOSAT-1 and UOSAT-2).

This satellite is an interesting example of 'after the fact' attitude determination. The spacecraft was necessarily controlled and 3-axis stabilised in order to do its task; but only to within ± 5 degrees on any axis. The high precision of 20 arc.sec (Abshire 1981) was achieved only in post-processing.

The satellite control used a momentum-bias design with the stability conferred by an internal momentum wheel spinning at 1500 rpm and aligned on the orbit normal. The rest of the structure was held nominally stationary within the Earth-referenced system. Pitch stability was conferred by closed-loop control of the wheel speed. This standard method was linked to an IR horizon-scanning system.

Instead of discussing attitude change in terms of roll and yaw the authors prefer to discuss the attitude of the spin axis using the astronomical terminology of *declination* (latitude in an attitude sphere defined by the celestial equator) and *right ascension* (longitude in this attitude sphere), both of which will be constant in the absence of disturbing torques. This is a more natural method of designating attitude for spin-stabilised satellites in this nominal attitude, than the attitude conventions adopted for the UOSATs.

Magnetorquing was intended for use in two important ways: first in momentum dumping. In this case the nominal speed of 1500 rpm could not be allowed to shift more than ± 200 rpm. Describing the spin axis as the Z axis then the use of spin-plane magnetorquers (X axis or Y axis) would create a torque in the Z direction and start to twist the spacecraft body; the wheel control system must then fight this torque by changing its angular momentum - and so drive back the speed to within the nominal range. On a polar orbit like this with the geomagnetic field lying close to the orbit plane it was always possible to create this

magnetic torque - it was just a question of switching on the right torquer at the right time. Note that unlike the spinning spacecraft (e.g UOSAT-1) there is no need to commutate this firing. The satellite was intended however to make magnetic measurements, and these the magnetorquer operation would obviously put at risk. Their account describes a satisfactory ad-hoc approach in which they were able to dispense with this momentum dumping. The remote magnetic measurement assembly trailed out on a scissors-like boom, backwards from the main body of the spacecraft. They found that they could dump momentum simply by setting a small pitch angle to this boom such that external aerodynamic disturbance torques were non-zero in pitch - making an imbalance between gravity gradient and aerodynamic torque.

At the right attitude the wheel control system would show no net drift. If the wheel speed showed signs of going outside certain limits then it was only necessary to advance or retard the pitch *requirement* to dump the momentum in the desired direction.

The other use of magnetics was to correct the attitude of the spin axis - the direction of the angular momentum. From their account the general principle can be demonstrated in a cross section through the orbital plane (fig.8.2.1-1). It involves correctly-timed operation of the pitch (spin axis) magnetorquer. The same idea has been widely adopted by others. Unlike the UOSAT requirement where the aim was to drive the spin axis up into the orbit plane (sec.5.1.3) the aim here is to keep it closely aligned on the orbit normal. Repeating arguments of sec.5.1.2 the fundamental rule is that turning on magnetic moment aligned on the angular momentum causes a precession around the B vector, meaning that the spinning axis will 'go round ' the current direction of the geomagnetic vector. The other basic is that the geomagnetic field can be said to lie largely in the plane of the orbit (IDPO model sec.4.3.1). When the satellite is immediately above the pole (A2 or A9) the roll measurement determines the declination there. It cannot however be corrected because the field direction is unfavourable. But within a few minutes of orbital advance the field has slewed.

Starting at a later point in the orbit (A3 or A10) the B vector has slewed through $\approx 90^\circ$ in inertial space, which is right for inducing a declination change by precessing the spin axis round it. Of course if the attitude was a pure roll over the pole it has now partly converted into a yaw. This is however irrelevant. Under the assumption of inertial stability the declination is invariant. I note that this is open-loop control. The amount of torquing must be estimated, and the results revealed when the spacecraft is back over the next pole.

Equally, over the equator the measured roll now directly estimates a right ascension. This can be measured and corrected simultaneously at this point in the orbit since the geomagnetic field is now favourable (starting from points A7 and A14 in fig.8.2.1-1).

8.2.2 Schmidt 1981

The second paper is a detailed analysis of the RCA approach which resulted in the series of TIROS satellites - polar orbiting in the latest series and sun-synchronous. Ignoring their first efforts which they called a 'pioneering success' (TIROS I) the design converged on the 'rolling drum' configuration, a spinning, barrel-shaped device with sensors noting the spin advance. With the spin axis nominally aligned on the orbit normal the first thing they had to do was to advance it constantly in right ascension in order to synchronise with the steady precession of the orbit. This could be done simply by leaving on the spin-axis magnetorquer. On the simplest IDPO model - everybody seems to use this - the orbit-averaged B field runs North-South and the spin-axis would tend to precess about this. From a nominal attitude it would therefore constantly advance in right ascension. They called this MBC (magnetic bias control), and used it to correct at intervals for precession of the sun-synchronous orbit. There was no control over declination here ; hence their pioneer efforts with QOMAC (quarter orbit magnetic attitude control). If the interacting magnetorquer has its sign reversed every quarter orbit then the average torque components exist in the plane of the orbit, pointing where wanted as a

result of the selected commutation. The mix of average equatorial component and North-South component can be controlled this way, determining the required amount of precession in declination and right ascension respectively.

RCA seemed reluctant to use any magnetometers. While the QOMAC principle requires a commutation only every quarter of an orbit, and cannot use a magnetic field measurement directly, any alteration to spin rate of the spinning satellite requires commutation every half-cycle of spin, which commutation is directly linked to the B vector as resolved on the body axes. They used Earth (horizon) sensors, and the knowledge that the B vector field was substantially up-down over the poles to commutate a spin-plane magnetorquer. The decision not to use magnetometers led to a certain complexity in the final series of spacecraft controlled by reaction wheels.

Their next development was to provide a 3-axis controlled platform and therefore install a flywheel inside the spacecraft to confer stability (again aligned on the orbit normal). As on the previous series they still used passive nutation dampers. No essentially new principle was applied. As on MAGSAT the pitch was controlled by closed-loop control of the wheel speed; and any tendency for this wheel speed to go outside bounds was prevented by dumping momentum using a spin-plane magnetorquer. As on MAGSAT no commutation was required since this torquer was attached to the non-rotating platform, and it was simply a matter of switching it on at the same point in the orbit (Atmospheric Explorer series)

Then they turned to geosynchronous satellites. The magnetic configuration is quite different here since the B vector field is roughly a constant on the equatorial orbit and is directed approximately on the orbit normal. A magnetorquer is aligned on the roll axis. This creates a controllable yaw torque which converts through the angular momentum to a ROLL change.

This they claimed gave control to an accuracy of 0.05 degrees. The problem, as also discussed by Goel (sec.8.5.3) with magnetic control at geosynchronous altitudes, is the tendency of the B field to disappear in the occasional storm, requiring therefore a back-up of thrusters (Satcom I and II). The account is of interest in that they also used constant d.c bias to various fixed magnetorquers to resist the torques of solar pressure on the arrays.

At some point in this series they adopted an ingenious form of nutation suppression - Phillips (1973). The problem with all attitude changes to a spinning body is that the imposed torques tend to induce nutation. This must either be damped out passively or as here 'semi-passively'. According to Phillips if the nutational error is sensed this can be fed back and added into the control loop which is controlling the speed of the momentum wheel (and is normally just controlling pitch). This ingenious method - for it does not require any external imposed torque - depends on constructing the satellite such that the spin axis is not aligned on a principal axis of inertia. Theory then shows that there exist cross products of inertia. This means that any change of momentum on the spin axis cross couples to any transverse (non-spin) axis, which Phillips's idea exploits it.

As a later development they *skewed* the active magnetorquer (in the roll-yaw plane). This means that whenever a yaw torque is created (tending to create a roll) there is also a roll torque (tending to create yaw). The detailed mathematics is complicated (Schmidt 1978) but the implication is that the Earth sensor still only measures roll deviation and the new torque component always has a damping function on current yaw rate of slew (which was induced as a roll correction 1/4 orbit earlier. This apparently enabled them to achieve much tighter loop control and control roll to 0.02 degrees.

Their third generation of spacecraft of polar orbiting (TIROS series and DSMP satellites) abandoned momentum wheels in favour of reaction wheels (mid 70 's).

As noted in the introduction to this chapter, the reaction wheel is essentially a device with a nominal average velocity of zero. Any one wheel must be unloaded when the magnitude of its spin exceeds a certain threshold. On the polar orbit as we know a torque can be created in any desired direction provided the correct position in the orbit is chosen. Again RCA did not attempt magnetometer measurements; on the other hand they needed to know the effective magnetic latitude. This they computed indirectly from stored data and the measurements from sun and Earth sensors. I feel that a reduction in system complexity would have resulted if they had broken with their own self-imposed tradition and used a 3-axis magnetometer as the data input to their 'unloading' magnetorquers. As I have tried to show in sec.5.4.2 there is immediate and direct relationship between the measurement by former and control by the latter. So the unloading of momentum could have been undertaken by an autonomous system control /calculation loop.

An interesting application of 'post-launch' system design was in a DMSP satellite whose control system failed after launch. They found that the gyros had failed. It was still possible to design a control system without them. With a static four-quadrant Earth sensor they could sense pitch and roll deviations - but not of course yaw. The solution then was to reconfigure the system to something approaching the momentum wheel approach based on the pitch flywheel.

Finally the authors remarked that for the series Satcom D and E of satellites based on momentum bias stabilisation they would dispense with passive or active nutation control on the Phillips pattern. They realised that it was always possible to time carefully the magnetorquing corrections such that nutation was simultaneously suppressed while the momentum was driven the way one wanted. This was of course our experience on UOSAT-1 (described in section 6.1) in the more difficult circumstances that it was based on a spin-plane magnetorquer rather than a spin-axis device.

Fig.8.2.1-1 Orbit Locations for Attitude Detection and Control

8.2.3 Emsley et al. 1981

This paper is an account of the work at the Ball Aerospace Systems Divisions. It opens with a description of the NASA orbiting Solar Observatory and an honest admission of an error: the second in the series OSO-2 had a permanently biased magnet supposed to assist the gas jet slewing systems in keeping the spin axis orthogonal to the sun line. It turned out to increase rather than decrease the gas consumption - from which they deduced that the magnetorquer was connected the wrong way round - and caused the satellite to exhaust its propellant in 9 months. From personal experience I know that this kind of error can occur very easily. We would now always stipulate that all torquers and sensors be qualitatively checked for channel assignment and polarities and compared with the nominal satellite body axes *just prior to launch*.

The account of the SOLWIND project is of interest. This was a 12.00 a.m / 12.00pm sun-synchronous polar-orbiting satellite with its spin axis nominally aligned on the orbit normal. And magnetorquers were used - just as in the RCA work - to keep the pitch axis slewed and also the QOMAC principle to adjust its orientation. The spacecraft was dual-spin so that the de-spun portion carrying the solar telescopes could keep pointing at the sun. Control was not autonomous and the pointing accuracy of the spin axis to about ± 5 degrees was maintained by monthly commands which ordered a restorative QOMAC sequence.

A comment of interest was that the magnetometer data by itself could determine the spin-axis attitude to 1 degree or better. The authors do not make it clear that this analysis requires (i) multiple samples over a significant fraction of an orbit and (ii) an assumption of constantly pointing spin axis, or some information on its likely trajectory

Their work on the Solar Mesosphere Explorer (SME) in Jan 1981 was interesting. This was claimed to be the first autonomous, single-spin magnetically-controlled satellite.

Details there are not, but it would seem to be the standard package based on a spin-axis magnetorquer, a spin-plane magnetorquer, a horizon sensor and a 3-axis magnetometer.

Their work on the Earth Radiation Budget Satellite (ERBS) is a useful illustration of spacecraft attitude control design based on a momentum wheel. The same standard approach was used - pitch control from the wheel, pitch axis magnetorquer to control roll and yaw and nutation; horizon scanners to give roll and pitch measurement. More sophisticated features were a gyro assembly to give yaw measurements with its tendency to drift being updated by occasional sun angle measurements. Nutation damping was automatic - by providing a control law on the pitch dipole which not only controlled precession but also controlled nutation. Identifying the spin axis with the Z axis then the essential measurement is simply the rate of change dB_z/dt (sec.5.1.4). The very slow nutation period meant that it was unwise to rely on the simplest control law based on dB_z/dt and instead they needed to determine the nutational motion from the gyros and/or horizon sensors.

Of special interest is their 'recovery mode ' in which the universal use of $M_i = -dB_i/dt$ (i - X,Y or Z) law kills all motion relative to the rotating B vector field and leaves the satellite with a residual pitch rate relative to orbital axes of ω_0 . With a spinning flywheel this simple control also tends to drive the spin axis into the orbit normal. This fact may first have been pointed out by Stickler and Alfrend (1974). I analyse the reason in sec.8.3.4.

Gravity-gradient control also features in this account. With a tilted solar array configuration the entire spacecraft suffered a gravity-gradient roll-torque. This was compensated out ingeniously by canting the momentum wheel, so causing a forced pitch precession which matches the orbital pitch rate.

It would seem that no microprocessor was used and that the entire control electronics was purpose-built.

8.2.4 Decanini et.al 1981

This paper describes the work of TRW Defence and Space Systems. They give an overview of basic control concepts and then divert to a general examination of momentum dumping - the most efficient use of magnetorquing coils to remove the momentum accumulating in very large spacecraft. They mention one of the first studies of control of a spinning satellite using a single spin-axis magnetorquer (Wheeler 1965).

The authors give an algebraic law for precession control from this law which gives a 'position error function' and a 'rate error function'. In the convention of their paper they treat the case of a single spinning satellite whose spin axis is aligned on the orbit normal, and consider the control laws for a spin axis magnetorquer which would induce attitude changes. Their algebraic treatment is less impressive than it would appear on first sight (fig.8.2.4-1). A positional error is defined and a rate error. The former is said to measure the pointing error from the current spin axis attitude to the orbit normal ; while the former measures the nutational state. On inspection their control law based on the positional error would seem to be essentially the same as the QOMAC principle.

The rate error parameter is un-impressive. As can be seen from the standard vector expansion this is no more than the negative of the rate of change of B_z within the nutating body in a static B field . And this is just the standard damping (denutation) law (sec.5.1.4).

As was analysed in detail in sec.5.1.5 the two requirements or precession and de-nutation can be combined linearly ; and one would choose a logical control law which simply switches on the magnetorquer when the two algorithms happen to agree.

More interesting results emerge from their study on momentum dumping. They first consider the CPL law (sec.5.4.1). Then a variant, in their terminology, is a 'bang-bang control law', BBL, which essentially consists of turning on full whichever magnetorquer has a magnetic

moment lying closest to the optimum direction computed by the above. A third option uses modern control theory and seeks to minimise the energy consumed by the magnetorquers while unloading a specific amount of momentum. This they call the Minimum Energy Desaturation Law (MEDL). An expanded discussion is given in the next reference with a fuller account of this concept.

The reassuring conclusion from their findings is that BBL - i.e the simple turning on/off of various magnetorquers at full power, is not greatly more inefficient than the MEDL option. Since the latter is much more complicated that is quite reassuring for users of the cross-product law and its derivatives.

8.2.5 Dougherty et al 1981

This fifth paper comes from the Lockheed Missile and Space Company . Their study is interesting on a number of counts. First because one of the proposed modes of controlling a certain spacecraft identical to the normal mode of UOSAT-2 - letting gravity gradient take control and damp out motion using magnetorquers. In their application this is a 'safe' state to which the payload - a telescope - would revert if something went wrong with its normal pointing mode (controlled by reaction wheels).

The final part of this study is of interest because of its presentation in terms of optimal control theory. The telescope is controlled by reaction wheels but there is a constant need to 'dump momentum' from them by using the magnetorquers. Given the requirement to dump momentum by a specific amount within a certain time then it is possible to formulate a minimum energy integral under a set of constraints (invoked by the usual variational methods and a vector of Lagrangian multipliers). One needs to know the history of this attitude over all of an interval, and the disturbance torque (gravity gradient in this case) , and the geomagnetic field in order to apply this theory.

Given all this knowledge then a precise requirement on the currents to feed to the torquers can be calculated and then implemented.

Such a procedure was out of the question for UQSAT operations in the past. The question is if we should be considering the more advanced theory in future mission design. There is one important difference between the telescope and the simple UQSAT type systems. The attitude during this momentum dump is being maintained by an independent system (the reaction wheels) so that the application of disturbance and magnetic torques does NOT affect the attitude. Whereas in the simpler UQSAT type this is absolutely not the case - since the magnetorquing is designed to CHANGE the attitude. What is of interest in their study for the telescope is how little is gained by this sophisticated control theory compared with the 'hit-it-now' cross-product magnetorquing which reacts to the instant measurement.

In comparing their 'minimum energy' law with the 'cross product' law (CPL) their detailed study showed that nothing is gained in the worst case with the telescope experiencing the highest possible gravity -gradient bias. In some instances it was even worse. Also the study showed that only a small percentage reduction in wheel speed after performing various manoeuvres with the space telescope. Only in settling times to get the wheel speeds down to near zero following a manoeuvre into a low-bias state was there some improvement (2/3 to 1/2 of the time).

The fact that their 'ME' law can give inferior results to the 'CP' law under some circumstances makes one question the 'optimality' of this minimum energy criterion. And I would question the point of it anyway. At least in our system the magnetorquer is not a major consumer of power - it is on for a very small duty cycle compared to the overall mission.

8.2.6 Gran and Proise 1981

This paper describes the magnetic control systems used in the Grumman's Orbiting Astronomical Observatory. Only a brief summary will be made here since geosynchronous satellites have not been part of this work. This study claimed to achieve pointing accuracy to ± 0.1 degrees using reaction wheels and momentum dumping using a single large magnetorquing coil wrapped round the solar array. It will be recalled that the problem in this instance is the low value of the B field (in the order of 100nT) its variability due to magnetic storms, and even its tendency to reverse. A complication of the study is that the spacecraft is a large flexible structure whose various modes of operation must be modelled. The main disturbing torque is assumed to be solar pressure.

8.2.7 Junkins et al 1981

The seventh and final paper of this set at the conference is an independent study by members of the Virginia Polytechnic Institute. This claims to be a state-of the art study in assessing the possibilities of controlling attitude to within the sort of accuracy to which it can be measured (a few arc secs using star trackers). The authors propose what looks like an ingenious method of linearising the fundamentally non-linear equations of motion under their control law which effectively linearises and decouples the equations of controlled motion in pitch, yaw and roll.

Some criticism is due for their supposedly 'novel' method of momentum dumping. They propose a ' criterion function' (fig.8.2.7-1 which copies almost exactly their equations 20 and 22). Thus L_1 to L_3 are components of momentum along three body axes which it is desired to reduce. M_1 to M_3 are component magnetorquer moments which are assumed to have values -1, 0 or +1 while B_1 to B_3 are component magnetic fields resolved along the three axes. The authors then propose an exercise in linear programming which searches for the best out of 27 possibilities in the moment vector M which will minimise this 'criterion function'.

This is not in any way novel: in fact it is exactly the cross product law all over again. As simply substituting Eq 22 into 20 shows that this so-called criterion function is nothing more than the scalar product of the magnetic torque $\underline{N}$ on the momentum error $\underline{L}$. Just one vector transformation makes this identical to the scalar product of $\underline{M}$ on $\underline{B} \wedge \underline{L}$. The last two quantities are 'given'. So the solution is still to direct $\underline{M}$ to point in the same direction as, or the closest approximation to, the cross -product. So much for this novelty.

Fig.8.2.4-1 TRW's magnetic control law

For Z axis nominally the spin axis pointing towards orbit normal then define

Position
error function $\sigma_P = a_{23}B_x - a_{13}B_y$

Rate
error function $\sigma_R = \omega_x B_y - \omega_y B_x$

On analysis does not work instantaneously : needs averaging over an orbit = QUOMAC .

Fig.8.2.7-1 Momentum dumping by minimising of 'criterion function

Criterion function

$$\begin{aligned} \phi = & w_1 \text{SGN}(L_1) (n_2 B_3 - n_3 B_2) \\ & + w_2 \text{SGN}(L_2) (n_3 B_1 - n_1 B_3) \\ & + w_3 \text{SGN}(L_3) (n_1 B_2 - n_2 B_1) \end{aligned} \quad \text{.....20}$$

where $\underline{L}$ is the error angular momentum

$\underline{n}$ is the polarity setting to X Y and Z magnetorquers

$\underline{w}$ is a vector weight computed according to

$$w_i = L_i / \text{MAX}(L_1, L_2, L_3) \quad \text{.....22}$$

But

$$\phi = \frac{1}{L_{\text{max}}} (\underline{L} \cdot (\underline{n} \wedge \underline{B})) = \frac{1}{L_{\text{max}}} (\underline{n} \cdot (\underline{B} \wedge \underline{L}))$$

Hence minimising ϕ simultaneously meets CPL criterion.

8.3 Miscellaneous Magnetorquing

Some additional papers are briefly looked at where they shed insight on the application of magnetorquers. In all cases the satellites concerned are spinners. The most innovative seem to be by Meinzer, Stickler and Alfrend and Teule

8.3.1 Stewart et al 1982

The satellite launched simultaneously with UOSAT-1 is described. It was designed to spin at 5 r.p.m with a nutational spin error of less than 0.1 degrees and the spin axes aligned on the orbit normal. Hardware included horizon sensors in a V-configuration (35 degrees on either side of the spin plane) and a 3-axis magnetometer. The control hardware consisted of one spin-axis magnetorquer and one spin-plane magnetorquer. The former was intended for attitude changes (including forced nutation damping). The latter was for controlling the spin rate. They also had on board a passive nutation damper. The designers intended to set the spin-axis magnetorquer at a number of levels; with small trickle currents to cancel out residual dipoles and/or to induce forced precession to match the orbital changes due to sun-synchronicity; and higher levels to give more violent forced changes.

They also mention the acquisition mode in which the spin axis is forced near the orbit normal (to an accuracy of only a few degrees) without horizon sensing data. This is a physical consequence of just applying damping through the spin axis magnetorquer (sensed by the rate of change in slope of the spin-axis component) and is discussed below (sec.8.3.4).

The paper contains much useful data on the problems of determining attitude from the horizon sensors and the satellite ephemeris. Accuracy requires the use of a Horizon Radiance Modelling Utility which was

provided by the Goddard Space Flight Centre. The oblateness of the Earth must be included. Indeed the whole account of precise attitude determination is potentially very useful. It is noteworthy that the ALTITUDE is also computed using the horizon sensors , with an accuracy achievable of 1 km.

8.3.2 Meinzer 1981

This work is of particular interest. His mission objective with his 'phase 3' satellites were to achieve a highly elliptical orbit from an equatorial launch. Two successful satellites have been launched - OSCAR 10 and very recently OSCAR 13 - requiring an onboard rocket motor to raise the satellite to a high-inclination orbit. His first spacecraft was launched after UOSAT-1 but before UOSAT-2. It therefore acquired the designation OSCAR10 because his satellites are designated within the same international program as our satellites.

The attitude determination system contained many novel elements but will not be discussed here. The engineering use of the on-board computer and its interaction with the rest of the spacecraft was based on an entirely different philosophy to UOSAT-1 and UOSAT-2.

For attitude control gravity gradient would be quite unworkable because of the eccentricity of the orbit. The most obvious choice was 1-axis spin stabilisation, with a need to point the spin axis in different directions during the control manoeuvres. To obtain this control Meinzer employed a trio of spin-plane magnetic solenoids mutually inclined at 120 degrees. I understand that his spacecraft incorporated passive nutation damping (although this was not mentioned in the article); so the aim with magnetorquing was to re-orientate the spin-axis of the spacecraft and alter the spin-state as necessary, through a variety of complicated manoeuvres. Any nutation build-up was assumed to damp out passively.

The fundamental mode of operation is that commutated currents are fed to the spin-plane magnetorquers in such a way that a STATIC magnetic moment is created relative to the current angular momentum vector and the current magnetic field. The technique has been reviewed and incorporated in sec.5.4.1 and Appendix G.

8.3.3 Goel et.al 1975

The paper is by an author with ISRO (the Indian Space Research Organisation). The proposal was for a geosynchronous satellite with 3-axis stability to be conferred by a momentum wheel fundamentally and the use of magnetorquers to ensure stability and damp out nutation.

A major difference between the geosynchronous altitude and the low-orbit altitude is that the geomagnetic field is very much weaker in the former case (they assume at best 1 milligauss = 100 nT compared to 20,000 nT) and is given to massive perturbation from magnetic storms.

The disadvantage to the overall approach is the turbulent weather-like nature of the magnetic field at this higher altitude. The geomagnetic field interacts with the solar wind forming a giant turbulent system called the magnetosphere (Hones 1986)- a system which can stretch down-wind of the sun to a length of a 1000 Earth radii. A solar flare initiates a shock in the solar wind and causes a magnetic storm. The engineer must contend with the fact that the B vector can drop to zero or even go into reverse, leaving him with no control or worse. Therefore a standby system involving control jets would need to be switched in. However the saving in fuel would presumably justify this operation.

8.3.4 Stickler and Alfrend 1974)

This earlier paper makes a comprehensive survey of control of low-altitude polar orbit using the common momentum wheel and

magnetorquers aligned on all three axes - for achieving full 3-axis control. Reviewing all the magnetorquing studies known to them previously they claim to have provided the first detailed study of using the aligned pitch axis magnetorquer simultaneously in precession control and in nutation suppression. They assume a horizon sensor for reading roll and pitch. Roll error is compensated out using the pitch magnetorquer, while the pitch error is compensated by closed loop variation of speed. Momentum dumping of this speed of course using yaw and roll axes magnetorquers.

