Dynamic shakedown and degradation of elastic reactions in laterally impacted steel tubes

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ABSTRACT

This paper reports some interesting observations on the response of laterally impacted steel tubes which in some respects have been considered to undergo an elastic shakedown. In an experimental study on the behaviour of axially compressed tubes under lateral impacts, it has been noticed that, after full development of plastic deformations in the impacted bodies, the structural system ceases to exhibit additional plastic responses. The impacted tubes then exhibit an elastic response. It has also been observed that the amplitude of the elastic excitations in the specimens becomes more restricted as the load configuration moves close to a dynamic failure state. With load conditions quite close to the dynamic failure limit, almost no elastic excitation has been perceived from the impacted specimens. Additional numerical and analytical investigations have been carried out on impacted tubes, frames and non-linear Single Degree of Freedom (SDOF) systems and similar results have been obtained. Despite the non-cyclic nature of the external loads in these impact cases, a phenomenon similar to elastic shakedown has been observed.

Keywords: dynamic shakedown, elastic shakedown, steel tube, lateral impact, simulation, dynamic failure, adaptation

NOMENCLATURE

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A	Acceleration
D	Tube outer diameter
F	Concentrated lateral load
F_o	Dynamic lateral step load
F_p	The minimum static concentrated lateral load required at mid-span of an encastre tubular beam to produce a three hinge plastic collapse mechanism ($8D^2t\sigma_y/L$)
K	Stiffness of a spring in its linear range
L	Tube length
M	Lumped mass of a SDOF system
n	Ratio between the applied step load and the plastic load of the spring
P	Lateral push over load (<i>Fig. 7</i>)
P_o	Impact step load on the non-linear spring (<i>Fig. 10</i>)
P_y	Plastic load in the non-linear spring (<i>Fig. 10</i>)
P_y	Axial squash load of the tube ($\pi Dt\sigma_y$)
t	Tube wall thickness
t	Time
u	Displacement
u_{st}	Displacement under static load
$\dot{u}$	Velocity
$\ddot{u}$	Acceleration
v	Velocity
ω	Radial frequency
σ_y	Material yield stress

1. INTRODUCTION

Classical ‘shakedown theory’ is related to the repetition of a quasi-static load in the structure and is restricted to linear geometrical problems and elastic, perfectly plastic, materials. The shakedown theory was promoted first by Bleich [1] and Melan [2] who gave the relevant criteria

for the 'static theorem of shakedown'. Koiter [3] provided a solid and rational basis for the theorem and proposed the so called 'kinematics theorem for shakedown'. Other contributors extended shakedown theory to more general structural and material models such as discrete or discretised models, including hardening effects and geometric non-linearity effects [4].

This paper reports some observations on the response of laterally impacted steel tubes [5] which in some respects have been considered to behave in a similar manner to elastic shakedown. To categorise the findings, first classical, dynamic and pseudo-shakedowns have been briefly reviewed. This is followed by experimental observations on the behaviour of impacted tubes, results from numerical models of impacted tubes and frames along with the response of a non-linear Single Degree Of Freedom (SDOF) system subjected to step loads have been presented.

2. BACKGROUND TO SHAKEDOWN AND PSEUDO-SHAKEDOWN

2.1. Static Classical Shakedown

An elasto-plastic structure subjected to repeated cycles of (quasi-static) loads, varying within a specific range, may eventually end in one of four typical states. In the first state, a purely elastic-reversible response occurs and the deformations of the structures remain bounded within elastic limits (*Fig. 1.a*). With the second state, some irreversible plastic deformation occurs in the structure but the accumulated plastic dissipated energy in the whole structure (after each cycle of loading) remains bounded. The structure eventually ceases to suffer further plastic deformation and thus responds to subsequent cycles of loads in a purely elastic manner (*Fig. 1.b*). This behaviour is called 'elastic shakedown' or 'adaptation' [6].

