Assessment of flat deck metallic plates – yield line and membrane analysis

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Abstract

A large number of metallic riveted bridges have been constructed using flat plates to form the deck of the structure. When assessed using traditional elastic methods these plates are routinely found to be under capacity for the application of the prescribed load even though they show zero signs of distress. This paper considers the use of alternative methods of assessment, namely yield line and membrane analysis, utilising the beneficial effects of plastic methods of analysis where appropriate to enhance the assessed capacity of these failing plates. The analytical formulation of both methods is presented considering the effects of the plate aspect ratio, support conditions, the presence of stiffeners, plate thicknesses and rivet sizes. By comparing the reassessed capacities of the plates obtained through the refined methods with the original assessed capacities it is shown that the former offer considerable enhancements in assessed capacity ranging between a factor of 1.3 to 7.

Keywords: metallic plates, riveted bridges, virtual work, assessment

1. Introduction

Many of the 19th century metallic bridges were constructed using metallic plates, made from wrought iron or mild steel, formed into riveted girders with flat metal plates spanning between them comprising the main bridge deck structure. Fig. 1 indicates a typical plate arrangement where the plate is spanning longitudinally between girders and transversely onto a stiffener or a secondary girder with a lower stiffness. These plates, when assessed using simple methods, tend to be found to be under capacity for the level of highway loading prescribed by modern assessment codes without, however, showing any signs of distress. For example, when considering highway loading, in general flat plates are assessed as inadequate to carry the required 40t assessment loading prescribed by the current UK assessment code [1]. These conservative assessment methods tend to be based on elastic bending theory and limit the use of internal mechanisms and hidden, usually difficult to quantify, strengths. One of the most extreme examples of conservative assessment is where a plate that is riveted on four sides is considered as a simply supported single spanning beam; this is unlikely to be representative of the in service loading conditions and structural stiffness of the real structure.

The aim of this paper is to consider alternative methods of analysis that can be employed by bridge engineers to assess the ultimate limit state capacity of flat deck plates. The beneficial effects of plastic behaviour are reviewed by considering the use of yield line analysis and the use of membrane analysis, both of which may offer enhancements to the assessed plate capacity. The theoretical background behind these two methods of analysis is first presented and the related formulation developed. Subsequently, the newly proposed methods are applied in three case study bridges having different plate configurations, and the enhancement of their assessed capacity, compared to traditional methods, is demonstrated.

Fig. 1. Typical flat deck plate bridge deck construction.

2. Yield Line Analysis

2.1. Theoretical background

Yield line analysis, which has been developed by Johansen [2, 3] for the design of reinforced concrete slabs, is applicable to the assessment of any material that can develop plastic behaviour. The method is based on the theory of virtual work where the work done by external loads is equated to the internal work generated by a small displacement within the failure mechanism.

By considering a strip of plate where the plate is fully restrained at the supports as shown in Fig. 2, the internal work E is defined as

$$E = \sum (M_p \cdot \phi \cdot L) \quad (1)$$

where M_p is the plastic moment capacity of the yield line, ϕ is the angle of rotation of the yield line and L is the length of the yield line.

The external work W is defined as

$$W = \sum(F \cdot \Delta) \quad (2)$$

where F is the force acting on the mechanism and Δ is the displacement at the centroid of the force.

(a) Internal work

(b) External work

Fig. 2. Yield line mechanism example.

Steel and wrought iron deck plates are suitable for the application of yield line due to the plastic nature of the material. Consideration must be given to the interaction of all elements to define a suitable model of the structural system. In order to identify a valid yield line pattern consideration must be given to the support conditions, loading conditions, plate geometry and to the relative stiffness within the structural system. Sagging yield lines will develop along

the lines of vertical deflection and hogging yield lines will develop at points of restraint. The level of this restraint will vary based on the support conditions. Yield lines will develop in the vicinity of supports where the reactions due to the generation of the plastic moment can be sustained; therefore, yield lines cannot be generated at simple supports.

