Shear forces in the contact patch of a braked racing tyre

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This article identifies tyre modelling features that are fundamental to the accurate simulation of the shear forces in the contact patch of a steady-rolling, slipping and cambered racing tyre. The features investigated include contact patch shape, contact pressure distribution, carcass flexibility, rolling radius variations and friction coefficient. Using a previously described physical tyre model of modular nature, validated for static conditions, the influence of each feature on the shear forces generated is examined under different running conditions including normal loads of 1500, 3000 and 4500 N, camber angles of 0° and -3° , and longitudinal slip ratios from 0 to -20%. Special attention is paid to heavy braking, in which context the aligning moment is of great interest in terms of its connection with limit-handling feel. The results of the simulations reveal that true representations of the contact patch shape, carcass flexibility and lateral rolling radius variation are essential for an accurate prediction of the distribution and the magnitude of the shear forces generated at the tread-road interface of the cambered tyre. Independent of the camber angle, the contact pressure distribution primarily influences the shear force distribution and the slip characteristics around the peak longitudinal force. At low brake slip ratios, the friction coefficient affects the shear forces in terms of their distribution, while, at medium to high slip ratios, the force magnitude is significantly affected. On the one hand, these findings help in the creation of efficient yet accurate tyre models. On the other hand, the research results allow improved understanding of how individual tyre components affect the generation of shear forces in the contact patch of a rolling and slipping tyre.

Keywords: tyre model, braking, rolling radius, shear forces, contact patch, racing tyre

1. Introduction

Tyre shear forces are central to vehicle motions. On the one hand, they affect the dynamic vehicle behaviour in terms of handling, stability, ride, traction and braking. On the other hand, shear forces provide information on the handling-limit of the vehicle [1], which is essential for the skilled driver to control the vehicle near or on the boundary of its performance envelope. For example, for the case of heavy braking as investigated here, the aligning moment (or the product of the magnitude and the location/distribution of the shear forces) varies with slip ratio. Most drivers can sense this variation through changes in the steering forces. However, the skilled driver can also relate these changes to the braking limit – that is, he can feel the braking-limit via the steering wheel. Owing to this close link with the controllability and, hence, the performance of the vehicle, knowledge on the distribution and the magnitude of the shear forces is of particular interest in racing.

Also, considering the rapidly increasing use of high fidelity simulation tools (e.g., full vehicle simulator) in the automotive/motorsport development process, tyre models that can accurately capture all facets of running characteristics are ever more important. In particular, physical models that build on the real structure of tyres to predict the rolling behaviour become more and more the focus of attention of researchers [3,4]. Physical tyre models range in complexity and accuracy [5], from highly detailed finite element (FE) models that replicate each component of a tyre (e.g., rubber compounds and reinforcement layers) to simpler models (e.g., brush model) that incorporate specific physical features. As shown in previous research [2,6], FE models allow extraction of finely detailed results, which are not easily obtainable through measurements on an actual tyre. On the downside, FE models require extensive information about the tyre structure such as geometry and material properties of each individual component, and demand considerable computational resources. These drawbacks limit the application of detailed FE models to dedicated tyre studies (e.g., vehicle dynamics studies are not possible). By being more-abstracted, simpler physical tyre models promise to compute rapid solutions while maintaining accuracy (over a specified operating range). However, to create efficient yet accurate simpler physical models an understanding of the influences of the different modelling features on the simulation output is required.

This article is an account of the identification of such modelling features in relation to the simulation of the shear forces in the contact patch of a steady-rolling, slipping and cambered racing tyre. Building on the results of previous research work [7], this study uses a physical tyre model of modular nature. A review of the model is presented in section 2.

For this investigation, five aspects common to most physical tyre models such as the brush model [5] or the Fiala model [8] are examined. These aspects are (see section 3): (i) contact patch geometry, (ii) contact pressure distribution, (iii) carcass flexibility, (iv) rolling radius variations and (v) friction coefficient.

Overall benefits of this research study include the identification of fundamental tyre model features, which allow the reduction of the development time of bespoke, accurate models by prioritising modelling features/parameters. In addition, the investigation improves understanding of the generation of shear forces in the contact patch of a rolling and slipping tyre. Also, the results aid better understanding of how individual tyre components influence the shear forces and how they are connected to limit-braking feel.

Within the following discussion, the right-hand orthogonal ISO axis system [9] is used. This has its origin at the intersection of the wheel centre plane with the vertical projection of the wheel spin axis onto the road plane, i.e., the centre of the contact patch.

