Dynamic Mass of a Reaction Wheel Including Gyroscopic Effects: an Experimental Approach

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Dynamic Mass of a Reaction Wheel Including Gyroscopic Effects: an Experimental Approach

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In the recent years, driven by the increasingly stringent stability requirements imposed by some satellites' payloads (e.g. the new generation of optical instruments), the issue of accurate on-board spacecraft microvibration modelling has attracted significant interest from engineers and scientists. This paper investigates the microvibration-induced phenomenon on a cantilever configured reaction wheel assembly including sub and higher harmonic amplifications due to modal resonances and broadband noise. A mathematical model of the reaction wheel assembly is developed and validated against experimental test results. The model is capable to represent each configuration in which the reaction wheel assembly will operate whether it is hard-mounted on a dynamometric platform or suspended free-free. The outcomes of this analysis are used to establish a novel methodology to retrieve the dynamic mass of the reaction wheel assembly in its operative range of speeds. An alternative measurement procedure has been developed for this purpose, showing to produce good estimates over a wide range of frequency using a less complex test campaign compared to typical dynamic mass setups. Furthermore, the gyroscopic effect influence in the reaction wheel assembly response is thoroughly examined both analytically and experimentally. Finally, to what extent the noise affects the convergence of the novel approach is investigated.

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Nomenclature

A	=	harmonic amplitude coefficient
$\mathbf{a}$	=	power spectral density matrix of accelerations
$\mathbf{C}$	=	damping matrix
c	=	generalized damping coefficient
$\mathbf{D}$	=	dynamic mass matrix
D	=	generalized dynamic mass coefficient
d	=	distance between system centre of mass and flywheel centre of mass
$\mathbf{f}$	=	force vector
$\mathbf{G}$	=	gyroscopic matrix
H	=	harmonic fraction
h	=	distance between system centre of mass and wheel-base centre of mass
IR	=	second moment of inertia
IZ	=	moment of inertia with respect to spin axis
$\mathbf{K}$	=	stiffness matrix
k	=	generalized stiffness coefficient
l	=	height distance from mass imbalance and flywheel centre of mass
$\mathbf{M}$	=	mass matrix
M	=	generalized mass coefficient
m	=	flywheel mass imbalance
$\mathbf{q}$	=	generalized vector of coordinates
R	=	cage radius
r	=	radial distance between flywheel mass imbalance and flywheel centre of mass
$\mathbf{T}$	=	transformation matrix
T	=	kinetic energy
t	=	generalized time unit
U	=	potential energy
v	=	distance between wheel-base and ground
W	=	broadband/random noise

x	=	generalized displacement vector
X, Y, Z	=	generalized inertial frame
x, y, z	=	generalized body frame
x', y', z'	=	coordinate system obtained rotating the inertial frame about the X-axis
Ω	=	flywheel steady-space angular rotation
ω	=	angular velocity vector
ω	=	modal angular frequency
Φ	=	power spectral density matrix of forces and moments
φ	=	rotation about the y-axis
ψ	=	rotation about the z-axis
θ	=	rotation about the x-axis

Sub/Superscripts

b	=	wheel-base
c	=	coupled configuration
ffs	=	free-free boundary condition
H	=	hermitian
hm	=	hard-mounted boundary condition
i	=	generalized index
m	=	flywheel mass imbalance
mp	=	mounting point
r	=	radial rotational mode
s	=	system
t	=	radial translational mode
w	=	flywheel
xz	=	xz-plane
yz	=	yz-plane
z	=	axial translational mode

I. Introduction

Some modern spacecraft demand stringent pointing requirements and platform stability which pose a significant challenge for the control and mitigation of microvibrations on-board spacecraft. The James Webb Space Telescope [1], the Solar Observatory Solar-C [2] but also the low-cost end market (e.g. SSTL300-S1 platform [3, 4] and Skybox [5]), all carry vibration sensitive instruments on-board (e.g. high resolution cameras aiming to achieve 1m ground resolution), thereby, even displacements in the order of micro-meters of the instrument mounting interface may significantly affect the light of sight of the instrument, dramatically reducing its performance. In this context we refer to microvibration as mechanical vibrations with amplitudes in the range of micro-g's (μg) acting in a range of frequency from a few Hz up to 1 kHz [6]. These disturbances are typically induced by the on-board satellite equipment, in particular fast rotating mechanisms such as wheel assemblies [7], cryo-coolers, and antenna and solar pointing mechanisms [8]. Other possible sources of microvibration are thrusters and switches [9]. Among the various sources, however, Reaction Wheel Assemblies (RWAs) are generally regarded as the largest and most important ones [10, 11]. The induced disturbances are transmitted through the satellite structure towards the payloads or sensitive instruments, impacting on their correct functioning. Moreover, due to coupled dynamics between the source and the supporting structure [12], estimates of the microvibration effects are even more challenging [13]. Therefore, the modeling of microvibration sources and their prediction have drawn considerable interest throughout the years and an extensive amount of work can be found in literature, i.e. [7, 14–19]. Moreover, their mitigation and control using passive or active damping systems have been thoroughly investigated, for example [16, 17, 20–22], with the final objective to estimate and minimise payload pointing errors [23, 24].