The most interesting result in this paper is the discussion of initial damping and detumbling by the usual application of $M_i = -dB_i/dt$ where B_i is one of the component fields. They point out that if the spinning wheel is ON (but not being controlled in speed) then there is an inherent tendency for the satellite's attitude to point the angular momentum in the direction of the orbit normal. The argument applies equally to a spinning satellite. The technique was used by Stewart et.al (1982) - discussed above.

If the algorithm repeated here, for a simple spinner (Z axis)

$$M_z = - dB_z/dt$$

...5.1.4.1

is used *continuously* without regard to the attitude then it has the subtle consequence is that not only would the application of this algorithm tend to damp out nutation but it would also drive the spinning satellite (or satellite with spinning wheel) to point along the orbit normal if the spin were positive or the antinormal if the spin were negative. This can be visualised from Fig.8.3.4-1. In this schematic, which assumes the IDPO model, one of the four instants in the course of the orbit has been selected when the B field is at right angles to the Z axis. It is assumed that the spacecraft is now not nutating and has a

positive Z-spin. The position is shown with dB_z/dt positive due to rotation of the geomagnetic field. The torque following from switch-on of the magnetorquer is such that it drives the the angular momentum vector $\underline{L}$ towards the orbit normal. Clearly if the spin were reversed then the Z axis would be driven the other way.

This equilibrium attitude is of course the only one in which NO change in dB_z/dt can be observed. The above argument shows how it gets there.

8.3.5 Teule 1979

The IRAS satellite achieved fame for its scientific results. It is of interest to study its control manoeuvres. This is only done partially here - in considering its initial 'detumble mode'. Magnetorquers were switched on in order to reduce the angular momentum to within the capacity of a second system of reaction wheels which would then deal with the fine control and give 3-axis stability. Magnetorquing control was the necessary choice (rather than the use of reaction jets) since all potential contamination of the payload had to be avoided. Some critical details are missing from this brief account, but as far as one can tell the proposal then - it may been modified subsequently - was the standard one, of using three magnetorquer coils , presumably aligned on each axis of the spacecraft, and then switch on whichever torquer has an axis in which the field component varies fastest. ie. measure dB_x , dB_y , dB_z and depending which has the largest magnitude then switch on M_x , M_y or M_z (in our notation) with the appropriate opposite sign.

The general requirement was to get the absolute spin rate (about some arbitrary axis) to less than 1 degree/sec; for spin rate less than this the reaction wheels could take over. As the author points out, it was highly desirable - if not essential in this early tumbling stage - to try and infer the state of the spin from the magnetometer measurements alone.

With magnetometer measurement alone there are two potential problems: only a vector component of the angular velocity can be measured ; and the field is rotating anyway. This short paper is somewhat cryptic, but it is possible to analyse the author's claims. First if one disregards the angular motion of the B field then given a measurement $\underline{B}$ this can be normalised by dividing by its magnitude i.e

$$\underline{b} = \underline{B}/B$$

and then the rate of change of this measured in a suitable time interval in order to derive $d\underline{b}/dt$.

The approach is to ignore the nature of the satellite motion and just assume some angular velocity $\underline{\omega}$ exists at some instant. He argues that this can only a component of this can be measured from the magnetic measurements where that component $\underline{\omega}_p$ is orthogonal to $\underline{B}$ at any particular instant. I incorporate this point along with others in Appendix G. In my notation

$$\underline{\omega}_p = [d\underline{b}/dt] \wedge \underline{b}$$

where $\underline{\omega}_p$ is the component of $\underline{\omega}$ perpendicular to $\underline{b}$.

As the author then implies the magnitude ω_p will therefore be less than the magnitude of the true angular velocity except when $\underline{\omega}$ is actually at right angles to $\underline{b}$. However for an inertially spinning (or nutating) spacecraft on a polar orbit the geomagnetic field describes two rotations per orbit and the coincidence will occur four times an orbit. The requirement then is to run an algorithm which checks that the component of angular velocity $\underline{\omega}_p$ is not only less than a pre-determined threshold, but also that it is maintained for a minimum time. It would seem that a minimum time of orbital period of (say) 100 minutes divided by 4 plus some margin would be enough. The author fixes on figure of 30 minutes as being the time in which the magnitude ω_p must be less than the preset threshold of 1 degree/sec.

A second aspect is that of rotation of the $\underline{B}$ field causing an error in the estimation of the angular velocity of the spacecraft. As discussed in section 5.3 this is a matter of current investigation during libration control. In the context here, and following the simplifying IDPO model (IRAS was also on a sun-synchronous orbit) two limiting states can be identified: the spin axis lying in the plane and spin axis orthogonal to the plane. In the former case the rotation of the $\underline{B}$ field induces no errors in measuring $\underline{\omega}$, but the true $\underline{\omega}$ is seen only four times an orbit. In the latter case the geomagnetic field stays at right angles to the spin (in practice of course it will be close) so that the magnitude ω_p is always the same (very nearly) as the true spin rate ω . But we then have to take account of the actual rotation of the magnetic field. This rotation rate is maximally in the order of $2 \omega_0$ in the Earth-referenced system and therefore $3 \omega_0$ in an inertially referenced system (IDPO model). The maximum error then is about 0.2 degrees per sec and is plus/minus error to be added to the measurement of ω . While this is small compared to the threshold of 1 degree/second the problem can be got round anyway by additionally setting a second, lower threshold in the order of this rate below which no torquer will be switched on. The author refers in his algorithm to 'unloading interrupted' when the magnitude of the apparent angular change is less than 0.4 degrees for 'improvement of the unloading efficiency'. He must also have realised that setting a lower threshold like this guarantees that when the magnetorquers are invoked the apparent rate of change must be in part due to true motion - which the magnetorquer will tend to damp out. This principle is used of course in the UDSAT de-libration algorithm (section 5.3).

8.3.6 Takezawa et al 1980

The paper concerns the simple magnetic stabilisation of satellites, in just aligning one of the axes permanently on the geomagnetic vector.

Adopting the Z axis as the one requiring to be controlled then there is assumed to be one permanent magnet pointing in this direction. There are the usual three orthogonal coils each driven independently by power amplifiers and electronics which process the component magnetic fields. The control law assumed is not quite the usual

$$\{ M_1 \} = - \{ dB_1 / dt \}$$

but is not far off. They claim that the speed of response of the system is such that variations of magnitude in the field does not degrade operations - which was definitely not our experience with UOSAT-2. Having stated their ideal requirement (or possibly having a linear region of operation) they then admitted to the difficulty that without an on-board computer, they could not reliably implement or detect the rates of change using low-cost analogue electronics. Instead they implemented in an electronic circuit the classic hysteresis loop. The text is somewhat obscure but apparently as the attitude damps down progressively smaller hysteresis loops are switched in. With this approach on two of their satellites 1977-12A and 1978-14A they managed to reduce residual oscillations to about ten degrees.

8.3.7 Summary and comparison

The history of magnetorquing as revealed in this selection of papers shows that it is frequently used for momentum dumping - a manoeuvre which is outside the scope of this thesis. It has been widely used in the control of the spin state - both magnitude and direction - for either simply spinning satellites or those with a single momentum wheel aligned on the orbit normal. Despite studies on their use at geosynchronous altitudes the sensible restriction would seem to be to LEO (Low Earth Orbit satellites). There is only one reference to its being used in conjunction with gravity-gradient control.

For attitude change in polar orbits a common theme is to assume a very simple model of the field and the orbit (IDPO model). The intimate direct connection between the measurement from a magnetometer and the control by magnetorquing has not been greatly exploited. Nobody has used a magnetorquer under active control for libration suppression under gravity gradient on a Low Earth Orbit under computer control.

Fig.8.3.4-1 Spin axis de-spin forces attitude to orbit normal

8.4 Gravity Gradient Developments

A brief review of principles of 3-axis control is given and then a summary of interesting case histories

Chapter 3 developed in considerable detail the main effects of gravity gradient in the various stages of UOSAT manoeuvres. The analysis was based on a simple model of a rigid body with equal moments of inertia I_x and I_y , and a mass deployed on the end of a boom along the Z axis. An exact theory and solutions were given for attitude change in the case of this axial symmetry in chapter 3 and appendices B through D. Useful linearised equations were presented. Because of the I_x and I_y symmetry the spacecraft can at best achieve 2-axis stabilisation from gravity gradient alone (but see section 7.5). Section 7.6 with Appendix K develops semi-linearised equations useful for the general 3-axis stabilised case, and leads to particular conclusions about past difficulties and why additional or complementary magnetorquing should cure the problem.

8.4.1 Simplified analysis of 3-axis control

A brief, less technical presentation is given here as a preliminary to reviewing past work. It may be read independently of section 7.6. The axis and attitude convention introduced in chapter 3 is maintained here.

A rigid body on a circular orbit, with unequal moments of inertia around its principal axes X Y and Z, experiences constraining torques maintaining its alignment in the gravity field. Not only is there the direct gravity gradient torque tending to maintain pitch and roll but there is an indirect effect tending to constrain it in yaw. There are in principle 24 different configurations which a rigid body could adopt (any one face pointing downward and 4 rotations each) in which the principal axes align with the orbit defined co-ordinates X_0 , Y_0 and Z_0 ; and stabilisation might follow. In theory and practice these reduce to just four.

To see this one can imagine a satellite which is 'squashed' into a flat plate or is modelled by two orthogonal dumb-bells defining its X and Z axes (fig.8.4.1-1)

The relation between moments of inertia of this body is simply

$$I_x = I_y + I_z \quad \dots 8.4.1$$

The advanced texts would agree that this structure tends to adopt one out of four stable, if undamped, attitude with body axes X, Y and Z aligning on the orbit-defined X_0 , Y_0 and Z_0 axes (or their inverses). Intuitively one would expect pitch stability only if the vertical (Z axis dumb-bells) dominated over the horizontal. Pitch stability is ensured then by

$$I_y > I_z. \quad \dots 8.4.2$$

From Eqs.8.4.1 and Eq.8.4.2 then

$$I_x > I_y > I_z \quad \dots 8.4.3$$

Whether the horizontal dumb-bell was present or not one would expect roll stability. Why yaw stability should exist is best understood as a 'major axis spin' or weak effect of spin stabilisation unconnected with gravity-gradient torque. The entire structure is rotating once every orbit in inertial space and this ' spin axis ' is the one with the maximum moment of inertia - as required by standard dynamics theory.

The linearised equations for this 'PLANARSAT' are particularly simple, predicting un-coupled undamped oscillations around all three axes, with rates shown in fig.8.4.1-1 (but these equations are deceptive - see section 7.6). Judging from the oscillation rates all the restoring torques are weak, with the weakest being the yaw oscillation.

The yaw 'stiffness' is won at the cost of weakening of pitch control. From a given Z axis dumb-bell configuration the addition of the Y axis dumb-bell creates a finite I_z (but not I_y) and so imposes the margin

of I_x over I_y (by Eq.8.4.1). The equations confirm however that the deployment of this dumb-bell weakens pitch control, because its torque contribution as a consequence of a pitch error is in the wrong sense.

In a model generalised by a third dumb-bell pair aligned on the X axis has both I_y and I_z (but not I_x) increased by its presence, so forcing the inequality

$$I_x < I_y + I_z$$

...8.4.4

and, according to the linearised equations, forces a coupled mixture of two different frequencies in roll and yaw. As I see it this weakens yaw stiffness (without restoring pitch), and if one visualises this equivalent X axis dumb-bell expanding in size there will come a point when axial symmetry (i.e UDSAT case) is regained with equal moments of inertia I_x and I_y ; at which point the potential 3-axis stabilisation is lost. So it is necessary generally for $I_x > I_y$, as implied already in the special case of Eq.8.4.1. Combined with the requirement for pitch stability this means that Eq.8.4.3 is generally true. It follows then that there are just 4 stable orientations of a general body (NOTE).

Even at this elementary level of appreciation it is clear that not only are the restoring torques are weak but some form of damping is required to bring about an 'engineering' stability. The niceties of the stabilisation process have long been recognised to be far more complicated than this, even within the assumptions of a rigid body analysis. I summarise, drawing mainly from Hughes (1986), the chief problems in sec.7.6.2. I take the argument a stage further there - there is no hint of this in Hughes - and identify what I believe is a fundamental problem of (intended) 3-axis stabilisation compared to (intended) 2-axis stabilisation. Basically, that 'damping' i.e rate control is not really good enough and needs re-inforcing by additional 'position' control, particularly about the yaw axis (which magnetorquing can simply provide).

NOTE This is the Lagrange region. The other apparent region of stability - the DeBra-Delp - does not work according to Hughes

Another way of overcoming this problem is to enhance the 'spin stability' effect by installation of a momentum wheel spinning on the pitch axis. The mathematics of this is highly complicated - with a mix of various resonant frequencies some of which can be very slow - and needing to be damped out (see Lerner et.al 1975). But in simple terms it must work by roll/yaw coupling. If there is a point in the orbit when the attitude error is all YAW then one quarter of an orbit later this yaw has converted - inertially - to a roll error which creates a restoring gravity gradient torque.

The two basic difficulties with gravity-gradient stabilisation acting alone are (i) it is a weak effect and (ii) it is inherently undamped. According to Hughes, attempts to maximise the effect by deploying masses on the end of increasing length booms are bound to run into problems of flexible-body dynamics which deteriorate performance; and the risk of being exposed to significant aerodynamic torques and even solar pressure. Ingenious methods of damping have been proposed, but some - based on magnetic anchoring for example - apparently introduced bias in proportion to the degree of damping they introduced. Even if the damping mechanism does not introduce bias the ultimate pointing direction in 'pure' passive gravity gradient is going to be the result of a competition between this wanted torque and any unwanted perturbing torques, putting the matter at its simplest.

8.4.2 El-Segundo Conference 1969

As a way of getting some insight into the real problems I briefly summarise the more interesting and relevant papers presented at this conference under the auspices of the Airforce System Command and the Aerospace Corporation (I was not able to find any later conference).

In a study of the control of very large vehicles (Nurre and Weygandt 1969) considered the SaturnI workshop at 400 km, based on the Saturn Launch Vehicle (22500 Kg) and the Lunar module positioned at geosynchronous altitude.

For the Saturn Workshop two configurations were possible: either main long axis of the satellite aligns on the radius vector (but then increased air drag) or it aligns on the velocity vector direction (but then needing to deploy booms). In both cases they invoked passive magnetic damping to assist the g.g control. For the long axis along the local vertical the spacestation has the right geometry. A damper can be adjusted for the rate damping coefficient and the magnetic moment but it induces error. They claimed to get pitch $\pm 0.7^\circ$, roll $\pm 3^\circ$, yaw $\pm 13^\circ$ with 9° bias in simulation. For the long axis along the orbital velocity they proposed to deploy two rods each with 225 pound tip mass at the end of 300 feet. Digital simulation expected $\pm 6.5^\circ$ $\pm 5^\circ$ and $\pm 19.0^\circ$ errors. Damper-induced error is the problem. Reference given to the effect of air-drag (Nurre 1968).

A geosynchronous package was based on the ascent stage of the lunar module. Expected a pointing accuracy to within $\pm 10^\circ$. With weak magnetic field at this high altitude they cannot use a magnetic damper; so the concept of the secondary mass and the dissipating spring is used connecting a damper-boom. There are four fixed booms and one damper-boom. They hoped to get $\pm 6^\circ$ in pitch, $\pm 0.5^\circ$ in roll, and $\pm 2.8^\circ$ in yaw after careful optimisation. Hardware was the tape on the drum unwinding onto a tube, which presumably is the same technology as we had to use on UOSATs.

It was pointed out that the requirement of communication satellites was going to be beyond gravity gradient control capability (Owens and Padraczky 1969)

Several interesting papers pointed out the problem with the tape-into-tube type boom which perforce had to be used on UOSATs. H.Foulke discussed experience with the GGSE-V and the OGO-IV spacecraft. For the former they used 3-axis control based on a swept back Y configuration with masses on end, one of them the magnetically anchored rate damper. The pitch roll yaw moments of inertia were 1600, 1380 and 470 respectively. Attitude information was provided by a digital sun sensor and magnetometer; there was the difficulty of attitude information only

in sunlight; and crude yaw momentum wheel control was employed. They achieved initially a good stability (3° in pitch 2° in roll and 10° in yaw) but then everything gradually fell apart. Yaw built up could not be cured. The satellite accumulated a high spin rate and the motion was now largely random.

On the OGO-IV satellite they experienced thermal flutter frequency with 35 second period corresponding to the fundamental natural frequency of the 40 foot deployable rod. Apparently well studied by Frisch and Hoffman of Goddard the boom would seem to be exactly the same as UOSAT type, only being silver plated to reduce thermal bending. The key fact is that the bending stiffness is OK but the torsional stiffness is 3 to 4 orders of magnitude less than a closed tube. Under sunlight the boom attempts to bend, then starts to twist, changes its profile, bends again, and so on. Oscillations force the rod into a coning type of motion rather than a 'back and forth'. It only happened in sunlight. They note that the rod is bi-stable and tends to flop. Their conclusion is that this overlay type of rod is a significant disturbance to the spacecraft. The authors note that not all gravity-gradient controlled spacecraft have behaved badly.

There was a follow-up paper (Kovall et al 1969) which analysed the effects seen on the OGO-IV spacecraft - a model of thermally excited boom oscillation which was 60 foot long and very similar to our boom; and a similar study (Connell 1969) of the OV1-10 spacecraft which apparently has 6 booms pointing along all principal axes (the horizontal ones are hinged, damping booms). In flight they experienced large yaw excursions as much as 180° and pitch and roll have exhibited little damping, sometimes being as much as 35° out. His simulation with boom flutter predicted nasty consequences. He recommended stiffer booms. Also analyses the effect of residual magnetic dipole in the spacecraft. Resonances are possible inducing unwanted motion. Yet another paper (Donohue and Frisch 1969) simulated boom flutter of a typical boom. OGO- IV simulation again predicts a typical 20° boom flutter amplitude connected to the OGO-IV satellite.

They showed that stability requires either zippering, perforating, but in any case some mechanism for stopping the twist in the boom.

An interesting paper (Longman 1969) analysed the rigid body with embedded gryostats aligned along principal axes. He pointed out that as these increased in speed the number of stable options declined to 4 with only the spin along the orbit normal allowing stability. He suggested that this analysis may be relevant to the phenomenon of previously stable gravity-gradient controlled satellites starting suddenly to tumble. He considered the boom problem in this context and suggested that if the boom produced an internal angular momentum which is equivalent to a rotor aligned with the axis of greatest or least moment of inertia and rotating at the critical rotor speed then the satellite would tumble. Cites some references (Longman and Roberson 1968) and (Roberson and Hooker 1967).

In the technology of damping systems there is the magnetically anchored eddy current damper and the the long thin rods of high magnetic permeability surrounded by copper sheath. This was analysed by Pettus (1969). He gave references to earliest work (Mobley and Fischell 1965), which I have mentioned also in section.8.2.1 under magnetic control.

The series of satellites controlled by the Naval Research Laboratory (Raymond et.al 1969) were perhaps the nearest to our current UQSATs. An early set of five satellites were '2-axis stabilised' using a single rod dumb-bell; four had eddy current dampers, one had viscous fluid. All dampers were magnetically anchored. The primitive attitude measurement used photo-tubes. Peak tilts of about 15° were suspected. The fifth satellite had inverted five times since its launch in spring 1967. The configuration rod and damper were essentially the same as two others on a common launch which had behaved correctly.

They then orbited three intended 3-axis stable satellites: GGSE III (viscous fluid), GGSE IV (eddy current torsion wire) and GGSE V (eddy current). The attitude was determined from sun and magnetometer measurements. GGSE III stabilised with $\pm 40^\circ$ and a bias 80° in yaw,

pitch and roll were $\pm 10^\circ$ (20° roll bias). It used three rods in a Y type configuration. GGSE IV's pitch and roll decayed to within 5° of nominal.

Yaw could be within 10° of nominal - then get strange behaviour when the yaw suddenly built up. Its behaviour was not regular. Again used a 3 rod configuration. GGSE V used the same configuration as GGSE III. Everything appeared to be fine with pitch roll and yaw stable within 3° to 5° (for nearly four weeks). Then the control fell apart. They could only suggest that the reason was solar driven thermal flutter.

Probably the most successful, detailed and spectacular satellite was the Radio Astronomy Experiment in which an X-shape antenna system was deployed which in part provided the gravity-gradient control (Blanchard et.al 1969). There was also a damper boom. The paper depicts the very detailed mathematics and a very complicated deployment sequence. Used concept of 'deadbeat deployment'. The antennas were first deployed to 450 feet then 600 feet and then 750 feet. It used magnetometers and sun sensors. UOSat-2 used the same trick of starting favourable deployment near North magnetic pole. Achieved a few degrees in roll and pitch and $\pm 10^\circ$ in yaw (with a bias).

An account was given of the early GEOSII satellite (Whisnant et.al 1969). Built by APL, it used an eddy current damper. They hoped for steady state libration of 5° . The boom bending phenomenon is again analysed. This one behaved itself.

One of the last papers cites perhaps the first proponent of gravity -gradient control (Roberson 1953)

Fig.8.4.1-1 Ideal geometry for gravity gradient 3-axis stability ?

'Planar' satellite $I_x = I_y + I_z$

$$\text{Pitch resonance} = 3 \omega_\theta \sqrt{\frac{I_y - I_z}{I_x}}$$

$$\text{Roll resonance} = 2 \omega_\theta$$

$$\text{Yaw resonance} = \omega_\theta$$

(uncoupled at small amplitude)

8.4.3 Anand and Whisnant 1973

A useful review of more satellites appeared four years later. All these spacecraft were developed at APL. Listing by satellite:

OSCAR -14 was an axially symmetric spacecraft and had therefore no control in yaw; with a boom mass of 2.8 pounds deployed on the end of a 100 foot boom. Damping was by hysteresis rods, using permeable magnetic material which magnetises/demagnetises in the variable field and dissipate energy. Their simulated results supported by flight data show quite large residual pitch (nearly 30°) and a small residual roll (5°)

GEOS II, the precursor of GEOS III, carried no momentum wheel, and relied on an end-mass which was also an eddy current damper. There was no yaw control again and they had thermal problems from an inadequate residual spin. In our terminology the inclination angle never exceeded 7° and was within 5° for 96% of the time.

DODGE was a satellite specifically designed to test gravity-gradient control at geo-synchronous altitudes on a geostationary orbit. It carried a wide range of hardware and could be used in a number of configurations. Four types of damping system were available. The satellite did not work in all the possible modes and reliable control was not always achieved. One learns that off-vertical angles were generally bounded by $\pm 10^\circ$ over complete passes. Nearly all experiments with the various boom configurations induced instability and the various dampers were ineffective. No doubt the reason for this was the much weaker magnetic field at this altitude (although one damper used the enhancing principle described by Mobley). This satellite also explored the only case I know of active damping by magnetorquing according to a committed 'sample and hold' electronic system - details of which I do not have (from Hughes this last information). The principle must be entirely different since nominally the magnetic field is always pointing in the same direction (except of course for problems with magnetic storms).

X-RAY SATELLITE was a spin stabilised low-altitude (500 km) satellite which used the spinning wheel in a lossy suspension system for damping out nutation. The account is too condensed for clarity and the achieved control of the nutation angle is not given.

LIDOS was intended to be launched into a moderately elliptic orbit and gravity gradient controlled. Computer studies found that a 60 foot boom length and a 5 pound end mass were optimum. Unfortunately this satellite never made it. The theoretical study was of interest because it showed that the high eccentricity would have severely limited the attitude windows available. But they hoped to get round the problem by combining this with the magnetorquing operation.

The theoretical difficulty with gravity-gradient control on eccentric orbits is apparently a well-known fact going back to the mid 60's (references given). In one paper cited (Lohfeld et al 1974) it is stated that that passive gravity-gradient capture is impossible when the orbital eccentricity exceeds 0.355. A possible remedy is a flywheel aligned on the orbit normal whose speed must be varied - making a semi-passive system.

8.4.4 Lerner (1975)

This paper describes the advanced spacecraft GEOS III. This interesting spacecraft used gravity gradient and the gyroscopic stability conferred by a spinning wheel to give passive three-axis stability. The passive damping technology depended on a magnetised ball floating in a viscous medium inside a sphere of 'pyrolytic graphite for diamagnetic centring of forced on the inner sphere and aluminium for energy dissipation through eddy currents' (from Wertz who summarises this on p632).

This damper was not supposed to be effective until deployed a metre or so from the main body on the end of the boom.

The attitude acquisition is described in detail. The spacecraft had been originally designed with a view to employing a Z-axis magnetorquer. The strategy then was almost exactly as with UOSAT-2 - acquire magnetic stability (but passively damped unlike UOSAT-2), and then deploy the boom over a polar region - provided the pointing error was not excessive. Then fire up the wheel carefully, and in such a way as to damp the pitch libration that inevitably builds up. In order to determine where the satellite was pointing they used three 2-axis sun sensors and a 3-axis magnetometer.