2. Overview of physical tyre model

This study makes use of the previously developed physical tyre model [7] that simulates a radial racing tyre with a plain tread containing four grooves under one of the most severe operating conditions, that is, heavy braking in a straight line. The developed model builds on the TreadSim model by Pacejka [5] and extends it by including novel mathematical expressions derived from a validated FE model [2,6]. Similar to TreadSim, the physical tyre model consists of three main structural components: a rigid wheel rim, a flexible carcass and a rubber tread that is discretised into several parallel rows consisting of brush-type bristles or tread elements (Figure 1). The shear forces generated by the steady-rolling tyre are computed using the tread-element-following method [5,7,10,11]. With this method the motion of one bristle per row is monitored as it travels through the contact patch. From the bristle's motion history, accounting for the prevailing friction coefficient, the deflection of each tread element is known. The deflection, in turn, allows computation of the contact forces. To capture the fundamental tyre-road contact behaviour, the friction coefficient is calculated based on the contact pressure and sliding speed at the bristle tip.

Figure 1: Top view of idealised contact patch of a cambered tyre, showing a single bristle row per contact region [7].

The major changes over TreadSim include a two-dimensional contact patch that varies in its dimensions with normal load and camber angle, and considers the tread pattern by dividing the contact patch in contacting and non-contacting (tread grooves) areas (see Figure 1). A flexible carcass is included that allows longitudinal and lateral deflections related to lateral contact patch shrinkage (or ‘waisting’ [2]), camber angle, ply-steer and horizontal tyre forces (see section 4.3). To capture tyre width effects, lateral rolling radius variations relative to the centroid of the contact patch are simulated. A contact pressure distribution is incorporated that is allowed to vary in the longitudinal direction and assumed to be uniform across the width of the individual tread ribs. This model will be referred to as the variable contact pressure model. Furthermore, ‘tweaking’ coefficients to enhance the accuracy of the results are avoided and the shear force computation algorithm is changed to allow non-isotropic bristle stiffness and to avoid approximation of the sliding distance of the tread elements.

The physical tyre model was developed with a modular framework in order to allow studying the features that are fundamental to an accurate representation of the contact forces (see section 1). This model structure comprises four individual sub-models that replicate the

different physical properties of the rolling, slipping and cambered tyre: 1) contact patch geometry model; 2) contact pressure model; 3) carcass model; and 4) contact (shear force) model. These sub-models will be described in more detail in section 4. As some of the variables used in the carcass model depend on the output of the tyre model (i.e., shear stress distribution and global tyre forces) an iterative loop is employed to compute an accurate solution (Figure 2).

Figure 2: Structure of previously developed physical tyre model outlining the four sub-models [7]

3. Overview of performed simulation

The influence of the five modelling features (see section 1) on the generation of the shear forces within the contact patch is investigated at three different normal loads (1500, 3000 and 4500 N) and two different camber angles (0° to -3°) for a variation of longitudinal slip from 0% to -20%. The ‘modelling importance’ of the individual features is examined with a sensitivity analysis. That is, the effect of changing the value of each relevant property or parameter on the shear forces generated within the contact patch (as described further below) is evaluated. The rationale for the selection of each feature and the particular variation of the feature is as follows:

i) **Contact patch geometry**

The shape of the contact patch changes significantly with the inclination of the tyre - from a rectangle with an upright tyre to a trapezoid with a cambered configuration

[6,10]. Commonly, simple physical tyre models do not account for this considerable variation in the contact patch shape. To evaluate the influence of the change in shape on the model output simulation results obtained with and without the camber induced variation in the contact patch geometry are compared (see section 4.1).

ii) ***Contact pressure distribution***

The contact pressure distribution of a freely rolling tyre is primarily influenced by the structure of the tyre (e.g., sidewall stiffness, belt stiffness and tread pattern). Several simulation models are proposed in the literature. These represent the contact pressure distribution with more or less precision [5,12]. To assess the importance of a true representation of the contact pressure distribution, simulation results obtained with the variable contact pressure model are compared with those assuming a parabolic pressure distribution which is less accurate, but commonly used in simple physical tyre models (e.g., brush model).

iii) ***Carcass flexibility***

A flexible carcass is usually considered essential for an accurate simulation of the lateral force characteristics of a side-slipping tyre [5]. Considering the substantial width of the investigated tyre in combination with the cambered running condition (-3°), the tyre produces a lateral force even while rolling in a straight line. To evaluate the influence of carcass flexibility on the model output, results obtained with a flexible carcass and with a rigid carcass are compared (see section 4.3).

iv) ***Rolling radius variation***

As shown by previous research [6,7], the lateral variation of the rolling radius is a special characteristic of wide tyres. To examine the importance of capturing this effect, simulations at different levels of lateral variation (see section 4.4) are evaluated.

v) ***Friction coefficient***

The friction properties at the tread-ground interface are dependent on various factors such as surface texture and, hence, may vary while the tyre is rolling. To gather an understanding on the influence of friction on the model results, simulations with three different levels of friction are analysed (see section 4.5).