Currently, the most common approach to evaluate the microvibrations induced by a RWA is to carry out experiments grounding (i.e. hard-mounted boundary condition) the RWA on a dynamometric table (e.g. Kistler table) and measure the forces and moments whilst the RWA is in operation [14, 15, 17, 25–27]. The data are subsequently used as input to the satellite Finite Element (FE) model to predict the induced effects on the payload. The concept of this methodology is, however, flawed because grounded RWA microvibrations measurements are not representative

of the real environment in which the RWA will operate as, in practice, it works in a zero “g” environment and is mounted on a flexible structure (i.e. the satellite). In fact, when a RWA is installed on a flexible supporting structure, its disturbances excite the modes of the structure which, in turn, perturb the RWA itself, thus creating coupled dynamics [12]. Hence the actual loads at the interface are different from those obtained in a hard-mounted boundary condition. For this reason, in order to accurately reproduce the dynamics when a RWA is mounted on its supporting structure, the apparent masses (also referred as dynamic masses or their inverse, the accelerances) of the source and the supporting structure at the interface mounting point between the two bodies have to be considered [15]. In [28], a method to analyse the RWA-structure coupled microvibrations was presented, considering both the RWA and the supporting structure as rigid bodies. The dynamic mass concept and importance, together with its dependence on the frequency were also introduced. The work was subsequently extended in [14, 29–31], where a practical method to retrieve the RWA dynamic mass was developed. Despite the outcomes provided better results than those obtained from the hard-mounted boundary condition, the RWA dynamic mass was measured with the flywheel not spinning (in this study referred as zero-speed condition). In contrast, when the flywheel is in operation (non-zero speed condition), it produces a dual effect: microvibration disturbances and variations of the RWA structural response as the speed increases. The latter is due to the gyroscopic effect which significantly influences the dynamic mass of the RWA. Its parameters will, in fact, not only change as a function of the frequency but also of the speed of the rotary flywheel. In [29, 32], analytical expressions of RWA dynamic mass including the gyroscopic effect based on a RWA hard-mounted microvibration model were derived and used for coupled microvibration analysis. Although the results were improved, the modes of vibration of the RWA were not included hence limiting the quality of predictions. These analyses were expanded in [33], where an experimental method to measure the driving point dynamic mass including the gyroscopic effect was discussed and validated. This involved a RWA-seismic mass coupled system suspended free-free using elastic cords and mini-shakers used to apply unit forces and moments at the interface mounting point location in one Degree of Freedom (DoF) at the time. The data were then used to estimate the coupled loads exchanged at the interface between the two bodies, including the internal

vibration modes of the RWA. The results showed some improvement with respect to previous works. However, the test configuration is quite challenging as several items have to be suspended by means of elastic cords; moreover, it is also important that the mini-shakers are perfectly aligned with the DoF they have to excite to not generate coupled responses, making this methodology less likely to be applicable.

In this article, an alternative and new approach to retrieve the dynamic mass of a RWA including the gyroscopic effect is presented. It involves measurements of the forces and moments derived from a hard-mounted boundary condition and the accelerations at the RWA mounting interface when the RWA is suspended free-free. The test setup is significantly simpler than that used in [33]. This involves the use of two dedicated platforms and common force sensors and accelerometers. A RWA model presented in [34] is used for this purpose. Characteristics of the RWA in hard-mounted and free-free boundary conditions are discussed in this article and both experimental and analytical results are used for dynamic mass studies. The dynamic mass of a RWA including the gyroscopic effect is initially obtained from its FE model and subsequently derived processing the experimental data. Coupled microvibration theory is also briefly introduced. Finally, discussions of the microvibration measurement method presented in this work are provided with emphasis given to the influence of the background noise on the convergence of the novel methodology.

II. RWA Analytical Model

In this study, a cantilever configured RWA supported by a semi-rigid suspension system is considered. The RWA disturbance model was developed in [17] and subsequently re-elaborated in [35]. An energy approach is here implemented to derive the generalized Equation of Motion (EoM) through Lagrange's equation [36]. Under linear assumptions, analytical expressions of kinetic energy, potential energy and work done can be expressed for the mass balanced flywheel case and subsequently for the mass imbalanced case. Finally, the linearized EoM of a fully mass imbalanced RWA are derived for both hard-mounted and free-free suspended boundary conditions. To clarify the terminology adopted in this article, *suspension* refers to the link between the flywheel and the rest of the mechanism; subscripts w and b refer to the flywheel and wheel-base, respectively. The model is subsequently implemented to predict the dynamic mass of the RWA in both static and

A. Wheel Assembly Modelling

7

hard-mounted or free-free). The suspension systems were modeled as a combination of five DoFs, including: two combinations of linear springs and dashpots, k_t and c_t respectively, each in one of the two radial translational DoFs (x and y); two pairs of torsional springs and dashpots, k_r and c_r respectively, each in one of the two radial rotational DoFs (θ and φ); a pair of linear spring and dashpot in the axial translational DoF (z), k_z and c_z respectively. The Z-convention is adopted and the RWA is assumed axisymmetric about its shaft. The generalized Lagrangian coordinates in the RWA model are ten (five associated with the flywheel and five describing the motion of the wheel-base): $x_w, y_w, z_w, \theta_w, \phi_w, x_b, y_b, z_b, \theta_b$, and φ_b whereas torque microvibrations are negligible due to domination of the flywheel steady speed rotation over angular speed perturbation in torque DoF and therefore ignored.

The angular velocity vector, ω_w , of the flywheel Centre of Mass (CoM) is defined by means of a Tait-Brian matrix rotation “1-2-3” (where “1” refers to a rotation about the X-axis, θ_w , “2” represents the rotation about the y'-axis, φ_w , and “3” is the rotation about the z-axis, ψ_w). Due to steady state rotation, it can be assumed $\dot{\psi}_w = \Omega$, where Ω is the constant angular speed of the flywheel. The angular velocity vector in the body coordinate system can be expressed as:

$$\omega_w = \begin{Bmatrix} \dot{\theta}_w \cos \psi_w + \dot{\varphi}_w \cos \theta_w \sin \psi_w \\ -\dot{\theta}_w \sin \psi_w + \dot{\varphi}_w \cos \theta_w \cos \psi_w \\ -\dot{\varphi}_w \sin \theta_w + \Omega \end{Bmatrix} \quad (1)$$

Considering small angles of rotations and small displacements, the linearized kinetic energy, T_w , of the balanced flywheel is derived as:

$$T_w = \frac{1}{2} \left[M_w (\dot{x}_w^2 + \dot{y}_w^2 + \dot{z}_w^2) + IR_w (\dot{\theta}_w^2 + \dot{\varphi}_w^2) - 2\Omega IZ_w \dot{\varphi}_w \theta_w \right] \quad (2)$$