If the amplitude of the load exceeds a threshold, either 'plastic shakedown' (third state of behaviour) or 'incremental collapse' (fourth state of behaviour) occurs. With plastic shakedown (also called alternating plasticity), the plastic energy after each cycle of loading still remains bounded but the plastic strain increments change their sign during the loading process (*Fig. 1.c*). Although with this kind of inadaptation, the plastic increment stays at zero in each cycle, local material failure will occur due to low cycle fatigue.

With incremental collapse (called also ratcheting), the plastic strains increase cycle after cycle so that, after a certain number of cycles, the net accumulation of plastic strains somewhere in

the structure will exceed the material ductility limit, or become unreasonably large for serviceability. This behaviour is shown in *Fig. 1.d* and the structure is seen to be accumulating a certain amount of energy during each cycle of loading which eventually leads to inadaptation of the structure [7].

Fig. 1: Four typical states of response in an elasto-plastic structure subjected to repeated cyclic loads, varying within a specific range.

There have been a number of practical observations of different types of shakedown in structural components under complex variable or cycling loadings. Halder *et al.* [8] reported that interaction of gravity and cyclic loads in soil-foundation components of offshore structures resulted in ratcheting settlements. Rosson and Boothby [9] noted the occurrence of elastic shakedown in a masonry arch bridge subjected to over loading which had developed irreversible deformations. Some nuclear reactor components [10, 11], silos [12], and structural components in turbines and aircraft [7] and components also in metallurgical industries [13] could become subject to shakedown. Field observations indicated that many pavements do in fact shakedown rather than deform continuously [14].

2.2. Dynamic Shakedown

Researchers have extended the static characteristics of the Bleich-Melan theorem to dynamic problems [15]. With this approach the concept of bounded total plastic energy, typical of quasi-static shakedown theory, was used to discriminate shakedown from non-shakedown cases. The basis of this approach was that, the total plastic energy dissipated within the structure might actually diverge as a result of the continued application of quasi-static loads, or indefinite repetition of cyclic dynamic loads. In another proposed approach for dynamic shakedown, repeated excitations of the structure were addressed instead of repeated loads. With this approach (called minimum adaptation time), the shortest time required for a continuous solid body with an elastic-plastic material to shakedown to an elastic state was defined theoretically [4].

Two classes of dynamic shakedown problems can be envisaged for the related load schemes and shakedown criteria; namely, restricted dynamic shakedown, in which the load is a specified load history of either finite or even infinite duration, and for which the adaptation time criterion is the most appropriate; and unrestricted dynamic shakedown, in which the load is an unknown sequence of short-duration excitations, and for which the classic bounded-plastic-work criterion is the most appropriate [4].

2.3. Pseudo-Shakedown

Some researchers reported that a phenomenon known as pseudo-shakedown could occur in a rigid, perfectly plastic rectangular plate which strengthens with the development of finite displacements when subjected to repeated dynamic impact loads having a triangular pressure time history [16, 17].

Pseudo-shakedown, which is different from classical shakedown, takes place in some structures when the permanent deformations created in the system by the first dynamic impact are less than the permanent deformations resulting from equivalent static loading.

3. ADAPTATION/DEGRADATION OF ELASTIC REACTIONS

In a number of experiments on steel tubes subjected to combinations of axial pre-compression and lateral impacts, there have been three distinctive and interesting observations. With one of the tubes, after development of plastic deformations, the tube ceased to respond with further

plastic deformation and predominantly exhibited elastic oscillations. With the second specimen, the amplitude of the elastic oscillations became more restricted when the applied loads moved closer to the dynamic failure limit. Very close to the dynamic limit load, the impacted tube only presented plastic deformations with no perceptible elastic reactions and the amplitude of the elastic oscillations almost decreased to zero. These observations have been supported by additional numerical and analytical investigations which are also reported in this section.