More detail on the five modelling characteristics is provided in the individual results sections (4.1 to 4.5).

As mentioned above, each of the five modelling features is examined in terms of its influence on the generation of the shear forces at the tread-road interface. To facilitate an objective evaluation, two ‘lumped-parameter’ criteria are primarily used here. The first criterion is the longitudinal tyre force (or braking force) vector. It is used to assess the influence of the relevant individual feature on the magnitude of the shear forces generated within the contact patch. The second evaluation criterion is the lateral coordinate of the longitudinal tyre force vector. This criterion is used to evaluate the influence of the relevant feature on the distribution of the shear forces within the contact patch. Also, the second criterion provides valuable information on the aligning moment, which is of great interest in the context of the investigated straight line braking manoeuvre due to its connection with limit-braking feel (see section 1). These two quantitative criteria are complemented by qualitative information in the form of shear stress vector plots for the entire contact patch region.

All simulations are performed with a constant longitudinal wheel velocity of 19.4 m/s, which is the speed used for the FE model investigation and is typical for in-door tyre tests [6]. To ensure accurate simulation results, the physical tyre model is run with a total of 3000 bristles within the contact patch area (200 bristles per row and three rows per tread rib).

4. Results obtained

The results obtained by varying the level of accuracy/detail of the five modelling features listed above are discussed and analysed in the following five sections.

4.1. *Contact patch geometry*

As mentioned in section 3, the contact patch dimensions vary with operating conditions and primarily with normal load and camber angle. Upon loading the tyre, the area of the contact patch increases – typically, linearly with the normal load [2,6]. Upon cambering the tyre, the shape of the contact patch changes from an almost symmetric, rectangular form to a trapezoidal one [2,6,7]. With the investigated tyre, these two characteristics are nearly independent from each other, i.e., the contact area of the tyre does not change when the tyre is inclined.

The shape and the area of the contact patch have direct influences on the force generation capability of the tyre, the area having a dominant effect due to the pressure dependency of rubber friction. For instance, assuming the same load a bigger contact area would allow the generation of greater shear forces because of a higher friction coefficient. Corresponding to the importance of the contact patch area, a variation of the area with normal load is commonly considered in simple physical tyre models, while the change in the shape of the tread-road interface is often neglected [e.g., 5,9,13]. The physical tyre model used for this study, considers changes of the area and the shape of the contact patch. Specifically, the contact area is considered proportional to the normal load. The contact patch shape is determined from a contact length model that uses radial tyre deflection and camber angle as inputs to compute the contact length of each tread rib.

To investigate the influence of the contact patch shape on the accuracy of the model output, the case of a cambered tyre is examined. In particular, simulation results obtained with and without the camber induced contact patch shape changes are compared. Within the physical tyre model, the change in the contact patch shape related to the tyre inclination is

suppressed by setting to zero the camber angle input to the contact length model. Camber influences on the carcass deflection behaviour, as will be described in section 4.3, are not suppressed. The resulting contact patch shapes with and without this camber angle input set to zero are shown in Figure 3.

Figure 3: Shear stress distribution of the free rolling tyre at -3° camber and 3000 N normal load. Left: physical model with default setup, i.e., camber-varying contact patch geometry; right: physical model without camber-varying contact patch geometry (camber angle input to the contact length model is set to zero). Both cases have the same contact patch area. For clarity, the number of displayed shear stress vectors is significantly reduced over the actual number of bristles used for the computation of the slip characteristics.

With the trapezoidal contact patch shape (Figure 3 left), the lateral shear stresses result from flexibility of the carcass (see also section 4.3). As reported in previous work [6,7] three superimposing ‘carcass deformation mechanisms’ can be observed with the investigated tyre for the free rolling condition. First, contact patch ‘waisting’ or lateral shrinkage of the contact patch induces lateral shear stresses that point away from the wheel centre plane (and are symmetrical about the wheel centre plane). Second, camber angle induces shear stresses which point in the direction of the inclination of the tyre (positive y-axis). This effect diminishes towards the unloaded contact patch side. Third, ply-steer, which is assumed to be constant across the contact patch (see section 4.3), causes lateral shear stresses that point in the direction of the negative y-axis. These lateral shear stresses oppose the camber induced

stresses throughout the contact patch, and augment the shear stresses resulting from tyre waisting on the unladen side and reduce them on the laden side. The longitudinal inclination of the shear stresses is caused by the lateral variation of the rolling radius as will be explained further below (see also section 4.4).