Similarly, the linearized kinetic energy, T_b , of the wheel-base is obtained:

$$T_b = \frac{1}{2} \left[M_b (\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) + IR_b (\dot{\theta}_b^2 + \dot{\varphi}_b^2) \right] \quad (3)$$

In order to reproduce the flywheel radial forces and moments due to rotor imperfections, a mass

imbalance m is added to the flywheel. The point mass is located at a radius r and distance l from the flywheel CoM and produces forces and moments when the flywheel is in operation. It is able to rotate about and move in the three linear DoFs. According to Fig. 1, the position of the imbalance mass with respect to (w.r.t) the inertial frame can be described as:

$$\mathbf{x}_m = \begin{Bmatrix} x_w \\ y_w \\ z_w \end{Bmatrix} + \begin{Bmatrix} r(-\cos \varphi_w \sin \psi_w + \sin \varphi_w \sin \theta_w \cos \psi_w) + l \sin \varphi_w \cos \theta_w \\ r(\cos \theta_w \cos \psi_w) - l \sin \theta_w \\ r(\sin \varphi_w \sin \psi_w + \cos \varphi_w \sin \theta_w \cos \psi_w) + l \cos \varphi_w \cos \theta_w \end{Bmatrix} \quad (4)$$

Differentiating Eq. (4) w.r.t time, the velocity vector can be computed and finally the kinetic energy associated with the mass imbalance derived:

$$T_m = -mr\Omega^2 [(1+l) \sin(\Omega t) + (1-l) \cos(\Omega t)] \quad (5)$$

Assuming:

- i. small displacements and angles (to neglect the second order terms),
- ii. the mass imbalance is significantly smaller than the mass of the flywheel and
- iii. the flywheel angular speed considerably larger than any rotational perturbation velocity about each axis

the kinetic energy related to the RWA imbalance system is given by:

$$\begin{aligned} T_s &= T_w + T_b + T_m \\ &\approx \frac{1}{2} \left[M_w (\dot{x}_w^2 + \dot{y}_w^2 + \dot{z}_w^2) + IR_w (\dot{\theta}_w^2 + \dot{\varphi}_w^2) - 2\Omega I Z_w \dot{\varphi}_w \theta_w \right] \\ &\quad + \frac{1}{2} \left[M_b (\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) + IR_b (\dot{\theta}_b^2 + \dot{\varphi}_b^2) \right] - mr\Omega^2 [(1+l) \sin(\Omega t) + (1-l) \cos(\Omega t)] \end{aligned} \quad (6)$$

The potential energy for the mass balanced RWA is expressed as function of the internal forces and moments due to the elastic reactions of the suspension systems. The total potential energy is the sum of the translational and rotational potential energies in the four radial DoFs and that

Fig. 2 Reaction wheel assembly model section view

associated with the axial translational DoF. Due to the connection of the flywheel and the wheel-base through the suspension systems, the relative displacements of the springs are required to derive the potential energy of the system. In addition, no dynamic coupling between axial and radial DoFs is considered and no potential energy is associated with the point mass imbalance.

In relation to Fig. 2, the process to retrieve the stiffness matrix of the RWA is divided in four stages: firstly, the potential energy in the xz-plane U_{xz} is calculated; secondly the potential energy in the yz-plane U_{yz} is evaluated; thirdly, the translational axial potential energy U_z is derived.

Finally, the six potential energies, three associated with the flywheel and three associated with the wheel-base, are summed and the linearized potential energy of the system U_s is obtained:

$$\begin{aligned}
U_s \approx & \frac{1}{2} [k_{t,w} (x_w^2 + y_w^2 + x_b^2 + y_b^2 - 2x_w x_b - 2y_w y_b) \\
& + (k_{t,w} d^2 + k_{r,w}) (\theta_w^2 + \varphi_w^2) + (k_{t,w} h^2 + k_{r,w}) (\theta_b^2 + \varphi_b^2) \\
& + k_{z,w} (z_w^2 + z_b^2 - 2z_w z_b) + 2k_{t,w} d (y_w \theta_w - \theta_w y_b - x_w \varphi_w + \varphi_w x_b) \\
& + 2k_{t,w} h (y_w \theta_b - \theta_b y_b - x_b \varphi_b + \varphi_b x_b) + 2(hdk_{t,w} - k_{r,w}) (\theta_w \theta_b + \varphi_w \varphi_b)] \\
& + \frac{1}{2} [k_{t,b} (x_b^2 + y_b^2) + k_{z,b} z_b^2 + (k_{t,b} v^2 + k_{r,b}) (\theta_b^2 + \varphi_b^2) + 2vk_{t,b} (y_b \theta_b - x_b \varphi_b)]
\end{aligned} \tag{7}$$

For completion, the work done by the linear and torsional dashpots must be evaluated. Assuming the damping of viscous type, the damping forces and moments generated by the dashpots can be expressed as the product of the viscous damping coefficient and the velocities of the DoFs. According to Fig. 2, the work done can be obtained applying virtual displacements to the flywheel and wheel-base CoMs and subsequently evaluating the forces and moments produced by the dashpots.