Typically, in flat plate highway bridges, the application of a single, centrally placed, wheel load on a single plate and also a pair of plates needs to be considered. Fig. 3 demonstrates the application of a single wheel load to a single plate, denoted here as Case 1 (Case 1a for simply supported plate edges; Case 1b for fully-fixed plates edges). The load is placed centrally to give the peak deflection and corresponding rotations. The figure indicates the location of the sagging and hogging yield lines. In the case of a plate simply supported on all four edges, only sagging yield lines need to be considered whereas in the case of fully-fixed supports both sagging and hogging lines need to be considered.

Fig. 3. Application of single wheel load to single plate (Case 1).

Fig. 4 demonstrates the application of a single wheel load to a pair of plates, denoted here as Case 2 (Case 2a for simply supported plate edges; Case 2b for fully-fixed plates edges). It is placed centrally above the transverse support to give the peak deflection and corresponding rotations. The figure indicates the location of the sagging and hogging yield lines, the latter being present in the case of fully-fixed plate edge conditions.

Cases 1 and 2 are considered here using the method of virtual work based on the notation shown in Fig. 5. The virtual work equation takes the following form

$$\Sigma(M_p \cdot \phi \cdot L) = M_p(2\phi_1 L_{bd} + \phi_2[L_{ab} + L_{bc} + L_{de} + L_{df}] + \phi_3[L_{ab} + L_{bc} + L_{de} + L_{df}] + \phi_4[L_{ac} + L_{ef}]) \quad (3)$$

Fig. 4. Application of single wheel load to pair of plates (Case 2).

Where stiffeners cross a yield line perpendicularly, as shown in Fig. 6, the capacity of the yield line can be enhanced to include the plastic moment capacity of the stiffener $M_{p-stiffener}$.

The capacity of yield line L_{bd} in Fig. 6 becomes

Plate Geometry and Yield Line Notation

Fig. 5. Virtual work notation.

Fig. 6. Stiffeners crossing yield lines.

$$[2M_p \cdot \varphi \cdot L_{bd}] + [2 \cdot 2M_{p-stiffener} \cdot \varphi] \quad (4)$$

which considers the effect of the two stiffeners crossing the yield line. During the assessment, it should be ensured that if the composite section of the plate and stiffener is used, the length of the yield line considered (L_{bd}) is reduced by the total width of the stiffeners crossing the yield line. If this is not taken into account then the capacity of the yield line will be overestimated by double counting the capacity of the plate.

If a plate has numerous closely spaced stiffeners, as shown in Fig. 6, and the diagonal yield lines cross this line of stiffeners, the moment capacity of this yield line should be resolved to give a capacity of

$$M_p \cdot L_{ab} \cdot \cos\beta \quad (5)$$

perpendicular to the yield line [4]. This capacity can be defined as

$$M_{ab} = M \cdot \sin^2\beta + m \cdot M \cdot \cos^2\beta \quad (6)$$

where m is the ratio of the transverse capacity to the longitudinal capacity.

Fig. 7 demonstrates the application of a single wheel load to a single plate generating a circular fan failure, denoted as Case 3. The internal work E can be obtained from [5]

$$E = 2p(m + m') \quad (7)$$

where m and m' are the plastic moment capacities of the plate in the longitudinal and transverse directions respectively and p is the value of the point load. For a plate without stiffeners, $m' = m$.

The external work W can be obtained as

$$W = q_1(\pi r_1^2) \frac{\Delta}{3} + \left[q_2(\pi r_2^2) \left(\frac{r_1^2 - 3r_2^2}{r_1} \right) \right] \Delta \quad (8)$$

where q_1 and q_2 are the dead and live loads (in kN/m^2), r_1 is the radius of the failure fan and r_2 is the radius of the dispersed wheel load. Equating equations (7) and (8) and solving for q_2 will give the maximum load that can be applied to the plate.

Fig. 7. Application of single wheel load producing a circular fan failure (Case 3).

Theoretically, there are an infinite number of yield line mechanisms that can be developed. Of this infinite number, there is a lower bound solution which gives the minimum failure load for the plate. The yield line patterns chosen and outlined in this paper are based on published solutions and can be considered as a reasonable starting point for the analysis. The circular fan failure mode (Case 3) tends to be the critical yield line pattern as it will almost certainly generate the lower bound solution. However, this pattern is only likely to be generated with a fully-fixed plate at its edges [6] and would tend to become an elliptical failure fan for a rectangular plate.