In general, by suppressing the camber-related change in the contact patch shape to obtain the rectangular contact patch shape (Figure 3 right), the influence of the three carcass deflection mechanisms (described above) on the shear forces is redistributed. Owing to the symmetric contact patch with respect to the wheel centre plane, the waisting effect becomes more prominent throughout the contact patch. At the same time, the superimposing influences of the camber-related carcass deflection and the ply-steer effect lead to lateral shear stresses that are slightly greater on the side of the negative y-axis than on the side of the positive y-axis. The influence of the rolling radius variation is equal and opposite on the laden and unladen sides because of the symmetry of the contact patch with the wheel centre plane.

Figure 4: Slip characteristics at -3° camber – solid lines denote physical model with camber varying contact patch geometry and dashed lines indicate physical model without camber varying contact patch geometry.

The observed differences in the shear stress distributions within the contact patch also affect the magnitude of the generated brake force (see Figure 4). Up to a certain low brake slip value (which varies with the applied vertical load), the consideration of a simplified rectangular contact patch leads to a slight under-prediction of the longitudinal force. At higher

brake slip ratios, the characteristics are reversed. Results obtained with the rectangular contact patch overestimate the tyre brake forces.

This behaviour primarily results from the influence of the contact length on the generation of shear forces at the tyre-road interface. At low slip ratio values, the majority of the shear forces are generated by adhesion. Accordingly, the shear forces increase linearly with distance from the first point of contact, starting from the leading edge and finishing at the trailing edge of the contact patch. Under these conditions, the shape of the contact patch in terms of its length-to-width-ratio has a significant influence on the force generation capability. Assuming a contact area of the same size, a long and thin tread will produce greater forces than a short and wide one. Consequently, the longer (and wider) ribs on the laden side (ribs 4 and 5) of the trapezoidal contact patch generate greater forces than the simplified rectangular contact patch (see Figure 5). Correspondingly, for the shorter (and narrower) tread ribs (ribs 1 and 2), the trapezoidal contact patch produces smaller shear forces. Up to a certain slip ratio, the increases in the shear force on the laden side outweigh the reductions on the unladen side so that the global tyre forces are greater when the camber-induced variation of the contact patch shape is considered.

Due to the greater differences in contact patch lengths at the lowest normal load (1500N), the specific brake slip value up to which the tyre forces are greater with the trapezoidal contact patch shape is markedly higher (approximately -12%) than for the other investigated loads (see explanation below).

Also, beyond this particular slip ratio value, the difference in the contact lengths accounts for the greater tyre forces predicted with the simplified symmetric tread-road interface geometry. At these elevated brake slips, a longer rib length (as found on the laden side of the trapezoidal contact patch) leads to a longer sliding region, which causes lower shear forces due to the velocity-dependent friction coefficient. This effect is augmented by normal load as the length of the outermost rib increases more quickly when the camber-

influence on the contact patch shape is included. As a result, the particular slip at which the force against slip curves cross, reduces with normal load: about -12% for 1500 N, -5% for 3000 N and -2% for 4500 N.

Increasing brake slip even further, the force against slip curves become independent of contact patch shape variations as the majority of the tread-road contact involves sliding. Therefore, the influence of contact length on the slip characteristics described before is reduced as the transition from adhesion to sliding is very close to the leading edge of the contact patch. As both models have the same contact patch area, the same amount of sliding shear force is generated.

Figure 5: Slip characteristics of each tread rib at -3° camber and 3000 N normal load. Left: physical model with default setup, i.e., camber varying contact patch geometry; right: physical model without camber varying contact patch geometry (camber angle input to the contact length model is set to zero).

With regard to the lateral locations of the longitudinal force vector, the two contact patch shapes yield similar characteristics but, quantitatively, lead to significantly different results (see Figure 6). In general, with increasing brake slip ratio the lateral offset of the longitudinal resultant force vector from the wheel centre plane reduces and approaches a constant value at high brake slip ratios. The migration of the F_x -vector with slip ratio is related to a combination of three factors: contact patch shape, lateral variation of the rolling radius (see section 4.4) and friction characteristics (see section 4.5).

