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A New Model for the Drag Coefficient of a Swarm of Condensing Vapour-Liquid Bubbles in a Third Immiscible Liquid Phase

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Abstract

A semi-analytical model for the drag coefficient of a swarm of two-phase bubbles, condensing in direct contact with an immiscible sub-cooled liquid, has been developed. The analysis used a cellular model configuration, assuming potential (but not inviscid) flow around the reference two-phase bubble in the cell. The effect of the condensation ratio within the two-phase bubbles was included using an approximate relation. The drag coefficient for a wide range of Reynolds numbers ($0.1 \leq Re \leq 1000$) has been found using the viscous dissipation integral method, and the effect of the liquid content within the two-phase bubble or the half opening angle (β), and the system void fraction(α) were examined. The drag coefficient has been found to increase with the condensation ratio and with the void fraction of the system. The present model agrees well with previously available experimental data and theoretical predictions for single bubbles or particles.

Keywords: Drag coefficient; Two-phase bubbles; Direct contact condensation.

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1. Introduction

Direct contact heat exchange with change of phase is an efficient heat transfer mechanism. Generally, it involves injection of drops or bubbles, as a dispersed phase, into a column of another immiscible liquid, as a continuous phase. The temperature of the continuous phase must be above the boiling point of the drops for evaporation, or less than the saturation temperature of the bubbles in the case of condensation. Using direct contact heat transfer between two immiscible fluids, utilizing a three – phase direct contact heat exchanger (Song et al., 1999), has many advantages over conventional configurations. For instance, it eliminates the metallic heat transfer surface between the fluids, which is prone to corrosion, fouling and also increases the heat transfer resistance. Direct contact heat exchangers can also be operated with a very low temperature driving force, and with smaller mass flow rates of the transferring fluids. Convenient separation of the fluids and a high heat transfer coefficient (about 20-100 times more than a single phase or surface type heat exchanger) (Peng et al., 2001) can also be achieved. Therefore, direct contact heat exchange can be used in several industrial applications, such as water desalination by freezing, geothermal power generation, crystallization, waste heat recovery, energy storage, and solar energy.

In order to obtain an optimal design of a direct contact boiler or condenser, a good understanding of the fluid mechanics and heat transfer characteristics in such systems is necessary. However, condensation of one or two-component bubbles in a cold liquid has proved difficult to study theoretically (Kalman and Mori, 2002; Kalman, 2003 and Kalman, 2006) and experimentally (Chen and Maynger, 1992). There are several factors which must be accounted for when considering the driving force for condensation (i.e. the temperature difference), between the condensation bubbles and the surrounding fluid. These are the external and internal resistances, and finally the resistance associated with the condensate

accumulated inside bubbles in the case of two components or two phase bubble condensation (Kalman, 2003).

There are a large number of experimental and theoretical investigations related to direct contact condensation of a single two-phase bubble in an immiscible liquid, (e.g. Sideman and Hirsch, 1965; Isenberg and Sideman, 1970; Moalem and Sideman, 1973; Higeta et al., 1983; Riana et al., 1984; Lerner et al., 1987; Wanchoo, 1993; Kalman and Mori, 2002 and Kalman, 2003) and bubble trains (Sideman and Moalem, 1974; Lerner and Kalman, 1990 and Kalman, 2006). More recently, Mahood et al., 2014a; Mahood et al., 2014b; Mahood et al., 2015) studied, experimentally and theoretically, the direct contact condensation of a swarm of two-phase bubbles in a three-phase condenser.

Only two investigations have addressed the drag coefficient of a single two-phase bubble condensing in an immiscible liquid; both of them were experimental studies. Higeta et al. (1979) have estimated the drag coefficient of a pentane bubble condensing in glycerol and a steam bubble condensing in silicon oil. They concluded that the two-phase bubble behaves as an inviscid fluid sphere at early stage of condensation (relatively high Re number), while it is approximated by a rigid sphere at the last stage of condensation (low Re number). The second study was carried out experimentally by Wanchoo et al. (1997). Three different dispersed phases, n – pentane, isopentane and furan were condensed in distilled water and in aqueous glycerol solutions of 75% wt and 98.3%. Their results for a very low Reynolds number ($Re < 0.1$) surprisingly fell under the drag coefficient results of an inviscid fluid sphere (bubble), which does not agree with other experimental results, for example (Higeta, et al., 1979). Wanchoo et al. (1997) justified these results by the citing the mobility of the condensate film surrounding the bubble surface and the strong internal circulation. This was rejected by Kalman and Mori (2002).