3.1. Experimental Observations

In the test program, which was carried out by the authors, tubular steel specimens were initially axially pre-compressed and then subjected to lateral impacts at their mid span. An overall view of the test rig is shown in *Fig. 2*. The specimens had one fixed and one free sliding support. A self-reacting system of disc springs was placed behind the free sliding support. These compressive springs allowed a pre-defined, axial compression to be applied to the specimen. They were also employed to maintain the compression loads during the axial shortening of the tube which is bound to happen during lateral impacts [18]. The specimens, each one meter long, were cut from 6-7m cold-drawn seamless tubes with a nominal outside diameter of 100mm and wall thickness of 2mm. Plates of 150×150×16mm were welded to the ends of each specimen. The specimens were instrumented, set up in the impact rig, axially pre-compressed and then impacted at mid-span by a dropping striker. The mechanical properties of the tube material are given in *Table 1*.

Table 1: Mechanical properties of the tubes material from tensile and stub-column tests.

	E (kN/mm ²)	σ_y (N/mm ²)	σ_u (N/mm ²)	σ_u / σ_y (%)	ϵ_u (μ ϵ)
Tensile test	200	516	538	104.3	11200
Stub column test	189	481	526	109.3	-

The striker, with an adjustable weight of 15 to 50kg, was able to travel within the vertical guides and hit the specimen at right angles to the tube axis (*Fig. 2*). The striker had a 90° toughened knife-edge indenter. The head of the indenter was sufficiently rounded to avoid the occurrence of local tearing in the specimen. In these impact tests, the velocity and mass of the striker were kept constant (7m/s and 25.45kg respectively) but the axial pre-compressions were different. The axial pre-compressions were varying within a range of 0, 25, 27, 50, 60, 65, 70, and 75% of the specimen squash load ($P_y = \pi D t \sigma_y$).

During the experiment the tubes performed in an identical manner. When the axial pre-compression exceeded $0.65P_y$, an instant dynamic failure was triggered in the specimen by the first impact. In these cases, the striker caused a dent in the specimen which grew deeper. The tube then buckled and moved downward in a 'dog leg' shape.

Fig. 2: Schematic view of the impact rig for testing of axially pre-compressed tubes subject to lateral impact.

When the axial pre-compression was less than $0.65P_y$, the specimen did not fail during the impact tests and remained stable. In these cases the first impact caused permanent local dents and dimples in addition to an overall bowing in the tube but the deformations remained limited. The first impact was followed by a number of rebounds. This was because no attempt was made to prevent the bouncing of the striker on the specimen after its first hit.

When the axial pre-compression was $0.65P_y$, the striker made a relatively deep dent in the specimen but it virtually stopped at a certain point and no bouncing was observed.

Table 2 provides some general results from the tests and *Fig. 3* shows an undamaged specimen together with post impact views of some other specimens. More details on the experimental programme can be found in Zeinoddini, et al. [5 and 19].

In addition to the useful data from the experiments on the behaviour of the impacted tubes (Zeinoddini, *et al.* [5 and 19]), it was noticed that the specimens elastic reactions was reducing as the loading configuration approached the dynamic failure conditions.

3.1.1. Rebounds characteristics

It was already mentioned that in the tubes which did not fail, the first impact was followed by a number of rebounds. The rebounds were the outcome of elastic reactions from the impacted specimen, which produced an upward initial velocity in the striker. The striker rose to a certain height due to this initial velocity and fell again for the next hit. The numbers of perceptible rebounds and the bouncing duration (the time interval between the striker separation upward from the specimen until its next contact) are given in *Table 2*. The table also shows the striker velocity recorded on its first touch with the specimen and the striker velocities in the succeeding hits.

Table 2: Specimens definition and some general impact test results.