Contact patch shape

With a cambered tyre, the contact area of the laden side is greater than that of the unladen side. Thus, as a bigger contact area produces greater shear forces, the resultant longitudinal force vector is shifted to the laden side. As will be explained below, this mechanism is responsible for the constant finite offset of the longitudinal resultant force at high brake slip conditions.

Lateral variation of the rolling radius

As will be described in section 4.4, the rolling radius varies across the width of the tyre and creates local differences in the slip ratio and, thus, in the longitudinal shear forces. These local shear force differences can be associated with the aligning moment of the tyre (moment about the z-axis) and are reflected in an offset of the F_x -vector with respect to the wheel centre plane. However, the effect of the rolling radius variation diminishes with increasing brake slip ratio because of the friction characteristics. Consequently, at high brake slip conditions the offset converges to a constant value and, when the contact patch shape influence is disregarded, approaches zero.

Friction characteristics

The friction properties at the tread-road interface influence the location of the shear force resultant at slip ratios between zero and approximately -10%. As described above, at very low slip conditions the majority of the contact region adheres to the ground so that the shear stresses increase linearly with distance along the contact length. Thus, as the contact on the laden side of the trapezoidal contact patch is longer than on the unladen side, light braking yields the highest shear stresses on the laden side. Correspondingly, the resultant longitudinal tyre force is furthest away from the wheel centre plane. With rising brake slip, sliding zones start to form in the tread ribs. In particular, on the laden side the sliding region grows more quickly than on the unladen side, again, because of the difference in contact length. Since the horizontal forces in the sliding region are limited by the product of the friction coefficient and

the normal load, the increase in shear forces with higher slip ratios peaks earlier on the laden side than on the unladen side (see Figure 5). Hence, with increasing brake slip the magnitude of the longitudinal forces generated within the unladen side of the contact patch increases, while the shear forces in the laden side stagnate or even reduce. Consequently, the resultant longitudinal force moves in the direction of the unladen side and thereby reduces the lateral offset from the wheel centre. With further increase in brake slip, the entire contact patch will eventually slide and the offset of the resultant longitudinal force reaches a constant level. As the contact pressure is fairly constant throughout the contact patch, the “steady-state” offset under high slip conditions approximately coincides with the lateral coordinate of the centroid of the contact patch. When the contact patch shape variations are disregarded, this friction-related influence is restricted to the effect of the formation of the ‘saturated’ sliding forces. The initial offset of the longitudinal force vector (around zero slip) results largely from the lateral variation of the rolling radius.

Figure 6: Lateral coordinate of longitudinal force vector against slip ratio. Tyre at -3° camber. Solid lines denote physical model with camber varying contact patch geometry and dashed lines indicate physical model without camber varying contact patch geometry.

In summary, in order to capture the correct shear stress distribution at the tread-road interface, the variation of the contact patch shape with camber angle is an essential model feature. Also, accounting for changes of the contact patch shape is beneficial to enhance the

accuracy of the tyre force prediction for low to medium slip ratio values, but it is less important for high slip investigations.

4.2. Contact pressure distribution

Figure 7: Contact pressure distribution (lines without markers) and static shear stress limit (lines with markers) along the centre tread rib calculated with the variable contact pressure model (solid line) and a parabolic pressure distribution (dashed line) at a vertical load of 3000 N and zero camber angle. The size of the adhesion and sliding zones with the two different pressure distributions at one particular slip ratio are indicated by the shaded areas.

Disregarding influences from the rolling conditions such as camber angle and normal load, the contact pressure distribution is affected primarily by the structure of the tyre (e.g., carcass and sidewall stiffnesses). In the literature, different simulation models capturing the real contact pressure distribution with varying accuracy are proposed. To obtain an understanding of the importance of the contact pressure distribution on the model output, simulation results computed with the variable contact pressure model (assumed to be most accurate) are compared to results obtained with a less accurate model, namely, a parabolic contact pressure distribution (Figure 7). Because of its simplicity, this parabolic contact pressure model is commonly employed in physically-based tyre models such as the brush model [5]. All other model details remain unchanged for this investigation.

Figure 8: Longitudinal force against slip ratio of an upright tyre (left) and a cambered tyre (right, $\gamma = -3^\circ$) at three different normal loads and with two different contact pressure models. Solid lines denote results obtained with the variable contact pressure model and dashed lines indicate results obtained with a parabolic pressure distribution.