All previous theoretical investigations have been carried out with an aim of studying the hydrodynamics and heat transfer of condensing two-phase bubbles in an immiscible liquid. These studies have largely relied on the expressions derived for gas bubbles or solid spheres. These models however, did not include the change in the two-phase bubble's contents due to the condensation. The condensate, of course, accumulates within the two-phase bubble and the vapour content decreases. The two-phase bubble therefore undergoes a continual change of viscosity because of the increased liquid content and the reduced vapour content. That leads to a change in the drag force on the two-phase bubble, which normally affects the two-phase bubble's velocity, and consequently the heat and the mass transfer. The lack of investigations considering the condensation of a swarm of two-phase bubbles in immiscible liquid is a clear obstacle facing the development of a full-scale direct contact condenser. The present investigation tries to remove part of the obstacle by studying the drag coefficient experienced by a swarm of two-phase bubbles condensing in an immiscible liquid. The effects of the ongoing condensation and the overall void fraction on the drag force on the two-phase bubbles will be discussed.

2. Modelling

Let us assume a spherical two-phase (vapour/liquid) bubble condensing in a Newtonian liquid which is immiscible with the liquid condensate and is completely free of surface active material. The surface tensions of the continuous fluid and the condensate are assumed to be high enough to keep the two-phase bubble spherical in shape, and the liquid condensate is confined within the mother bubble throughout the condensation process. Whilst this configuration is a simplification, there is experimental evidence to support the existence of such two-phase bubbles (e.g. Sideman and Hirsch, 1965; Isenberg and Sideman, 1970). Of course, if the balance between body forces and surface tension changes significantly, different bubble shapes may emerge, or the vapour and condensed liquid could detach

forming vapour bubbles and liquid droplets. Nevertheless, for the development of an initial model, such as proposed below, this is a reasonable assumption. This is particularly true for the fluids and injection rates that are typical of direct contact condensers. A viscous-potential flow is assumed around the reference two-phase bubble with a cell configuration model.

In addition, the following assumptions are made:

- There is a sufficient (constant) temperature difference between the two-phase bubble and the continuous phase (cooling phase) along the column to complete condensation of the two-phase bubble.
- The direct contact condensation process ends in a spherical liquid drop, which then rises to the top of the vessel to mix with the bulk accumulated liquid condensate.
- The direct contact condensation process forms a liquid-liquid interface, and this interface can be treated as a rigid or immobile. Isenberg and Sideman (1970) showed experimentally that when a single bubble condenses in an immiscible liquid, the surface of the bubble is much smoother when compared to a bubble condensing in its own liquid. They therefore inferred that the surface of the two-phase bubble tends to be immobile.
- The effect of internal circulation within the condensate layer is neglected. This can be attributed to the small bubble size and relative immobility of the liquid-liquid interface.
- The two-phase bubble has the shape depicted schematically in Fig. 2b, which is consistent with the literature and has been observed experimentally by, e.g. Sideman and Hirsch, 1965. In such a configuration, the condensate will form at the surface of the bubble and be transported rapidly via a thin film to the bottom of the bubble, where it accumulates.

Whilst these assumptions limit the generality of the model somewhat, they are consistent with conditions within a direct contact condenser and, as mentioned above, are generally supported by empirical evidence.

Using the cell model shown in Fig. 1, the velocity potential of the two-phase bubble in the swarm was given by Milne-Thomson (1965) and used by Cai and Wallis (1994) as

$$\phi = \frac{1}{b^3 - a^3} \left[(a^3 U - b^3 v) r + \frac{a^3 b^3}{2r^2} (U - v) \right] \cos\theta \quad (1)$$

where U , v , a , b and θ represent the velocity of the inner cell, the velocity of the outer cell, the radius of the inner cell, the radius of the outer cell and the angular coordinate respectively.