Substituting Eq. (6), Eq. (7), and the work done in the Lagrange's equation [36], the linearized EoM of the RWA system in matrix form can be articulated as:

$$\mathbf{M}_s \ddot{\mathbf{q}}_s + (\mathbf{C}_s + \mathbf{G}_s) \dot{\mathbf{q}}_s + \mathbf{K}_s \mathbf{q}_s = \mathbf{f}_s \tag{8}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent the mass, damping and stiffness matrices of the system, respectively, $\mathbf{G}$ includes terms describing the gyroscopic effect, and $\mathbf{q}$ is the vector of generalized coordinates. All matrices in Eq. (8) are expanded and illustrated in Appendix A. Note that in both stiffness and damping matrices there are non-zero off-diagonal elements which are representative of the coupled motion of the radial translational and rotational modes due to the RWA cantilever configuration [17]. The terms on the right-hand-side of Eq. (8) represent the excitations in the system due to the imbalance mass. Although Eq. (8) captures the RWA structural modes, the gyroscopic effect, the fundamental harmonics and their amplifications, the model does not include neither sub- and higher harmonics nor the broadband noise excitations. Parameters for harmonic excitations modelling include the amplitude coefficients A_i and harmonics' fractions H_i , where i is the number of harmonics considered in each DoF. The harmonic fraction value is defined in [37] as function of the ball bearing geometry, described by the internal radius of the cage, R_{int} , external radius of the

cage, R_{ext} , and ball radius, R_{ball} . Three different velocities can be quantified: the velocity of the balls and the cage w.r.t. the stator, $\Omega_{cage,stator}$; the velocity of the balls and the cage w.r.t. the rotor, $\Omega_{cage,rotor}$; and the velocity of the balls on the two raceways, Ω_{ball} :

$$\Omega_{cage,stator} = \frac{\Omega R_{ext}}{R_{int} + R_{ext}} \quad \Omega_{cage,rotor} = \frac{\Omega R_{int}}{R_{int} + R_{ext}} \quad \Omega_{ball} = \frac{\Omega R_{int} R_{ext}}{R_{ball}(R_{int} + R_{ext})} \quad (9)$$

Broadband random excitations on the other hand are expressed as $W(\Omega)$ at this stage. Liu *et al.* [38] observed that higher harmonics and broadband noise can be superimposed to fundamental harmonics. Therefore, including these in the excitation vector on the right hand side of Eq. (8), the complete disturbance vector can be obtained:

$$f_s = \begin{pmatrix} \sum_{i=1}^{n_t} A_{t,i} \Omega^2 \sin(H_{t,i}t) + W(\Omega) \\ \sum_{i=1}^{n_t} A_{t,i} \Omega^2 \cos(H_{t,i}t) + W(\Omega) \\ \sum_{i=1}^{n_a} A_{a,i} \Omega^2 \sin(H_{a,i}t) + W(\Omega) \\ \sum_{i=1}^{n_r} A_{r,i} \Omega^2 \cos(H_{r,i}t) + W(\Omega) \\ \sum_{i=1}^{n_r} A_{r,i} \Omega^2 \sin(H_{r,i}t) + W(\Omega) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

B. Hard-mounted Configuration

Assuming the suspension system connecting the wheel-base to the ground is extremely rigid, i.e. representative of a hard-mounted boundary condition (also referred to as “blocked” configuration), only five DoFs of the RWA are left in Eq. (8) and the system stiffness and damping matrices reduce to a simpler form. Mathematically, this assumption consists in giving infinite values to the stiffness coefficients of the wheel-base to ground suspension system $\mathbf{K}_b$:

$$\lim_{\mathbf{K}_b \rightarrow \infty} \mathbf{K}_s = \mathbf{K}_{hm} \quad (11)$$

where hm denotes hard-mounted boundary condition. The resulting system of EoM can be solved to derive the displacement of the flywheel CoM. Finally, the forces and moments at the RWA mounting point can be computed as:

$$\mathbf{f}_{hm} = \mathbf{T}_{hm} \mathbf{K}_{hm} \mathbf{x}_{w,hm} \quad (12)$$

where $\mathbf{T}_{hm}$ is the transformation matrix from the flywheel CoM to the RWA mounting point. Complex coordinates can be used to describe the RWA motion and estimate its speed dependent modal frequencies. Assuming small damping and solving the homogenous system, the characteristic equation can be written as:

$$\begin{aligned} \omega_w^4 - \Omega \frac{IZ_w}{IR_w} \omega_w^3 - \left(\frac{k_{t,w}}{M_w} + \frac{k_{t,w}d^2 + k_{r,w}}{IR_w} \right) \omega_w^2 \\ + \Omega \frac{k_{t,w}IZ_w}{M_w IR_w} \omega_w + \frac{k_{t,w}k_{r,w}}{M_w IR_w} = 0 \end{aligned} \quad (13)$$

$$\omega_{z_w} = \sqrt{\frac{k_{z_w}}{M_w}}$$

Coupling between the two radial and two rotational DoFs can be observed whereas the axial mode is uncoupled and does not change with speed.

C. Free-free Configuration

Similarly to the hard-mounted case, the stiffness value of the suspension system connecting wheel-base and ground can be tuned to represent a free-free boundary condition. This is achieved by setting the coefficients with appendix b to zero in both stiffness and damping matrices in Eq. (8).

$$\lim_{\mathbf{K}_b \rightarrow 0} \mathbf{K}_s = \mathbf{K}_{fis} \quad (14)$$

where fis refers to free-free boundary condition. The resulting system of EoM can be solved to derive the acceleration vectors at the flywheel and wheel-base CoMs. For the purpose of this study,

we are interested in the accelerations at the mounting point of the RWA. Thereby, a rigid body transformation matrix $\mathbf{T}_{mp}$ has to be applied and the acceleration vector at the RWA mounting point can be computed as:

$$\ddot{\mathbf{x}}_{mp} = \mathbf{T}_{mp} \ddot{\mathbf{x}}_{fis} \quad (15)$$

where mp denotes the RWA mounting points. The EoM can be re-written in terms of complex coordinates in order to retrieve the structural modes of the RWA. These can be calculated solving the characteristic polynomial w.r.t ω_s as shown in Eq. (16).