General data			Rebounds	Impact velocities				Bounce duration			
Name	P/P _y (%)	Test Result	Recorded Number	1 st (m/s)	2 nd (m/s)	3 rd (m/s)	6 th (m/s)	1 st (ms)	2 nd (ms)	3 rd (ms)	5 th (ms)
PD0	75	Failed	0	7.014	0	0	0	0	0	0	0
PD1	0	Stable	15	7.006	2.620	1.810	0.730	514	355	249	143
PD2	27	Stable	9	6.998	2.410	1.650	0.690	473	324	226	135
PD3	50	Stable	7	6.995	1.840	1.030	0.405	361	202	157	79
PD4	60	Stable	5	7.012	1.215	0.850	0.135	238	167	106	26
PD5	62	Stable	3	7.002	0.560	0.375	0	110	74	51	0
PD6	70	Failed	0	7.006	0	0	0	0	0	0	0
PD7	65	Failed	0	6.988	0	0	0	0	0	0	0
PD8	25	Stable	8	6.991	2.495	1.715	0.705	490	336	232	138
PD9	0	Stable	14	7.004	2.590	1.790	0.720	508	351	253	141

Table 2 shows that, in each test, as the number of striker hits increases, the impact velocity and the bouncing duration (corresponding to the striker rising height) decreased. This was apparently caused by dissipation of the external input energy through the system damping.

From *Table 2* it can also be noticed that the number of discernible rebounds decreased as the axial compression increased. Similarly, with a certain impact number, for instance the second impact, when the axial pre-compression increased, shorter bouncing duration (accordingly a shorter striker rise) and consequently less impact velocity for the next hit were recorded.

Fig. 3: View of an undamaged specimen together with post impact views of PD1, PD2, PD3, PD4, PD6 and PD7 specimens (in left to right order).

The rebounds were created by the elastic (flexural) reactions from the impacted specimens. Therefore, reductions in the number of recorded rebounds and similar parameters such as bouncing duration and rebound velocities (Table 2) indicate a degradation of the elastic reaction from the tubes. With the tests listed in Table 2, all parameters such as the tube dimensions, velocity and mass of the striker (say the external input energy) remained primarily constant and just the axial compression varied. The striker itself was made as rigidly as practically possible, so it can be concluded that the elastic reaction from the specimens was gradually decreasing out as the axial pre-compression increased towards the dynamic failure limits.

3.1.2. Reduction in the amplitude of elastic oscillations

In *Fig. 4* the time histories of the impact load for specimens with different levels of axial pre-compression are given. This figure displays snapshots of the first impact duration. It should be noted that the specimen with 70% axial pre-compression (PD6), failed as a result of the impact test, but the other specimens remained stable.

Fig. 4: Variation of the impact load during the first impact by the change in the specimens' axial pre-compression.

With each specimen, *Fig. 4* demonstrates two types of fluctuations. One nearly has a half sine shape with its duration close to the specimen half period in its first natural mode of vibration (exhibiting a bowing mode shape measured to be around 31.6ms). The second type of fluctuation has a period around 2ms which is related to other modes of vibration, possibly in the tube wall, in the striker or in the disk spring system.

The impact loads in *Fig. 4* were recorded using load cells placed inside the striker. So, they inevitably report the (elastic) reactions from the specimens imparted to the striker. The half sine fluctuation, most probably, represents the flexural reactions from the specimens. As it can be noticed the maximum impact load (or the tube elastic reaction) decreases as the axial pre-compression increases (or in other words the load conditions move closer to the dynamic failure status). *Fig. 4.* shows that the amplitude of the second type of oscillations also decreases as the axial pre-compression increases particularly after the initial impact. The response of the failed specimen appears almost as if it is free from these oscillations. These observations, once more, underline that the elastic reactions from the impacted specimens reduced as the loading configuration approached the dynamic failure limits.

3.1.3. Failure case

When the striker hit the specimen PD7, which had an axial pre-compression of $65\%P_y$, the tube was severely damaged by the first impact but nevertheless remained stable. Interestingly with this specimen no distinct rebound occurred after the first impact. The striker remained virtually stationary on the damaged specimen. After about 1.6 seconds from the first impact (a relatively long time in the experiment time scale), the tube suddenly failed making a large noise.

The ways in which the tube with $65\%P_y$ axial pre-compression failed, firstly indicated that the load combination was particularly close to the specimen's exact minimum failure load. With this particular load configuration, the tube remained briefly in a stable but critical condition. In this critical condition, the structural system required only a slight additional perturbation to make the specimen unstable. This additional effect could have been any change in the system, such as a small stress relief in the axial springs.