For $v = 0$, Eq. (1) reduces to the expression given by Lamb (1945) and used by Kendoush (2004):

$$\phi = \frac{U}{b^3 - a^3} \left[a^3 r + \frac{a^3 b^3}{2r^2} \right] \cos\theta \quad (2)$$

and for $\alpha = \left(\frac{a}{b}\right)^3$, Eq. (2) becomes:

$$\phi = \frac{U}{(1-\alpha)} \left[\alpha r + \frac{1}{2} \left(\frac{a^3}{r^2} \right) \right] \cos\theta \quad (3)$$

Using Eq. (3), the velocity components can be found:

$$V_r = -\frac{\partial\phi}{\partial r} = \frac{U}{(1-\alpha)} \left[\alpha - \left(\frac{a}{r} \right)^3 \right] \cos\theta \quad (4)$$

$$V_\theta = -\frac{1}{r} \frac{\partial\phi}{\partial\theta} = \frac{-U}{(1-\alpha)} \left[\alpha + \frac{1}{2} \left(\frac{a}{r} \right)^3 \right] \sin\theta \quad (5)$$

where V_r and V_θ are the radial and tangential velocity components. The equations found above for the velocity components are slightly different from those derived by Mariucci (1965) and Kendoush (2001) when they analyzed the problem of gas bubble swarms.

Fig. 1. Schematic representation of the cell model

Generally, the viscous drag for the spherical bubbles regime has been modelled by using the dissipation method, and an assumption of potential flow around the particles. This confines the viscous effect to a thin layer surrounding the particles (Padrino and Joseph, 2009). The viscous dissipation can be found by using the method that suggested by Kendoush (2001), as follows:

$$E = \int_0^\pi [\tau_{r\theta} V_\theta]_{r=a} (2\pi a^2) \sin\theta \, d\theta \quad (6)$$

where $\tau_{r\theta}$ represents the tangential shear stress at the surface of the two-phase bubble. It is given by

$$\tau_{r\theta} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{V_\theta}{r} \right) + \frac{1}{r} \frac{\partial V_r}{\partial \theta} \right]_{r=a} \quad (7)$$

Substituting Eq. (4) and Eq. (5) into Eq. (7) results in

$$\tau_{r\theta} = -3\mu \frac{U \sin \theta}{a(1-\alpha)} \quad (8)$$

Substituting Eq. (5) and Eq. (8) into Eq. (6) and evaluating the integral leads to

$$E = -8\pi\mu a U^2 f(\alpha) \quad (9)$$

where

$$f(\alpha) = \frac{(\alpha+1/2)}{(1-\alpha)^2} \quad (10)$$

The drag force can be found as:

$$F_D = \frac{E}{U} \quad (11)$$

Using Eq. (9) and Eq. (11), results in:

$$F_D = -8\pi\mu a U f(\alpha) \quad (12a)$$

For $\alpha \rightarrow 0$ (i.e. when the the system tends toward a single bubble in an infinite medium), Eq. (12a) reduces to:

$$F_D = -4\pi\mu a U \quad (12b)$$

The drag force in Eq. (12a) therefore represents that for a single bubble, modified by a correction factor, which is a function of void fraction.

To date, no exact expression has been derived for the drag force acting on a two-phase condensing bubble in an immiscible liquid. The relevant studies used approximate formula, which were derived for gas bubbles or solid spheres. The condensation process considered here is similar to the evaporation of volatile liquid drops in an immiscible liquid (both are consist of a stable two-phase bubble shape, with vapour at the top of the bubble and liquid at the bottom due to the effect of gravity). This configuration was first observed experimentally by Sideman and Hirsch (1965) for an isopentane bubble condensing in water. They noted that the condensation progresses until a liquid drop is produced. This observation was confirmed by Higeta et al. (1979) and Higeta et al. (1983) who observed a similarity between the

condensation of bubble and the evaporation of drop in an immiscible liquid medium. This mechanism has been used after that by different investigators, (e.g. Wanchoo et al., 1997).