Acting apparently on the belief that this plan would all take too long they identified a fast but higher-risk scheme, which had to be rapidly revised as events unfolded. Now the intention was to abandon the idea of first stabilising the attitude magnetically. Instead they proposed to just wait for a suitable attitude opportunity when the pitch and roll were seen to be small and then then deploy the boom at once, but only a limited amount enough to ensure capture and simple 1-axis stability. Assuming gravity gradient capture had happened, and assuming that the libration was starting to damp down sufficiently the next intention was to deploy the boom more and then fire up the wheel and start to impose yaw stability.

Immediately after launch they discovered that the arbitrary tumbling motion was damping far quicker than they expected even though the boom was not extended. This meant that it was heading for a possible inverted capture with 50% probability on gravity gradient. Possibly alarmed by this (it was only 5 hours after launch) they pushed the boom out - intending for about 2 metres but getting in fact about 3/4 metres. Subsequent motion was distressing, showing very large roll and yaw variations (roll near the limits and yaw as much as 100°). This was because they missed a command window when they should have deployed at zero pitch (because of 'command difficulties') and decided to go at a pitch angle of -36° and a roll angle of 50° - which might be thought to be a risky procedure.

Despite the yaw oscillations being way beyond their previous estimate of a safe limit they deployed the boom further and spun up the wheel, and achieved capture in all components, with the required pointing accuracy in the order of a half a degree in pitch and roll and one degree in yaw.

8.4.5 Hughes 1986

Hughes reviews much of the above ground and in greater detail. His pessimistic conclusion is that gravity-gradient control is not really a feasible engineering option, as far as 3-axis stabilisation is concerned (unless re-inforced by a spinning pitch axis momentum wheel). It is intimidating to read, on the question of attempted 3-axis stabilisation by gravity gradient, that 'the best people in the business tried many configurations and many types of damping, yet their success was quite limited ' (p.345). On the other hand he does wonder ' why there have not been more attitude stabilisation experiments in the past 25 years ' (p 336). However, it is clear now that the rules of the game have changed. Hughes had not considered the interactions between gravity gradient and other control, particularly magnetorquing reinforced by attitude determination. The role of signal-processing does not enter what is essentially a classical study in dynamics.

The analysis in the book is presented at a very high theoretical level. I have drawn considerably from it in my proposals for 'complementary' magnetorquing in section 7.6

8.4.6 Siahpush and Sexton 1987

This study may be a typical product of the recent renewed interest in putting up small satellites - in this case for store-and-forward communications. The study by these authors decides on the UOSAT-type configuration with axial symmetry, the only essential difference being that they hope to determine attitude by Sun and Earth measurement. It is not clear from their account whether they propose also to use a

magnetometer or not. Indeed the study is vague on most important matters of practical control.

For example, on the need for initial near-vertical attitude needing to be achieved before deployment of the boom, they realise that active magnetorquing control will be needed to force the orientation. But there is no indication that they have realised how to achieve it.

The authors would seem to mis-interpret the basic theory of libration, once gravity-gradient stabilisation has been achieved, starting with the crucial point that they do not seem really to understand the basic angular motion of UOSAT2-type spacecraft (i.e having axial symmetry).

Their derivations show that they may have been deceived by adopting the standard linearised equations of the libration state (see section 7.6), which only applies to the general body with unequal moments of inertia, and experiencing small roll, pitch and yaw deviations from a nominal 3-axis stable state.

The seemingly general analysis is not useful in the important limiting case of axial symmetry when $I_x = I_y$ (NOTE) . In this case - which describes UOSAT and also their proposed satellite - it is not easily spotted that there is a constant Z-spin rate (obvious though it may seem on physical grounds). My analysis in section 3.7 was intended to clarify that point.

They state that the 'dumb-bell' satellite provides 'two-axis stability (in roll and pitch)'; which is correct. They then run a numerical integration on the linearised equations for the UOSAT-type configur-

NOTE I have suggested in section 7.6 that the linearised set of equations are not so useful even in the case $I_x \neq I_y$, suggesting that more insight is gained from semi-linearised equations which allow an arbitrary yaw state.

-ation. As a result of which simulation they state that the 'yaw axis is unstable' and will 'precess about the local vertical'. That is very unclear. What they must mean is that the angular rotation of the rigid satellite body is not constrained about the yaw axis.

As their figure 7 shows quite clearly, the actual yaw angle can decrement continually. However this is simply due to the constant rate of spinning of the satellite about its yaw axis, which in their simulation is a negative Z-spin. But this state is not 'unstable' nor is it a 'precession' in the common meaning of these terms (NOTE1). Their plot, from an angle of +0.2 radians to -1.0 radians (NOTE2), shows that the yaw angle does not decrease uniformly, but exhibits an oscillation around a general decrement. My analysis and my Eq.3.7.2.4 shows that such an effect is simply a geometrical projection of the roll oscillation on the constant Z spin.

Since they propose in fact to build a UQSAT-type satellite with near-equal moments of inertia in I_x and I_y their analysis is singularly useless.

Their study at least may have brought into the open an underlying confusion generated by the terminology '2-axis stability' as compared with what I believe is a more logical terminology: '1-axis stability'. In the case of an axially symmetric satellite under the control of gravity gradient just one axis is nominally stabilised (NOTE1 again).

NOTE1 The yaw axis is itself in principle stable - the vector identified with this axis is constrained relative to the local vertical with its direction at any instant *defined* by the roll and pitch angles.

NOTE2 It is of course quite illegal in general to run an integration of what should be linearised equations over this range of yaw angle. A more detailed analysis (from section 3.7) shows how they have 'got away with it'.

8.4.7 Martel, Pal and Psiaki 1988

A joint study by the ITHACO company and Cornell University shows a pursuit of some of the same goals which have been investigated in this thesis.

They have yet to fly any satellite. They have produced two studies quite independently of myself; in the same critical area of magnetorquing combined with gravity gradient control, assuming that the attitude is fully determined; and of deriving this attitude determination in all 3 axes using only a magnetometer and a Kalman filter. These are both matters I have considered in detail in chapter 7. It is of interest to compare and contrast their approach.

On attitude determination (1st paper) the authors model two 'spacecraft', one of which is effectively the UOSAT-2 case.

They have followed the orthodox (i.e extremely complicated) matrix formulations of an 'extended' Kalman filter. The results can be compared qualitatively with mine. They test orbit inclinations of 43° and 57° (rather than 99°) at between 1.1 and 1.2 earth radii (i.e between altitudes 640 to 1280 km). They use a simple dipole geomagnetic field and ignore the rotation of the earth in orbital progression.

Personally I would consider that a major omission; since in the course of the empirical development, I naturally started with this simplest model; and found that my earlier ideas simply would not work when dealing with considerable variations encountered as the orbit plane 'scans' across the real-world IGRF model.

They consider two models; one with a spinning momentum wheel. They also consider a UOSAT-2 type with axial symmetry. This model they leave in a state which is 'neutrally stable in yaw' - i.e not spinning. In my view that is not good enough - it is essential to demonstrate tracking capability with the spacecraft performing wide yaw excursions - i.e slowly spinning.

They admit that their system simply does not work from an arbitrary initial start up (all angles initially must be known within $\pm 60^\circ$). Sometimes, under circumstance which I do not understand, their estimator does not converge at all. In contrast my estimator must start with small initial pitch and roll angles; but will converge from any initial yaw state. On speed: their figures quote a time of 3 minutes to calculate an orbit's worth of 50 samples. Whereas my calculation (see sections 7.1 through 7.4) always converges and takes just 8 seconds per orbit of 80 samples, not including the time taken to generate the IGRF field. That is 36 times faster, in computer processing time, per data point (NOTE).

On *attitude control* (2nd paper) the authors have plumped for what might be called an 'obvious' approach, in combining magnetorquing with passive gravity-stabilisation. They construct an error vector, which is actually a desired torque $\underline{N}$, from a *position plus rate* vector, derived from all three axes.

Their algorithm then calculates a *generally pointing* magnetic moment $\underline{M}$ from this required $\underline{N}$, following the well-known Cross-Product law - the apparently 'general' way of directing a magnetic moment to do most good. I have critically reviewed this ancient method in sec.5.4.1 where I point out that it does not generally do what is wanted.

NOTE Both run on fast personal computers. They used apparently an 8 MHZ 8088/8087 μ -processor where the '8087' represents, I understand, dedicated floating-point hardware. I used a RISC-based Acorn 'Archi-medes' computer, which is a 32 bit machine and runs at 8 MHZ. It has no other special arithmetic processor. Presumably they use a compiled language (no information); while mine runs in interpretative BASIC. In the absence of detail on just what they compute it is not possible to make a more scientific comparison. Even if my machine runs 10 times faster my algorithm is much faster (nearly 4 times)

The simple reason is that the actual torque does not point, in general, in the required direction. Further, in the study here, because they try to control all motions at once at every point on the orbit, they end up, by their analysis, with a characteristic polynomial of sixth order, in which the magnetic control now couples every motion together (which is not the case in the natural libration state).

One consequence of this strategy, on their own admission, is a general instability if the loop gains are too high. While their method would seem to work, and they do a useful analysis of the effect of various biases and perturbations, their model simulation takes *hours* to converge on the yaw angle, because the loop gains have to be backed off, to a suitably low value (NOTE)

As a minor matter, they fail to demonstrate acquisition from an arbitrary prior yaw-spinning state. And philosophically, they do not emphasise the deep inter-dependence that a complete stabilisation requires between magnetorquing and the 'background' gravitational gradient. If the latter effect did not exist the overall convergence on three axes could not work.

NOTE Whereas my quasi-orthogonal approach, which effectively separates the system into independent control systems, achieves the result *in a few minutes* (in predicted real time for the satellite - see section 7.5).

9 SUMMARY AND CONCLUSIONS

In this final chapter I sum up, draw general conclusions, and try to predict the future. As one main achievement I refer to the unique method of 2-axis stabilisation, combining passive gravity-gradient control with active magnetorquing. The stabilisation can be combined with other functions (such as controlling the spin rate). I have developed a general principle of improved 2 -axis stabilisation and 3-axis stabilisation based on magnetorquing control and actual attitude determination. In my theoretical efforts I believe I have identified a basic problem in past attempts to achieve 3-axis stabilisation on asymmetric satellites by gravity gradient alone, which additional magnetorquing should overcome and tested in simulation, algorithms which add magnetorquing in such a way as to overcome this problem. The results look sufficiently promising as to identify a new way of low-cost control of small satellites on high-inclination Low Earth Orbits. Future work will test this claim.

9.1 Summary

I summarise under appropriate headings the various aspects of my work which seem to me to be significant and/or original. Cross references back to the main chapters and Appendices are given.

9.1.1 Overall mission control

The work on the two satellites has, I believe, identified and justified the simplest and most practical way of putting a high-inclination LEO satellite into the 2-axis (or 3-axis) stabilised state; with the minimum of software and ground support. We now know what, in my view, is the best method of putting a spacecraft of our type, from an arbitrary tumbling state into a reasonable attitude, during which a boom is deployed and gravity-gradient capture is automatic. Substantial flight experience with both satellites established and identified the practical controls required, particularly in regard to ground station design and data collection (section 6.2 and 6.3). The thesis gives

some indication of this knowledge - which may not be known to the various groups currently planning small satellites.

9.1.2 Theoretical studies on gravity gradient

The theory of transition to gravity-gradient stabilisation was extensively studied; and librational limits established by a novel vectorial analysis (section 3.5 and Appendix C). These formulations are believed to be new.

Some simple mathematics (section 3.7) identifies a roll/pitch coupling in an adaption of the linearised dynamic equations for the symmetric satellite. The relevant terms do not appear in the standard equations and the existence of this coupling effect may not be well-known. It comes about because of a maintained Z-spin (spin about the nominal vertical axis). The result is important for the de-libration algorithm, because the coupling persists down to infinitesimal libration amplitudes.

Experience with UOSAT-1 made me aware of the importance of the gravity-gradient effect even if the boom is not deployed and there is no capture or stabilisation. Some new theory was developed (section 3.4 and Appendix B) to account for observed perturbations on a spinning and nutating spacecraft.

I have re-considered the role of 'passive' 3-axis control and stabilisation which an asymmetric satellite should exhibit. From this I believe I have made original conclusions (summarised sec.9.1.6).

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9.1.3 Magnetorquing manoeuvres

The attitude of the satellites had to be stabilised from an arbitrary initial tumbling state. I exploited nearly all the possible ways of manipulating the attitude. Flight experience with UOSAT-1 showed what

can be done with a spin-plane magnetorquer only, in controlling the state of a nominally spin-stabilised spacecraft, before making the transition to gravity-gradient capture. While UOSAT-2 demonstrated how to achieve a magnetically stabilised state, using a spin-axis magnetorquer, before making the transition. Some of the particular methods described here are believed to be new (see chapter 5).

I would emphasise the relative operational and computational simplicity of the magnetorquing algorithms - involving in most cases no more than a vector measurement of the local magnetic field and the testing of various inequalities within the algorithms. In most cases the attitude does not need to be determined for the algorithms to function. I believe these techniques are useful outside the UOSAT context - for example in emergency 'recovery' modes for LEO spacecrafts which are nominally controlled by other means.

9.1.4 Practical 2- axis stabilisation - threshold comparison algorithm

The method of 2-axis stabilisation under gravity-gradient control, initiated by deploying a tip mass away on a boom, has proved successful in the past. We have duplicated this work and results in the order of accuracy ($\sim 10^\circ$). Unlike the past use of various kinds of passive dampers, we continue using the magnetorquers, in order to control libration. A unique damping or *de-libration* algorithm has been tested and shown to work. The technique requires the excitation of the spin-axis magnetorquer (nominal alignment up/down). The attitude is not known during this continued manoeuvre; and the control is a form of 'damping' inferred indirectly from the magnetometer measurements. I believe that the algorithm which achieves this result (see section 5.3), can be greatly improved, in both accuracy and consistency. The improved version has a more sophisticated measure of the libration rate, a variable fire time in the magnetorquing; and other features (see from sec. 5.3.3).

The 'de-libration' is achieved by a direct reduction of the pitch libration, by magnetorquing. There is then an indirect suppression of the roll libration, which is made possible by the roll/pitch coupling.

9.1.5 Stabilisation plus spin control

One advantage of magnetorquing control is that other functions can be combined with the basic de-libration algorithm.

Our current software runs two algorithms simultaneously, providing spin rate control along with the de-libration control. The maintenance of a spin rate is important in minimising thermal gradients, as well as maintaining the coupling mechanism (the spacecraft is routinely held in a spin period of about 5 minutes).

The spin control is applied by excitation of a spin-plane magnetorquer. Both activities take place at a very low duty cycle, generally in the vicinity of an equatorial crossing of an orbit. I believe this approach has not been tried by anybody before.

9.1.6 Problem of 3-axis stabilisation

In the beginning there was much American experimentation on 3-axis stabilisation, still exploiting the gravity-gradient effect. I have studied the situation further, in the low-cost UOSAT context, seeking low-cost solutions as before. Although it seemed to be a 'free' and straightforward method, with the necessary addition of passive rate dampers, problems were almost always encountered in the past (see discussion in section 7.6 and 8.4). Often it was not just a question of low pointing accuracy but difficulty in maintaining any kind of stability.

I argue, from some 'quasi-linearised equations ' (see Appendix K) developed by myself, that the asymmetric body is prone to instability,

in a practical engineering sense, if and when the yaw angle happens to drift from nominal pointing direction. If some disturbance torque should push yaw oscillations out by more than a few degrees then increasingly any rate-damping mechanism will be ineffective in restoring the yaw state. In other words, although the whole body may be 'stable' in an academic sense, in the special case when the attitude is close already to its desired state, such 'stability' is unattainable if the yaw attitude should depart by a few tens of degrees from one of the two nominal stable directions.

My analysis shows that the problem is a *direct consequence of promoting an asymmetric structure*. In other words the more one designs the satellite to give it inherent yaw stiffness (broadly by enhancing the asymmetry and difference between I_x and I_y moments of inertia) the more likely it is to 'tumble' if, for any reason, the yaw angle has drifted appreciably away from one of the two stable values.

I believe that this effect is not well-known; there is no indication, for example, that a recent (and advanced) text-book by Hughes (1986), has understood the point.

9.1.7 3-axis stabilisation of symmetrical satellite with magnetorquing

One solution to this problem is to impose magnetorquing on the *symmetric satellite* i.e one with the UDSAT-type shape. In principle magnetorquing can be used to force the required 'yaw stiffness'.

Such a point may seem obvious and trivial until the matter is analysed in detail. In fact it is a difficult problem. It will be recalled that *no other* control system is involved - no passive dampers, momentum wheels, reaction wheels, CMG (NOTE), or thrusters; just magnetorquing.

CMG = Control Moment Gyro

The chief difficulty is that magnetorquing can only complement the gravity-gradient control. The magnetic torque can hardly ever be directed in the exact direction it is wanted; as a vector it can only be directed within a plane in space orthogonal to the geomagnetic vector.

There are only '2 degrees of freedom' available, in a control problem which requires three (independent control of pitch roll and yaw). Given this considerable limitation what to do ?

My solution involves spin-plane magnetorquers operating in a mode of pulse width modulation, and a principle of control alternation. Libration control (roll/pitch damping) operates over near-polar latitudes, while position plus rate yaw control operates over the equator. This solution exploits the cyclical nature of the geomagnetic field, as encountered in a high-inclination orbit. Effectively I impose a quasi-orthogonality on the overall control system (see section 7.5). Gravity gradient exists as an essential 'background' -providing an averaging environment to the magnetorquing (NOTE).

One practical advantage of this technique should be a low-cost method of azimuth pointing of a small spacecraft. While the main Z axis can still only be stabilised on the main local vertical (zenith/nadir axis), any desired rotation 'around' this axis can be initiated and held. No doubt many useful applications could be found for such a facility.

In simulation the method would seem to work well. It cannot be tried at present on UOSAT-2 because real-time 3 axis attitude determination is required (but see sec.9.1.12).

NOTE The UOSAT magnetorquers have three states - forward, reverse or off. With X and Y torquers available, and a p.w.m mode they are switched on alternatively for a variable time ΔX and ΔY every constant T seconds. The effective moment created is directed wherever wanted in the X-Y plane

9.1.8 3-axis stabilisation of an asymmetric satellite with magnetorquing

Almost the same algorithm and mode of magnetorquing can be used to impose magnetorquing on the asymmetric satellite (see section 7.6).

In principle, magnetorquing overcomes the instability problem (summarised in sec.9.1.6), by applying position-plus-rate control to the yaw angle, forcing it to one of the two stable values permitted by the gravity-gradient equations. The magnetorquers simultaneously provide roll/pitch damping in the quasi-orthogonalised manner described in the preceeding section.

The disadvantage of an asymmetric rather than a symmetric structure is that the yaw angle can be forced on only one of the two possible stable values. However the advantage is an enhanced yaw stiffness persisting over all the orbit, provided indirectly by the gravity-gradient control ' in the background '.

Within the overall scheme, the gravity-gradient effect still plays an essential, if passive, role. It provides a continuity to the stabilisation, since at any instant, the magnetorquing inherently cannot supply an arbitrary torque vector to cancel some unwanted motion.

Since neither gravity gradient nor magnetorquing can do the job independently of the other, the two control mechanisms are essentially *complementary*.

Simulations show that the convergence on the stable state takes place in a graceful and well-behaved manner, with noise introduced on all three axes proving to be contained. Again this technique needs full 3-axis attitude determination to guide it.

Depending on the final accuracy which can be achieved (see sec.9.2.2) this could be conceivably become a significant, and new way of controlling low Earth orbiting satellites

9.1.9 Attitude determination problem

The basic (practically demonstrated) 2-axis stabilisation uses a magnetometer but does not make an attitude determination measurement as such. As a test of its effectiveness however, and to support many payloads, attitude determination is of course essential, even if 'off-line'. To repeat, my proposed methods of 3-axis stabilisation routinely require real-time attitude determination

I have pursued attitude determination using solely a 3-axis magnetometer (Acuna 1982). The practical reason is that the in-house sun and horizon sensors on UOSAT-2 failed to work satisfactorily (a power supply problem - not the sensors themselves). Confronted with this situation over the 5 years since UOSAT-2 has been in orbit I have come to the conclusion that a solution to the problem was not just of local interest but has more general applicability .

One reason is that proprietary attitude sensors may cost individually as much as our whole satellite put together. For small satellites of course, the absence of any other sensor frees valuable power requirements and volume, which can then be utilised by revenue-earning payloads.

Another reason is that a magnetometer measurement uniquely provides *continuous* measurement, in all attitudes of the spacecraft. I believe that no other type of sun or horizon sensor can do this.

9.1.10 1-axis determination in 2-axis stabilised state

I have identified a novel method of 'bounding' the attitude of the nominally vertical Z axis of the satellite. I call this the $\alpha \beta \gamma$ method (see from section 4.4). It determines upper and lower limits to the libration angle at various (arbitrary) points in the orbit. At various times these bounds come together and provide a 'fix' of the

current libration angle (see section 3.1 for attitude definitions). The method requires knowledge of the IGRF field and the location of the satellite over the Earth at that time. The method works best on a high-inclination orbit, relying on the fact that the Z-axis and the geomagnetic vector on occasions point close to each other (but do not usually co-incide). Since the librating state is uncoupled to the orbit period one can expect to build up statistics of the librating state from extended observation.

This method has been tested in full and shown to work.

It may be noted that 2-axis control is the natural complement to 1-axis determination. Control of the angular rotations about the reference roll and pitch axes determines where the Z axis is pointing. The determination of the attitude of that axis is the goal here in '1-axis' determination

9.1.11 Magnetometer auto-calibration

A problem experienced over the life time of UOSAT-2 has been a general drift in offset and scaling of the magnetometer measurement, on all of its three channels. I have devised a method which establishes the best values to give the six, independent scaling and offset parameters, as needed in the calibration. The method expects a 2-axis stabilised, Earth centre pointing attitude and a residual spin ; but does not in fact need to 'know' the attitude, at any time. The method makes a least squares-fit between the magnitude of the measured geomagnetic field (which should be independent of attitude) and that predicted by the IGRF model.

It again uses a 'quasi-orthogonal' principle; no large matrix interactions are required; the calculations are simple; and convergence is rapid within a few iterations. Section 4.5 gives examples of the marked improvement obtainable, from a badly drifted initial state. Typically the r.m.s error has been reduced to 0.75 μ T in every channel.

Part of this residual error is attributable to a known non-linearity in the electronics which read the magnetometer channels. Without that problem the accuracy would be better (the model IGRF field is now quite old and any discrepancy between it and the true geomagnetic field is automatically included in the gross error). An r.m.s. error of 0.75 μT creates a pointing error of 1.5° in a field magnitude of 40 μT .

9.1.12 3-axis attitude determination

A method of real-time 3-axis attitude determination, based solely on a magnetic vector measurement, has been investigated. Although not enough work has been done to decide on the eventual accuracy, the results look promising. My solution is what I would call a 'Kalman-like' estimator. The proposed solution works well in simulation (sections 7.1 through 7.5). On comparison with the conventional literature it may look unusual. I was looking for the simplest possible structure; and the essence of Kalman filtering can be conveyed in a much simpler mathematical structure than the standard discrete, matrix formulations (see Appendix I); the measurement problem here, in trying to infer incomplete attitude information from the geomagnetic B field, is highly non-linear; and I aimed to keep the number of computational steps to a minimum. My 'first-principles' analysis generates a solution which is effectively a controlled coupling to a PLL estimator (in yaw) and quasi-independent coupling to harmonic or cyclic estimators (in roll and pitch).

It has been tested on the most difficult continuously-spinning 2-axis stabilised state (rather than the 3-axis stabilised state). It can 'pick-up ' and lock onto an arbitrary initial yaw (slewing) state without prior knowledge of the attitude.

It adopts non-standard ideas such as *innovation interrupts*, during which intervals no information is admitted into the estimators from external sources; and a concept I call *double innovation*.

9.2 Future Work

The facilities exist to allow continuing work. UOSAT-2 is still operational and UOSAT-D and UOSAT-E are scheduled for launch in the autumn of 1989 (NOTE). Satellite UOSAT-D will carry the same ADCS hardware as UOSAT-2, and will allow continued operation in the event of the latter's failure.

9.2.1 New hardware

Satellite UOSAT-E will carry additional, in-house developed sun and Earth horizon sensors. The aim is to cover our bets by separating the control problem from the determination problem. The complementary gravitational and magnetorquing control principle is conceptually separate from the magnetic Kalman attitude filter. Having the additional determination facilities will test the relative success of each independently.

Each horizon sensor (designed with P. Wright) is an experimental, triple-beam device based on three inclined photo-tubes. The sensor generates a spatial discriminator-type response on intersection of the illuminated horizon. The sun sensors are simple analogue devices based on triangular-masked photo-cells.

9.2.2 New software

Only since January 1989, after a long program of development by the UOSAT team, is there now installed an on-board multi-tasking combined operating system and language (POLYFORTH). Fig.6.3.2-17 and 6.3.2-18 were first fruits of this system, providing the remarkable detail of

NOTE An interim satellite in the series UOSAT-C had been designed is currently on hold while waiting for a suitable launcher.

measurements every 5 seconds over 4096 data points. Collected on-board by the computer the data is then down-linked from memory perfectly to ground using a packet data protocol. UOSAT-D and UOSAT-E will be able to use either this software or run a completely independent multi-tasking C-based operating system on an independent computer. We are now embarrassed for choice, with no lack of opportunities on future missions to try out ideas in ADCS.

9.2.3 Research aims- accuracy

One fundamental issue, on which much more effort is needed, is to establish the limits to accuracy in our approach to determination and control of the attitude.