The cases outlined above consider that the support conditions are the same on all sides of the plate. However, where varying support conditions exist, such as rivet arrangements that cannot sustain the applied hogging moment or a mixture of simple and restrained supports, there is potential for the failure mechanism indicated in Fig. 8, denoted as Case 4.

Fig. 8. Plate with simple and fixed supports (Case 4).

The work equation for this case is

$$E = 2\pi(m + m') \frac{\varphi}{180} + 4m \cdot \varphi_1 r_1 \quad (9)$$

where m and m' are as defined above, r_1 is the radius of the failure fan and φ is the angle of the failure fan.

The external work W is given by

$W =$

$$\left(q_1 (\pi r_1^2) \frac{\Delta}{3} \left(\frac{2\varphi}{360} \right) \right) + \left(q_1 L a \frac{\Delta}{3} \right) + \left[q_2 (\pi r_2^2) \left(\frac{2\varphi}{360} \right) \left(\frac{r_1 - \frac{2}{3} r_2}{r_1} \right) \right] \Delta + \left[q_2 (\pi r_2^2) \left(\frac{360 - 2\varphi}{360} \right) \left(a - \frac{2}{3} r_2 \right) \right] \quad (10)$$

The cases above should be used to estimate the lowest failure load of the plate for the support conditions identified. For plates with riveted connections it is reasonable to consider that the supports are fixed-ended and the capacity of the yield line should be limited to the capacity that can be generated by the rivets in tension.

2.2. Assessment method

The first stage of the assessment is to ensure that the wheel load on the plate is distributed through the surfacing and fill, as shown in Fig. 1. BD21/01 allows this distribution through the surfacing down to the plate at a spread to depth ratio of 1:2 [1]. The support conditions are then identified; if the connection between the plate and supporting elements is riveted it can be considered to be fixed provided that the connection can be proven to be able to sustain the moment generated at the support. Any stiffeners that could enhance the capacity should be considered by modifying the yield line capacity as described in the previous section. The analysis of the plate can then be carried out using the theory of virtual work or modified published solutions.

Once the plate capacity has been established, the supporting elements must be assessed to ensure that the load effects generated by the imposed loading on the supporting elements can be sustained. Plastic section properties can be used to derive the capacities of these sections where appropriate, provided that the section can be shown to be able to generate the plastic moment capacity. Care should be taken to ensure that the capacity of the supporting element is not used in the yield line capacity.

In cases where the support conditions are considered to be fully-fixed, this assumption can be validated by considering the capacity of the riveted connection to resist the applied moment. Typically, the moment capacity of the connection can be calculated by considering the tensile capacity of the rivet multiplied by the distance between the centreline of the rivet to the outside edge of the flange plate.

3. Membrane Analysis

When a plate deflects by a distance larger than one half of the thickness of the plate, the middle surface of the plate is subjected to significant strains and associated stresses which have a significant effect on the behaviour of the plate [7]. Whilst it is conservative to ignore this effect in assessment, this behaviour cannot be represented using simple beam theory as this assumes that the element under loading is not subject to significant deflection and that there is zero strain at the middle surface. Flat plates used in riveted bridges usually have large span to depth ratios hence they are likely to deflect significantly under load and will reach their yield stress under the application of the Ultimate Limit State (ULS) wheel loading. The plates are restrained by the riveted connections at the extremes of the plate, hence the edges cannot move towards each other. This restraint results in axial stresses throughout the plate that reduce the bending stresses and associated deflection and enable the plate to carry more load. The axial forces that develop in the plate have to be resisted by the riveted connections at the plate edges.

For a uniformly loaded rectangular plate with simply supported edges, shown in Fig. 9, the bending moment M is given by [8]

$$M = \frac{ql}{2}x - \frac{qx^2}{2} - Sw_x \quad (11)$$

where q is the applied load intensity, l is the plate span, x is the distance to point under consideration from support, S is the axial force at the restraints and w_x is the plate deflection. This is based on the transverse deflection of a plate with simply supported edges that can rotate but are restrained in the plane of the plate. The theory is based on elastic analysis, hence when applied to the heavily loaded plates considered in this paper results in the computation of stresses that cannot be sustained elastically. Equation (11) can be modified to consider the plastic behaviour of the plate. In order to do that, the interaction between the different elements needs consideration. The applied load is distributed through the road construction onto the plate and then transferred from the plate through the riveted connections into the supporting structure. The transfer of the axial force through the rivets is resisted by the plate bearing on the rivets and shear transfer through the rivets into the plates forming the main and secondary girders.