As indicated by Figure 8, a reduction in the modelling accuracy for the contact pressure distribution results in lower brake force magnitudes at low and high slip values, and a greater peak longitudinal force, mainly at the highest investigated normal load. These characteristics are caused by the influence of the contact pressure distribution on the formation of the sliding and adhesion zones within the tyre contact patch and their effect on the generation of shear forces, as explained in the previous section. At low slip values, the parabolic distribution creates a shorter adhesion region and, accordingly, smaller shear forces than are obtained with the more accurate variable contact pressure distribution. Around the longitudinal force peak, the consideration of a simplified pressure distribution yields an adhesion region which is extended over a greater area, because of a higher peak pressure and, thus, a greater frictional shear stress limit (i.e., the product of the friction coefficient and the normal load as shown by the lines with markers in Figure 7). This extended adhesion region is highlighted graphically in Figure 7 by the differences in the shaded areas. As a tyre with a bigger adhesion region generates greater forces than with a smaller one, the simplification of the pressure distribution gives greater longitudinal peak forces. Yet, because of the pressure dependency of the friction coefficient, this trend is reversed at higher brake slip ratios when the majority of the contact patch is taken up by the sliding region.

Figure 9: Lateral coordinate of longitudinal force vector against slip ratio of a cambered tyre ($\gamma = -3^\circ$) at three different normal loads. Results computed with two different contact pressure models: solid lines denote results obtained with the variable contact pressure model and dashed lines indicate results obtained with a parabolic pressure distribution.

In terms of the distribution of the shear stresses within the contact patch of the cambered tyre, the accuracy level of the contact pressure distribution influences the simulation results primarily at low to medium slip values (Figure 9). At these slip conditions, the lateral offset of the F_x -vector from the wheel centre plane is larger with the less accurate parabolic pressure distribution because of the aforementioned slip-dependent influence of the contact pressure on the size of the adhesion zones created within the contact patch. In particular, due to the lateral rolling radius variation (i.e., brake slip increases on the laden side and reduces on the unladen side), the adoption of the parabolic pressure distribution results in adhesion regions that are shorter on the unladen side and longer on the laden side. Correspondingly, the resultant longitudinal force predicted with the parabolic contact pressure distribution is offset further from the wheel centre plane.

At slip ratios greater in magnitude than approximately -6%, the distribution characteristics of the contact pressure are less influential and only small differences are present. The difference can be attributed to small variations in the carcass deformations which are caused by the different tyre force magnitudes generated at higher slip ratios, as will be described in section 4.3.

In summary, a true representation of the contact pressure distribution is important for the accurate computation of the peak longitudinal force and the shear stress distribution at low to medium slip ratios.

4.3. Carcass flexibility

As mentioned in section 2, the physical tyre model replicates the compliance of the tyre structure by including a flexible carcass model, which considers the following four belt deformation characteristics (see also section 4.1): a) lateral shrinkage (or contact patch waisting) upon vertical loading; b) lateral belt deflections induced by camber angle (due to the change in radial tyre deflection across the width of the cambered tyre, the extent of lateral belt deflections varies laterally); c) belt rotation upon vertical loading because of ply-steer; and d) lateral and longitudinal carcass deflections due to horizontal forces generated by the rolling and slipping tyre. These deformations include global contact patch displacements and local/individual belt deflections.

To study the influence of carcass flexibility on the simulation accuracy of the straight line braking manoeuvre, model results obtained with the flexible carcass and with a rigid carcass (that only considers the ply-steer effect) are compared and analysed. The rigid carcass is simulated by assuming a rigid foundation for the bristle rows. The ply-steer effect is considered to be constant throughout the entire contact patch and is included by rotating the rigid bristle base about the z-axis by a small fixed amount. For the case of the upright tyre (Figure 10 left), the ply-steer effect can be easily observed by the shear stress vectors that point in the lateral direction and increase linearly with longitudinal distance from the front of the contact patch. With the cambered tyre, these lateral shear stresses are superimposed by local driving and braking shear stresses (due to the variation of the rolling radius), creating a ‘whirl’ pattern (Figure 10 right).

Figure 10: Shear stress distribution computed with the physical model and a rigid carcass at 0° camber (left) and -3° camber (right), 3000 N and zero slip. The shear stresses arise from the ply-steer effect and, for the case of the cambered tyre, also from the rolling radius effect. For clarity, the number of displayed shear stress vectors is significantly reduced over the actual number of bristles used for the computation of the slip characteristics.