On attitude control is too early to say precisely what fundamental accuracy will be achievable by our general approach. A useful 'baseline' from which we can expect to improve was given in fig.6.3.2-17. We have found empirically that steady operation of just the spin control tends to keep the libration more or less steady - with an error $\sim 15^\circ$ shown in the γ plot - this is without running the delibration algorithm. The best earlier result obtained with the threshold comparison algorithm should be compared (fig.6.3.2-16), achieving $\sim 10^\circ$. I have presented a variety of proposals for improving on this basic result. But what pointing accuracy can be expected at best?

The answer would seem to depend partly on the boom technology, as much research in the past revealed. A practical problem with the UOSAT-2 boom was revealed in the fine detail of fig.6.3.2-18. The oscillations seen in the α and β plots are almost certainly due to a boom flexing (it is known to be prone to thermal distortion) which shifts the line of the Z axis and couples in the associated Z spin to the measurement (the meaning of these plots is explained in section 4.4). That problem would presumably be overcome in future missions by a more rigid boom (see section 2.2 for discussion of the one currently employed).

A second limitation is that the threshold comparison algorithm, however much it is improved, is inherently limited by its pure damping property; no account can be taken of residual bias or disturbance torques, the nature of which have been considered briefly in section 3.8. Residual air pressure and solar pressure would seem to be dominating disturbances. In the circumstances a reasonable design goal would seem to be a consistent 5° r.m.s error by this method, operating on UOSAT-2. The residual disturbances and the boom flexing effect will probably not allow anything better.

With a working 3-axis attitude determination system (on UOSAT-C or later satellites) and a better engineered boom the situation surely improves. In principle the disturbing, bias torques can be compensated by magnetorquing in a control loop.

Given that capability, at an educated guess, a 2° r.m.s error is likely to be achievable in pitch and roll error (with some larger yaw error if attempting full 3-axis control), if relying on the magnetic Kalman filter. We may have to wait for the as yet undesigned UOSAT-F to demonstrate this point - or even later satellites in the series - which hopefully will have better-engineered booms. And in principle even better stabilisation would be achieved if there were at least *TWO* booms. If we stay with the symmetric geometry it is clear that the residual air resistance and solar pressure can be cancelled by a balanced structure - i.e with a boom deployed in two opposing directions on the vertical axis. However this might be thought to be moving out of the 'low-cost ' concept.

We also need to re-investigate multiple boom configurations *in combination with the magnetorquing concept* on possible asymmetric satellite structures. The aim here is to see if the enhanced natural yaw stiffness will achieve greater accuracy in yaw pointing

Whatever is being attempted we need much more elaborate modelling. There is a need to incorporate all dominant disturbance torques; and to allow for flexible mechanical modes in the deployed boom.

I am very conscious of limitations in making a rigid body assumption in the theory (NOTE)

9.2.4 Development plan

If events go favourably then the plan is to

1) Quantify the definite limits of performance available from the currently-employed threshold comparison algorithm and its improvements (as sec.9.1.3). Test on satellite (UOSAT-2 and UOSAT-D).

2) Justify the 3-axis attitude determination method of chapter 7 -- the magnetic Kalman filter, by off-line processing of real magnetic data from the satellite, using IGRF field model.

3 Implement code which will calculate the IGRF field on demand at any point in the orbit in real time on board the space-craft.

4 Given 3: now implement 3-axis attitude determination in real time on the satellite.

5 given 4: check from other attitude sensors (from UOSAT-E inclusive).

6 further analysis of full 3-axis 'complementary gravitational magnetorquing ' and if the promise seems fulfilled start gradual introduction (from UOSAT-E).

9.3 Conclusions

A low-cost small satellite requires a low-cost solution to those sub-systems which perform ADCS - attitude determination control and stabilisation. I believe that I have identified and demonstrated in

NOTE. I would excuse making that assumption up to the present time because it has been surprisingly fruitful despite its limitations. It has taken me to this date to see all the control implications in the rigid-body model. However it is now time to move on

orbit successful stabilisation and control techniques using various combinations of gravity-gradient control and magnetorquing, mediated through a computer. There would seem to be considerable novelty in one or more aspects. Various attitude determination techniques have been tried, and using just a magnetometer and a computer. These again would appear to be new. All the methods look promising for future missions. Substantial computing power and memory on small satellites is not now a significant cost factor. One or more computers will be provided anyway in support of data collection from on-board experiments and sensors, maintaining the space-craft autonomy, and in general house-keeping. The emphasis is therefore on exploiting DSP techniques of sufficient mathematical complexity to achieve the desired aims, while minimising hardware.

Considerable time and effort was needed to develop software which would support on-board ADCS. All operations on UOSAT-1 had to be carried out with the crudest software, usually ground-based and operating in real-time over the links when the satellite was in view. The early stabilisation of UOSAT-2 was slightly easier, in having working autonomous control on board; but all the operations were developed in small machine-code routines. Only recently is there a fully-fledged operating system running on the spacecraft.

We have matched past pointing accuracies with an active damping system in practice, in the course of which we have surely demonstrated the simplest possible arrangement for controlling and stabilising an Earth pointing LED satellite (2-axis control), as well as demonstrating many other features of mixed combinations of gravity-gradient and magnetorquing control, which have been itemised above and in the text. In theory, one principle conclusion is that magnetorquing will convert a nominal 2-axis stabilised symmetrical satellite into a 3-axis stabilised system; and will make a nominally 3-axis asymmetric satellite stabilise properly. Novel methods of achieving these control aims have been identified. Subject to the eventual pointing accuracies achieved then hopefully our work will lead to a re-appraisal of the control options wanted on low Earth orbiting satellites.

A VECTOR DERIVATION OF NUTATION

The derivation of the nutation equations for symmetric rigid body is a standard piece of dynamics Pars (1965) Thomson (1963) Kaplan (1976) Wertz (1978) Hughes (1986) follow the same approach. The different technique here of all-vectorial analysis eliminates the need to invoke a co-ordinate system and Euler angles which is a common feature of the standard analysis.

The 'classical' approach always starts from the (non-linear) Eulerian set of equations. In this study I am interested only in the case of axial symmetry for which this set of equations give simple-looking algebraic answers. It is shown in the usual approach that the angular motion about two of the axes is harmonic, and the angular velocity about the remaining axis is a constant. A deduction can then be made that there is a conical kind of motion. But only an internal description of the motion has been achieved. As Pars says (p222) "this is hardly a complete solution of the problem because it does not tell us about the phenomena we actually observe during the motion; it does not determine the actual configuration of the system at time t ". Quite. To arrive at this essential point the author (and other authors) then have to invoke the system of Eulerian angles. These are often awkward to handle - and artificial in the sense that there 12 different possible conventions.

An alternative analysis is given here which arrives at this external description directly and avoids both the Eulerian equation and the system of Eulerian rotation angles. The essential equations are found in a relatively simple manner.

The spacecraft must be assigned an angular momentum $\underline{L}$. This is related to its angular motion through a well-known equation

$$\underline{L} = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k} \quad \dots A.1a$$

where (Fig. T7.1-1)

$\underline{i}$, $\underline{j}$, $\underline{k}$ are unit vectors specifying the directions of the x, y, and z axes respectively

I_x , I_y , and I_z are the moments of inertia about the X, Y, and Z axes respectively

ω_x , ω_y , and ω_z are the components of the angular velocity $\underline{\omega}$ resolved along the x, y, and z axes of the body

The axes must be the *principal axes*; these exist for any rigid body, but can only be identified with the nominal axes of symmetry assigned to the body if it has been balanced. Normally a satellite is balanced in dynamic tests.

The general angular motion of a body in principle can be found from the physics relating any applied torque $\underline{N}$ to the rate of change of the angular momentum $\underline{L}$ i.e

$$\underline{N} = d\underline{L}/dt \quad \dots A.1b$$

In the case of strict nutation this torque is by definition zero.

In vector formulation then the angular velocity $\underline{\omega}$ resolves as

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \quad \dots A.2$$

where it is understood that these angular velocity components are in general time-varying. A first step to solution is that two of the components of moment of inertia can be assumed to be the same i.e

$$I_x = I_y = I_T \quad \dots A.3$$

where I_T is a common transverse moment of inertia. This is the case of axial symmetry.

Then define a transverse vector component of angular velocity (T-spin)

$$\underline{\omega}_T = \omega_x \underline{i} + \omega_y \underline{j} \quad \dots A.4a$$

as well as identifying an vector in-line component (Z-spin)

$$\underline{\omega}_Z = \omega_z \underline{k} \quad \dots A.4b$$

so that a substitution into Equation A.1a yields a vector equation

$$\underline{L} = I_T \underline{\omega}_T + I_z \underline{\omega}_Z \quad \dots A.5$$

Analogous to Eq.A.4b we may define a unit vector $\underline{t}$ such that

$$\underline{\omega}_T = \omega_T \underline{t} \quad \dots A.6$$

where vector $\underline{t}$ lies in the plane defined by the X and Y axes. The combined angular motion can be written as

$$\underline{\omega} = \omega_z \underline{k} + \omega_T \underline{t} \quad \dots A.7$$

and may be visualised as a simultaneous slewing of (i) the $\underline{k}$ vector (the Z axis) about the $\underline{t}$ vector with a speed ω_T , and (ii) of the $\underline{t}$ vector about the $\underline{k}$ vector with a speed ω_z (Fig.A-1).

Under zero torque Eq.A.5 differentiates simply to

$$I_T d\underline{\omega}_T/dt + I_z d\underline{\omega}_Z/dt = 0 \quad \dots A.8$$

and by the usual rules of vector differentiation this becomes, after substitution of Eq.A.4b

$$I_T \frac{d\omega_T}{dt} + I_z \frac{d(\omega_z)}{dt} \underline{k} + I_z \omega_z \frac{d\underline{k}}{dt} = 0 \quad \dots A.9a$$

or

$$I_T \frac{d\omega_T}{dt} + I_z \frac{d(\omega_z)}{dt} \underline{k} + I_z (\underline{\omega} \wedge \underline{k}) \omega_z = 0 \quad \dots A.9b$$

because the differentiation of the constant magnitude $\underline{k}$ is a slewing motion by the current angular velocity. Therefore

$$I_T \frac{d\omega_T}{dt} + I_z \frac{d(\omega_z)}{dt} \underline{k} + I_z (\underline{\omega} \wedge \underline{\omega}_z) = 0 \quad \dots A.9c$$

However $\underline{\omega}_T$ is varying its rate of change it always by definition lies in the spin plane defined by the X and Y axes. And the cross-product $\underline{\omega} \wedge \underline{\omega}_z$ is always orthogonal to $\underline{\omega}_z$. Hence the term in this equation containing vector $\underline{k}$ is inherently orthogonal to the other terms. Therefore the acceleration $d(\omega_z)/dt$ is zero and the magnitude of the z spin ω_z is a constant.

The square of the angular momentum

$$L^2 = \underline{L} \cdot \underline{L}$$

From Eq.A.5 it follows that

$$= (I_T \omega_T)^2 + (I_z \omega_z)^2 \quad \dots A.10$$

Under zero torque the angular momentum vector $\underline{L}$ does not change. From Eq.A.10 it follows that the magnitude of the transverse t-spin ω_T is also a constant, since ω_z has been found to be a constant. And the scalar product

$$\underline{\omega}_z \cdot \underline{L} = \underline{\omega}_z \cdot (I_z \underline{\omega}_z + I_T \underline{\omega}_T) \quad \dots A.11a$$

$$= \omega_z^2 I_z \quad \dots A.11b$$

is also a constant since ω_z is a constant. Therefore the angle between the $\underline{k}$ vector and the angular momentum vector $\underline{L}$ is a constant. This leads to a definition of the angle between the $\underline{k}$ vector (z-axis) and the $\underline{L}$ vector.

nutation angle θ

By a similar argument, from inspection of the scalar product $\underline{\omega}_T \cdot \underline{L}$ it follows that the angle between the $\underline{t}$ vector and the $\underline{L}$ vector is a constant (but not the same constant).

Since the scalar acceleration $d(\underline{\omega}_z)/dt$ is zero it follows that the changing value in $\underline{\omega}_z$ is due to its slewing in space i.e

$$d\underline{\omega}_z/dt = \underline{\omega} \wedge \underline{\omega}_z \quad \dots A.12a$$

$$= \underline{\omega}_T \wedge \underline{\omega}_z \quad \dots A.12b$$

A dummy term may be introduced into Eq.A.12b so that, after multiplying both sides by I_T

$$I_T d\underline{\omega}_z/dt = (I_T \underline{\omega}_T + I_z \underline{\omega}_z) \wedge \underline{\omega}_z \quad \dots A.13a$$

because $\underline{\omega}_z \wedge \underline{\omega}_z$ is inherently zero. Then

$$d\underline{\omega}_z/dt = (\underline{L}/I_T) \wedge \underline{\omega}_z \quad \dots A.13b$$

Since $\underline{L}$ is invariant and the nutation angle θ is a constant this identifies a component angular velocity of constant direction which slews the $\underline{k}$ vector around $\underline{L}$. We define

$$\text{inertial nutation rate } \underline{\omega}_B = \underline{L}/I_T$$

Hence

$$d\underline{\omega}/dt = \underline{\omega}_B \wedge \underline{\omega}_z \quad \dots A.13c$$

and from the definition of the nutation angle which is substituted back in Eq.A.11b

$$\omega_z L \cos(\theta) = \omega_z^2 I_z$$

and given the definition of $\underline{\omega}_B$ it follows that

$$\omega_B I_T \cos(\theta) = \omega_z I_z \quad \dots A.13d$$

Rearranging from Eq.A.8 we have

$$d\omega_T/dt = - (I_z/I_T) d\omega_z/dt$$

and from Eq.A.12b

$$= (I_z/I_T) (\omega_z \wedge \omega_T) \quad \dots A.14$$

and by a similar argument using a dummy variable it is found that

$$d\omega_T/dt = (\underline{L}/I_T) \wedge \omega_T \quad \dots A.15a$$

$$= \omega_B \wedge \omega_T \quad \dots A.15b$$

showing that the t vector is slewed at the same rate as the k variable.

These last two results have a simple geometrical interpretation (Fig.A-2). Vectors k, t, and L must be co-planar. The plane containing all three vectors rotates around the (fixed) L axis at the inertial nutation rate.

By the above simple arguments it has been shown that

* there is a constant z-spin ω_z

* the z axis describes a cone in space of constant angle to the L vector at a constant inertial nutation rate ω_B

It remains to describe exactly the motion of the X and Y axes. The approach here is to define the difference between the total angular velocity and the inertial nutation rate:

$$\text{body nutation rate } \omega_A = \omega - \omega_B$$

and then give this a physical interpretation.

It may first be noted that as a vector it is co-linear with ω_z .

Proof follows simply from

$$\begin{aligned} \omega_A \wedge \omega_z &= (\omega_z + \omega_T - \omega_B) \wedge \omega_z \\ &= (\omega_T - \omega_B) \wedge \omega_z \\ &= 0 \end{aligned} \quad \dots A.16$$

(check back from Eq.A12b and Eq.A.13c). Fig.A-3 helps to fix this relationship between all these vectors. Essentially the total angular velocity $\underline{\omega}$ is seen to be the vector sum of either the z-spin $\underline{\omega}_x$ and the t-spin $\underline{\omega}_T$; or it is the vector sum of the inertial spin rate $\underline{\omega}_B$ and the body nutational rate $\underline{\omega}_A$. The two descriptions are complementary.

Following the usual convention in vector differentiation we write

$$d\underline{\omega}_T/dt = [d\underline{\omega}_T/dt] + \underline{\omega} \wedge \underline{\omega}_T \quad \dots A.17$$

where

$$[d\underline{\omega}_T/dt] = \underline{i} d(\omega_x)/dt + \underline{j} d(\omega_y)/dt \quad \dots A.18a$$

is the 'local' differentiation showing the movement of the $\underline{t}$ axis relative to the x and y axes. Since the magnitude of $\underline{\omega}_T$ is a constant it follows that here is a pure rotation. The inverse of this describes the rotation of the X and Y axes relative to the $\underline{t}$ vector i.e

$$\begin{aligned} - [d\underline{\omega}_T/dt] &= - d\underline{\omega}_T/dt + \underline{\omega} \wedge \underline{\omega}_T \\ &= - \underline{\omega}_B \wedge \underline{\omega}_T + \underline{\omega} \wedge \underline{\omega}_T \end{aligned} \quad \dots A.18b$$

having substituted from Eq.A.15b. The definition of $\underline{\omega}_A$ then leads to

$$= \underline{\omega}_A \wedge \underline{\omega}_T \quad \dots A.19$$

It can be easily verified that this means that the component angular velocities are harmonic at the frequency ω_A . If we set

$$\omega_x = \omega_T \sin(\omega_A t)$$

then

$$\omega_y = \omega_T \cos(\omega_A t)$$

The magnitude of $\underline{\omega}_A$ can be related to the previously defined parameters by making a different substitution into Eq.A.17 using Eq.A.14

$$\begin{aligned}
 - [d\omega_T/dt] &= - (I_Z/I_T) (\omega_Z \wedge \omega_T) + \omega \wedge \omega_T \\
 &= - (I_Z/I_T) (\omega_Z \wedge \omega_T) + \omega_Z \wedge \omega_T \\
 &= (1 - I_Z/I_T) (\omega_Z \wedge \omega_T)
 \end{aligned}$$

Given that ω_Z is co-linear with ω_A , substituting Eq.A.19 into the LHS of this equation, and recognising that ω_Z and ω_A are both orthogonal to ω_T , it follows that

$$\omega_A = (1 - I_Z/I_T) \omega_Z \quad \dots A.20$$

and by substituting across from Eq.A.13d

$$\omega_A = (I_T/I_Z - 1) \omega_B \cos(\theta) \quad \dots A.21$$

and finally, from appeal to Fig.A-3

$$\omega_T = \omega_B \sin(\theta) \quad \dots A.22$$

Fig. A-1 The concept of z-spin and t-spin

Angular velocity with arbitrary magnitude and direction may be resolved along the Z axis of the body. What is left is resolved orthogonally to this axis and is termed the current transverse angular velocity ω_t which resolves then to ω_x and ω_y . Unit vector $\underline{t}$ is defined in the direction of ω_t .

(ω in plane of paper)

(Z axis out of paper)

Fig. A-2. Vector relations in nutational state

Vectors shown are
coplanar and all
rotate at inertial
nutation rate.

Fig. A-3 Angular velocity vectors in nutational state

$$\begin{aligned}\underline{\omega} &= \underline{\omega}_a + \underline{\omega}_b \\ &= \underline{\omega}_z + \underline{\omega}_t\end{aligned}$$

a_n = nutation angle

B GRAVITY GRAVITY INDUCED PRECESSION

This appendix shows how the secular averaged gravity-gradient torque can be found in a simple manner and semi-exact expressions for the precession rate of the spin-stabilised satellite. The useful generality includes the effect of nutation

B.1 Torque Derivations

This first step establishes a mean torque due to gravity gradient when the satellite is nutating.

Repeating basic torque equation Eq.3.3.2

$$N_{gg} = 3 \omega_0^2 (I_T - I_z) (\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r}) \quad \dots B.1.1$$

The orientation of the $\underline{z}$ vector can be described at any instant within a temporary cartesian system

$$\begin{aligned} \underline{z} = & \sin(\theta) \cos(\omega_B t) \underline{i} \\ & + \sin(\theta) \sin(\omega_B t) \underline{j} \\ & + \cos(\theta) \underline{z}_0 \end{aligned}$$

where

$\underline{z}_0$ is identified as a unit vector with a direction which is the average of the $\underline{z}$ vector

ω_B is inertial nutation rate

$\underline{i}, \underline{j}$ are unit vectors completing the description of the (temporary) co-ordinate system.

Since the $\underline{z}$ axis is constantly changing in direction then so must the g.g torque. Being interested only in the average value which by definition must be a constant, substituting Eq.B.1.2 into Eq.B.1.1 and taking the time average then

$$\begin{aligned}
 \underline{N}_{gg} = & 3 \omega_o^2 (I_T - I_z) \\
 & ((1/2) \sin(\theta)^2 (\underline{i} \cdot \underline{r}) (\underline{i} \wedge \underline{r}) \\
 & + (1/2) \sin(\theta)^2 (\underline{j} \cdot \underline{r}) (\underline{j} \wedge \underline{r}) \\
 & + \cos(\theta)^2 (\underline{z}_o \cdot \underline{r}) (\underline{z}_o \wedge \underline{r}))
 \end{aligned}
 \quad \dots B.1.3$$

Now the choice of co-ordinate vectors $\underline{i}$ and $\underline{j}$ is arbitrary provided they are orthogonal to each other and to $\underline{z}_o$. The vector $\underline{i}$ can be set orthogonal to vector $\underline{r}$; in which case the scalar product $\underline{i} \cdot \underline{r}$ will be zero, and $\underline{j}$ will be co-planar with $\underline{r}$ and $\underline{z}_o$.

Then without elaborate vector operations it is easy to see that in all circumstances

$$(\underline{i} \cdot \underline{r}) (\underline{i} \wedge \underline{r}) = - (\underline{z}_o \cdot \underline{r}) (\underline{z}_o \wedge \underline{r}) \quad \dots B.1.4$$

hence the result that the average torque

$$\underline{N}_{gg} = 3 \omega_o^2 (I_T - I_z) (1 - (3/2) \sin(\theta)^2) (\underline{z}_o \cdot \underline{r}) (\underline{z}_o \wedge \underline{r}) \quad \dots B.1.5$$

which is similar to Eq.3.3.2 with the difference that the average direction $\underline{z}_o$ vector replaces the instantaneous direction $\underline{z}$; and there is this additional factor which reduces the torque depending on the nutation angle θ . It is clear that a critical value of $\theta = 54.7^\circ$ will eliminate the g.g torque. For a zero nutation angle Eq.B.1.5 reverts to Eq.B.1.1

It has been assumed that in the course of a nutational cycle that the effect of the g.g torque would be small - in the sense that the direction of the momentum vector - and therefore the average direction $\underline{z}_o$ - would not significantly change.

We extend this assumption here over the period of an orbit - typically 100 mins. In other words the assumption is that the angular momentum is there will be no significant advance in the precession. This assumption needs of course to be checked, in any practical application, by numerical substitution.

Taking Eq.B.1.2 as a starting point (Eq. B.1.5 to include nutation) the aim is to find the average value of the term $(\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r})$

- or $(\underline{z}_0 \cdot \underline{r}) (\underline{z}_0 \wedge \underline{r})$ in Eq.B.1.5 - where the vector $\underline{r}$ is constantly rotating in the course of an orbit. For simplicity the nutation free-case will be assumed initially.

Write

$$\underline{r} = \underline{i}' \cos(\omega_0 t) + \underline{j}' \sin(\omega_0 t) \quad \dots B.1.6$$

where $\underline{i}'$ and $\underline{j}'$ are unit orthogonal vectors lying in the plane of the orbit. It follows that

$$\begin{aligned} (\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r}) &= ((\underline{z} \cdot \underline{i}') \cos(\omega_0 t) + (\underline{z} \cdot \underline{j}') \sin(\omega_0 t)) \\ &\quad ((\underline{z} \wedge \underline{i}') \cos(\omega_0 t) + (\underline{z} \wedge \underline{j}') \sin(\omega_0 t)) \end{aligned} \quad \dots B.1.7$$

And the time average:

$$\begin{aligned} \langle (\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r}) \rangle &= \\ &\quad (1/2) (\underline{z} \cdot \underline{i}') (\underline{z} \wedge \underline{i}') + (1/2) (\underline{z} \cdot \underline{j}') (\underline{z} \wedge \underline{j}') \end{aligned} \quad \dots B.1.8$$

It is permitted to choose the condition

$$\underline{z} \cdot \underline{j}' = 0 \quad \dots B.1.9$$

In which case the time average simplifies

$$\langle (\underline{z} \cdot \underline{r}) (\underline{z} \wedge \underline{r}) \rangle = -(1/2) (\underline{z} \cdot \underline{n}) (\underline{z} \wedge \underline{n}) \quad \dots B.1.10$$

This leads then to an expression for the secular average torque. For the straight spin case a substitution into Eq.B.1.1 yields

$$\langle \underline{N}_{gg} \rangle = - (3/2) \omega_0^2 (I_T - I_Z) (\underline{z} \cdot \underline{n}) (\underline{z} \wedge \underline{n}) \quad \dots B.1.11$$

Inspection of this equation shows that the torque is always orthogonal to the $\underline{z}$ direction. There is therefore no tendency to change the magnitude of the angular momentum vector; there is however going to be a precessional effect - analysed in the next section.

For the general nutation case, a substitution into Eq.B.1.5 with $\underline{z}_0$ rather than $\underline{z}$ as the constant direction, yields

$$\langle \underline{N}_{aa} \rangle = - \frac{(3/2)}{(\underline{z}_0 \cdot \underline{n})} \omega_0^2 (I_T - I_z) (1 - (3/2) \sin^2(\theta)) (\underline{z}_0 \wedge \underline{n}) \quad \dots B.1.12$$

B.2 Precession equations

The next step is to evolve steady-state equations which account for a precession of the angular momentum (average $\underline{z}_0$ direction). The analysis first assumes that the spacecraft is not nutating - just has a pure z-spin. A co-ordinate -free derivation of the fundamental precession equation will be argued assuming a steady-state exists. The approach is justified by a more detailed analysis in the next chapter, where the analysis is quite general.