Fig. 9. Membrane analysis notation.

For the case of a plate having simple supports along its edges, Equation (11) can be re-written as

$$M_c = M_x - S \cdot w_x \quad (12)$$

where M_c is the moment capacity of the section (consideration must be given for the reduction in bending capacity due to the direct axial stress), M_x is the applied moment at the

section being considered, S is the axial force restraining the plate acting in the plane of the plate, taken as the capacity of the riveted connection and w_x is the deflection of the plate. Rearranging Equation (12), the axial force S in the plate due to a given deflection can be calculated as

$$S = \frac{M_x - M_c}{w_x} \quad (13)$$

M_c can then be equated to the moment at which the plate will first yield, prior to the onset of plastic behaviour by making use of simple bending theory. The radius of curvature R will be given by

$$R = \frac{EI}{M} \quad (14)$$

where EI is the bending stiffness of the plate. R can be approximated to the deflection w_x by considering the geometry of a circular arc having the relationship indicated in Fig. 10 as

$$R = \left(w_x^2 + \frac{l^2}{4} \right) \frac{1}{2w_x} \quad (15)$$

Fig. 10. Circular arc geometry.

The deflection w_x that relates to the first yield of the plate can then be found by trial and error by iteration to identify the radius of curvature and deflection that corresponds to the span length l .

As w_x is limited to the deflection of the plate at first yield based on elastic analysis then S becomes that axial force in the rivets at first yield of the plate. This axial force can be considered to act across the width of the plate and transfer through the riveted connection. If the riveted connection can be shown to resist the effects of this axial force then the plate is shown to be capable of resisting the applied loading as the deflection has been limited to that at first yield and therefore has additional capacity available when considering plastic behaviour. Alternatively, for plastic analysis, if we consider S to be limited to the capacity of the riveted connection we can calculate the deflection w_x required to generate this force and consider if this is within acceptable limits.

4. Results and discussion

4.1. Case study bridges

Both the yield line analysis and the membrane analysis have been used to derive the capacity of the flat deck plates for three case study bridges, denoted, in this paper, as Bridge A, B and C. Bridges with plates with different aspect ratios have been chosen to demonstrate the applicability of the proposed methods as described below.

Bridge A is a riveted steel structure where the plates of the bridge span 1.27m between longitudinal girders and 1.22m between the transverse stiffeners. The bridge has been originally assessed as limited to 7.5 tonnes assessment live load due to the capacity of the flat deck plates. The BD21/01 [1] assessment had reported the capacity based on the plate and stiffener arrangement in the service bay using a Pucher chart [9] for the assessment. Fig. 11

shows the connection detail at the transverse stiffeners for Bridge A. For the reassessment using the refined methods developed in this paper and when considering the fixed end case, the hogging yield lines were considered to pass through the line of the riveted connection. The capacity of the yield line at these locations was reduced to take into account the rivet holes and ensure that the rivet capacity could provide the fixed end moment capacity. For the simply supported case, the support was considered at the line of rivets.

Fig. 11. Connection detail at transverse stiffener for Bridge A.

Bridge B is a riveted wrought iron structure and its flat deck plates span 1.24m between longitudinal girders and 0.60m between transverse stiffeners. The bridge had originally been assessed as limited to 3 tonnes assessment live load due to the capacity of the flat deck plates. The assessment of the plate was based on the application of the wheel load as a point load at midspan of the plate which was considered as simply supported on two parallel edges. Fig. 12 shows the connection details both at the main longitudinal girders as well as at the transverse stiffeners for the bridge. For the purposes of the plate reassessment, the hogging yield lines were considered in a similar manner to those generated at Bridge A. For the simply supported case, the support was considered at the line of rivets.

Fig. 12. Connection details at main longitudinal girder (top) and transverse stiffener (bottom) at Bridge B.