Secondly, as mentioned earlier, after the first impact on PD7, the striker showed no distinct rebound. Considering that the rebounds are the outcome of elastic reactions from the impacted body, absence of distinct rebounds indicated that almost no elastic reaction existed in the tube to force the striker up from the specimen. Lack of the elastic response in this case, yet again, indicated that the elastic reactions of the impacted tubes decreased towards zero as the level of the applied load approached the dynamic failure states.

It is worth noting that the response of specimen PD7 was a rare physical observation. Out of 10 experiments, quite by chance, the impact conditions for PD7 (the striker velocity, tube conditions, pre-compression level, etc) coincided with the minimum dynamic failure load. It is clear that in these kinds of physical experiments the possibility of coming across this precise point is quite low. This is because the minimum failure load can be regarded as a point in a space made by different variables such as the striker weight, shape, velocity, axial compression etc. With merely a slight deviation from this point, the specimen response would have moved to become either stable or in a collapse condition.

3.2. Numerical examinations

Degradation of elastic reactions and behaviour similar to the adaptation or elastic shakedown have also been observed in numerical models of lateral impacts on tubular members. The

simulations have been carried out using the ABAQUS non-linear finite element program (Hibbit, *et al.* [20]). An implicit direct integration dynamic approach based on Newark's constant average acceleration method has been employed to solve the non-linear equations of the motion in the impact analysis.

In the numerical models reported in this section, the impact has been introduced as a lateral dynamic step load applied at the mid-span of the tubes. The axial pre-compression has been kept constant at $50\%P_y$ while the impact load has been varied. No structural damping has been incorporated into the finite element models. The von Mises yield criterion has been utilised to model the inelastic material properties. An elastic, perfectly plastic material property has been used. No strain rate effects have been considered for the steel material.

3.2.1. Individual tubes

Individual tubes have been numerically studied under lateral impact loads. The encastre end tubes have been modelled using twenty four shell elements (*S4R*) in the circumference and fifty in the longitudinal direction. These models allow for local deformations. The tubes have also been modelled using 20 beam elements (type *PIPE31*) which exclude local deformations. The modelled tubular member has an outer diameter of 356mm, a wall thickness of 12.7mm and a length of 5700mm with a yield stress of 350 N/mm^2 .

Figs. 5, and *6* show the numerical responses of the individual tubes under different dynamic step lateral loads applied at their mid-span, with local deformations excluded or included. The plastic load $F_p (=8D^2t \sigma_y/L)$ corresponds to a static concentrated lateral load required at the mid-span of an encastre tubular beam member to produce a three hinge plastic collapse mechanism. Time histories of non-dimensional lateral displacement at the impact position in the mid-span of the tubes are displayed in these figures.

Figs. 5 and *6* indicate that some responses have remained bounded. This indicates that the corresponding structural systems remained dynamically stable. By increase in the lateral impact load certain responses have become unbounded, indicating the occurrence of a dynamic failure in the structural system as a result of the impact load.

Figs. 5 to 6 also show that with an increase in the level of lateral impact load, the mean deformation in the stable responses has increased but after development of plastic

deformations, the asymptotic oscillations remained elastic. The amplitude of these elastic oscillations (which have a frequency close to the main bowing natural frequency of the tube) has become more restricted with an increase in the level of impact load. With responses close to the dynamic limit load, the asymptotic oscillations have almost faded out.

Fig. 5: Numerical time histories of lateral displacement at the impact position in the individual tubes subjected to lateral step loads at their mid-spans (local effects excluded).

Fig. 6: Numerical time histories of lateral displacement at the impact position in the individual tubes subjected to lateral step loads at their mid-spans (local effects included).

3.2.2. Impacted frames

Benchmark and large scale tubular frames were tested under static push over loads (*Fig. 7*) by other researchers (Nichols *et al.* [21] and Bolt *et al.* [22]). These tubular frames have been numerically modelled by the authors to study the behaviour of laterally impacted frames. The numerical models of the frame have shown very good correlation with the experimental results under quasi static push over loading.