Given the fundamental formula relating change of angular momentum to an applied torque.

$$\underline{N}_{aa} = d\underline{L}/dt \quad \dots B.2.1$$

then in a steady state there will be no changes in the magnitude of $\underline{L}$. Hence any changes will be due to its slewing in space. We identify a precession angular velocity $\underline{\omega}_N$ which is assumed to be invariant in direction

$$d\underline{L}/dt = \underline{\omega}_N \wedge \underline{L} \quad \dots B.2.2$$

Referring to fig.8.4-1 and the arguments of the previous section, that the torque $\underline{N}_{aa}$ is orthogonal to both $\underline{z}$ and $\underline{n}$, it follows that the torque $\underline{N}_{aa}$ can be maintained orthogonal to $\underline{\omega}_N$ only if the latter is co-linear with the orbit normal $\underline{n}$.

Now taking the fundamental equation, repeated here

$$\underline{L} = I_T \underline{\omega}_T + I_z \underline{\omega}_z \quad \dots B.2.3$$

and differentiating yields

$$d\underline{L}/dt = I_T d\underline{\omega}_T/dt + I_z d(\underline{\omega}_z)/dt + I_z (\underline{\omega}_T \wedge \underline{\omega}_z) \quad \dots B.2.4$$

Since the torque $\underline{N}_{gg}$ is always orthogonal to $\underline{k}$ and for the same reasons given in the nutation analysis it follows that there is no speed change ω_z . Hence

$$\underline{\omega} = \underline{\omega}_T + \underline{\omega}_z$$

$$\underline{N}_{gg} = I_T d\underline{\omega}_T/dt + I_z (\underline{\omega} \wedge \underline{\omega}_z) \quad \dots B.2.5a$$

$$= I_T d\underline{\omega}_T/dt + I_z (\underline{\omega}_T \wedge \underline{\omega}_z) \quad \dots B.2.5b$$

since the cross product of the component $\underline{\omega}_z$ in $\underline{\omega}$ with $\underline{\omega}_z$ is zero.

If Eq.B.2.2 is substituted into Eq.B.2.3 then

$$\underline{N}_{gg} = I_T (\underline{\omega}_T \wedge \underline{\omega}_T) + I_z (\underline{\omega}_T \wedge \underline{\omega}_z) \quad \dots B.2.6$$

then comparison of Eq.2.5b with Eq.B.2.6

$$\begin{aligned} \underline{\omega}_T \wedge \underline{\omega}_z &= \underline{\omega}_T \wedge \underline{\omega}_z \\ \omega_T \omega_z \sin 90^\circ &= \omega_T \omega_z \sin \delta \end{aligned} \quad \dots B.2.7$$

And establishes the scalar relationship (

$$\omega_T = \omega_N \sin(\delta) \quad \dots B.2.8$$

where δ has been already identified as a tilt angle (it should be clear then that in the course of the precession the tilt angle is maintained). Substituting B.2.8 into Eq.B.2.6 and putting in scalar form then

$$N_{gg} = -I_T \omega_N^2 \cos(\delta) \sin(\delta) + I_z \omega_N \omega_z \sin(\delta) \quad \dots B.2.9$$

and then from the scalar version of Eq.B.1.1

$$\begin{aligned} (3/2) \omega_0^2 (I_T - I_z) \cos(\delta) \sin(\delta) \\ = -I_T \omega_N^2 \cos(\delta) \sin(\delta) + I_z \omega_N \omega_z \sin(\delta) \end{aligned} \quad \dots B.2.10$$

where the signs have been checked from reference to the appropriate figures.

Now it is a matter of interest to calculate the precessional rate ω_N only when it is a fraction of the spin rate ω_z . It is proposed then to

disregard the first term on the R.H.S of Eq.B.2.10 which has a square law dependence on the rate ω_N . This then yields the approximation

$$(3/2) \omega_0^2 (I_T - I_Z) \cos(\delta) \sin(\delta) = I_Z \omega_N \omega_Z \sin(\delta) \quad \dots B.2.11$$

and dividing both sides out by $\sin(\delta)$

$$(3/2) \omega_0^2 (I_T - I_Z) \cos(\delta) = I_Z \omega_Z \omega_N \quad \dots B.2.12$$

This equation yields the believable result that the precess rate ω_N is inversely proportional to the spin rate ω_Z . Also it effectively states that all the angular momentum is expressed in the product $\omega_Z I_Z$ with negligible contribution from the slewing motion of the precession). If this approximation were exact it could have been obtained in one step. It follows that one can generalise to the nutating body by substituting for the the angular momentum of nutating spacecraft on the R.H.S instead of $\omega_Z I_Z$. Hence

$$\begin{aligned} (3/2) \omega_0^2 (I_T - I_Z) (1 - (3/2) \sin^2(\theta)) \cos(\delta) \sin(\delta) \\ = I_Z \omega_N \omega_Z \sin(\delta) / \cos(\theta) \end{aligned} \quad \dots B.2.13$$

or dividing out $\sin(\delta)$ from both sides

$$\begin{aligned} (3/2) \omega_0^2 (I_T - I_Z) (1 - (3/2) \sin^2(\theta)) \cos(\delta) \\ = I_Z \omega_N \omega_Z / \cos(\theta) \end{aligned} \quad \dots B.2.19$$

which after re-ordering terms establishes the chief result of Eq.3.4.3.5

Overall these equations are valid provided the period of the precession is long compared to an orbit.

C LIBRATION BOUNDS FORMULAE

Some unique formulae establish the limits or bounds on libration state as a function of the librational energy. The all-vectorial approach given here exploits the spacecraft symmetry having $I_x = I_y$, without recourse to advanced dynamics theory.

The derivation starts from a set of equations which were used to analyse the nutational state (Appendix A). A convenient starting point is the equation Eq.A.5 previously derived in the nutational analysis: that

$$\underline{L} = I_T \underline{\omega}_T + I_z \underline{\omega}_z \quad \dots A.5$$

which expresses the current angular momentum as the vector sum of a momentum component associated with the Z-spin and a momentum component associated with a transverse spin

It is convenient to describe the true angular velocity $\underline{\omega}$ in inertial space as the vector sum of the angular velocity with respect to the orbit-rotating frame of (apparent motion within this frame) and the angular velocity of this frame in inertial space. So

$$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_0 \quad \dots C.1$$

where

$\underline{\omega}_1$ is the relative angular velocity of UQSAT at some instant within this frame

$\underline{\omega}_0$ is the frame's angular velocity (and is a vector whose magnitude ω_0 is the orbit rate and which points in the direction of the orbit normal $\underline{n}$).

Then each of the two velocities defined above can be resolved into components: one directed along the z-axis of the space-craft; the other at right angles to this.

In which case one may define

$$\underline{\omega}_1 = \underline{\omega}_{z1} + \underline{\omega}_{T1} \quad \dots C.2a$$

$$\underline{\omega}_0 = \underline{\omega}_{z0} + \underline{\omega}_{T0} \quad \dots C.2b$$

and it follows that

$$\underline{\omega}_T = \underline{\omega}_{T1} + \underline{\omega}_{T0} \quad \dots C.2c$$

$$\underline{\omega}_z = \underline{\omega}_{z1} + \underline{\omega}_{z0} \quad \dots C.2d$$

In this division of the total angular velocity into four components it should be noted that $\underline{\omega}_{z1}$ and $\underline{\omega}_{z0}$ are necessarily co-linear with $\underline{k}$ (the Z-axis), while $\underline{\omega}_{T1}$ and $\underline{\omega}_{T0}$ are at some arbitrary angle to each other depending on UOSAT's spinstate (fig.C-1). UOSAT is relatively stationary within the 'PYR' system if and only if $\underline{\omega}_{T1} = 0$; meaning that the direction of the Z-axis is constant in this system. Although 'stationary' in this sense there will in general be a finite Z-spin i.e in our notation the magnitude of $\underline{\omega}_z$ is non-zero.

The orbit spin rate $\underline{\omega}_0$ is a constant; but this does not mean that its components are a constant; this is because components $\underline{\omega}_{z0}$ and $\underline{\omega}_{T0}$ are resolutions of $\underline{\omega}_0$ on the Z axis and T axis of the spacecraft; and these axes will generally be changing in direction.

Substituting Eq.C.2c into Eq.7.1.5 then gives

$$\underline{L} = I_T (\underline{\omega}_{T1} + \underline{\omega}_{T0}) + I_z \underline{\omega}_z \quad \dots C.3$$

The rate of change of angular momentum will be then found by differentiating this expression to

$$d\underline{L}/dt = I_T (d\underline{\omega}_{T1}/dt + d\underline{\omega}_{T0}/dt) + I_z d\underline{\omega}_z/dt \quad \dots C.4$$

But the angular velocity $\underline{\omega}_0$ of the PYR system is constant. From Eq.C.2b therefore

$$d\underline{\omega}_{T0}/dt = - d\underline{\omega}_{z0}/dt \quad \dots C.5$$

The vector $\underline{\omega}_{zo}$ is the component of $\underline{\omega}_o$ resolved along the z axis.
Therefore

$$\underline{\omega}_{zo} = (\underline{\omega}_o \cdot \underline{k}) \underline{k} \quad \dots C.6$$

and differentiating this

$$d\underline{\omega}_{zo}/dt = (\underline{\omega}_o \cdot \underline{\omega} \wedge \underline{k}) \underline{k} + (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega} \wedge \underline{k}) \quad \dots C.7$$

because the time change of the unit vector $\underline{k}$ is a pure rotation

After substituting Eq.C.5 into Eq.C.7

$$d\underline{\omega}_{zo}/dt = - (\underline{\omega}_o \cdot \underline{\omega} \wedge \underline{k}) \underline{k} - (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega} \wedge \underline{k}) \quad \dots C.8a$$

and some manipulation using Eqs.C.2 (see NOTE_1) this can be reduced to

$$\begin{aligned} d\underline{\omega}_{zo}/dt = & - \underline{\omega}_{zo} \wedge \underline{\omega}_{T1} - (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega}_o \wedge \underline{k}) \\ & - (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega}_{T1} \wedge \underline{k}) \end{aligned} \quad \dots C.8b$$

and therefore, after substituting in Eq.C.4

$$\begin{aligned} d\underline{L}/dt = & I_T d\underline{\omega}_{T1}/dt - I_T (\underline{\omega}_{zo} \wedge \underline{\omega}_{T1}) \\ & - I_T (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega}_o \wedge \underline{k}) - I_T (\underline{\omega}_o \cdot \underline{k}) (\underline{\omega}_{T1} \wedge \underline{k}) \\ & + I_z d\underline{\omega}_z/dt \end{aligned} \quad \dots C.8c$$

Looking at Eq.C.4 and then at Eq..A.1b any z-component N_z of an applied torque $\underline{N}$ can only affect the term $d\underline{\omega}_z/dt$. This is because the vector derivatives $d\underline{\omega}_{T1}/dt$ and $d\underline{\omega}_{zo}/dt$ are inherently orthogonal to the Z-axis ($\underline{k}$ vector), and cannot be affected by any N_z component. Further, by the rules of vector derivatives it can be shown that

$$d\underline{\omega}_z/dt = d\omega_z/dt \underline{k} + \underline{\omega} \wedge \underline{\omega}_z \quad \dots C.9a$$

which means that the magnitude of the spin ω_z is affected by N_z and nothing else. This is of course consistent with common sense which would demand that a 'twisting' type torque must exist if the magnitude

of the Z-spin is to change. In the derivation to follow the gravity-gradient is always orthogonal (transverse) to the Z-axis and there is never any such a twisting torque. Therefore the Eq.C.9a simplifies to

$$d\omega_z/dt = \omega \wedge \omega_z \quad \dots C.9b$$

Eq.C.9b can be expanded in terms of component angular velocities

$$d\omega_z/dt = \omega_o \wedge \omega_z + \omega_1 \wedge \omega_z \quad \dots C.10a$$

$$= \omega_o \wedge \omega_z + \omega_{T1} \wedge \omega_z \quad \dots C.10b$$

The result of this substitution then makes this rather long expression

$$\begin{aligned} dL/dt &= I_T d\omega_{T1}/dt - I_T (\omega_{T0} \wedge \omega_{T1}) \\ &- I_T (\omega_o \cdot k) (\omega_o \wedge k) - I_T (\omega_o \cdot k) (\omega_{T1} \wedge k) \\ &+ I_z (\omega_o \wedge \omega_z) + I_z (\omega_{T1} \wedge \omega_z) \end{aligned} \quad \dots C.11$$

But after scalar multiplying both sides of this equations by the angular vector ω_{T1} , a simple expression emerges

$$\begin{aligned} dL/dt \cdot \omega_{T1} &= I_T (d\omega_{T1}/dt \cdot \omega_{T1}) \\ &- I_T (\omega_o \cdot k) (\omega_o \wedge k) \cdot \omega_{T1} \\ &+ I_z (\omega_o \wedge \omega_z) \cdot \omega_{T1} \end{aligned} \quad \dots C.12$$

because all the terms in Eq.C.11 which contain the factor (vector $\wedge \omega_{T1}$) become identically zero after scalar multiplying by the vector ω_{T1} .

It may then be shown that this last equation can be reformulated as

$$dL/dt \cdot \omega_{T1} = d(KE)/dt \quad \dots C.13a$$

where

$$\begin{aligned} KE &= (I_T/2) (\omega_{T1} \cdot \omega_{T1}) + (I_T/2) (\omega_o \cdot k)^2 \\ &- I_z (\omega_o \cdot \omega_z) \end{aligned} \quad \dots C.13b$$

is introduced as an effective kinetic energy. This is easily tested by differentiating Eq.C.13b and noting that ω_z is a constant, that $\dot{\omega}_z = \omega_z \underline{k}$, and that $\underline{\omega}_0 \wedge \underline{k} = \underline{\omega}_{T0} \wedge \underline{k}$ (NOTE_2).

An exact expression for the gravity-gradient torque on body of the UOSAT type was given in Appendix B. Repeating Eq.B.1.1 with a slight change of symbolism, replacing unit vector $\underline{z}$ with $\underline{k}$ it reads

$$\underline{N}_{gg} = 3 \omega_0^2 (I_T - I_z) (\underline{k} \cdot \underline{r}) (\underline{k} \wedge \underline{r}) \quad \dots C.14$$

where ω_0 is the magnitude of the vector $\underline{\omega}_0$. In any orientation the torque $\underline{N}$ is always orthogonal to the $\underline{k}$ vector.

If this expression is multiplied by the angular vector component $\underline{\omega}_{T1}$ it is readily shown that

$$\underline{N}_{gg} \cdot \underline{\omega}_{T1} = d(-PE)/dt \quad \dots C.15a$$

where

$$-PE = (3/2) \omega_0^2 (I_T - I_z) (\underline{k} \cdot \underline{r})^2 \quad \dots C.15b$$

is introduced as an effective negative potential energy (NOTE_3).

Combining Eq.C.13b and Eq.C.15b then leads to the important equation

$$\begin{aligned} (3/2) \omega_0^2 (I_T - I_z) (\underline{k} \cdot \underline{r})^2 \\ = (I_T/2) (\underline{\omega}_{T1} \cdot \underline{\omega}_{T1}) + (I_T/2) (\underline{\omega}_0 \cdot \underline{k})^2 \\ - I_z \omega_z (\underline{\omega}_0 \cdot \underline{k}) + U \end{aligned} \quad \dots C.16$$

where U is an arbitrary constant

This general equation is not an equation of motion as such. Its use is merely to determine the bounds on angular position and angular transverse velocity.

Inspection of Eq.C.16 shows that it consists of products of the $\underline{r}$ vector with the $\underline{k}$ vector, and the $\underline{\omega}_0$ vector with the $\underline{k}$ vector. This leads to a natural angular system for defining the attitude of UOSAT at any instant.

Two angles have been defined in sec.3.1.1: the *inclination angle* y and the *tilt angle* δ

Reference to fig.3.1.1-5 shows that the inclination angle exists between the radius vector $\underline{r}$ and the $\underline{k}$ vector (the direction of the Z-axis); while the tilt angle is between the vector $\underline{r}$ and the $\underline{k}$ vector.

These are natural choices simply because of their close relation to the scalar and vector products constituting. Thus

$$(\underline{k} \cdot \underline{r}) = \cos(y) \quad \dots C.17a$$

$$(\underline{\omega}_0 \cdot \underline{k}) = \omega_0 \cos(\delta) \quad \dots C.17b$$

and

$$ABS(\underline{k} \wedge \underline{r}) = ABS(\sin(y)) \quad \dots C.18a$$

$$ABS(\underline{\omega}_0 \wedge \underline{k}) = ABS(\omega_0 \sin(\delta)) \quad \dots C.18b$$

substituting then in Eq.C16 leads to

$$\begin{aligned} & 3/2 \omega_0^2 (I_T - I_Z) \cos(y)^2 \\ & = (I_T/2) \omega_{T1}^2 \\ & + (I_T/2) \omega_0^2 \cos(\delta)^2 \\ & - I_Z \omega_Z \omega_0 \cos(\delta) + C \end{aligned} \quad \begin{aligned} & \dots C.18 \\ & \dots 3.5.3.1 \end{aligned}$$

NOTE_1

The manipulations are as follows. Taking the first term in Eq. C.7

Because $\underline{\omega}_z \wedge \underline{k} = 0$ then

$$\begin{aligned} (\underline{\omega}_0 \cdot \underline{\omega} \wedge \underline{k}) \underline{k} &= (\underline{\omega}_0 \cdot \underline{\omega}_T \wedge \underline{k}) \\ &= (\underline{\omega}_{T0} \cdot \underline{\omega}_T \wedge \underline{k}) \underline{k} \\ &= \underline{\omega}_{T0} \wedge \underline{\omega}_T \end{aligned}$$

The last step is not obviously a well-known identity but clear from inspection of fig.C-1. First the direction is o.k - that of the $\underline{k}$ vector. Secondly the dot product of $\underline{\omega}_{T0}$ on $\underline{\omega}_T \wedge \underline{k}$ must be the same in magnitude as the vector product of $\underline{\omega}_{T0}$ on $\underline{\omega}_T$

$$= \underline{\omega}_{T0} \wedge \underline{\omega}_{T1}$$

The second term

$$\begin{aligned} (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega} \wedge \underline{k}) &= (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_0 \wedge \underline{k}) + (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_1 \wedge \underline{k}) \\ &= (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_0 \wedge \underline{k}) + (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_{T1} \wedge \underline{k}) \end{aligned}$$

NOTE_2

Starting with the term

$$(1/2) (\underline{\omega}_0 \cdot \underline{k})^2$$

then differentiating yields

$$\begin{aligned} (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_0 \cdot d\underline{k}/dt) &= (\underline{\omega}_0 \cdot \underline{k}) (\underline{\omega}_0 \cdot \underline{\omega} \wedge \underline{k}) \end{aligned}$$

and it remains to show that

$$\underline{\omega}_0 \cdot \underline{\omega} \wedge \underline{k} = - (\underline{\omega}_0 \wedge \underline{k}) \cdot \underline{\omega}_{T1}$$

Now by vector identity the LHS

$$\begin{aligned} &= - \underline{\omega} \cdot \underline{\omega}_0 \wedge \underline{k} \\ &= - (\underline{\omega}_{z1} + \underline{\omega}_{T1} + \underline{\omega}_0) \cdot \underline{\omega}_0 \wedge \underline{k} \\ &= - \underline{\omega}_{T1} \cdot (\underline{\omega}_0 \wedge \underline{k}) \end{aligned}$$

since by the usual vector identities $\underline{\omega}_{z1} \wedge \underline{k}$ must be zero and $\underline{\omega}_0 \wedge \underline{\omega}_0$ must be zero.

Starting with the term

$$\underline{\omega}_0 \cdot \underline{\omega}_z$$

then differentiating yields

$$= - \underline{\omega}_0 \cdot \underline{\omega} \wedge \underline{\omega}_z$$

since we have argued that is only being slewed - not changed in magnitude

$$\begin{aligned} &= \underline{\omega} \cdot \underline{\omega}_0 \wedge \underline{\omega}_z \\ &= (\underline{\omega}_{z1} + \underline{\omega}_{T1} + \underline{\omega}_0) \cdot (\underline{\omega}_0 \wedge \underline{\omega}_z) \\ &= \underline{\omega}_{T1} \wedge (\underline{\omega}_0 \wedge \underline{\omega}_z) \end{aligned}$$

The usual vector identities $\underline{\omega}_{z1} \wedge \underline{\omega}_z$ must be zero and $\underline{\omega}_0 \wedge \underline{\omega}_0$ must be zero.

NOTE_3

Starting with the term

$$(3/2) (\underline{k} \cdot \underline{r})^2$$

then differentiating yields

$$3 (\underline{k} \cdot \underline{r}) (\underline{dk}/dt \cdot \underline{r} + \underline{k} \cdot \underline{dr}/dt)$$

The need is to show that the second term is the same as

$$\underline{\omega}_{T1} \cdot (\underline{k} \wedge \underline{r})$$

in order to establish the identity.

Taking

$$\begin{aligned} & \left(\frac{d\underline{k}}{dt} \cdot \underline{r} + \underline{k} \cdot \frac{d\underline{r}}{dt} \right) \\ &= (\underline{\omega} \wedge \underline{k}) \cdot \underline{r} + \underline{k} \cdot (\underline{\omega}_0 \wedge \underline{r}) \end{aligned}$$

since the $\underline{k}$ vector is being slewed by whatever is the current angular motion of the satellite while the $\underline{r}$ vector is constantly rotating at the orbital rate ω_0 .

By the usual vector theorem

$$= (\underline{k} \wedge \underline{r}) \cdot \underline{\omega} - (\underline{k} \wedge \underline{r}) \cdot \underline{\omega}_0$$

And since $\underline{\omega} - \underline{\omega}_0$ is the relative velocity in the rotating reference frame

$$\begin{aligned} &= (\underline{k} \wedge \underline{r}) \cdot \underline{\omega}_1 \\ &= (\underline{k} \wedge \underline{r}) \cdot (\underline{\omega}_{T1} + \underline{\omega}_{Z1}) \\ &= (\underline{k} \wedge \underline{r}) \cdot \underline{\omega}_{T1} \end{aligned}$$

Since a first draft of this thesis the book by Hughes (1986) has appeared which approaches the problem on similar lines for the more general case (unequal moments of inertia on all 3 axes).

Fig. C-1 Assignment of vector components

D ANGULAR EQUATIONS OF MOTION

This Appendix first derives the exact differential equations of motion under the gravity gradient torque for the axial symmetry case of UDSAT2. The model is extended to allow for boom deployment and also the effect of magnetorquing

The reference system used here and the given meaning to the various angles have been defined in section 3.1.2.

The main concern is the attitude of the Z axis, which depends on the roll angle r and pitch angle p only. These angles also determine the orientation of the intermediate vectors $\underline{l}$ and $\underline{m}$; the yaw angle Φ is only required if the directions of the X and Y body axes must be specified.

D.1 Kinematic Definitions

The first requirement is to represent the relation between the angular velocity of the body and rates of changes of these angles. Actually this is most easily done in reverse. The effect of rates of change can be taken in any order. So the velocity of rotation of the specified axes can be found by considering the effect of rates of change dp/dt , dr/dt , and $d\Phi/dt$ separately.

The angular velocity components ω_x , ω_y and ω_z resolved along the body axes may be related to these rates of change in the angles; however it simpler to resolve the effects of the rates of change not along the body axes but on the intermediate axis set as defined in section 3.1.2

$$\begin{aligned} \text{Due to 'dp/dt':} \quad \omega_L &= (\omega_0 + dp/dt) \cos(r) \\ \omega_M &= 0 \\ \omega_Z &= (\omega_0 + dp/dt) \sin(r) \end{aligned} \quad \dots D.1.1$$

$$\begin{aligned} \text{Due to 'dr/dt':} \quad \omega_L &= 0 \\ \omega_M &= dr/dt \\ \omega_Z &= 0 \end{aligned} \quad \dots D.1.2$$

$$\begin{aligned} \text{Due to 'd}\phi\text{/dt':} \quad \omega_L &= 0 \\ \omega_M &= 0 \\ \omega_Z &= d\phi/dt \end{aligned} \quad \dots D.1.3$$

$$\begin{aligned} \text{Combining:} \quad \omega_L &= (\omega_0 + dp/dt) \cos(r) \\ \omega_M &= dr/dt \\ \omega_Z &= d\phi/dt + (\omega_0 + dp/dt) \sin(r) \end{aligned} \quad \dots D.1.4a$$

which set defines the component of velocity of the INTERMEDIATE system based on these component vectors. As a further step then the component velocities ω_X and ω_Y in the body axes could be found; but is not needed in this analysis. However a partial Z rotation needs to be defined

$$\begin{aligned} \Omega_Z &= \omega_Z - d\phi/dt \\ &= (\omega_0 + dp/dt) \sin(r) \end{aligned} \quad \dots D.1.4b$$

whose physical meaning is that component of the Z spin which would exist if there were no rotation with respect to the l and m intermediate vectors.