Additionally, the two-phase bubble condensing in another immiscible liquid forms a liquid-liquid interface. This interface is likely to be immobile and considerably reduces the internal circulation within the two-phase bubble. Therefore, the two-phase bubble interface can be considered as a rigid, especially at the liquid-liquid interface, as Isenberg and Sideman (1970) showed experimentally.

The structure described above can be likened to the case when a clean bubble or drop is injected into a fluid medium with surface-active components or other impurities. In this case, the contaminants can attach themselves to the bubble or drop interface. They move downward and accumulate at the bottom of the bubble or the drop. With time, the contaminant layer will grow and could eventually cover the entire surface of the bubble or drop (Loth, 2010).

To study this phenomena theoretically, the stagnant-cap model was proposed by Griffith (1962). In this model, the bubble or drop is separated into two parts: a clean part, which the accumulated contaminant has not yet reached (this part of the drop therefore still has a mobile surface), and an immobile surface for the part covered by the surface-active contaminants. The separation of the two sections can be characterised by θ_c (see Fig.2a), termed the ‘clean angle’ whose value is limited to $0 \leq \theta_c \leq 180^\circ$.

Sadhal and Johnson (1983) developed an exact solution for the stagnant cap bubble, including the effect of viscosity and the fraction of contaminants for the case of creeping flow:

$$f(\mu, \theta_c) = \left\{ \frac{\mu_o}{4\pi(\mu_o + \mu_i)} \left[2\theta_c + \sin(\theta_c) - \sin(2\theta_c) - \frac{1}{3}\sin(3\theta_c) + \frac{\mu_o + \frac{3}{2}\mu_i}{\mu_o + \mu_i} \right] \right\} \quad (13)$$

This factor modifies the drag force (e.g. Eq. (12a)) to account for the effects of the contaminant. When no contaminants are present on the bubble or drop surface then Eq. (13) reduces to unity, and for complete coverage of the surface by the contaminant, Eq. (13)

reduces to $3/2$. Sadhal and Johnson (1983) developed this expression whilst considering a system wherein external circulation still occurred due to the mobile, uncontaminated surface, but was reduced due to the ‘rigid’ stagnant cap.

In case of a two-phase bubble condensing in an immiscible liquid, the condensate is swept down and accumulates at the base of the bubble, while the vapour is concentrated at the top due to the buoyancy (Sideman and Hirsch, 1965; Higeta et al., 1979 and Higeta et al., 1983). The progress of the condensation is indicated by a half opening angle β , which describes the separation of the vapour phase at the top of the bubble from the condensate or the liquid phase at the bottom (see Figure 2b). As for the clean angle above, $0 \leq \beta \leq 180^\circ$.

Fig. 2. Similarity between (a) the stagnant-cap bubble model and (b) the condensation of a two-phase bubble in an immiscible liquid, both shown in the frame of reference of the moving bubble

The similarity between the condensation of vapour bubbles in an immiscible liquid, and the contaminant absorbed by a clean bubble or drop surface (the stagnant-cap model) is quite

obvious. For a clean vapour bubble condensing in a clean liquid medium (impurity $< 10^{-6} \text{ g/L}$), the immobile interface is produced as a result of the accumulation of the condensate inside the bubble. The half opening angle is analogous to the clean angle, and clearly indicates the progress of condensation within a two-phase bubble. The accumulated liquid is assumed to have no internal circulation and therefore to have a similar effect on the drag of the two-phase bubble as the rigid interface has on the drag of a stagnant cap bubble. Of course, in reality, there will always be some degree of circulation within the liquid but it is assumed to be of secondary importance in a direct contact condenser.