Bridge C is a riveted wrought iron structure where the flat deck plates span 1.22 between longitudinal girders and 2.29m between transverse girders. The bridge has been originally assessed as limited to dead load only; this theoretically indicates that the structure cannot carry any vehicle loading. The plates had been originally assessed assuming fixed support conditions using Pounder's plate theory [10] for the dead load and Roark's formulae for centre point loads for the application of the wheel load [7]. Fig. 13 depicts the connection details at the main longitudinal girders and at the transverse stiffeners for the bridge. The plates have a single transverse tee stiffener midway between the transverse girders. It is riveted to the top of the plate which limits its use for plastic analysis as the web of the T would buckle before the plastic section capacity could be reached. For the plate reassessment,

the support conditions for this plate had to consider yielding of the 3/8" (9.5mm) flange plate at the longitudinal beam for the fully fixed case. The hogging yield lines at the transverse girders were considered in the same manner as the other two structures. For the simply supported case the support was considered at the line of rivets.

Fig. 13. Connection details at main longitudinal girder (bottom) and transverse stiffener (top) at Bridge C.

The original assessment methods used to derive the capacities of the deck plates identified above have used analyses considered to be conservative for a number of reasons. The assessment of Bridge B ignored the beneficial effects of applying a uniformly distributed load as well as ignoring the effects of the plate spanning in two directions. Similarly, Bridge A has

been assessed as single spanning although load distribution is included through the use of a Pucher chart [9]. Bridge C has been assessed in the least conservative manner, when considering elastic methods, consideration has been given for the effects of built in construction due to the stiffness of the riveted connection as well as ensuring that the plate is assessed as supported on all four edges. The effects of load distribution have also been considered as the dead load was applied as a uniformly distributed load and the wheel loading was applied as a circular patch load.

4.2. Yield line analysis

Table 1 presents the results obtained by using the yield line analysis for the reassessment of the plates of the three case study bridges. The Ultimate Limit State (ULS) plate capacities for different assumed support conditions presented in the previous sections are shown. The results for the single plate and the double plate cases are based on a β angle value of 45 degrees in Equations (5) and (6). Whilst this is a good starting point, in further analysis it may be possible to demonstrate that modifying the β angle reduces the capacity of the plate. In practice this may not be necessary as it can also be seen from Table 1 that the fan failure mode theoretically governs the plate capacity. The actual result would lie between these values and will be reliant on the support conditions.

It can be seen in Table 1 that Bridge A has been reassessed as being capable of withstanding a 150kN wheel load which equates to a 40t assessment rating whereas the original assessment had given a 7.5t assessment rating. The capacity of Bridge C has also been reassessed as 40t provided the surfacing is maintained to a high standard, evidently a significant increase to the original zero assessment load. The final structure, Bridge B has been shown able to withstand an 82.5kN wheel load, equivalent to 7.5t assessment load, which is higher than the original 3t assessment live load.

Yield line analysis – Based on application of 150 kN wheel load (ULS)					
	Single Plate		Double Plate		Fan failure
Structure	Case 1a	Case 1b	Case 2a	Case 2b	Case 3
Bridge A	98.4 kN	170.3 kN	94.5 kN	181.7 kN	157.6 kN
Bridge B	121.1 kN	238.9 kN	50.3 kN	97.7 kN	82.5 kN
Bridge C	102.6 kN	168.5 kN	N/A – Transverse girders will not yield before plate due to significant stiffness		137.7 kN

Table 1. Plate capacities based on the application of yield line analysis.

Table 2 shows the comparison of the plate capacities obtained from yield line analysis to the original capacities obtained from the traditional assessment methods. The potential increase in capacity obtained by the use of yield line analysis is evident as compared to the conservative nature of the original assessments. The reassessed capacities obtained through yield line analysis range between 1.9 to 6.6 times higher than their original counterparts.

Structure	Original assessment rating	Original wheel load capacity	Yield line analysis wheel load capacity	Revised assessment rating	Increase in capacity
Bridge A	7.5 tonnes	57.3 kN	157.6 kN	40 tonnes	2.75
Bridge B	3 tonnes	44.0 kN	82.5 kN	7.5 tonnes	1.88
Bridge C	0 tonnes	20.8 kN	137.7 kN	40 tonnes	6.62

Table 2. Comparison of yield line assessment results with original assessments.