Differentiating Eqs.D.1.4a creates

$$\begin{aligned} d\omega_L/dt &= d^2p/dt^2 \cos(r) - dr/dt (\omega_0 + dp/dt) \sin(r) \\ d\omega_M/dt &= d^2r/dt^2 \\ d\omega_Z/dt &= d^2\phi/dt^2 + d^2p/dt^2 \sin(r) + dr/dt (\omega_0 + dp/dt) \cos(r) \end{aligned} \quad \dots D.1.5$$

D.2 Modified Euler Equations

The standard approach to the dynamical analysis takes as a starting point an expression for the angular momentum in terms of angular velocity components. Repeating

$$\underline{L} = \omega_x I_x \underline{x} + \omega_y I_y \underline{y} + \omega_z I_z \underline{z} \quad \dots A.4a$$

The modified Euler equations can be found following Thomson (1963). The angular momentum can be described alternatively as

$$\underline{L} = \omega_L I_T \underline{l} + \omega_M I_T \underline{m} + \omega_z I_z \underline{z} \quad \dots D.2.1$$

where the common transverse moment of inertia

$$I_T = I_x = I_y \quad \dots D.2.2$$

Differentiating D.2.2 and substituting Eq. A.1b. for the torque yields

$$\begin{aligned} \underline{N} = & d\omega_L/dt I_T \underline{l} + \omega_L I_T d\underline{l}/dt \\ & + d\omega_M/dt I_T \underline{m} + \omega_M I_T d\underline{m}/dt \\ & + d\omega_z/dt I_z \underline{z} + \omega_z I_z d\underline{z}/dt \end{aligned} \quad \dots D.2.3$$

and the modified Euler equations

$$\begin{aligned} = & d\omega_L/dt I_T \underline{l} + d\omega_M/dt I_T \underline{m} + d\omega_z/dt I_z \underline{z} \\ & + \begin{vmatrix} \underline{l} & \underline{m} & \underline{z} \\ \omega_L & \omega_M & \Omega_z \\ \omega_L I_T & \omega_M I_T & \omega_z I_z \end{vmatrix} \end{aligned} \quad \dots D.2.4$$

where Ω_z is the partial Z-spin of the 'l,m,z' system, and accounts for the the slewing of the $\underline{l}$ and $\underline{m}$ vectors in space.

In the motions of interest no torque vector is applied in the $\underline{z}$ direction. From inspection of D.2.4 therefore $d\omega_z/dt = 0$ and ω_z is a constant.

From D.2.4

$$N_L = d\omega_L/dt \ I_T + \omega_L (\omega_z \ I_z - \Omega_z \ I_T)$$

and then substituting from Eqs. D.1.5

$$\begin{aligned} &= d^2p/dt^2 \ \cos(r) \ I_T \\ &- (\omega_0 + dp/dt) \ \sin(r) \ dr/dt \ I_T \\ &+ dr/dt \ \omega_z \ I_z \\ &- (\omega_0 + dp/dt) \ \sin(r) \ dr/dt \ I_T \end{aligned}$$

It will be noted that the second and fourth terms of the RHS are the same. Therefore

$$\begin{aligned} N_L &= d^2p/dt^2 \ \cos(r) \ I_T \\ &+ dr/dt \ \omega_z \ I_z \\ &- 2 (\omega_0 + dp/dt) \ \sin(r) \ dr/dt \ I_T \end{aligned}$$

...D.2.5a

Similarly

$$\begin{aligned} N_M &= d\omega_M/dt \ I_T + \omega_L (\omega_z \ I_z - \Omega_z \ I_T) \\ &= d^2r/dt^2 \ I_T \\ &- (\omega_0 + dp/dt) \ \cos(r) \ \omega_z \ I_z \\ &+ (\omega_0 + dp/dt)^2 \ \sin(r) \ \cos(r) \ I_T \end{aligned}$$

...D.2.5b

D.2.1 Gravity gradient torque

Repeating Eq. B.1.1. with a change of notation of axes (from $\underline{r}$ to $\underline{z}_0$)
we have

$$\underline{N}_{gg} = 3 \ \omega_0^2 \ (I_T - I_z) \ (\underline{z} \wedge \underline{z}_0) \ (\underline{z} \cdot \underline{z}_0)$$

...

Now the reference vector $\underline{z}_0$ can be resolved as

$$\underline{z}_0 = (\underline{z}_0 \cdot \underline{l}) \ \underline{l} + (\underline{z}_0 \cdot \underline{m}) \ \underline{m} + (\underline{z}_0 \cdot \underline{z}) \ \underline{z}$$

...D.2.1.1

Then

$$\begin{aligned} \underline{z} \wedge \underline{z}_0 &= (\underline{z} \wedge \underline{l}) \ (\underline{z}_0 \cdot \underline{l}) + (\underline{z} \wedge \underline{m}) \ (\underline{z}_0 \cdot \underline{m}) \\ &= (\underline{z}_0 \cdot \underline{l}) \ \underline{m} - (\underline{z}_0 \cdot \underline{m}) \ \underline{l} \end{aligned}$$

...D.2.1.2

Then

$$\begin{aligned}\underline{z}_0 \cdot \underline{l} &= -\sin(r) \cos(p) \\ \underline{z}_0 \cdot \underline{m} &= \sin(p) \\ \underline{z}_0 \cdot \underline{z} &= \cos(r) \cos(p)\end{aligned}\quad \dots D.2.1.3$$

Hence expressions for the gravity gradient torque

$$N_L = -3 \omega_0^2 \sin(p) \cos(r) \cos(p) (I_T - I_Z) \quad \dots D.2.1.4a$$

$$N_M = -3 \omega_0^2 \cos(p)^2 \sin(r) \cos(r) (I_T - I_Z) \quad \dots D.2.1.4b$$

D.2.2 Effect of boom deployment

Combining Eqs.D.2.5 with Eqs.D.2.1.4 is almost but not quite sufficient to describe fully the UOSAT angular motion under gravity gradient. A missing factor is the effect of deploying the boom (which is the point of this section). Since I_T is now time varying a term $\omega_L d(I_T)/dt$ needs to be added to the RHS of Eq.2.5a and a term $\omega_M d(I_T)/dt$ to the RHS of Eq.D.2.5b with appropriate substitution for ω_L and ω_M .

D.2.3 Effect of magnetorquing

Ignoring the effect of scaling and the necessary subsidiary calculations it may be assumed that the geomagnetic field is known in component form resolved on the $\underline{l}$ axis as B_L and the $\underline{m}$ axis as B_M . Only the effect of switching on the Z- axis magnetorquer need be considered - so the corresponding torques are

$$\begin{aligned}N_L &= -M_Z B_M \\ N_M &= +M_Z B_L\end{aligned}\quad \dots D.2.3.1$$

D.2.4 Final equations

With the necessary substitutions made for ω_L and ω_M , which are the angular rotations around the $\underline{l}$ and $\underline{m}$ axes respectively, the suitably modified Eqs. D.2.5 and combined with Eqs D.2.1, 4 now read

$$\begin{aligned} d^2p/dt^2 \cos(r) I_T = & - dr/dt \omega_z I_z \\ & + 2 (\omega_0 + dp/dt) \sin(r) dr/dt I_T \\ & - (\omega_0 + dp/dt) \cos(r) d(I_T)/dt \\ & - 3 \omega_0^2 \sin(p) \cos(r) \cos(p) (I_T - I_z) \\ & - M_z B_M \end{aligned} \quad \dots D.2.4.1a$$

$$\begin{aligned} d^2r/dt^2 I_T = & + (\omega_0 + dp/dt) \cos(r) \omega_z I_z \\ & - (\omega_0 + dp/dt)^2 \sin(r) \cos(r) \\ & - dr/dt d(I_T)/dt \\ & - 3 \omega_0^2 \cos(p)^2 \sin(r) \cos(r) (I_T - I_z) \\ & + M_z B_L \end{aligned} \quad \dots D.2.4.1b$$

D.3 Computer Coding

The requirement of the usual numerical algorithms is that equations of second order or higher need to be recast as a set of first order equations, with a normalised time scale. Defining

$$\begin{aligned} \text{pitch velocity} & y[1] = dp/dt/\omega_0 \\ \text{pitch displacement} & y[2] = p \\ \text{roll velocity} & y[3] = dr/dt/\omega_0 \\ \text{roll displacement} & y[4] = r \end{aligned} \quad \dots D.3.1a$$

and the first order derivatives

$$\begin{aligned} \text{pitch acceleration} & f[1] \\ \text{pitch velocity} & f[2] = y[1] \\ \text{roll acceleration} & f[3] \\ \text{roll velocity} & f[4] = y[3] \end{aligned} \quad \dots D.3.1b$$

Practical test was only required for zero Z-spin.

Modifying Eqs.D.2.4 these now read

```
f[1] := - (3/2)*(1-Iz/It)*SIN(2*y[2])
        - (1 + y[1])*It_dot/It
        + 2 (1 + y[1])*y[3] TAN(y[4])
        - y[3]*nz*(Iz/It)/COS(y[4])
        - N_scale*Mz*B[2]/It/COS(y[4]);

f[2] := y[1];

f[3] := - (3/2)*(1-Iz/It)*SIN(2*y[4])*COS(y[2])*COS(y[2])
        - y[3]*It_dot/It
        - (1/2)*SIN(2*y[4])*(1+y[1])*(1+y[1])
        + (1 + y[1])*COS(y[4])*nz*(Iz/It)
        + N_scale*Mz*B(1)/It;
```

f[4] := y[3];

why call it It_dot?
...D.3.2

where

It = transverse moment of inertia
Iz = Z-axis moment of inertia
It_dot = rate of change of It (during boom deployment)
nz = normalised z-spin (= w_z/w_0) (NOTE 2) — *where?*
N_scale = generalised parameter encompassing magnetorquer coil

and

assumptions about the magnetic field
Mz = 1, 0, or -1 (magnetorquer state)
B(1), B(2) = normalised magnetic field component within generalised
body co-ordinates (= B_L and B_H respectively)

The practical calculation is kicked off with some a-priori values given to y[1] to y[4]. With standard Runge-Kutta Merson computation a start and finish time are declared; the values y[1] to y[4] passed in and the suitably modified value passed back by the computation.

This is unnecessary

D.3.1 Boom deployment details

Tautologous
During boom deployment both It and It_dot are functions of time during the interval of boom deployment. Assuming the magnetorquer is not on (i.e. $M_z = 0$), the evaluation of N_scale, B(1) and B(2) can be ignored. A simple model (Fig.3.6.1-1) represents the spacecraft as 2 'boxes' - one the spacecraft proper, the other the boom end.

An expression for the transverse moment of inertia is simply derived

$$I_t := I_{t0} + (m*M/(m + M))*D^2 \quad \dots D.3.1.1$$

where

I_{t0} = transverse moment of inertia of the main body only
 plus transverse moment of inertia of the boom mass only
 M = mass of the main body
 m = mass of the boom end
 D = distance between the separate centres of mass of these two bodies

The boom itself is assumed weightless.

By differentiating the above obtain

$$I_{t_dot} := 2*(m*M/(m + M))*D*boom_vel \quad \dots D.3.1.2$$

where *boom_vel* is the velocity of boom deployment

It should be noted that it is necessary to go to all this trouble because on both UOSAT1 and UOSAT2 the deployment time was very slow - the ultimate decision was to take 15 minutes. And in this time the attitude of the spacecraft can change significantly.

As written in the computer program this became

$$\begin{aligned} boom_dist &:= boom_vel*time \\ I_t &:= I_{t0} + eff_mass*(boom_dist + 0.302)^2 \end{aligned} \quad \dots D.3.1.3$$

and

$$I_{t_dot} := 2*eff_mass*(boom_dist + 0.302)*boom_vel \quad \dots D.3.1.4a$$

and

$$eff_mass := m*M/(m+M) \quad \dots D.3.1.4b$$

while $D = 0.302$ is a dimension of the spacecraft between assumed mass centres of the main body and the boom mass before deployment starts.

The parameters values substituted for UOSAT2

M	= 62
m	= 2.63
$lt0$	= 2.06
Iz	= 1.09
$boom_vel = 0.462 * orbit_period / 2 / PI$	

(the boom deployed 0.462 metres/min - the orbit period in minutes)

what does this mean ?

D.3.2 Magnetorquing Details

The effect of magnetorquing is not complicated if instead of the IGRF model, the field is idealised as due to a dipole. No great complexity follows from tilting the dipole by 8° (TDIO model) . The orbit is simulated with the actual inclination of 98° of the real UOSAT2. Then in the course of repeated orbits the satellite sometimes passes close to the pole and sometimes not.

In Eqs.D.3.2 the value of N_scale is determined by UOSAT values

N_scale	= no of turns in spin axis torquer	200
	* current passed	0.18
	* edge dimension	0.337
	* edge	0.337
	* magnitude of equatorial field	20 E-6
	/ (orbital rate) ²	

...D.3.2.1

where

$$orbital_rate := orbital_period (mins) * 60 / 2 / PI$$

...D.3.2.2

In Eqs.D.3.2 the values of $B(1)$ and $B(2)$ are angular transforms of field magnitude model varying between a minimum of 1 (if over the magnetic equator) to a maximum of 2 (if over a magnetic pole). It is evaluated in the following way (PASCAL notation). Basic to the calculation is the simple procedure of rotation ie

$ROTATE(x,y,angle);$

...D.3.2.3

which has the meaning that two values x and y are 'passed' into the function along with the instruction to rotate by the specific angle.

Then the values x , y are passed back suitably transformed

Starting with an dipole inclined at an angle $inclin_m$

```
xM = COS(inclin_m);
yM = 0;
zM = SIN(inclin_m);
```

...D.3.2.4

these dipole values are rotated to the current local co-ordinate by successive rotations in this order

```
ROTATE(yM,zM,time*omega_E);
ROTATE(zM,xM,inclin_o);
ROTATE(yM,zM,time*omega_o)
```

...D.3.2.5

where

```
time      = current time
omega_o   = 1 ( time normalised to orbital rate )
omega_E   = orbit period ( mins )/60/24 ( rotation of the earth )
inclin_o  = fixed orbital inclination ( say 98 ° )
```

...D.3.2.6

Then the normalised magnetic field in local reference co-ordinates from the classic dipole model is

```
B[1] := - xM
B[2] := - yM
B[3] := 2*zM
```

...D.3.2.7

This must be rotated to the generalised body co-ordinates by the rotations which again must be in the right order

```
ROTATE(B[2],B[3],y[2])
ROTATE(B[3],B[1],y[4])
```

...D.3.2.8

and the values $B[1]$, $B[2]$ now returned may be substituted in Eqs.D.3.2

E MAGNETIC OBSERVATIONS ON NUTATION

A rigid body with axial symmetry nutating with nutation angle θ in a static magnetic field $\underline{B}$ generates characteristic waveforms B_x , B_y , B_z . This appendix provides supporting mathematics to section 4.2

Following a common practice it is convenient to define a special reference co-ordinate system which will be given the designation X_0 , Y_0 and Z_0 but is NOT the same as the usual local reference system adopted from section 3.1.

Here the Z_0 axis is the current direction of the angular momentum vector $\underline{L}$. The B field makes some angle X to $\underline{L}$ which can be anywhere in the range 0° to 180° and serves to define always the positive direction of a Y_0 axis (see fig.3.1.5-1). The reference axis X_0 completes the orthogonal set. Therefore by definition

$$\begin{aligned} B_{x0} &= 0 \\ B_{y0} &= B \sin(X) \\ B_{z0} &= B \cos(X) \end{aligned} \quad \dots E.1$$

B_{y0} is always positive while B_{z0} can have either sign

As is well known the spinning/nutating state most readily matches the 3-1-3 Eulerian angular description. From Wertz p.418 and Eqs. 12-20 it follows that

$$\begin{aligned} B_x &= B_{y0} (\cos\psi \sin\phi + \cos\theta \sin\psi \cos\phi) + B_{z0} \sin\theta \sin\psi \\ B_y &= B_{y0} (-\sin\psi \sin\phi + \cos\theta \cos\psi \cos\phi) + B_{z0} \sin\theta \cos\psi \\ B_z &= -B_{y0} \sin\theta \cos\phi + B_{z0} \cos\theta \end{aligned} \quad \dots E.2$$

and without imposing special constraint one may set

$$\psi = \omega_A t = 2 \pi f_A t \quad \dots E.3a$$

$$\phi = \omega_B t = 2 \pi f_B t \quad \dots E.3b$$

where t is flow of time

$$\begin{aligned}\sin(\phi+\psi) &= \sin\phi\cos\psi + \sin\psi\cos\phi \\ \sin(\phi-\psi) &= \sin\phi\cos\psi - \sin\psi\cos\phi\end{aligned}$$

$$\frac{1+\cos\theta}{2} = \cos^2\frac{\theta}{2}$$

$$\frac{1-\cos\theta}{2} = \sin^2\frac{\theta}{2}$$

Substituting Eqs.E.1 in Eqs.E.2 then the kinematic magnetic waveforms (and setting $B = 1$) are

$$\begin{aligned} B_x &= \sin X (\cos \psi \sin \phi + \cos \theta \sin \psi \cos \phi) + \cos X \sin \theta \sin \psi \\ B_y &= \sin X (- \sin \psi \sin \phi + \cos \theta \cos \psi \cos \phi) + \cos X \sin \theta \cos \psi \\ B_z &= - \sin X \sin \theta \cos \phi + \cos X \cos \theta \end{aligned}$$

...E.4

where it is understood that ψ and ϕ are time varying according to Eqs.E.3 while the nutation angle θ is a constant; and setting $B = 1$.

The usual harmonic expansion identifies 3 component frequencies in B_x and B_y .

From the expansion of

$$\begin{aligned} &\cos \psi \sin \phi + \cos \theta \sin \psi \cos \phi \\ &= (\sin(\phi + \psi) + \sin(\phi - \psi) + \cos \theta \sin(\phi + \psi) - \cos \theta \sin(\phi - \psi)) / 2 \end{aligned}$$

Then the three components in B_x are

	waveform	FREQUENCY
B_{x1}	$= \cos^2(\theta) \sin(X) \sin((\omega_B + \omega_A)t)$	$f_B + f_A$
B_{x2}	$= \sin^2(\theta) \sin(X) \sin((\omega_B - \omega_A)t)$	$f_B - f_A$
B_{x3}	$= \sin(\theta) \cos(X) \sin(\omega_A t)$	f_A

...E.6

(similarly in B_y but different phases for all the frequencies)

The discussion of section 4.1 identified type I and type II situations It is not so clear from this algebra but is clear from the geometry that the borderline conditions are

either

$$\theta = X$$

or

$$\theta = 180^\circ - X$$

where

$$0^\circ < \theta, X < 180^\circ$$

The borderline case is identified when BOTH B_x and B_y are zero instantaneously. In type I case the B vector points outside the nutational cone and in consequence the mean rate of zero crossings

$$f_B = f_A + f_B$$

which corresponds to term B_x , dominating in Eq.E.6

The type II case has the B vector pointing inside the nutational cone.

Suppose that the nutation angle were $\theta = 90^\circ$. Going back to the original equations E.4 with appropriate substitution for $\cos(\theta)$ and $\sin(\theta)$ then body nutation rate $f_A = 0$ and ψ is ~~any constant~~ some constant.

$$\begin{aligned} B_x &= \sin(X) \cos(\psi) \sin(\omega_B t) + \cos(X) \sin(\psi) \\ B_y &= \sin(X) \sin(\psi) \sin(\omega_B t) + \cos(X) \cos(\psi) \\ B_z &= -\sin(X) \cos(\omega_B t) \end{aligned}$$

...E.7

For $\psi = 0^\circ$ or 180° then B_x is seen to oscillate at the inertial nutation rate and B_y is a constant. But if $\psi = \pm 90^\circ$ then B_x is a constant while B_y oscillates at the inertial nutation rate. The explanation geometrically is simply that in a flat spin the Z axis is being swept round in the X_0 and Y_0 plane (at the inertial nutation rate which also equals the transverse T-spin rate in this limiting case). In general all axes intercept the B vector at the inertial nutation rate f_B . If no modulation is seen on one of the body axes that is simply because it happens to be parallel or anti-parallel with the B vector to that body axes lining up with the Z_0 axis.

If now it is envisaged that the nutation angle is not exactly 90° then there will be a residual body nutation rate ; and the X and Y axes will interchange modulations as ψ increments (or decrements) steadily. This effect was shown in fig.4.2-1f. For both B_x and B_y there will be intervals in which a 'crop' of zero crossings are generated at the inertial nutation rate f_B . The the body nutation rate f_A can be inferred reliably only by trying to ignore these and measure the time interval between peaks in the B_x or B_y waveforms - a measurement which is necessarily less precise.

F QUASI-ORTHOGONAL ESTIMATOR

Details of a method for compensating for drifts in offset and scaling of the output of a 3-axis magnetometer are given

Magnetic field measurements are read from telemetry in arbitrary units and scaled and shifted as required to measurements B_x , B_y and B_z . If there is an error in this conversion then there will be a disagreement between the field magnitude

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

and magnitude B_0 calculated from IGRF model.

With a sufficient record extending over many orbits correction may be computed automatically. For convenience a change of symbolism

$$\begin{aligned} B_x &\rightarrow X \\ B_y &\rightarrow Y \\ B_z &\rightarrow Z \end{aligned}$$

F.1 Phase I correction

The aim is to find corrective coefficients t_1 , c_0 and c_1 such that the mean square error E_M is minimised in the expression

$$E_M = X^2 (1 - t_1)^2 + Y^2 (1 - t_1)^2 + (Z - c_0 - c_1 Z)^2 - B_0^2 \quad \dots F.1.1$$

where X , Y and Z are current measurements of the magnetic field components and B_0 is the computed reference field. Differentiating with respect to c_0 , c_1 and t_1 in that order this is equivalent to the quasi-orthogonal estimate E_M

$$\begin{aligned} \langle E_M Z \rangle &= 0 \\ \langle E_M Z^2 \rangle &= 0 \\ \langle E_M T^2 \rangle &= 0 \end{aligned}$$

...F.1.2a

where

$$T^2 = X^2 + Y^2 \quad \dots F.1.2b$$

By 'quasi-orthogonal' I mean that Eqs F.1.2 are truly orthogonal only for small corrections

$$\begin{aligned} |t_1| &\ll 1 \\ |c_1| &\ll 1 \\ |c_0| &\ll |Z| \end{aligned}$$

If E is the error *without* applying the corrections c_0 , c_1 and t_1 then

$$E = E_M + 2 T^2 t_1 + 2 Z c_0 + 2 Z^2 c_1 \quad \dots F.1.3$$

Then substituting in Eqs.F.1.1

$$\begin{aligned} \langle E Z \rangle &\approx 2 \langle T^2 Z \rangle t_1 + 2 \langle Z^2 \rangle c_0 + 2 \langle Z^3 \rangle c_1 \\ \langle E Z^2 \rangle &\approx 2 \langle T^2 Z^2 \rangle t_1 + 2 \langle Z^3 \rangle c_0 + 2 \langle Z^4 \rangle c_1 \\ \langle E T^2 \rangle &\approx 2 \langle T^4 \rangle t_1 + 2 \langle Z T^2 \rangle c_0 + 2 \langle Z^2 T^2 \rangle c_1 \end{aligned} \quad \dots F.1.4$$

This simplifies if the ideal dipole model is adopted.

$$\begin{aligned} Z &\propto 2 \cos(\lambda) \\ T^2 &\propto \sin^2(\lambda) \end{aligned} \quad \dots F.1.5$$

where λ is the advance in the course of an orbit.

Then averaged over a number of orbits on average

$$\begin{aligned} \langle T^2 Z \rangle &\approx 0 \\ \langle Z^3 \rangle &\approx 0 \end{aligned} \quad \dots F.1.6$$

simplifying Eqs.F.1.4 to

$$\begin{aligned} \langle E Z \rangle &\approx 2 \langle Z^2 \rangle c_0 \\ \langle E Z^2 \rangle &\approx 2 \langle T^2 Z^2 \rangle t_1 + 2 \langle Z^4 \rangle c_1 \\ \langle E T^2 \rangle &\approx 2 \langle T^4 \rangle t_1 + 2 \langle Z^2 T^2 \rangle c_1 \end{aligned} \quad \dots F.1.7$$

Because the calculation is iterated it is not necessary that the last set of equations should be exact

F.2 Phase II Correction

The aim is to find coefficients d_1 , a_0 and b_0 such that the mean square error E_M is minimised in the expression

$$E_M = (X - a_0 - d_1 X)^2 + (Y - b_0 + d_1 Y)^2 + Z^2 - B_0^2 \quad \dots F.2.1$$

where X , Y and Z are current estimates of the magnetic field components and B_0 is the computed reference field

Differentiating with respect to a_0 , b_0 and d_1 in that order ~~that~~ is equivalent to the quasi-orthogonal estimate

$$\begin{aligned} \langle E_M X \rangle &= 0 \\ \langle E_M Y \rangle &= 0 \\ \langle E_M D^2 \rangle &= 0 \end{aligned} \quad \dots F.2.2$$

where

$$D^2 = X^2 - Y^2 \quad \dots F.2.3$$

By 'quasi-orthogonal' I mean that Eqs F.1.2 are truly orthogonal only for small corrections

$$\begin{aligned} |d_1| &\ll 1 \\ |a_0| &\ll |X| \\ |b_0| &\ll |Y| \end{aligned}$$

If E is the error *without* applying the corrections a_0 , b_0 and d_1 then

$$E \approx E_M + 2 X a_0 + 2 Y b_0 + 2 D^2 d_1 \quad \dots F.2.4$$

Then substituting in Eqs.F.2.2

$$\begin{aligned} \langle E X \rangle &= 2 \langle X^2 \rangle a_0 + 2 \langle X Y \rangle b_0 + 2 \langle X D^2 \rangle d_1 \\ \langle E Y \rangle &= 2 \langle X Y \rangle a_0 + 2 \langle Y^2 \rangle b_0 + 2 \langle Y D^2 \rangle d_1 \\ \langle E D^2 \rangle &= 2 \langle X D^2 \rangle a_0 + 2 \langle Y D^2 \rangle b_0 + 2 \langle D^4 \rangle d_1 \end{aligned} \quad \dots F.2.5$$

Assuming a constant Z spin it may be assumed that

$$\begin{aligned} X &\propto \cos(\omega_z t) \\ Y &\propto \sin(\omega_z t) \end{aligned} \quad \dots F.2.6$$

Then averaged over a number of cycles on average

$$\langle X Y \rangle \approx \langle X D^2 \rangle \approx \langle Y D^2 \rangle \approx 0 \quad \dots F.2.7$$

Simplifying Eqs.2.5 to

$$\begin{aligned} \langle E X \rangle &\approx 2 \langle X^2 \rangle a_0 \\ \langle E Y \rangle &\approx 2 \langle Y^2 \rangle b_0 \\ \langle E D^2 \rangle &\approx 2 \langle D^4 \rangle d_1 \end{aligned} \quad \dots F.2.8$$

F.3 Computation

It will be seen that the combination of corrections d_1 and t_1 as sum and difference combinations allow independent re-scaling in X and Y. This unusual combination arose out of the need to find quasi-Fourier relations which exploited the cyclic nature of the B_z field (varying as one cycle per orbit, and the faster quadrature relationship between B_x and B_y . The method depends on the continuous spinning in Z to work. Empirically it is found that the algorithm converges in a few iterations by repeated, and alternating application of phase I and phase II. Because the calculation is iterated it is not necessary that the last set of equations should be exact.