Equation (12a) now can be re-written in a manner similar to that suggested by Loth (2010), to include the effect of the change of viscosity inside two-phase bubbles by introducing a further correction factor, which is given by Eq. (13):

$$F_D = -8\pi\mu aU f(\alpha)f(\mu, \beta) \quad (14)$$

where $f(\mu, \theta_c)$, (Eq.(13) for contaminated bubbles or drops) becomes

$$f(\mu, \beta) = \left\{ \frac{\mu_o}{4\pi(\mu_o + \mu_i)} \left[2\beta + \sin(\beta) - \sin(2\beta) - \frac{1}{3}\sin(3\beta) + \frac{\mu_o + \frac{3}{2}\mu_i}{\mu_o + \mu_i} \right] \right\} \quad (15)$$

for condensing or evaporating drops. Here μ_i represents the viscosity of the vapour and liquid (condensate) within a two-phase bubble, i.e.

$$\mu_i = \mu_{i,mixture} \quad (16)$$

It can be found simply as:

$$\mu_{i,mixture} = (1 - x)\mu_v + x\mu_l \quad (17)$$

where x represents the condensation ratio, which can be calculated depending on β , using the expression given by Tochitani et al. (1977) as:

$$x = \frac{\rho_{dv}(2-3\cos\beta+\cos^3\beta)}{\rho_{dl}(2+3\cos\beta-\cos^3\beta)+\rho_{dv}(2-3\cos\beta+\cos^3\beta)} \quad (18)$$

where ρ_{dl} and ρ_{dv} represent the density of liquid and vapour within a two-phase bubble. The use of the weighted average viscosity here again represents a simplification on reality. Rather than introducing further complexity into the model by considering separately the drag on the liquid and vapour parts of the bubble, the weighted average approach has allowed the development of a simple model, which, as shown below, is consistent with previous experimental and theoretical works.

Now, in Eq. (15), $\beta = 0$ corresponds to a pure vapour bubble, whilst $\beta = 180^\circ$ represents a drop which is entirely liquid. Therefore, the value of the drag force on the condensing bubbles in immiscible liquid will be located between these two-limits. This is consistent with the experimental results of Higeta et al. (1979).

Additionally, the drag coefficient of a swarm of drops in a fluid of density ρ_c can be written as:

$$C_D = \frac{F_D}{\frac{1}{2}(1-\alpha)\rho_c U^2 \pi a^2} \quad (19)$$

Using Eq. (14), results in

$$C_D = \frac{32}{Re} \frac{(\alpha+\frac{1}{2})}{(1-\alpha)^3} \left\{ \frac{\mu_o}{4\pi(\mu_o+\mu_i)} \left[2\beta + \sin(\beta) - \sin(2\beta) - \frac{1}{3}\sin(3\beta) + \frac{\mu_o+\frac{3}{2}\mu_i}{\mu_o+\mu_i} \right] \right\} \quad (20)$$

Equation (20) when $\beta \rightarrow 0$ (i.e. no condensation) and when $\alpha \rightarrow 0$ (i.e a single bubble), reduces to Hadamard's (1911) formula , i.e.:

$$C_D = \left(\frac{16}{Re} \right) \left(\frac{\mu_o+\frac{3}{2}\mu_i}{\mu_o+\mu_i} \right) \quad (21)$$

which gives the drag coefficient of a single gas bubble in a clean liquid ($\mu_i \rightarrow 0$), as:

$$C_D = \left(\frac{16}{Re} \right) \quad (22)$$

When $\mu_i \rightarrow \infty$ i.e. a solid sphere:

$$C_D = \left(\frac{24}{Re} \right) \quad (23)$$

In addition, when $\beta \rightarrow 180^\circ$ (complete condensation) and when $\alpha \rightarrow 0$ (single bubble), Eq. (20) reduces to the solid sphere expression, Eq.(23) above.

The drag coefficient, of course, also depends on Re . Three distinct regions are evident. Firstly, at $Re < 1$, (the Stokes regime), an exact drag coefficient was derived for both the creeping flow and the Stokes flow ranges. Secondly, the intermediate Re regime, $1 < Re < 2000$, in which (i) a separated laminar wake, (ii) an unsteady transitional wake, and finally (iii) a turbulent wake can exist. No exact theoretical or analytical solution is available for the drag coefficient in this intermediate region. Finally, for large Re , ($2000 < Re < 300,000$), which is often termed the Newton regime, the drag coefficient has an approximately constant value of 0.4 - 0.45.