$$\begin{aligned} \omega_s^5 - \Omega \frac{IZ_w}{IR_w} \omega_s^4 - \left[\frac{(M_b + M_w) k_{t,w}}{M_b M_w} + \frac{k_{r,w} + k_{t,w} d^2}{IR_w} + \frac{k_{r,w} + k_{t,w} h^2}{IR_b} \right] \omega_s^3 \\ + \Omega \left[\frac{(M_b + M_w) k_{t,w} IZ_w}{IR_w M_b M_w} + \frac{(k_{r,w} + k_{t,w} h^2) IZ_w}{IR_b IR_w} \right] \omega_s^2 \\ + \left[\frac{(IR_b + IR_w) (M_b + M_w) k_{r,w} k_{t,w}}{IR_b IR_w M_b M_w} + \frac{k_{r,w} k_{t,w} (h + d)^2}{IR_b IR_w} \right] \omega_s \\ + \Omega \frac{(M_b + M_w) IZ_w k_{r,w} k_{t,w}}{IR_b IR_w M_b M_w} = 0 \end{aligned} \quad (16)$$

$$\omega_{zs} = \sqrt{k_{zw} \frac{M_b + M_w}{M_b M_w}}$$

Likewise the hard-mounted case, the axial mode is decoupled from the remaining DoFs and can be solved independently. On the other hand, the four radial and four rotational DoFs are all coupled together and speed dependent, as expected. The natural frequencies can subsequently be represented as Campbell diagram as function of the angular speed of the flywheel. It can also be noted that if the mass and the inertia of the wheel-base, M_b and IR_b , respectively, tend to infinity (i.e. as to reproduce a hard-mounted boundary condition), Eq. (16) reduces to Eq. (13), as expected.

D. Dynamic Mass

The dynamic mass (at times also referred to as apparent mass) is defined as a frequency dependent ratio of the forces imparted on a body and its resulting accelerations measured at the same location where the force is applied. When dealing with rotary mechanisms, the dynamic response also varies with the speed due to the gyroscopic effect.

Generally speaking, the dynamic mass is a fully populated 6x6 matrix. Earlier works on RWAs have shown that the diagonal coefficients ($D_{w_{ii}}$ with $i = 1, 2, 3, 4, 5, 6$) are generally the most important ones [29, 32]. Moreover, the four cross-DoF terms D_{w15} , D_{w24} , D_{w42} , and D_{w51} have also to be considered, in particular for non-symmetric configurations (i.e. cantilever configured RWA). Concerning the off-diagonal elements (i.e. those correlating the rocking mode of the x and y axes), in all previous studies [17, 29, 32, 33], they were assumed negligible and set to zero. However, when a flywheel is in operation, the gyroscopic effect significantly modifies the RWA internal dynamics, and some of the off-diagonal terms assume values (in absolute value) comparable to the other coefficients in the dynamic mass matrix [34]. In this article, a validation of the analytical results reported in [34] is presented. Due to symmetry properties of the dynamic mass about its diagonal and RWA axisymmetry, the dynamic mass matrix can be re-formulated as:

$$\mathbf{D}_w = \begin{bmatrix} \textcircled{D_{w11}} & \textcircled{D_{w12}} & 0 & \textcircled{D_{w14}} & \textcircled{D_{w15}} & 0 \\ -D_{w12} & D_{w11} & 0 & -D_{w15} & D_{w14} & 0 \\ 0 & 0 & \textcircled{D_{w33}} & 0 & 0 & 0 \\ -D_{w14} & -D_{w15} & 0 & \textcircled{D_{w44}} & \textcircled{D_{w45}} & 0 \\ D_{w15} & -D_{w14} & 0 & -D_{w45} & D_{w44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \textcircled{D_{w66}} \end{bmatrix} \quad (17)$$

The 36 original elements are now reduced to only 8, circled in Eq. (17), all of which are speed and frequency dependent. As introduced in Section I, a methodology based on measurements of the forces and moments grounding the RWA on a dynamometric platform and the accelerations at the RWA mounting point when this is suspended free-free, is used to retrieve the 8 most significant elements in the RWA dynamic mass matrix. The derivation process involves a re-arrangement of the coupling theory developed in [14] and later expanded in [12, 33]. The coupled loads exchanged at the interface between a source and its supporting structure are function of the inertial properties of the bodies (described by the dynamic mass) and the disturbances generated by the source. In particular, it can be said that the coupled forces and moments can be expressed as a portion of the

forces and moments measured in a hard-mounted condition:

$$\mathbf{f}_c = \mathbf{f}_{hm} - \mathbf{D}_w \ddot{\mathbf{x}}_c \quad (18)$$

where c denotes coupling, and $\mathbf{f}_c$ and $\ddot{\mathbf{x}}_c$ are 6x1 vectors representing the coupled loads and the coupled accelerations, respectively. Nevertheless, when the RWA is suspended free-free but not attached to any supporting structure, the coupled forces vector is null. The coupled accelerations, in contrast, can be re-written in terms of the source free-free accelerations $\ddot{\mathbf{x}}_{fis}$, yielding to:

$$\mathbf{f}_{hm} = \mathbf{D}_w \ddot{\mathbf{x}}_{fis} \quad (19)$$

Equation (19) can be re-formulated in terms of Power Spectral Density (PSD). The dynamic mass is the transfer function between the loads and the accelerations calculated at the same location (i.e. RWA interface mounting point) therefore, when the PSD is applied, its Hermitian (conjugate and transpose) value has to be considered.

$$\Phi_{hm} = \mathbf{D}_w \mathbf{a}_{fis} \mathbf{D}_w^H \quad (20)$$

Finally, Eq. (20) can be solved for $\mathbf{D}_w$ at each speed and each frequency using an iterative process thus to define all the elements in Eq. (17) over the speed and frequency ranges of interest.