The figures given on each member are D , t and σ_y of the tube in mm , mm and N/mm^2 respectively.

Fig. 7: Elevation and properties of the tubular frame, used in the benchmarking exercise (Nichols *et al.* [22]).

Fig. 8: Numerical time histories of lateral displacement in the impacted tubular frame (local effects excluded).

Two identical numerical models of the benchmark frames have been developed. In the first numerical model, all frame members have been modelled using up to 20 beam elements (type *PIPE31*) for each member. Using beam elements excludes modelling local deformations. In the second numerical model the impacted chord member has been modelled using twenty four shell elements (*S4R*) in the circumference and fifty in the longitudinal direction but the remaining members have been modelled with beam elements. This model allows for local deformations to be considered in the impacted member. Both models have been loaded vertically which produced an axial compression in the chord members equal to 50% of their squash load.

Degradation of elastic reactions and behaviour similar to dynamic shakedown have also been observed in the benchmark frames when subjected to a lateral impact at mid-span of the upper chord member (see *Fig. 7*). Once more, in both numerical models, with an increase in the level of lateral impact load, the mean deformations increased but with responses close to the dynamic limit points, the amplitude of the ensuing oscillations has become more restricted (*Fig. 8*). *Fig. 9* shows a post impact view of the numerical model of the tubular frame (with local effects included) when subjected to a dynamic failure.

Fig. 9: Post impact view of the numerical model of the tubular frame, subjected to a dynamic failure as a result of lateral impact (local effects included).

3.3. Analytical Model

The response of a non-linear SDOF system to a dynamic step impact load has also been analytically studied. The system is shown in *Fig. 10a* and consists of a lumped mass M and a non-linear spring. A step load of P_o is applied to the system (*Fig. 10b*). A simplified elastic-perfectly plastic behaviour has been considered for the spring (*Fig. 10c*). With $0 \leq u(t) \leq u_1 = u_y$ the spring, having a linear stiffness of K , remains elastic. Beyond u_1 the spring is perfectly plastic but possesses an elastic retardation path. No damping is included in the system and at $t=0$, it has a zero initial condition.

In phase one of the response, $0 \leq u(t) \leq u_1 = u_y$ (or $0 \leq t \leq t_1$), the equilibrium equation is:

$$M \ddot{u} + Ku - P_o = 0 \quad 1$$

With zero initial conditions $u(0) = \dot{u}(0) = 0$, $\omega = \sqrt{K/M}$ and $n=P_o/P_y$, the solution for the differential equation (Eq. 1) is:

$$u(t) = nu_1[1 - \cos(\omega t)] \quad 2$$

At $t = t_1$:

$$u(t_1) = P_y / K = u_1 \quad 3$$

From Eqs. 2 and 3:

$$t_1 = \frac{1}{\omega} \cos^{-1}\left(\frac{n-1}{n}\right) \quad 4$$

$$\dot{u}(t_1) = u_1 \omega \sqrt{2n-1} \quad 5$$

a) Dynamic model of the SDOF system.

b) The applied dynamic step load.

c) Plastic incremental law for the non-linear spring.

Fig. 10: Characteristics of the non-linear SDOF system.

Eqs. 4 and 5 indicate that the system only reaches the non-linear part of the behaviour if $n \geq 0.5$.

In that case in phase two, $u_1 \leq u(t) \leq u_2$ (Fig. 10b) or $t_1 \leq t \leq t_2$ the equilibrium equation is:

$$M \ddot{u} + P_y - P_o = 0 \quad 6$$

With initial conditions from Eqs. 3 and 5, the solution of the above differential equation is:

$$u(t) = u_1 \left[\omega^2 \frac{n-1}{2} (t-t_1)^2 + \omega \sqrt{2n-1} (t-t_1) + 1 \right] \quad 7$$