For the intermediate Re regime, the drag coefficient is generally obtained from an empirical expression. For instance, Clift and Gauvin (1970) proposed an expression for drag coefficient ($Re < 2 \times 10^5$) and it agreed with experimental data to within about 6%. Schiller-Naumann (1933) proposed an expression for the drag coefficient when $Re < 800$. Many other different empirical expressions for the drag coefficient in this flow regime can be found in the literature (e.g. Clift et al., 1978). All these expressions can be characterized by a general form (Loth, 2010; Loth, 2008) as:

$$C_D(Re \gg 1) = C_D(Re < 1) \cdot f_{Re} \quad (24)$$

where f_{Re} represents the drag correction factor for a high Re value, and $C_D(Re < 1)$ is the Stokes drag coefficient. Accordingly, to obtain the drag coefficient for the transient or intermediate region, Eq. (20) can be re-expressed

$$C_D(\alpha, \beta, Re \gg 1) = \left(\frac{32}{Re}\right) \cdot f(\alpha) \cdot f(\mu, \beta) \cdot f_{Re} \quad (25)$$

By using the Schiller-Naumann (1933) high Re correction factor (Loth, 2008)

$$f_{Re} = 1 + 0.15Re^{0.687} \quad (26)$$

Eq. (25) for high Re, now can be written as:

$$C_D = \left(\frac{32}{Re}\right) \left(\frac{(\alpha+1/2)}{(1-\alpha)^3}\right) \left(\frac{\mu_o}{4\pi(\mu_o+\mu_i)} \left(2\beta + \sin(\beta) - \sin(2\beta) - \frac{1}{3}\sin(3\beta) + \frac{\mu_o+\frac{3}{2}\mu_i}{\mu_o+\mu_i}\right)\right) (1 + 0.15Re^{0.687}) \quad (27)$$

Eq. (27) for a single bubble ($\alpha \rightarrow 0$), reduces to:

$$C_D(0) = \left(\frac{16}{Re}\right) \left(\frac{\mu_o}{4\pi(\mu_o+\mu_i)} \left(2\beta + \sin(\beta) - \sin(2\beta) - \frac{1}{3}\sin(3\beta) + \frac{\mu_o+\frac{3}{2}\mu_i}{\mu_o+\mu_i}\right)\right) (1 + 0.15Re^{0.687}) \quad (28)$$

The ratio of the drag coefficient of the two-phase bubble swarm to that of a single one is:

$$\frac{C_D}{C_D(0)} = 2 \left(\frac{(\alpha+1/2)}{(1-\alpha)^3}\right) \quad (29)$$

when $\alpha \rightarrow 0$, this yields:

$$\frac{C_D}{C_D(0)} = 1 \quad (30)$$

as expected.

3. Results and discussion

The theoretical results of the present analysis are tested by comparison with previous published experimental and theoretical data. The merit of the present model is that it is a simple model which includes the effect of the change in the two-phase bubble's apparent

viscosity, as a result of the on-going condensation process, by introducing the half opening angle. This effect is associated with the alteration of the density of the two-phase bubble and the liquid-vapour content within it. These changes can significantly affect the drag force acting on the two-phase bubble.

As discussed already, all previous studies, which are relevant to the investigation of a hydrodynamics of condensation or evaporation of the two-phase bubble in an immiscible liquid, assumed the drag force to be that of a solid sphere or an inviscid bubble. It is more appropriate to include the change in the drag force values by introducing the condensation progression, which can be found by using a half opening angle (β), which indicates the liquid - vapour content within the mother bubble or the progression of the condensation.