Figure 11: Longitudinal force against slip ratio at three different normal loads. Left: tyre at 0° camber; right: tyre at -3° camber. Solid lines denote physical model with flexible carcass and dashed lines indicate physical model with rigid carcass.

When carcass flexibility is disregarded, the lateral shear stress level at the tread-road interface is lowered (see also section 4.1). At the same time, considering the concept of the friction circle, the ability of the tyre to generate longitudinal shear stresses is increased. In other words, as the magnitudes of the shear forces at the tyre-road interface are limited by the maximum rubber friction forces (i.e., the product of normal load and the prevailing friction coefficient), additional lateral shear stresses, as introduced by carcass flexibility, reduce the maximum attainable longitudinal force. As a result, the magnitude of the tyre brake force

increases with reducing carcass compliance. This effect is most pronounced with high lateral shear stress conditions such as when the tyre is cambered and subjected to the highest normal loads (see Figure 11).

Figure 12: Lateral coordinate of longitudinal force vector against slip ratio of a cambered tyre ($\gamma = -3^\circ$) at three different normal loads. Solid lines denote physical model with flexible carcass and dashed lines indicate physical model with rigid carcass.

The flexibility of the carcass also influences the migration of the lateral location of the longitudinal resultant shear force of the cambered tyre. For the investigated slip ratio range and at all three vertical loads, the simplified modelling assumption of a rigid carcass increases the lateral offset for the F_x -vector from the wheel centre plane. As with the slip characteristics, this change can be ascribed to differences in the sliding zones within the contact patch which arise due to higher shear stress levels caused by lateral belt deformations.

In summary, consideration of carcass flexibility is primarily important for the case of the cambered and heavily loaded tyre in terms of the distribution and the magnitude of the longitudinal shear stresses within the contact patch.

4.4. Rolling radius variation

The ratio between the longitudinal speed and the angular velocity of a freely rolling tyre is referred to as the rolling radius (RR). The RR changes with many factors such as rolling conditions (e.g., normal load, velocity and camber angle) and tyre construction [5,14,15]. As

highlighted by previous work [6,7] and shown by other researchers [5,14], the RR also varies within the contact patch because of its dependence on the deformation behaviour of the tyre structure¹. This variation requires the definition of a local rolling radius [6]. Specifically, a local rolling radius can be defined for any lateral location in the tyre contact patch (a rib) as the ratio of the forward velocity of the tread surface of the rib and the wheel speed for the condition when no longitudinal force is generated by the rib. For the studied racing tyre, the concept of the local rolling radius is incorporated in the simulation model by allowing the rolling radius to vary linearly across the width of the tyre and relative to the centroid of the contact patch when the tyre is cambered. As mentioned before, this variation creates local differences in the slip ratio, which lead to the generation of locally varying longitudinal shear stresses. For instance, at zero slip (free rolling condition) the cambered tyre produces braking forces on the laden side and driving forces on the unladen side (see Figure 5).

To investigate influences caused by the RR characteristics, cambered tyre simulations at 0%, 100% and 200% of the default model value for the variation of the rolling radius across the tyre width are compared. Specifically, the different levels are achieved by changing the gradient of the linear RR variation across the width of the tyre.

Figure 13: Longitudinal force against slip ratio (left) and lateral coordinate of longitudinal force vector against slip ratio (right) of the cambered tyre (-3°) at three different normal loads and three different rolling radius (RR) variation levels. The solid lines denote the 100% RR variation, the dotted lines indicate an RR variation raised to 200% and the dashed lines represent 0% RR variation.

¹ In this respect, Genta [15] considers the rolling radius as the radius of an equivalent rigid wheel that rolls at the same speed as the pneumatic wheel.

As indicated by Figure 13 left, by raising the amount of lateral variation of the rolling radius, the differences of the shear forces across the width of the tyre are increased and the overall brake force reduced, and vice versa. This characteristic is maintained up to a brake slip ratio of about -10%. Beyond this slip value, the brake force is primarily generated by sliding friction forces that have reached the friction limit and, thus, are less dependent on the prevailing slip ratio. As a result, the longitudinal force/slip curves are nearly independent of the RR variation level. Due to its direct influence on the magnitude of the shear stresses, the RR variation level also affects the location of the longitudinal resultant force vector. At low slip ratios of up to about -9%, increasing the RR variation further offsets the resultant brake force from the wheel centre plane (Figure 13 right) and, thereby, raises the aligning moment. At slip ratios greater than approximately -9%, these characteristics are reversed (only small difference in simulation results), because of differences in the behaviour of the transition from adhesion to sliding regions across the width of the contact patch with changing RR levels, as described in section 4.1.