The motion in phase two proceeds until the system velocity comes to zero at $t = t_2$:

$$\dot{u}(t_2) = 0 \quad 8$$

$$t_2 = \frac{\sqrt{2n-1}}{\omega(1-n)} + t_1 \quad 9$$

$$u(t_2) = u_1 \frac{1}{2(1-n)} \quad 10$$

Eqs. 9 and 10 indicate that the response of the non-linear SDOF system will only remain bounded if n is less than 1. In that case in phase three ($t \geq t_2$), an elastic unloading in the spring takes place. The equation of equilibrium in this phase is:

$$M \ddot{u} + Ku + P_y - P_o = 0 \quad 11$$

With initial conditions from Eqs. 8 and 10, the solution of Eq. 11 becomes:

$$u(t) = u_1 \left\{ (1-n)(\cos[\omega(t-t_2)]) + \frac{4n-2n^2-1}{2(1-n)} \right\} \quad 12$$

Eq. 12 indicates that for a non-linear and bounded response ($0.5 < n < 1$), the displacements in phase three would be confined to $0 \leq u(t) \leq u_1$ indicating that in phase three, the displacements are restricted to the elastic limits and the non-linear system will confine itself to an elastic behaviour (comparable to elastic shakedown).

With $u_{st} = P_o/K$, Eq. 12 could be rewritten as:

$$u(t) = u_{st.} + \frac{2n-1}{2(1-n)} u_1 + (u_1 - u_{st.}) \cos[\omega(t - t_2)] \quad 13$$

Eq. 13 shows that the adapted response of a non-linear SDOF system subjected to a step load can be divided into three distinct components. The first one, an elastic time independent displacement ($u_{st.}$), is equal to the response of the system to a static load of P_o and monotonically increases with an increase in the applied external step load (P_o). The second component, the plastic time independent displacement $[u_1(2n-1)/2/(1-n)]$, is zero when $n \leq 0.5$ or $P_o \leq P_y/2$ (a pure elastic response) and becomes infinite when $n=1$ or $P_o=P_y$ (a dynamic failure). The magnitude of this plastic displacement depends on the coefficient n (the ratio between the applied load and the plastic load) and monotonically increases with an increase in the applied external step load (P_o). The third component is the transient time dependent deformation response $(u_1 - u_{st.}) \cos[\omega(t - t_2)]$. This oscillation has its maximum amplitude of u_1 when $n=0.5$ (maximum elastic response). In post elastic response, the amplitude of the oscillations decreases monotonically with increases in the applied impact load. With $n=1$, (a dynamic failure) the amplitude of the elastic adaptation becomes equal to zero.

Displacement time histories of the non-linear SDOF model subjected to step impact loads are shown in Fig. 11. The characteristics of the spring model (ω , P_y , K and P_o) have been adapted to represent those from the previously studied individual tube (see Fig. 5). The displacements in Fig. 11 are non-dimensional. The figure displays the purely elastic ($P_o=0.50P_y$) and the elasto-plastic responses (with $P_o=0.80, 0.90, 0.935$ and $0.95 P_y$). Fig. 11 shows similar behaviour as presented in Figs. 5, 6 and 8. It can be seen that under these excitations the system remains dynamically stable but the non-linear SDOF system eventually elastically shakes down. The amplitude of the adopted elastic responses diminishes away as the system approaches its dynamic failure limit ($P_o = P_y$). These responses can also be clearly recognized from Figs. 12 to 14.

Fig. 12 gives plots for the (non-dimensional) displacements versus the velocities in the nonlinear SDOF system subjected to increasing step impact loads. Once again, when $P_o \leq 0.50P_y$ the system remains elastic, For $P_o > P_y$ the response becomes unbounded and a dynamic failure happens. With $0.50P_y \leq P_o \leq P_y$, as it can be seen that large plastic displacements are developed

in the system, however, they end up in an adaptation phase. In this range, as the level of applied load increases, the amplitude of the oscillations of these elastically shaken down responses grow smaller.

Fig. 11: Time history of the displacement in the non-linear SDOF model subjected to a step load of P_0 .

Fig. 12: Elastic shakedown/adaptation and degradation of the elastic response in a nonlinear SDOF system subjected to increasing step loads of P_0 .