Fig. 3 shows the variation of the drag force with the half angle of condensation at different values of the viscosity ratio. It can be seen from Fig. 3, that regardless of the viscosity ratio, the drag force is at its minimum value when $\beta = 0$ i.e. when the bubble is entirely vapour. As condensation progresses, the liquid content in the bubble increases and so consequently does β . As a result, the internal circulation pattern is changed and the drag force increases to the solid sphere value when $\beta = 180^\circ$ (i.e. on complete condensation to a liquid bubble). Interestingly, the drag force is approximately constant at the solid sphere value for $\beta > 120^\circ$. It is also obvious that the drag force increases with increasing viscosity ratio, as a result of increasing the immobility of the two-phase bubble wall and the consequential decrease of the circulation inside the bubble.

Therefore, the value of the drag force of the two-phase bubble is located between the inviscid bubble and a solid sphere values. These results are consistent with the experimental observation of Higeta et al. (1979).

Fig. 4 shows a comparison between the present model for two different viscosity ratios (0.0307 and 0.1689, calculated via Eqs. (17) and (18)), which correspond to two different

values of β , (60° and 120° respectively), and the empirical correlation of the experimental data of Higeta et al. (1979), which was derived by Kalman and Mori (2002) for $0.1 \leq Re \leq 200$. The values of the drag coefficients from Higeta et al. (1979) show excellent agreement with the present model when $\beta = 120^\circ$ over the whole range of Re considered. For $\beta = 60^\circ$ the present model predicts lower values for the drag coefficient than Kalman and Mori (2002) for low values of Re ; however, for $4 \leq Re \leq 200$, the predictions of the two models are virtually indistinguishable for any β .

As can be seen from Fig. 5, there is some difference between our results and the empirical correlation of Haas et al. (1935), for a solid sphere, which was used by Kalman and Mori (2002) when they validated their results. For $Re > 8$, Haas et al.'s (1935) correlation suggests a lower value of drag coefficient than does our model. At large Re the probability of circulation within the bubble is increased. This means the drag coefficient will be lower than in the case of a solid sphere, and also the reduction in the drag coefficient with Re will be more significant than the case of a solid sphere, where no internal circulation can occur. Fig. 6 includes the comparison of the present model with an empirical correlation for a solid sphere given by Clift et al. (1978). This correlation is used commonly in the study of two-phase bubble condensation in an immiscible liquid (e.g. Moalem et al., 1973 and Raina et al., 1984). A good agreement between two expressions can be obtained at $Re > 1$ and for $\beta = 120^\circ$, as shown clearly in the figure. The greatest discrepancy clearly occurs for very small values of Re .

Appearing in Fig. 7 is the comparison of the present model with another two-phase bubble model called the “encapsulated bubble”. This model considers a gas bubble surrounded by a liquid shell. This structure is similar to that assumed by Mahood (2008) when studying the evaporation of a two-phase bubble in immiscible liquid media. A good agreement is evident, for $0.1 \leq Re \leq 45$, between Eq. (28) and the experimental results of Kawano and

Hashimoto (1992) for the encapsulated bubble. For $Re > 100$, the models diverge somewhat. The main reason for this divergence between the present model and those of the encapsulated two-phase bubble may be due to the assumptions made about the nature of such a bubble. The bubble size appears to be constant along its path and it has a constant rise velocity, which differs in our case.

Fig. 8 shows a comparison of the model proposed herein, with the experimental measurements of two-phase bubble condensation in an immiscible liquid, given by Higeta et al. (1979) for a pentane bubble condensing in glycerol. A good agreement is shown over the range of the Re considered for $\beta = 60^\circ$. Kalman and Mori (2002) measured the drag coefficient for a different fluid pair system (Fig. 9) over a wide range Re ($100 \leq Re \leq 1500$). The experimental results of Kalman and Mori (2002) show a satisfactory agreement with the present model (Eq. (28)) for $Re \leq 700$. The reason for the discrepancy at higher Re may be the limitation of the Re correction factor (Eq. (26)), which is valid for $Re \leq 800$.