III. RWA Model Validation

Microvibration measurements were carried out to verify the analytical predictions for both the hard-mounted and free-free boundary conditions. The former involved the use of a dynamometric measurement platform whereas the latter required the specimen to be hung using elastic cords such to reproduce a free-free configuration. Model parameters such as stiffness and natural frequencies were extracted from the test results and compared to the theoretical values. The RWA consisted of a Brass-made rotor, an Aluminum 6082-T6 housing, and a Polyactic acid thermoplastic motor holder (acting as suspension system), resulting in an overall mass of 1.16 kg. A brushless DC-motor was used to run the rotor providing an angular momentum at 4800 revolutions per minute (rpm)

of 0.553 Nms. The RWA was spun from 600 rpm to 4800 rpm with a 60 rpm step increase and the data were recorded for 8 seconds at each speed. The entire test campaign was performed overnight in an extremely quiet environment to maintain the level of the noise as low as possible. In addition, the functioning of electrical devices such as air ventilation was halted. For instance, the typical background noise for axial measurements compared to RWA disturbances generated at 600 rpm is graphed in Fig. 3 for both hard-mounted and free-free boundary conditions.

Fig. 3 Background noise compared to the excitation level at 600 rpm

A. Hard-mounted Microvibration Testing

The hard-mounted RWA-induced disturbances were measured by means of a microvibration measurement platform developed and validated in [31]. A total of three test configurations was needed to achieve the full characterisation of the RWA-induced loads. The measurement setups in the wheel y-axis and z-axis are illustrated in Fig. 4. Measurements along the wheel x-axis were obtained by rotating the RWA y-axis test configuration by 90 degrees anti-clockwise.

In addition, a stripe of high reflective tape (e.g. Kapton tape) was embedded onto the flywheel with the purpose of synchronizing the signals acquired from different measurements (i.e. hard-mounted and free-free configurations), facilitating their processing and increasing the reliability of the results.

The predicted speed dependent modal frequencies ω_w obtained from Eq. (13) are plotted as function of the flywheel angular speed and superimposed to the experimental results as red circles, as shown in Fig. 5. Among the four speed dependent modal responses, two are identified as the Forward Whirls (FWs), starting at 51 Hz and 158 Hz and increasing with speed, and two represent the Backward Whirls (BW), starting at 44 Hz and 154 Hz decreasing with speed. The axial mode,

Fig. 4 Hard-mounted boundary condition test configuration

a) PSD of lateral force, $F_{hm,x}$

b) PSD of axial force, $F_{hm,z}$

Fig. 5 Hard-mounted loads from the tests superimposed to the RWA predicted structural modes (red circles)

in contrast, is at 117 Hz and remains constant as speed increases, as expected. A good correlation between the experimental and the simulated results can be observed. In particular, the system displays amplifications due to interaction between the angular speed of the flywheel and the speed dependent modes, matching the theoretical predictions. On this basis, the hard-mounted RWA model was verified and validated.

B. Free-free Microvibration Testing

The RWA was suspended using elastic cords aiming to reproduce an unconstrained boundary condition. The subsystem was attached to a steel frame which is able to isolate the specimen from

the surroundings, limiting any external interaction to a minimum. A set of seven accelerometers was installed at the RWA mounting interface. In particular, two accelerometers were placed in the xy -plane for measuring the radial translations along x , two accelerometers were set at 90 degrees w.r.t the former accelerometers in the xy -plane allowing measurements of the radial translations along y , and three accelerometers were placed on top of the interface plate retrieving the axial translations of the RWA, as shown in Fig. 6. The accelerometers' responses were then combined to compute the in- and out-of-plane rotations.

The predicted speed dependent modal frequencies ω_w obtained from Eq. (16) are plotted as function of the flywheel angular speed and superimposed to the experimental results as red circles, as shown in Fig. 7.

Similarly to the hard-mounted case, also here the axial mode is not influenced by speed variations and remains constant at a value of 171 Hz. In addition, two modes corresponding at the FWs start at 75 Hz and 291 Hz, respectively, and increase with speed whereas the two modes corresponding to the BWs start at 58 Hz and 290 Hz, respectively, and decrease as speed increases. It can be

Fig. 6 Free-free boundary condition test configuration

a) PSD of axial acceleration, $\ddot{z}_{fis}$

b) PSD of in-plane acceleration, $\ddot{\varphi}_{fis}$

Fig. 7 Experimental free-free accelerations superimposed to the RWA predicted structural modes (red circles)

observed that amplifications of the system responses occur at values where the angular speed of the flywheel and the speed dependent modes cross, as predicted. Overall, the experimental and analytical outcomes display a good correlation allowing the validation of the free-free RWA model.

IV. Dynamic Mass Measurements and Results

A. Analytical Prediction

The importance to evaluate the effect of the gyroscopic effect on the dynamic mass when the RWA is in operation (i.e. flywheel spinning at speed different from zero) can be observed comparing the RWA theoretical system response in either static or dynamic configurations.

a) element D_{w11} b) element D_{w45}

Fig. 8 Comparison between static and in-operation dynamic mass

Figure 8 highlights the main differences and, in particular, shows how the two initial structural resonances split in four modal resonances due to the gyroscopic effect. In addition, some of the off-diagonal terms, which are null in the static configuration, are no longer negligible when the RWA is in operation and, therefore, must be included for the complete characterization of the RWA. It is worth to note that a linear scale, rather than a logarithmic one, has been used here to show the line representing the analytical response when the flywheel is not operating in Fig. 8b due to its zero constant value throughout the whole range of frequency.

For simplicity only the elements D_{w11} , D_{w33} , and D_{w45} are shown in Fig. 9. The dynamic mass

Fig. 9 Predicted dynamic mass behaviour as function of frequency and flywheel angular speed

elements are plotted in a range of frequency from 20 to 300 Hz and in a range of flywheel speed from 600 to 4800 rpm with 60 rpm interval. The axial mode remains constant as speed increases, as expected, whereas all the other coefficients display a strong dependance from both frequency and speed. Moreover, they present coupled effects between each DoF whereas the axial mode remains uncoupled, typical of cantilever-configured RWAs.