G MAGNETIC CONTROL THEORY

The basic relations justifying the magnetometer-magnetorquer rules of section 5 are proved formally here. The basic assumption is that the rate of angular motion is faster than the rate of change of the magnetic field.

G.1 Basic relation

All basic relations in section 5.1 can be justified by appeal to general vector expansion:

$$[d\mathbf{B}/dt] + \underline{\omega} \wedge \mathbf{B} = d\mathbf{B}/dt \quad \dots G.1.1$$

where [] means an 'in-place' differentiation

Assuming a negligible absolute rotation of field then

$$[d\mathbf{B}/dt] + \underline{\omega} \wedge \mathbf{B} = 0 \quad \dots G.1.2$$

Expanding in terms of X, Y and Z components

$$[dB_x/dt] + \omega_y B_z - \omega_z B_y = 0 \quad \dots G.1.3a$$

$$[dB_y/dt] + \omega_z B_x - \omega_x B_z = 0 \quad \dots G.1.3b$$

$$[dB_z/dt] + \omega_x B_y - \omega_y B_x = 0 \quad \dots G.1.3c$$

It is proposed to abandon the [] notation on the components since there is no ambiguity. For example dB_x/dt (strictly $[dB_x/dt]$) is what the magnetometer measures. The basic torque relation

$$\mathbf{N} = \mathbf{M} \wedge \mathbf{B} \quad \dots G.1.4$$

Expanded in component form

$$N_x = M_y B_z - M_z B_y \quad \dots G.1.4a$$

$$N_y = M_z B_x - M_x B_z \quad \dots G.1.4b$$

$$N_z = M_x B_y - M_y B_x \quad \dots G.1.4c$$

The notation $\{ \}$ is used to mean the ' sign of ' whatever is enclosed. It has the values of +1 or -1.

The symbol $s = +1$ or -1 . The general purpose constant S is used such that

$$s = \{ S \}$$

G.2 Spin Up/Down

Rule:

$$\{ M_x \} = \{ S \} \{ dB_x/dt \} \text{ spin down/up, } (B_x = 0) B_z \approx 0 \quad \dots G.2.1$$

Proof: if $\omega_y = 0$ (no significant nutation) then from Eq.G.1.2

$$\{ dB_x/dt \} = \{ \omega_z \} \{ B_y \} \quad \dots G.2.2$$

and since Eq.G.1.4a

$$\{ M_x \} = \{ N_z \} \{ B_y \} \quad \dots G.2.3$$

and since we want

$$\{ N_z \} = S \{ \omega_z \} \quad \dots G.2.4$$

Then Eq.G.2.1 is proved, by comparing signs,

G.3 Denutate -Spin Axis

Rule:

$$\{ M_z \} = - \{ dB_z/dt \} \quad \dots G.3.1$$

This is formally similar to Eq.G.2.1 with $S = -1$.

Formal proof as follows:

Resolved

$$\underline{\omega} = \omega_z \underline{k} + \omega_T \underline{t} \quad \dots G.3.2a$$

where

$\underline{k}$ is along Z axis and $\underline{t}$ is orthogonal

Similarly

$$\underline{B} = B_z \underline{k} + B_T \underline{t} \quad \dots G.3.2b$$

therefore, substituting these relations in G.1.2

$$dB_z/dt \underline{k} + (\underline{\omega}_T \wedge \underline{B}_T) = 0 \quad \dots G.3.3$$

all other components cancel. But

$$\underline{N} = \underline{M}_z \wedge \underline{B}_T$$

Therefore

$$\begin{aligned} \underline{\omega}_T \cdot \underline{N} &= \underline{\omega}_T \cdot (\underline{M}_z \wedge \underline{B}_T) \\ &= \underline{M}_z \cdot (\underline{B}_T \wedge \underline{\omega}_T) \\ &= M_z \cdot dB_z/dt \end{aligned} \quad \dots G.3.4$$

Therefore the denutation rule always makes the scalar product $\underline{\omega}_T \cdot \underline{N} < 0$ and removes transverse energy from the system

G.4 Denutate Spin Plane

Rule:

$$\{ M_x \} = \{ B_z \} \{ B_x \} \{ dB_z/dt \}, \quad B_y = 0 \quad \dots G.4.1$$

Proof: if $B_y = 0$ then M_x creates N_y torque only

$$\begin{aligned} \{ N_y \} &= - \{ M_x \} \{ B_z \} \\ \text{or} \\ \{ M_x \} &= - \{ N_y \} \{ B_z \} \end{aligned} \quad \dots G.4.2$$

$$\text{De-nutation requires } \{ N_y \} = - \{ \omega_y \}. \quad \dots G.4.3$$

From Eq.G.1.3c and setting $B_y = 0$

$$\{ dB_z/dt \} = \{ \omega_y \} \{ B_x \} \quad \dots G.4.4$$

Combining G.4.2 , G.4.3 and G.4.4 establishes G.4.1

G.5 Precession Spin Axis Control

Rule:

$$\{M_z\} = -S \{ \omega_z \} \quad \dots G.5.1$$

Proof: From geometry

$$\underline{N} = S \underline{B} \wedge \underline{L} \quad \dots G.5.2a$$

where

$\underline{L}$ = angular momentum
 $S > 0$ is a positive precession
 $S < 0$ is a negative precession

But

$$\underline{N} = \underline{M} \wedge \underline{B} = - \underline{B} \wedge \underline{M} \quad \dots G.5.2b$$

Momentum $\underline{L}$ is either parallel or antiparallel to $\underline{M}$. From G.5.2a and G.5.2b

$$S \{ \underline{L} \} = - \{ \underline{M} \} \quad \dots G.5.3$$

set

$$\underline{M} = M_z \underline{z}$$

and

$$\underline{L} = \omega_z I_z \underline{z}$$

then

$$M_z = -S \omega_z I_z$$

which establishes G.5.1 since $I_z > 0$

G.6 Precession Spin-Plane Control (using M_x)

$$\text{Rule: } \{ M_x \} = \{ S \} \{ \omega_z \} \{ B_x \} \{ B_z \}, \quad B_y = 0 \quad \dots G.6.1a$$

Alternative rule:

$$\{ M_x \} = - \{ S \} \{ [dB_y/dt] \} \{ B_z \} \quad \dots G.6.1b$$

Proof: use index unit vector system $\underline{i}$, $\underline{j}$ and $\underline{k}$ then general torque expression

$$\begin{aligned} \underline{N} &= M_x \underline{i} \wedge \underline{B} \\ &= M_x B_y \underline{i} \wedge \underline{j} + M_x B_z \underline{i} \wedge \underline{k} \\ &= M_x B_y \underline{k} - M_x B_z \underline{j} \end{aligned} \quad \dots G.6.2$$

Require $\underline{N} \cdot \underline{k} = 0$ (to be orthogonal to $\underline{z}$), so that there is no spin-up or down. Therefore $B_y = 0$ is a necessary condition and

$$N_y = - M_x B_z$$

Comparing spin-axis and spin plane control in these conditions and disregarding constants

$$\begin{aligned} \underline{N} &= M_z \underline{k} \wedge \underline{B} \\ &= M_z B_x \underline{k} \wedge \underline{i} \\ &= M_z B_x \underline{j} \end{aligned}$$

...G.6.3

therefore

$$\{ M_x \} = - \{ M_z \} \{ B_x \} \{ B_z \}$$

...G.6.4

establishes the same torque as in the spin AXIS condition. Substitute G.6.4 into G.5.1 establishes G.6.1a

The alternative rule starts from Eq.G.1.3b statement

$$\{ dB_y/dt \} = - \{ \omega_z \} \{ B_x \} \quad \text{if } \omega_x = 0$$

substituting into G.6.1a establishes Eq.G.6.1b

G.7 Direct Cross Product Law

Rule: if $d\underline{L}$ is the required change in momentum then set

$$\underline{M} \propto \underline{B} \wedge d\underline{L} \quad (\text{White 1961})$$

...G.7.1

The criterion is to maximise scalar product

$$p = \underline{N} \cdot d\underline{L}$$

...G.7.2

then

$$\begin{aligned} p &= (\underline{M} \wedge \underline{B}) \cdot d\underline{L} \\ &= (\underline{B} \wedge d\underline{L}) \cdot \underline{M} \end{aligned}$$

Therefore pointing $\underline{M}$ in the same direction as $\underline{B} \wedge d\underline{L}$ will maximise p .

G.8 Projected CPL Law (Meinzer version)

If $\underline{M}$ cannot be pointed directly as in the direct law then point it at nearest possible angle. In this proof $\underline{M}$ is confined to the spin plane.

Rule: If $d\underline{L}$ is the required change in momentum then calculate

$$\underline{v} = \underline{B} \wedge d\underline{L}$$

...G.8.1

The criterion is again to maximise scalar product

$$p = \underline{N} \cdot d\underline{L}$$

Calculate

$$\underline{v} = \underline{B} \wedge d\underline{L}$$

and rotate $\underline{M}$ until it coincides with projection of $\underline{v}$ on the spin plane

Proof:

$$\begin{aligned} p &= (\underline{M} \wedge \underline{B}) \cdot d\underline{L} \\ &= (\underline{B} \wedge d\underline{L}) \cdot \underline{M} \\ &= \underline{v} \cdot \underline{M} \end{aligned}$$

resolve

$$\underline{v} = \underline{v}' + \underline{v}''$$

...G.8.2

where

$\underline{v}'$ is aligned on the $\underline{L}$ vector - the spin axis
 $\underline{v}''$ is orthogonal residual aligned on the spin plane

Since $\underline{M}$ is always orthogonal to $\underline{v}'$ therefore

$$p = \underline{v}'' \cdot \underline{M}$$

...G.8.3

so that a simple rotation of $\underline{M}$ until it coincides with direction of $\underline{v}''$ will maximise p - which completes proof.

G.9 Reciprocity Principle

The rule is $\underline{M} = S [d\underline{b}/dt]$

...G.9.1

which will maximise rate of change of angular kinetic energy

The analysis starts with Teule (1979) and first proves that the apparent $d\mathbf{B}/dt$ responds only to that component of the angular velocity which is normal to $\mathbf{B}$.

Normalise magnetic vector

$$\mathbf{b} = \mathbf{B}/B \quad \dots G.9.2$$

Repeating G.1.2 in normalised form

$$d\mathbf{b}/dt = [d\mathbf{b}/dt] + \boldsymbol{\omega} \wedge \mathbf{b} = 0 \quad \dots G.9.3$$

Resolve general angular velocity $\boldsymbol{\omega}$ into two components: one parallel to $\mathbf{b}$; the other orthogonal

$$\boldsymbol{\omega} = \boldsymbol{\omega}_p + (\boldsymbol{\omega} \cdot \mathbf{b}) \mathbf{b} \quad \dots G.9.4$$

Therefore

$$[d\mathbf{b}/dt] = - \boldsymbol{\omega}_p \wedge \mathbf{b} \quad \dots G.9.5$$

And since $\boldsymbol{\omega}_p$ is orthogonal to $\mathbf{b}$ and $|\mathbf{b}| = 1$

$$\boldsymbol{\omega}_p = [d\mathbf{b}/dt] \wedge \mathbf{b} \quad \dots G.9.6$$

($\boldsymbol{\omega}_p$, $[d\mathbf{b}/dt]$ and $\mathbf{b}$ are mutually orthogonal). Now

$$\mathbf{N} = \mathbf{M} \wedge \mathbf{B}$$

And from rule G.9.1 compare G.9.6

$$\boldsymbol{\omega}_p = S \mathbf{N} \quad \dots G.9.7$$

Therefore depending on the sign S the torque is either parallel or anti-parallel to the component angular velocity.

Now the scalar product of the torque $\mathbf{N}$ with the current angular velocity $\boldsymbol{\omega}$ = rate of change of KE. Since the velocity and the torque are parallel (Eq.G.9.7) the rate of change must be maximum.

Alternative argument is as follows. If $\underline{M}$ is pointed in any other direction than $d\underline{b}/dt$ then it must have a component in the $\underline{b}$ direction and/or a component in the $\underline{\omega}_p$ direction (the other two orthogonal directions). With the former it can create no torque; with the latter it creates a torque orthogonal to $\underline{\omega}_p$. Hence only the component of $\underline{M}$ in the $d\underline{b}/dt$ direction affects $d(KE)/dt$. Hence the effect is maximised by pointing $\underline{M}$ actually in the direction $d\underline{b}/dt$.

Note that $\underline{L}$ and $\underline{\omega}$ do not point in the same direction instantaneously for rigid body. It is pointing the torque in the same direction as $\underline{\omega}$ rather than $\underline{L}$ which changes energy maximally.

H EXACT PRECESSION/NUTATION

This appendix clarifies the relation between precession
(magnetically induced) and the nutational state.

The conventional description states that a spacecraft (for example) spinning has a vector angular momentum $\underline{L}$ pointing either in the direction of the Z (spin) axis or the opposite (depending on the sign of the spin). There is a vector field $\underline{B}$ assumed to be some angle χ to the direction of the Z axis. Assuming a spin axis magnetorquer (section 5.1) a magnetic moment M_z creates a magnetic torque $\underline{N}$ orthogonal to both the Z axis and the local $\underline{B}$ vector. The torque is therefore orthogonal to the angular momentum and cannot change the latter's magnitude; instead the spacecraft will be slewed in such a direction that the angular momentum maintains the current angle χ to the $\underline{B}$ vector. Hence the Z axis describes a cone in space. That is precession as normally described (with equations given in section 5.1.2).

There is more to it than that. The orthodox explanation is incomplete. It fails to account for a quite possible situation where the existing angular momentum is relatively weak. I envisage the spacecraft spinning slower and slower; in the limit of no spin then obviously the torque tries to slew the spacecraft *towards* the magnetic field and subsequently can set up a rocking harmonic motion in a plane (not a cone) passing forward and backward through the field. This effect of course we used in the transitional magnetic capture of UOSAT2 (see section 5.2). And there is clearly an angular motion which is neither one thing nor the other.

I now envisage the satellite having an initial state with a sufficiently slow Z-spin and the Z axis directed at right angles to the $\underline{B}$ vector. If the magnetorquer were switched on (with positive polarity) the Z axis will accelerate towards the $\underline{B}$ vector initially.

However because of the spin it will never actually pass through the B vector but will pass to one side or the other. The precise trajectory prior and subsequent to this would be interesting to work out. It cannot be a pure conical motion.

The pure conical motion is a special case and strictly depends on precisely the right start-up conditions with some unseen hand giving the motion a kick in the right direction which the magnetorquing then maintains. In practice if the precession rate is small compared to the spin rate we are supposed not to worry because departures from the conical motion are deemed not to be significant.

What actually is the precession rate ?

Adapting the analysis from Appendix B (Eq.B.2.9) then a torque N drives a response

$$N = I_T \omega_p^2 \cos(X) \sin(X) + I_z \omega_p \omega_z \sin(X) \quad \dots H.1$$

where

$$N = M_z B \sin(X) \quad \dots H.2$$

An exact analysis of this special state yields

$$\omega_p = - \frac{M_z B_M}{I_z \omega_z} + \frac{I_T \omega_p^2}{I_z \omega_z} \cos(X) \quad \dots H.3$$

where

B_M = magnitude of the magnetic field
 I_z = Moment of inertia about the Z axis
 ω_z = spin rate about the Z axis
 I_T = common transverse moment of inertia (X or Y)
 X = cone angle

The formula frequently quoted for the precession rate (E.g Wertz P 636 -638) includes only the first term of the RHS and is valid only if the second term is relatively small.

Note that the full equation predicts pure conical motion *even if the magnetorquer is switched off*. Using then the second term only and re-arranging terms implies an exact relation:

$$\omega_z = I_T/I_z \omega_p \cos(X)$$

...H.4

which can be confirmed more quickly by an analysis of the state of nutation for a body with axial symmetry. It is exactly the same as Eq.3.2.2.2 with $\omega_p \rightarrow \omega_B$ (the inertial nutation rate); and $X \rightarrow \theta$.

I STATISTICALLY OPTIMUM ESTIMATORS

A simplified approach to Kalman filtering for a real continuous-time model is possible without any matrix or discrete analysis. First applied to the simplest possible RC system it is then used to give statistically optimum solutions to optimum gains for both the spinning (PLL) estimator and the cyclic (harmonic) estimators discussed in chapter 7.

The aim is to estimate continuously the spacecraft angular motion, based on the partial information supplied by the on-board magnetometer. These estimators will 'fill in the gaps' when (or where) the attitude information is not available, yet are always updated when the attitude information is available. Because the 3-axis attitude is now continuously determined by a mixture of measurement and prediction it should be possible to apply magnetorquing in the most effective manner (see from section 7.5).

These estimators were embedded in a control structure which would seem identical to the formulations of modern control theory - see for example 'non-statistical estimator design with full estimator dimension' (from p.149 of Anderson and Moore (1971)).

The spinning (PLL) type filter and the cyclic or harmonic filters were developed deterministically in order to realise a desirable transient behaviour. In the background however was the notion of *Kalman filtering*. Both my estimators are capable of being analysed statistically. Optimum values can be found for the feedback gains which will minimise the overall error. The conventional pedagogy developing the Kalman filter - see for example Anderson and Moore (1979) is intellectually formidable for the non-specialist.

A step in the direction of comprehensibility comes from Kailath (1971). He may have been the first to point out that it all follows from constructing a mathematical analogue copy of the system or 'plant' as the control people insist on calling it, to which are fed 'innovations'.

The error signals created in this note would all seem to be 'innovations' which terminology I have adopted. Following Kailath the Kalman filter can be visualised, discussed and analysed within an *analogue* or continuous time model; it is not true that a Kalman filter is inherently a discrete estimator (although it was first described and is normally presented as such). Nor is there any necessary connection with control theory. The Kalman filter is just another mean square estimator. With an analogue approach, and treating the filter as just another approach to estimation by minimising the mean square error, the mathematics greatly simplifies.

I.1 Kalman RC Filter

Fig.II-1 is my reworking of the Kailath/Kalman filter reduced to its simplest form (simplifying Kailath). The 'system ' or 'plant' is modelled by a white noise source *system noise* $q(t)$ - driving an RC filter of time constant τ (the resistor itself is assumed to be noiseless). The output $x(t)$ is not directly observable because of the presence of additive *measurement noise* $r(t)$. An estimator only gets to see their sum $y(t)$ and aims to make a best estimate $\hat{x}(t)$ in the mean square sense of the (non-observable) $x(t)$ in *real time*. This means that it is allowed to look back over the past but not into the future. The estimator knows the time constant RC of the system. It also knows the power spectral density of both system and measurement noise. What it does not know is the detailed structure of the two noise processes. So what does it do?.

The analogue Kailath/Kalman filter consists of an embedded model obeying exactly the same equations of the real system with matching inputs and outputs. The input to the filter is the noisy $y(t)$. The filter's output $\hat{x}(t)$ is the estimate of the system $x(t)$. Within the filter the difference between $x(t)$ and $\hat{x}(t)$ is fed as an innovation v via a Kalman gain K to the internal model which generates $\hat{x}(t)$.

The objective is a best estimate in the mean square sense of $x(t)$ by $\hat{x}(t)$. Intuitively it is clear that if the measurement noise were low relative to the system noise one would make the Kalman gain high so that the model closely follows $y(t)$. While if the measurement noise were high relative to the system noise one would make the Kalman gain low and let the model do 'its own thing'. In the steady state the optimum Kalman gain is a constant which is rather easily calculated here assuming that both system and measurement noise are stationary (having a constant power spectral density) and *white* (flat spectral density). The only knowledge of stochastic theory required is the mean square noise at the output of a noise-less RC filter (i.e a filter with a simple pole) having a white noise input. It is possible to cut a swathe through large quantities of mathematics which the conventional approach generates

From the figure and by inspection

$$\begin{aligned} v &= x + r - \hat{x} \\ \tau \, dx/dt + x &= q \\ \tau \, d\hat{x}/dt + \hat{x} &= K \, v \\ e &= x - \hat{x} \end{aligned}$$

...I.1.1

where

$$\begin{aligned} v &= \text{innovation} \\ K &= \text{Kalman gain} \\ e &= \text{the actual error in the estimate} \\ \tau &= \text{time constant of equiv RC filter} \end{aligned}$$

then

$$\tau \, de/dt + e \, (K + 1) = q + K \, r$$

...I.1.2

from which is easily solved

$$\langle e^2 \rangle = \frac{S_q}{4 \, \tau \, (K + 1)} + \frac{K^2 \, S_R}{4 \, \tau \, (K + 1)}$$

...I.1.3

where

$$\begin{aligned} S_q &= \text{white system noise density (driving the plant)} \\ S_R &= \text{white measurement noise} \end{aligned}$$

The optimum gain follows

$$K = -1 + \sqrt{1 + y} \quad \dots I.1.4$$

where

$$y = S_n/S_e$$

confirming the qualitative point that the higher the y value the higher is the desirable Kalman gain. The performance is perhaps best indicated by an output signal to noise ratio

$$\frac{\langle x^2 \rangle}{\langle e^2 \rangle} = \frac{y}{K} \rightarrow \frac{\sqrt{y}}{2} \quad \text{for large } y \quad \dots I.1.5$$

This result is identical, as a special case, to that obtained by rigorous analysis from Kailath's paper. His more general theory allows amongst other things for a transient effect, and introduces the well-known effect of a time-varying gain K in order to optimise the system immediately after switch-on.

With only one variable - so not requiring a matrix analysis - and a continuous analogue representation - the basic idea of innovation and updating of the model through a 'Kalman gain' stands out clearly. The filter itself functions in real-time, using knowledge of the past but having no access to the future. The optimum gain balances the two noise effects in order to minimise the overall error. Intuitively one can also see that it may not be optimal if the system or measurement noise are not white but at least will work (in this simple case). In particular if the measurement noise $r(t)$ should be non-stationary one can easily visualise turning down the Kalman gain when the noise m.s value is high and turning the gain down if and when the m.s of $r(t)$ decreases.

This is all thanks to Professor Kailath. The question remains : are the two different filters used in this work - the spinning PLL filter and the cyclic filters - actually Kalman filters and/or are they optimal ?

The true Kalman filter has the 'same structure as a class of deterministic estimators' Anderson and Moore (1979) P 44 .

This means that it is always possible to identify a *copy* of the signal-generating system or model within the estimator which is turn surrounded by a further feed back which generates an innovation returning to and driving the copy system through a gain.

In the same spirit the PLL filter and the harmonic filter can be seen to be embedded copies of the plant to which are fed innovations. I intend to duck the question whether either or both systems can be termed true Kalman filters - the main point is that both can be analysed statistically in a reasonably simple manner, by supposing, in both cases that system noise $q(t)$ and measurement noise $r(t)$ may be introduced at the points shown, and an optimum gain calculated which minimises the mean square error between $x(t)$ and $\hat{x}(t)$. To avoid argument I shall refer if necessary to the spinning PLL filter and the cyclic filter as as 'Kalman-like'.