It should be noted that the results achieved through yield line analysis should be considered an upper bound solution although can be considered to provide a safe estimation of the plate capacity. This is because there are other factors that are not considered in the analysis that will enhance the capacity of the plate. These include the arching action of the fill and the

tensile membrane action of the plate, which is also being considered in this paper through membrane analysis.

4.3. Membrane analysis

Table 3 presents the results obtained by using membrane analysis for the reassessment of the plates of the three case study bridges, compared to the original capacities obtained from the traditional assessment methods. The deflection required to mobilise the axial force S at the connection and corresponding relieving moment $S \cdot w_x$ are discussed below.

The reassessment of the flat deck plate of Bridge A suggests that the deflection required to mobilise the axial force and relieving moment is 41mm in the transverse direction (1.147m between transverse supports) for the application of a 150kN wheel load. This is over three times the thickness of the plate and equivalent to span/28, therefore it is beyond the elastic limit of the plate material and would certainly require considerable plastic behaviour of the plate material to be mobilised. It has been found that it would be reasonable to consider the safe wheel load capacity of this plate to be 75 kN, equating to approximately 50% of the deflection required for a 150kN wheel load, as the wheel load is the dominant load on the plate.

Structure	Original assessment rating	Original wheel load capacity	Membrane Analysis wheel load capacity	Revised assessment rating	Increase in capacity
Bridge A	7.5 tonnes	57.3 kN	75 kN	7.5 tonnes	1.31
Bridge B	3 tonnes	44.0 kN	150 kN	40 tonnes	3.41
Bridge C	0 tonnes	20.8 kN	150 kN	40 tonnes	7.21

Table 3. Comparison of membrane analysis results with original assessments.

The reassessment of the flat plate of Bridge C through membrane analysis suggests that with a deflection of 21mm in the shorter span direction (1.122m between transverse girders) the plate and rivets have sufficient capacity to resist the axial load required to resist the load at this deflection. This has resulted in a potential assessed capacity of 150kN wheel load (equivalent to 40t assessment live load) for this plate. The deflection here is approximately 1.65 times the thickness of the plate and equivalent to span/53 so whilst plastic behaviour is required this would be in reasonable limits. Similarly, the reassessment of the flat plate of Bridge B suggests that the plate may be able to sustain a 150kN wheel load if the plate is able to deflect by 12.4mm in the transverse direction (600mm between transverse stiffeners). This is 1.15 times the thickness of the plate and equivalent to span/48, so whilst plastic behaviour is required this would be in reasonable limits.

The potential enhancement of the assessed capacity is clear from the reassessment of Bridge B and Bridge C. Both bridges have been reassessed to 40t assessment live load through membrane analysis, although originally they had been assessed to 3t and 0t assessment load. However, the reassessment of Bridge A shows that there may be limitations due to the significant deflections required to mobilise the restraint force and corresponding relieving moment. There are specific differences between the three bridge structures that are considered to be significant factors in the outcome of the reassessments; these are the plate thicknesses, the rivet diameter and the longitudinal connection length and are discussed in the following.

The plate thickness at Bridge C is 12.7mm whereas the plate thickness at Bridge B is 10.7mm. As the thickness of the plate reduces, the capacity of the connection also reduces due to bearing and shear of the rivets. This effect is demonstrated in Fig. 14. The plate itself is also weaker in direct axial tension. In terms of the rivet diameters, the diameter at Bridge C is 7/8" (22.2mm) whereas the rivet diameter at Bridge B and Bridge A is 3/4" (19.1mm). As

the rivet diameter reduces, the capacity of the connection reduces due to the shear capacity of the riveted connection, as shown in Fig. 15. On the other hand, the effect of the length of the longitudinal connection becomes evident by comparing the results of Bridge C and Bridge A whose transverse span is similar but their longitudinal connections have a length ratio of approximately 1:2, hence, the required deflection to mobilise the restraint force and corresponding moment is also at this ratio.

Combining the above factors will result in a significant variation in the outcome of assessments using membrane analysis. This is demonstrated by comparing the results for Bridge B which has the thinnest plate coupled with the smallest rivet diameter and shortest span and Bridge C which has the largest diameter rivets and has a longitudinal connection twice the length of the other plates.