B. Experimental results

The outcomes derived in section III were combined to evaluate the speed and frequency dependent dynamic mass of the RWA. Due to the dynamic mass imaginary complex nature (intended as composed by a real and an imaginary number) and therefore, the importance of the phase between the signals, a synchronisation procedure was required before their direct implementation in Eq. (20). This was accomplished by means of a reflective tape (e.g. Kapton tape) on the flywheel and an optical sensor which recorded any passage during the tests. The optical sensor signals measured in both the hard-mounted and the free-free tests were then cross-correlated and subsequently opportunely shifted to obtain the desired synchronisation. An example of the optical sensor response for the flywheel spinning at 600 rpm during the hard-mounted test is presented in Fig. 10.

Fig. 10 Optical sensor signal superimposed to a force transducer response as the flywheel spins at 600 rpm.

The synchronised loads and accelerations were post-processed and reformulated in terms of PSD and cross-PSD to form two 6x6 complex matrices, Φ_{hm} and $\mathbf{a}_{fis}$. The off-diagonal elements of the forces' and accelerations' matrices $\Phi_{hm,ji}$ and $a_{fis,ji}$, (with $i \neq j$), respectively, are the conjugate of the elements $\Phi_{hm,ij}$ and $a_{fis,ij}$. The matrices were subsequently implemented in Eq. (20) to retrieve the dynamic mass matrix.

The equation was solved for each frequency of interest and for each angular speed, allowing the full description of the RWA dynamic response. It must be observed that due to the non linear nature of the system (72 equations of the second order, 36 for the real part and 36 for the imaginary part, and 16 unknowns, 8 real and 8 imaginary), for the system to converge to an acceptable solution, a reasonable estimate of the initial value of the coefficients circled in Eq. (17) was mandatory. In particular, the knowledge of the main diagonal elements (e.g. the total mass of the RWA and the second moment of inertia calculated at the RWA mounting interface) allows to achieve accurate results. In practice, these are also the simplest values that can be measured (i.e. by means of a scale). For the RWA under investigation these are 1.16 kg and $5.9 \times 10^{-3} \text{ kgm}^{-2}$, respectively. The outcomes of the analysis were subsequently compared to the theoretical predictions and are here illustrated in Fig. 11 for an angular speed equal to 660 rpm.

An overall acceptable agreement can be observed, in particular the axial translation mode is correctly captured throughout the frequency range. The experimental results display an initial poor correlation (between 20 Hz and 50 Hz) for the radial mode in Fig. 11b. Despite the trend of the curve is similar to the predicted one, its amplitude is lower. Nevertheless, the outcomes become more accurate as frequency increases, specially after the first backward and forward whirls. Moving towards the second backward and forward whirls, these show a better agreement with the theoretical values. The curve then decreases with the same pattern of the FE model estimate. However, a series of spikes between 230 Hz and 260 Hz can be observed which are not present in the analytical prediction. Considering the axial mode only, the experimental and predicted data correlate well in all the range of frequency and the modal resonance at 117 Hz is correctly captured.

The mismatches occurring at both low and high frequencies were thoroughly investigated as to determine the causes which led to values differing from the predicted ones. The dominant reason was

a) element D_{w33} b) element D_{w44}

Fig. 11 Comparison between predicted and experimental dynamic mass at 1320 rpm

identified in the different characteristics of the broadband noises, in terms of spectral contribution, measured in the hard-mounted and free-free configurations. In section IV C, a demonstration of the influence of the broadband noise is given by means of the analytical model. The outcomes are then used against test results to validate the aforementioned thesis.

C. Influence of broadband noise

The noise spectra measured during the two test campaigns (hard-mounted and free-free configurations) may severely affect the convergence of Eq. (20). If the spectral contribution of the noise obtained in the two different configurations is significantly different (hence the RWA excitation vectors are different) then the iterative process to calculate the dynamic mass of the RWA using Eq. (20) may fail or lead to inaccurate calculations at some frequencies in the spectrum. This matter was investigated analytically using the model described in section II and the outcomes are shown in Fig. 12.

A set of random noise signals was generated and applied to the models. A noise ratio was calculated between the magnitude of the noise applied to the model in hard-mounted boundary condition and that given as input to the model in free-free configuration. Due to the nature of this analysis, for which only the effect of the background noise is emulated, a zero-speed condition was assumed. The results displayed a remarkable difference as the noise ratio increases. It can also be observed that the noise ratio has less influence on the axial DoF compared to the other DoFs. Moreover, the variation of dynamic mass coefficient D_{w33} as function of the noise ratio is linear. Furthermore, Fig. 12b shows that even small differences in the noise spectra in the range between 270 Hz and 300 Hz may lead to considerable miscalculations in the dynamic mass estimation.

The spectral contribution of the noise must thereby be similar between the hard-mounted case and the free-free case. The background noise spectrum derived from the hard-mounted configuration was compared to that obtained in the free-free configuration. The latter required to be re-elaborated using Eq. (20), where the term $\mathbf{a}_{fis}$ represents the PSD of the background noise in the free-free boundary condition and the term $\mathbf{D}_w$ represents the RWA dynamic mass from FE model. The outcomes are shown in Fig. 13.