In summary, the variation of the rolling radius is essential for an accurate description of the distribution and magnitude of the tyre shear forces and the associated aligning moment for low to medium brake slip ratio values. At high brake slip ratios, when the shear force generation is dominated by sliding friction characteristics, the RR variation is of lesser importance to obtain accurate simulation results.

4.5. Friction coefficient

As indicated by the vast extent of current research and the associated published literature [e.g.,16,17], an accurate replication of the friction properties at the tread-ground interface is one of the most difficult topics in tyre modelling. The friction coefficient depends on contact pressure, sliding velocity, temperature and surface texture, among other things. In order to

gain an insight into the importance of the correct prediction of the friction characteristics, the sensitivity of the simulation results to $\pm 10\%$ changes in the friction conditions is assessed.

Figure 14: Longitudinal force against slip ratio at three different normal loads and as a function of friction level. Left: tyre at 0° camber; right: tyre at -3° camber. The solid lines denote the original friction level, the dashed lines indicate a friction level of 90% and the dotted lines represent a friction level of 110%.

As indicated by Figure 14, the longitudinal slip characteristics at three different normal loads exhibit the expected characteristics with the varied friction conditions. That is, the slip stiffness is independent of the friction coefficient. By lowering the friction coefficient, the maximum achievable shear forces within the contact patch are reduced and, as a consequence, the generated longitudinal tyre forces are consistently lower than at higher friction conditions. Inversely, increasing the friction level raises the generated brake forces.

Figure 15: Lateral coordinate of longitudinal force vector against slip ratio of a cambered tyre ($\gamma = -3^\circ$) at three different normal loads and as a function of friction level. The solid lines denote the original friction level, the dashed lines indicate a friction level of 90% and the dotted lines represents a friction level of 110%.

As shown by the lateral coordinate of the longitudinal resultant force against slip ratio curves (see Figure 15), the distribution of the shear forces within the contact patch is influenced by the friction level to a much lesser extent than the brake force magnitude. A reduction of the friction coefficient causes the resultant longitudinal force vector to move closer to the wheel centre at small brake slip values, as the growth characteristics of the sliding region within the contact patch are changed. In particular, by lowering the friction level the onset of sliding within the contact patch is reached at smaller brake slip values. Thus, considering the shear force mechanisms described in section 4.1, a change in the friction coefficient can be seen as causing a shift of the lateral F_x -vector coordinate against slip ratio curve along the abscissa. Nonetheless, as the friction-related change of the sliding region characteristics are rather small, the shear force distribution itself may be considered to be mostly independent of the friction conditions.

In summary, the friction level is essential for an accurate description of the magnitude of the tyre shear forces at medium to high brake slip ratio values. The shear force distribution is hardly influenced.

5. Conclusions

A previously developed and validated physical tyre model [7] that simulates contact patch characteristics of a braked racing tyre has been used successfully to identify fundamental tyre modelling features. The simulation results obtained reveal that the following five features are crucial for replicating certain contact patch characteristics accurately:

- **Contact patch shape:** a true representation of the two-dimensional contact patch geometry is key for the simulation of the shear forces (magnitude and distribution) of the cambered tyre.
- **Contact pressure distribution:** an accurate description of the contact pressure distribution is required for the exact prediction of the shear stress distribution within the contact patch of a lightly braked tyre. Also, the slip characteristics around the peak longitudinal force are influenced by the distribution of the contact pressure.

- **Carcass flexibility:** inclusion of belt deformations caused by vertical load, horizontal tyre forces and camber angle is essential for an accurate prediction of the shear stress distribution and magnitude within the contact patch of a braked, cambered tyre.
- **Rolling radius:** accounting for variation of the rolling radius across the width of the tyre (and relative to the centroid of the contact patch) is required to capture tyre width effects that are necessary to accurately simulate the magnitude and location of the longitudinal resultant force (and thus, the aligning moment) generated by a cambered tyre.
- **Friction coefficient:** an accurate friction coefficient value is important for the correct simulation of the magnitude of the tyre force at medium to high slip ratios. The distribution of the shear forces as indicated by the lateral location of the longitudinal force vector is influenced to a lesser extent.

These findings help in the creation of dedicated tyre models that are efficient yet accurate. Also, the research results allow insights on the impact of individual tyre components on the rolling behaviour of the tyre in general and, specifically, on the characteristics of the contact patch.

Current work focuses on better understanding of friction properties at the tread-road interface.

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