Fig. 14. Bearing capacity of plate per mm thickness for varying rivet diameters.

Fig. 15. Rivet capacity in single shear for varying rivet diameters.

Whilst, for the purposes of this study, consideration for the application of membrane analysis has been limited to modification of existing standard equations available in the literature, it would be possible to further refine the study by computerised methods. By using finite element analysis and nonlinear material properties, true plastic behaviour could be modelled. It would then be possible to consider the variations identified above quickly and efficiently and, potentially, with greater accuracy. This further work could be used to produce a set of empirical rules based on fill depths, connection details and plate thickness to allow swift assessment of the plate capacity utilising membrane action. It could also include further justification for the use of yield line analysis which, as mentioned in the previous section, does not consider the effects of membrane action.

It should be noted that, for membrane analysis, this paper considered one-way spanning plate cases. However, methods for solving two-way spanning plate problems are also available in

the literature [7,8]. These could be applied if necessary although, due to their complexity, it is considered more practical to use computer modelling techniques should this case require consideration.

5. Conclusions

In this paper, new refined assessment methods to potentially increase the assessed capacity of metallic flat deck plates used in bridges have been presented. Such plates are generally assessed by engineers as incapable of withstanding the wheel loads suggested by traditional methods of assessment [1]. Whilst these methods produce a safe estimation of the plate capacity, they tend to be overly conservative due to the potentially significant structural actions ignored by these simplistic methods and their reliance on elastic behaviour which limits the deflection of the plate. Unlike beams, these plates have significant span to depth ratios and therefore would be subject to significant deflections and are likely to benefit from the use of plastic analysis.

The application of yield line analysis has led to significant increases in the assessed capacity of three case study bridges with flat deck plates. Whilst this is encouraging, it should be reiterated that yield line analysis produces an upper bound solution; hence any reported capacities must be considered to be the absolute limit of the plate capacity. The reassessed plates showed significant increases in their capacity with allowable wheel load capacities ranging from 1.8 to 6.6 times those assessed originally. Similarly, the application of membrane analysis has also resulted in significant enhancement of the plate capacities with increase in assessed capacities affording the application of between 1.3 and 7 times the original assessed capacity.

This paper has shown that, by using yield line analysis and membrane analysis, there is the potential to enhance the assessed capacities of flat deck plates. This would prove beneficial in terms of avoiding unnecessary strengthening or repair actions on old metallic bridges with low original assessment ratings. Further work is recommended to realise the full potential of both of these methods; this could be achieved by undertaking parametric studies considering the support conditions, plate thicknesses, rivet diameters and spacing and aspect ratio of the plates. Acceptable plastic deflection limits should also be considered and nonlinear finite element analysis can be employed for this purpose.

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References

- [1] The Highways Agency. *BD21/01. The assessment of highway bridges and structures*. Norwich: The Highways Agency; 2001.
- [2] Johansen KW. *Yield Line Theory*. London: Cement and Concrete Association; 1962.
- [3] Johansen KW. *Yield Line Formulae for Slabs*. London: Cement and Concrete Association; 1972.
- [4] Moy SSJ. *Plastic methods of analysis for steel and concrete structures*. 2nd ed. London: Macmillan Press Ltd; 1996.

- [5] Bhatt P, MacGinley TJ, Choo BS. Reinforced concrete design; theory and examples. London: Taylor and Francis; 2006.
- [6] Jones LL, Wood RH. Yield-line analysis of slabs. London: Thames & Hudson; 1967.
- [7] Roark RJ. Formulas for stress and strain. 4th ed. New York: McGraw-Hill; 1965.
- [8] Timoshenko SP, Woinowsky-Krieger S. Theory of plates and shells. 2nd ed. New York: McGraw-Hil; 1959.
- [9] Pucher A. Influence surfaces of elastic plates. 5th ed. Austria: Springer-Verlag; 1977.
- [10] Stainsby D. Chapter 30: Floors and orthotropic decks. In: Davidson B, Owens GW, editors. Steel Designers Manual, 6th ed. Oxford: Blackwell; 2003.