I.2 The Spinning Estimator

Fig I.2-1 is a development of fig.7.1.3-1 with noise sources inserted. From inspection and using Laplace notation

$$\begin{aligned} N &= X + R - \hat{X} \\ E &= X - \hat{X} \\ \hat{X} &= N (K_1^2 \omega_0/s + K_2) \omega_0/s \\ X &= (Q_1 \omega_0/s + Q_2) \omega_0/s \end{aligned} \quad \dots I.2.1$$

where

Q_1 and Q_2 = assumed noise sources in the plant
 K_1^2 and K_2 = gains to the PLL equivalent estimator

Then

$$\begin{aligned} E (s^2 + \omega_0 K_2 s + K_1^2 \omega_0^2) \\ = Q_2 \omega_0 s + Q_1 \omega_0^2 - R (\omega_0 K_2 s + K_1^2 \omega_0^2) \end{aligned} \quad \dots I.2.2$$

Note change of notation compared to sec.7.1.3

The error E can be interpreted as the sum of the output of three equivalent filters which have statistically independent inputs R , Q_1 and Q_2 . So

$$E = E_1 + E_2 + E_3 \quad \dots I.2.3$$

$$E_1 = Q_1 \frac{\omega_0^2}{\omega_1^2 + \omega_2 s + s^2} \quad \dots I.2.4a$$

$$E_2 = Q_2 \frac{\omega_0 s}{\omega_1^2 + \omega_2 s + s^2} \quad \dots I.2.4b$$

$$E_3 = R \frac{\omega_1^2 + \omega_2 s}{\omega_1^2 + \omega_2 s + s^2} \quad \dots I.2.4c$$

where

$$\begin{aligned} \omega_1 &= \omega_0 K_1 \\ \omega_2 &= \omega_0 K_2 \end{aligned} \quad \dots I.2.5$$

Assigning noise densities and back substituting Eqs.I.2.4 it is easily shown (list of integrals at the end of this appendix) that

$$\begin{aligned} \langle e^2 \rangle &= \frac{\omega_0}{4 K_2} \left\{ \frac{S_{Q1}}{K_1^2} + S_{Q2} \right\} \\ &\quad + \frac{1}{4} \left\{ \frac{\omega_1^2}{\omega_2} + \omega_2 \right\} S_R \end{aligned} \quad \dots I.2.6$$

where

$$\begin{aligned} S_{Q1} &= \text{one-sided noise power density of } Q_1 \\ S_{Q2} &= \text{one-sided noise power density of } Q_2 \\ S_R &= \text{one-sided noise power density of } R \end{aligned}$$

from which an optimum choice of K_2 and K_1 could be established (but keeping a reasonable damping factor) provided there was information on the relative levels of S_R , S_{Q1} and S_{Q2} .

1.3 The harmonic estimator

The theoretical system used to model pitch and roll variations is a simple extension of an RC filter to a simple cyclic oscillator (fig.1.3.1). The upper block is an oscillator consisting of two integrators (internally scaled) and a negative feedback loop. This is the system producing an output state $x(t)$. White system noise $q(t)$ is introduced at the point shown. The oscillator now shows random variations in amplitude and phase over both short and long term. A real oscillator of course has some non-linearity which controls the amplitude, which is not modelled here (but the instability in phase remains).

The output $x(t)$ of the oscillator is then corrupted by measurement noise $r(t)$ to the observable $y(t)$ and is fed to the filter, within which, having first the estimate $\hat{x}(t)$, the innovation is fed to the model embedded oscillator. It up-dates the model through a gain K .

The aim is to make the model track the system. The deterministic analysis of section 7.1.4 showed that it will do so in a well-behaved manner for civilised values of gain. The model will get itself 'into line' with the system and at a rate which is directly controllable through the feedback gain K . Because of noise present the fit or match is not perfect.

Now the smaller the error gain K the less responsive is the model to measurement noise $r(t)$. But the larger the K value the more responsive is the model to actual changes in the system - the better it is able to track the fluctuations in the imperfect oscillator $x(t)$. An optimum value of K reconciles these claims.

From inspection again and in Laplace notation the equations are

$$\begin{aligned} N &= X + R - \hat{X} \\ E &= X - \hat{X} \\ X &= (- X \omega_0/s + Q) (\omega_0/s) \\ \hat{X} &= (- \hat{X} \omega_0/s + N K) (\omega_0/s) \end{aligned}$$

...1.3.1

Then

$$E (s^2 + s K \omega_0 + \omega_0^2) = s \omega_0 Q - s \omega_0 K R \quad \dots 1.3.2$$

Using results for single tuned circuit and assuming one-sided noise densities S_Q and S_R again then

$$\langle e^2 \rangle = \frac{\omega_0 K}{4} \left\{ \frac{S_Q}{K^2} + S_R \right\} \quad \dots 1.3.3$$

From which an optimum value can be established

$$K = \sqrt{S_Q/S_R} \quad \dots 1.3.4$$

Because the system is an oscillator the amplitude of its output $x(t)$ is independent of the 'driving' system noise. However it does not oscillate in error e (if it did of course the system would be unworkable).

It is not possible then to define an output signal-to-noise ratio as is possible in the naive Kalman system. However the mean square error $\langle e^2 \rangle$ is determinate and when minimised is seen to be proportional to the geometric product of the two noise densities.

Equivalent bandwidths

1 The function

$$H = \frac{\omega_1^2}{s^2 + \omega_2 s + \omega_1^2}$$

is an equivalent low pass (2-pole filter) with a bandwidth

$$B = \frac{1}{4 \pi j} \int_{-j\infty}^{+j\infty} \frac{\omega_1^2 ds}{(s^2 + \omega_2 s + \omega_1^2) (s^2 - \omega_2 s + \omega_1^2)}$$

$$= \frac{\omega_1^2}{4 \omega_2}$$

2 The function

$$H = \frac{\omega_2 s}{s^2 + \omega_2 s + \omega_1^2}$$

is an equivalent single tuned filter and has a bandwidth

$$B = \frac{1}{4\pi j} \int_{-j\infty}^{+j\infty} \frac{\omega_2^2 s^2 ds}{(s^2 + \omega_2 s + \omega_1^2)(s^2 - \omega_2 s + \omega_1^2)}$$

$$= \frac{\omega_2}{4}$$

3 The function

$$H = \frac{\omega_2 s + \omega_1^2}{s^2 + \omega_2 s + \omega_1^2}$$

is another low filter (characteristic of a PLL filter) and has a bandwidth

$$B = \frac{\omega_1^2}{4\omega_2} + \frac{\omega_2}{4}$$

FIG I.1-1 BASIC KALMAN FILTER

FIG I.2-1 SPINNING SYSTEM WITH ESTIMATOR

FIG I.3-1 CYCLIC SYSTEM WITH ESTIMATOR

J DIRECT VECTOR ROTATION IN ORBIT

The specific application is to geomagnetic field measurement, and offers a fast method of converting the geomagnetic vector from POLAR co-ordinates to ORBIT co-ordinates.

Given a circular orbit then assume that orbit always passes through equator at $y = 0$ $z = 0$. Point on orbit of unit radius is known from set (i, Ω) where

i = inclination of orbit
 Ω = orbit advance (α to time since RA)

Then convert to (θ, ϕ) in spherical co-ordinates via cartesian (x, y, z) and compute B components in spherical co-ordinates. Remains the problem of converting back to orbital. This appendix shows it can be done in one step.

From

$B_s = B$ (" South ") (θ increasing)
 $B_e = B$ (" East ") (ϕ increasing)

require a rotation to

$B_v = B$ ("velocity") (along roll axis)
 $B_n = B$ (" normal ") (along orbit normal axis)

(Only the vertical component $B_n = B$ ("up") is unchanged)

(Geocentric system - not converted to terrestrial geodetic or oblate earth)

Orbital system

$\underline{v}$ (roll) = $(-\sin(\Omega), \cos(\Omega) \cos(i), \cos(\Omega) \sin(i))$
 $\underline{n}$ (pitch) = $(0, \sin(i), -\cos(i))$
 $\underline{r}$ (yaw) = (x, y, z)
 $= (\cos(\Omega), \sin(\Omega) \cos(i), \sin(\Omega) \sin(i))$

From inspection

$\cos(\Omega) = x$ $\sin(\Omega) = \rho_x$
 $\cos(i) = y/\rho_x$ $\sin(i) = z/\rho_x$

and

$$\begin{aligned}\underline{v} &= (-\rho_x, xy/\rho_x, xz/\rho_x) \\ \underline{n} &= (0, z/\rho_x, -y/\rho_x)\end{aligned}$$

where

$$\rho_x = \sqrt{y^2 + z^2}$$

Validity of the above for the range of $0 < \Omega < 180^\circ$

Spherical system

$$\underline{S} \text{ (South)} = (\cos(\theta) \cos(\phi), \cos(\theta) \sin(\phi), \sin(\theta))$$

$$\underline{E} \text{ (East)} = (-\sin(\phi), \cos(\phi), 0)$$

$$\begin{aligned}\underline{r} = \text{(up)} &= (x, y, z) \\ &= (\rho_z \cos(\phi), \rho_z \sin(\phi), z) \\ &= (\sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta))\end{aligned}$$

$$\begin{aligned}\text{From inspection} \quad \cos(\theta) &= z & \sin(\theta) &= \rho_z \\ \cos(\phi) &= x/\rho_z & \sin(\phi) &= y/\rho_z\end{aligned}$$

where

$$\rho_z = \sqrt{x^2 + y^2} \quad (\text{conventional cylindrical radius})$$

and

$$\begin{aligned}\theta \uparrow = \underline{S} \text{ (South)} &= (z x/\rho_z, z y/\rho_z, \rho_z) \\ \phi \uparrow = \underline{E} \text{ (East)} &= (-y/\rho_z, x/\rho_z, 0)\end{aligned}$$

Then by algebraic manipulation in x, y, z

$$-\underline{S.v} = \underline{E.n} = \frac{xz}{\rho_z \rho_x} = \cos(\phi) \sin(i)$$

$$\underline{S.n} = \underline{E.v} = \frac{y}{\rho_x \rho_z} = \frac{\cos(i)}{\sin(\theta)}$$

Since

$$\begin{aligned}B_v &= \underline{B.v} = B_\theta \underline{S.v} + B_E \underline{E.v} \\ B_n &= \underline{B.n} = B_\theta \underline{S.n} + B_E \underline{E.n}\end{aligned}$$

$$B_v = -\cos(\phi) \sin(i) B_\theta + \frac{\cos(i)}{\sin(\theta)} B_E$$

$$B_n = \frac{\cos(i)}{\sin(\theta)} B_\theta + \cos(\phi) \sin(i) B_E$$

K 3-AXIS GRAVITY GRADIENT DYNAMICS

This analysis first re-derives the standard small angle libration equations for unequal I_x , I_y and I_z moments of inertia. It is then shown how a simple transformation yields useful equations in the case of arbitrary yaw angle. The result shows how instability easily sets in for passive nominally stabilised systems if the yaw deflections start to exceed a few tens of degrees.

A starting point from Euler's equations in body axes

$$N_x = d\omega_x/dt I_x - \omega_y \omega_z (I_y - I_z) \quad \dots K.1a$$

$$N_y = d\omega_y/dt I_y + \omega_z \omega_x (I_x - I_z) \quad \dots K.1b$$

$$N_z = d\omega_z/dt I_z - \omega_x \omega_y (I_x - I_y) \quad \dots K.1c$$

Gravity-gradient torque standard formulae for small pitch and roll angle when pitch and roll are always defined as small rotations around the body axes

$$N_x = -3 \omega_o^2 (I_y - I_z) p \quad \dots K.2a$$

$$N_y = -3 \omega_o^2 (I_x - I_z) r \quad \dots K.2b$$

In order to deal with wide yaw angle a different Eulerian rotation system is needed (3-1-2 or 3-2-1). Then approximate formulae are

$$\omega_x = dp/dt + \omega_o \cos(\phi) \quad \dots K.3a$$

$$\omega_y = dr/dt - \omega_o \sin(\phi) \quad \dots K.3b$$

$$\omega_z = d\phi/dt + (r \cos(\phi) + p \sin(\phi)) \omega_o \quad \dots K.3c$$

K 1 Small pitch p and roll r and yaw angles ϕ

Small angle approximation

$$\begin{aligned} \omega_x &= dp/dt + \omega_o \\ \omega_y &= dr/dt - \omega_o \phi \\ \omega_z &= d\phi/dt + \omega_o r \end{aligned} \quad \dots K.1.1$$

Combining K.1, K.2 and K.1.1 has solutions

$$I_x d^2p/dt^2 + 3 \omega_o^2 (I_y - I_z) p = 0 \quad \dots K.1.2a$$

$$I_y d^2r/dt^2 + \omega_o (I_x - I_y - I_z) d\phi/dt + 4 \omega_o^2 (I_x - I_z) r = 0 \quad \dots K.1.2b$$

$$I_z d^2\phi/dt^2 - \omega_o (I_x - I_y - I_z) dr/dt + \omega_o^2 (I_x - I_y) \phi = 0 \quad \dots K.1.2c$$

Which confirms Wertz p609/610 (after interchanging I_x I_y) to suit different axis convention, and change of symbols.

A 'planar' satellite can be defined with the property

$$I_x = I_y + I_z \quad \dots K.1.3$$

Then

$$d^2p/dt^2 (I_y + I_z) + 3 \omega_o^2 (I_y - I_z) p = 0 \quad \dots K.1.4a$$

$$4 \omega_o^2 I_y r + d^2r/dt^2 I_y = 0 \quad \dots K.1.4b$$

$$\omega_o^2 I_z \phi + d^2\phi/dt^2 I_z = 0 \quad \dots K.1.4c$$

which is particularly simple

K 2 General Pitch derivation

Substituting Eqs.K.1a,K.2a and K.3, and ignoring second order terms

$$\begin{aligned} - 3 \omega_o^2 (I_y - I_z) p &= I_x d^2p/dt^2 - \omega_o \sin(\phi) I_x d\phi/dt \\ &+ (\omega_o \sin(\phi) - \dots) ((r \cos(\phi) + p \sin(\phi)) \omega_o + d\phi/dt) (I_y - I_z) \end{aligned} \quad \dots K.2.1$$

Re-arranging terms

$$\begin{aligned} & (I_Y + I_Z) d^2p/dt^2 - \omega_0 \sin(\phi) (I_X - I_Y + I_Z) d\phi/dt \\ & + \omega_0^2 \sin(\phi) \cos(\phi) (I_Y - I_Z) r \\ & + (3 + \sin(\phi)^2) \omega_0^2 (I_Y - I_Z) p = 0 \end{aligned}$$

...K.2.2

The planar satellite then yields (introducing disturbance torque)

$$\begin{aligned} & I_X d^2p/dt^2 - 2 \omega_0 \sin(\phi) I_Z d\phi/dt \\ & + \omega_0^2 \sin(\phi) \cos(\phi) (I_Y - I_Z) r \\ & + (3 + \sin(\phi)^2) \omega_0^2 (I_Y - I_Z) p = N_X' \end{aligned}$$

...K.2.3

K 3 Roll derivation

Substituting Eqs.K.1b, K.2b,K.3 and ignoring second order terms

$$\begin{aligned} - 3 \omega_0^2 (I_X - I_Z) r &= I_Y d^2r/dt^2 - \omega_0 \cos(\phi) I_Y d\phi/dt \\ &+ (\omega_0 \cos(\phi) + \dots) ((r \cos(\phi) + p \sin(\phi)) \omega_0 + d\phi/dt) (I_X - I_Z) \end{aligned}$$

...K.3.1

Re-arranging terms

$$\begin{aligned} & I_Y d^2r/dt^2 + \omega_0 \cos(\phi) (I_X - I_Y - I_Z) d\phi/dt \\ & + \omega_0^2 \sin(\phi) \cos(\phi) (I_X - I_Z) p \\ & + \omega_0^2 (3 + \cos(\phi)^2) (I_X - I_Z) r = 0 \end{aligned}$$

...K.3.2

The planar satellite then yields (introducing disturbance torque)

$$\begin{aligned} & I_Y d^2r/dt^2 + \\ & + \omega_0^2 \sin(\phi) \cos(\phi) I_Y p \\ & + \omega_0^2 (3 + \cos(\phi)^2) I_Y r = N_Y' \end{aligned}$$

...K.3.3

4 Yaw derivation

From Eqs.K.3a ,K.3b then (ignoring second order terms ..dp/dt dr/dt ..)

$$\begin{aligned}\omega_x \omega_y &= (\omega_o \cos(\phi) + dp/dt) (-\omega_o \sin(\phi) + dr/dt) \\ &= - (1/2) \omega_o^2 \sin(2\phi) + \omega_o \cos(\phi) dr/dt - \omega_o dp/dt \sin(\phi)\end{aligned}$$

...K.4.1

Differentiating Eq.K.3c and again ignoring second order terms

$$d\omega_z/dt = d^2\phi/dt^2 + \omega_o \cos(\phi) dr/dt + \omega_o \sin(\phi) dp/dt$$

...K.4.2

Therefore

$$\begin{aligned}d^2\phi/dt^2 I_z - \omega_o \cos(\phi) (I_x - I_y - I_z) dr/dt \\ + \omega_o \sin(\phi) (I_x - I_y + I_z) dp/dt \\ + (1/2) \omega_o^2 (I_x - I_y) \sin(2\phi) = 0\end{aligned}$$

...K.4.3

The planar satellite then yields (introducing disturbance torque)

$$\begin{aligned}d^2\phi/dt^2 I_z \\ + 2 \omega_o \sin(\phi) I_z dp/dt \\ + (1/2) \omega_o^2 I_z \sin(2\phi) = N_z'\end{aligned}$$

...K.4.4

FIGURES

Chapter 2

- 2.1-1 UOSAT-2 perspective
- 2.1.1-1 Block diagram UOSAT-1 system
- 2.1.2-1 Block diagram UOSAT-2 system
- 2.1.2-2 UOSAT-2 internal connections
- 2.1.2-3 Partial list of telecommands
- 2.1.2-4a UOSAT-2 telemetry (no error correction)
- 2.1.2-4b UOSAT-2 telemetry (with error correction)
- 2.1.2-5 Serial communication net

- 2.2.1-1 Photo of UOSAT-1 boom
- 2.2.3-1 Magnetorquer geometry
- 2.3-1 Ground station
- 2.5.1-1 Autonomous ADCS
- 2.5.2-1a/b Capture sequence

Chapter 3

- 3.1.1-1 Attitude conventions (i)
- 3.1.1-2 Attitude conventions (ii)
- 3.1.1-3 Attitude of pitch only
- 3.1.1-4 Absolute pitch convention
- 3.1.1-5 Attitude definitions
- 3.1.2-1 Related reference systems
- 3.1.2-2 Rotations in ref system
- 3.1.3-1 Attitude Sphere
- 3.1.3-2 Projected attitude circle
- 3.1.5-1 Magnetic reference system

- 3.2.1-1 Mechanical model of nutation
- 3.2.1-2 Angular velocity vectors in nutational state
- 3.2.1-3a/b Two limiting cases of nutation -Z spin and T spin

- 3.4.1-1 Precession torque geometry
- 3.4.3-1 Precession rate to spin period UOSAT-2
- 3.4.4-1 Average roll to Z spin

- 3.5.1-1 Pitch and rate of pitch conventions
- 3.5.2-1 Equilibrium states -earth reference
- 3.5.3-1 Predicted bounds from instant deployment

- 3.6.1-1 Dynamic model of UOSAT-2
- 3.6.2-1/4 Boom deployment simulation
- 3.6.2-5/6 Libration with static analytic bounds

Chapter 4

- 4.1-1 Ambiguity in attitude determination
- 4.1-2 Attitude uncertainty cone

- 4.2-1a/f Plot of B field components in nutating spacecraft
- 4.2-2 The nutational cone

- 4.4-1 Attitude measurements II
- 4.4-2a B_z and B_r plots
- 4.4-2b Magnitude plots
- 4.4-3 Plot of B to vertical α and B to Z axis β
- 4.4-4 Bounds on attitude Z to local vertical y_1, y_2
- 4.5-1 First stage of flight calibration
- 4.5-2 Second stage of flight calibration
- 4.5-3a B_z and B_r plot
- 4.5-3b Magnitude plots
- 4.5-4 Plot of B to vertical α and B to Z axis β
- 4.5-5 Bounds on attitude Z to local vertical y_1, y_2

Chapter 5

- 5.1.1-1 Optimal magnetorquer firing for Z despin
- 5.1.2-1 Precession control
- 5.1.2-2a/d Optimal attitudes for precession
- 5.1.3-1 Polarities for required precession sign

- 5.1.4-1 Optimal instant for denutation
- 5.1.5-1 Denutation with spin-plane torquer + precession
- 5.2.1-1 Simplest algorithm for magnetic stabilisation
- 5.2.2-1 Improved simple algorithm
- 5.2.3-1/3 Angular magnetic stabilisation with active damping

- 5.3.1-1 Favoured orientation for effective delibration
- 5.3.2-1 Example sequence in anticipation mode
- 5.3.2-2 Example sequence in delay mode
- 5.3.2-3/9 Threshold algorithm test

- 5.3.4-1/3 Refined libration control $B_z \approx 0$
- 5.3.4-4 Refined libration control $B_z \approx B_T$
- 5.3.5-1 Table of orbit parameters IGRF model

- 5.4.1-1/2 Momentum change CPL law

Chapter 6

- 6.1.1-1 ASCII MAGPLOT
- 6.1.4-1 First ever successful de-spin
- 6.1.4-2 Initial and final states from first de-spin sequence
- 6.1.5-1 Magnetometer plot for orbit 6071M

- 6.1.6-1 Projection of sun and UOSAT-1 ascending node
- 6.1.6-2 Magnetometer plot for orbit 6353M
- 6.1.6-3 The cooling of UOSAT-1's base
- 6.1.6-4 Illumination conditions
- 6.1.6-5 Magnetometer plot for orbit 6466M
- 6.1.6-6 Magnetometer plot for orbit 6519M
- 6.1.6-7 Magnetometer plot for orbit 6891
- 6.1.6-8 Magnetometer plot for orbit 7734/5
- 6.1.6-9 Magnetometer plot for orbit 7802
- 6.1.7-1 B_z mag reading as boom deployed
- 6.1.7-2a B_x mag reading as boom deployed
- 6.1.7-2b B_y mag reading as boom deployed
- 6.1.7-3 Plot of nav magnetometer for orbit 7826
- 6.1.7-4 Plot of nav magnetometer for orbit 7848

- 6.2.2-1 Orientation of UOSAT-2 for magnetic calibration
- 6.2.2-2 Magnetometer calibration values
- 6.2.4-1 Mag data on orbit 1076
- 6.2.4-2 Mag data on orbit 1090
- 6.2.7-1 Signal processing in boom deployment phase
- 6.2.7-2a/d B_z data on orbits after deployment

- 6.3.2-1 Tumbling state at start of manoeuvres - B_z values
- 6.3.2-2 Slower tumbling from 'manual' attempts - B_z values
- 6.3.2-3 Drifting state after first application of algorithm
- 6.3.2-4 Near magnetic capture
- 6.3.2-5 Nearly captured state α and β plots
- 6.3.2-6 Roughly captured state α and β plots
- 6.3.2-7 Reducing libration
- 6.3.2-8 Increasing libration
- 6.3.2-9 Attitude now lost
- 6.3.2-10 Lost attitude first y_1 and y_2 plots
- 6.3.2-11 Cause from insufficient firing B_x and B_y plots
- 6.3.2-12 Recovery achieved - α and β plots
- 6.3.2-13 Plots showing mag firing on ascending B_z slope
- 6.3.2-14 Check on poor calibration - field magnitude
- 6.3.2-15 Improvement after autocalibration routine
- 6.3.2-16 Best plot obtained on y_1 and y_2 bounds
- 6.3.2-17 α and β plots - fine detail plots (4.8 secs int)
- 6.3.2-18 y_1 and y_2 bounds with de-spin only

Chapter 7

- 7-1 Attitude determination IDPO model small attitude angles
- 7.1.3-1 Analogue spinning estimator
- 7.1.4-1 Analogue cyclic (pitch) estimator
- 7.1.5-1 Analogue cyclic with offset (roll) estimator
- 7.1.6-1 Sampled data conversion of yaw estimator
- 7.1.6-2 Sampled data conversion of pitch estimator
- 7.1.6-3 Sampled data conversion of roll estimator
- 7.4-1/7 Acquisition sequences by State Estimator on IGRF field
- 7.5.3-1 Alternating estimators
- 7.5.6-1 3-axis controller (low yaw gain)
- 7.5.6-2 3-axis controller (high yaw gain)
- 7.6.5-1 3-axis controller (high gain)
- 7.6.5-2 3-axis controller (damping only)
- 7.6.5-3 3-axis controller (noise all axes)

Chapter 8

- 8.2.1-1 Orbit Locations for Attitude Detection and Control
- 8.2.4-1 TRW's magnetic control law
- 8.2.7-1 Momentum dumping by minimising of 'criterion function '
- 8.3.4-1 Spin axis de-spin forces attitude to orbit normal
- 8.4.1-1 Ideal geometry for gravity gradient 3-axis stability ?

Appendices

- A-1 The concept of Z-spin and T-spin
- A-2 Vector relations in nutationional state
- A-3 Angular velocities in nutational state
- C-1 Assignment of vector components
- I.1-1 Basic Kalman filter
- I.2-1 Spinning system with estimator
- I.3-1 Cyclic system with estimator

ACRONYMS

ADCS	Attitude Determination Control and Stabilisation
ADC	Analogue to Digital Conversion
CMG	Control Moment Gyro
CPL	Cross Product Law
EIRP	Equivalent Isotropic Radiated Power
G.G	Gravity Gradient
IDPO	Ideal Dipole Polar Orbit
TDIO	Tilted Dipole Inclined Orbit
IGRF	International Geomagnetic Reference Field
LEO	Low Earth Orbit
LSB	Least Significant Bit
MNS	Martin N. Sweeting
MODEM	MODulator DEModulator
OBC	On-board Computer
PCPL	Projected Cross Product Law
PLL	Phase-locked Loop
QOMAC	Quarter Orbit Magnetic Control
SHM	Simple Harmonic Motion
UOSAT1	University of Surrey Satellite - 1
UOSAT2	University of Surrey Satellite - 2

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