Fig. 13 shows plots of the (non-dimensional) velocities of the SDOF system subjected to increasing step loads against its accelerations. Zones for the purely elastic behaviour, dynamic failure and elastic shakedown are also shown (for the first quarter of the coordinate system). When the external load grows closer to the failure point, adaptation and degradation of the elastic responses can be recognized.

It should be mentioned that with a step load type excitation, the dynamic failure in the non-linear SDOF system will be characterised by unbounded responses, so there will be no ratcheting type failure. With other excitations, e.g. a harmonic type, a nonlinear SDOF system may experience alternating plasticity or a ratcheting type of failure [23]. As it was already mentioned, no damping was considered so the elastically shaken responses have closed loop forms (Figs. 12, and 13). If damping is introduced in the system, these closed loops will turn into spirals in which the loop size steadily decreases until it vanishes.

Fig. 13: Elastic shakedown/adaptation and degradation of the elastic response in a nonlinear SDOF system subjected to increasing step loads of P_0 .

Fig. 14 gives the load displacement curves for the nonlinear spring in the SDOF model subjected to increasing external step excitations of P_0 . The occurrence of adaptation and elastic shakedown can be obviously recognized from the figure (see also Fig. 1b). After undergoing nonlinear behaviour the system behaves in purely elastic cycles of loading and unloading. The

range of these cycles decreases as the excitation to the SDOF system approaches the dynamic failure point.

Fig. 14: Load displacement curves for the nonlinear spring in the SDOF model subjected to a step load of P_0 .

With the SDOF models studied, the external load had no cyclic nature (with no repetition), however, the impacted systems presented behaviours similar to the elastic shakedown of structures which basically happens under cyclic loads. This is because in a dynamically excited structure, even with time independent external loading, the structure becomes subject to cycles of acceleration (inertia), velocity (damping), and displacement (stiffness) based on internal loading and unloading. With this internal cyclic loading, the structure could become prone to various types of shakedown.

The rationale provided in this section has been outlined for a time independent external load.

In the SDOF models studied in this section an elastic perfectly plastic behaviour has been considered for the system. The study may be further enriched by integrating plastic hardening (isotropic, kinematic or mixed hardening), as studied by Savi and Pacheco [24], or extending it to MDOF systems. The external load has had a time independent nature. The study may be extended to examining the adaptation, degradation of the adapted response, alternating plasticity, ratcheting and dynamic failures in both the SDOF and MDOF systems subjected to time varying excitations.

4. CLOSING REMARKS

This paper is devoted to some observations on the response of laterally impacted steel tubes, which in some respects have been considered to be similar to shakedown. In these experiments on steel tubes subjected to combinations of axial compression and lateral impacts, three distinctive and interesting behaviours were noticed. With one response, after the development of plastic deformation, the tubes ceased to exhibit further plastic deformation and reverted to a purely elastic response. With the second behaviour, the amplitude of the elastic oscillations became further restricted when the applied load approached the dynamic failure limits. Very close to the dynamic limit load, the impacted tube only exhibited plastic deformation with no perceptible elastic reactions and the amplitude of the elastic oscillations almost decreased to zero.

Additional numerical and analytical investigations have been carried out on impacted tubes, frames and non-linear SDOF systems to further examine the experimental observations. These models have indicated similar results. They have substantiated that outside of the purely elastic/unbounded response zones, the models studied exhibit behaviour similar to elastic shakedown and adaptation. This means that although the impacted structure experiences nonlinear deformations, the asymptotic responses remain elastic. The amplitude of these adopted elastic oscillations becomes more restricted with increases in the level of impact load. With responses close to the dynamic limit load, the oscillations almost die away.

Degradation of the elastic reactions in the response of the dynamically exited systems could have important effects on the post damage behaviour or the failure of structures subjected to dynamic loads. In the current study the issue of the adaptation and degradation of elastic reactions have been studied for systems subjected to impact loads. Considering up the subject for other structures and other types of dynamic excitations remains subject to further investigation.

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