It can be observed that the spectral contribution is different in the range of frequency between 20 Hz and 50 Hz and above 230 Hz between the two configurations. This therefore, proves that the dominant reason which led to a inaccurate calculation of the dynamic mass in the aforementioned ranges of frequency, as depicted in Fig. 11, lies in the different noise spectra between the hard-mounted and the free-free tests, validating the thesis herein stated.

a) element D_{w33} b) element D_{w44}

Fig. 12 Effect of different noise inputs on the calculation of the wheel assembly dynamic mass

V. Conclusions

In this paper, a mathematical model able to characterize the microvibrations produced by a mass imbalanced cantilever-configured RWA in both hard-mounted and free-free boundary conditions, including the gyroscopic effect, has been developed and validated. This work also presents a novel

Fig. 13 Spectral contribution of the background noise in the hard-mounted and free-free configurations

methodology for the measurement of the dynamic mass, which is derived from a re-elaboration of the coupling theory. This represents an alternative approach to the current practice which involved the use of mini-shakers in a challenging test setup (e.g. free-free suspended RWA with the flywheel spinning while forces are exerted by mini-shakers). The procedure presented herein has proved to

produce accurate outcomes implementing an extremely simpler test configuration. This involves the use of the reaction loads at the RWA mounting interface measured in a hard-mounted configuration and the accelerations at the RWA mounting interface measured in a free-free boundary condition. In addition, compared to the traditional approach (where the flywheel is not in operation), the method allows the complete characterization of the RWA and express the dynamic mass as function of both frequency and flywheel angular speed. A good agreement was observed between the mathematical model and the experimental data and the forward and backward whirls due to the gyroscopic effect were correctly captured. The main differences between the static and in-operation dynamic mass were highlighted, showing the significant influence of the angular speed on the RWA dynamic response. In addition, it was observed that the off-diagonal terms typically neglected in the previous approaches, become important when the flywheel is spinning and, therefore, require to be included for the complete characterization of microvibration sources. Finally, a brief investigation on the influence of the noise on the derivation of the dynamic mass using the novel approach was conducted. Different set of noise signals were implemented in both the hard-mounted and free-free analytical models. The outcomes showed that different levels of noise between hard-mounted and free-free tests can significantly deteriorate the quality of the results.

Appendix

With reference to Eq. (8), all matrices are as follows:

$$\mathbf{q}_s = \{x_w, y_w, z_w, \theta_w, \varphi_w, x_b, y_b, z_b, \theta_b, \varphi_b\}^T$$

$$\mathbf{M}_s = \text{diag} \{M_w, M_w, M_w, IR_w, IR_w, M_b, M_b, M_b, IR_b, IR_b\}$$

$$\mathbf{G}_s = \begin{cases} G_{s45} = \Omega I Z_w \\ G_{s54} = -\Omega I Z_w \\ 0, \quad \text{elsewhere} \end{cases}$$

Table 1 Damping coefficients in compact format

$c_{11} = c_{t,w}$	$c_{15} = -c_{t,w}d$
$c_{33} = c_{z,w}$	$c_{44} = c_{t,w}d^2 + c_{r,w}$
$c_{29} = c_{t,w}h$	$c_{49} = c_{t,w}hd - c_{r,w}$
$c_{66} = c_{t,w} + c_{t,b}$	$c_{88} = c_{z,w} + c_{z,b}$
$c_{79} = -(c_{t,w}h - c_{t,b}v)$	$c_{99} = c_{t,w}h^2 + c_{r,w} + c_{t,b}v^2 + c_{r_v}$

$$\mathbf{C}_s = \begin{bmatrix} c_{11} & 0 & 0 & 0 & c_{15} & -c_{11} & 0 & 0 & 0 & -c_{29} \\ 0 & c_{11} & 0 & -c_{15} & 0 & 0 & -c_{11} & 0 & c_{29} & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 & 0 & -c_{33} & 0 & 0 \\ 0 & -c_{15} & 0 & c_{44} & 0 & 0 & c_{15} & 0 & c_{49} & 0 \\ c_{15} & 0 & 0 & 0 & c_{44} & -c_{15} & 0 & 0 & 0 & c_{49} \\ -c_{11} & 0 & 0 & 0 & -c_{15} & c_{66} & 0 & 0 & 0 & -c_{79} \\ 0 & -c_{11} & 0 & c_{15} & 0 & 0 & c_{66} & 0 & c_{79} & 0 \\ 0 & 0 & -c_{33} & 0 & 0 & 0 & 0 & c_{88} & 0 & 0 \\ 0 & c_{29} & 0 & c_{49} & 0 & 0 & c_{79} & 0 & c_{99} & 0 \\ -c_{29} & 0 & 0 & 0 & c_{49} & -c_{79} & 0 & 0 & 0 & c_{99} \end{bmatrix}$$

Table 2 Stiffness coefficients in compact format

$k_{11} = k_{t,w}$	$k_{15} = -k_{t,w}d$
$k_{33} = k_{z,w}$	$k_{44} = k_{t,w}d^2 + k_{r,w}$
$k_{29} = k_{t,w}h$	$k_{49} = k_{t,w}hd - k_{r,w}$
$k_{66} = k_{t,w} + k_{t,b}$	$k_{88} = k_{z,w} + k_{z,b}$
$k_{79} = -(k_{t,w}h - k_{t,b}v)$	$k_{99} = k_{t,w}h^2 + k_{r,w} + k_{t,b}v^2 + k_{r_v}$

$$\mathbf{K}_s = \begin{bmatrix} k_{11} & 0 & 0 & 0 & k_{15} & -k_{11} & 0 & 0 & 0 & -k_{29} \\ 0 & k_{11} & 0 & -k_{15} & 0 & 0 & -k_{11} & 0 & k_{29} & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 & 0 & -k_{33} & 0 & 0 \\ 0 & -k_{15} & 0 & k_{44} & 0 & 0 & k_{15} & 0 & k_{49} & 0 \\ k_{15} & 0 & 0 & 0 & k_{44} & -k_{15} & 0 & 0 & 0 & k_{49} \\ -k_{11} & 0 & 0 & 0 & -k_{15} & k_{66} & 0 & 0 & 0 & -k_{79} \\ 0 & -k_{11} & 0 & k_{15} & 0 & 0 & k_{66} & 0 & k_{79} & 0 \\ 0 & 0 & -k_{33} & 0 & 0 & 0 & 0 & k_{88} & 0 & 0 \\ 0 & k_{29} & 0 & k_{49} & 0 & 0 & k_{79} & 0 & k_{99} & 0 \\ -k_{29} & 0 & 0 & 0 & k_{49} & -k_{79} & 0 & 0 & 0 & k_{99} \end{bmatrix}$$

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