The model proposed in Eq. (28) clearly shows good agreement with previous experimental and numerical studies. A more detailed analysis of some features of the model can now be performed with greater confidence. To this end an examination to the effect of the half opening angle (β) on the drag coefficient, as a function of Re , has been made. This is shown in Fig. 10. As expected, the drag coefficient increases as the half opening angle β increases. The minimum value occurs when $\beta = 0$ (inviscid bubble) whilst the maximum occurs when $\beta = 180^\circ$ (solid sphere). Whilst β does not affect the order of magnitude of the drag coefficient, its effect is still significant. Finally, the effect of a void fraction of the system on the drag coefficient has been tested, as shown in Fig. 11, for a constant value of $\beta = 120^\circ$. It is clear that the drag coefficient is increased by increasing the void fraction, which is consistent with other investigators' results (e.g. Marrucci, 1965; Kendoush, 2001 and Ishii and Mishima, 1984). It is also clear, through inspection of Figs. 10 and 11 that the presence

of other bubbles (i.e. the void fraction) has a much greater effect on the drag coefficient than does the degree of condensation (i.e. β).

Fig. 3. The drag coefficient as a function of β at different viscosity ratios

Fig. 4. A comparison of the drag coefficient with the empirical correlation of Kalman and Mori (2002) for a single two-phase bubble condensing in an immiscible liquid.

Fig. 5. A comparison of the drag coefficient with the empirical correlation of Haas et al. (1935) for a solid sphere

Fig. 6. A comparison of the drag coefficient with the empirical correlation of Clift et al. (1978) for a solid sphere

Fig. 7. A comparison of the drag coefficient with the experimental results of Kawano et al.(1992) for an encapsulated bubble

Fig. 8. A comparison of the drag coefficient with the experimental results of Higeta et al. (1979) for a single two-phase condensing bubble

Fig. 9. A comparison of the drag coefficient with the experimental results of Kalman and Mori (2002) of a single two-phase bubble

Fig. 10. The effect of the condensation ratio inside a two-phase bubble on the drag coefficient

Fig. 11. The effect of the void fraction on the drag coefficient for $\beta = 120^\circ$

4. Conclusions

A simple semi-analytical model for the drag coefficient of a two-phase bubble condensing in an immiscible liquid medium has been developed. In this model, the effect of the change of the liquid-vapour content has been introduced and its effect on the drag coefficient has been modelled and validated through comparison with experimental measurements and theoretical predictions for single particles and multi-phase bubbles. According to the results, the following conclusions can be drawn:

- The assumption of the similarity between the stagnant-cap bubble mechanism and the liquid content development inside the mother bubble, seems reasonable for modelling the drag coefficient of a two-phase bubble condensation or evaporation in an immiscible liquid medium.

- The use of a weighted average bubble viscosity in this model produces results which are consistent with experimental measurements of drag coefficient
- The drag coefficient increases with increasing liquid content within the two-phase bubble. This is the first time such results have been reported.
- The drag coefficient increases with increasing progress of condensation as well as with void fraction.
- This model agrees with previously reported models for single bubbles or particles, but importantly can also describe the behaviour in a swarm of bubbles. This model is therefore more general than those reported previously.

Nomenclature

a	drop radius (m)
b	cell radius (m)
C_D	drag coefficient
$C_D(0)$	drag coefficient of a single two-phase bubble
E	viscous dissipation function
F_D	drag force (N)
$f(\alpha)$	function appearing in eq.(10)
$f(\mu, \beta)$	function appearing in eq.(15)
$f(\mu, \theta_c)$	function appearing in eq.(13)
f_{Re}	function appearing in eq.(25)
r	radial coordinate (m)
Re	Reynolds number

U velocity of a continuous phase (m/s)

v inner boundary velocity (m/s)

Greek symbols

α void fraction

β half opening angle (degrees)

θ_c angular angle (degrees)

μ dynamic viscosity ($N \cdot s \cdot m^{-2}$)

ρ_c continuous phase density ($kg \cdot m^{-3}$)

ρ_{dl} dispersed phase liquid density ($kg \cdot m^{-3}$)

ρ_{dv} dispersed phase vapour density ($kg \cdot m^{-3}$)

$\tau_{r\theta}$ tangential shear stress ($N \cdot m^{-2}$)

Subscripts

c clean part of bubble surface

i dispersed phase

o continuous phase

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