should be noted that for target trajectories which are not the UFP of the original system, the control objective is achieved with a nonzero error, consistent with the feedback control theory.

- Some guidelines and procedures for determining the feedback gains can be derived under our framework based on the Lyapunov first method.

Finally, we point out that the bifurcation control method described here relies on just a few easily tuned gains which can be selected based upon short-term observation of the system under study. The associated controller is simpler to implement than the OGY method for example, which requires that the locations of saddle-type UFP's be found using the method of delay-coordinate embedding. There is no guarantee that finding these UFP's will require less samples of the process than the present method, or even that its success is ensured for arbitrary nonlinear systems such as with multiple attractors, or those with nonsaddle-type UFP's. Thus, the controller described here may possibly be implemented in future clinical pacemaker designs, an issue we are currently addressing.

REFERENCES

- [1] L. Glass, "Dynamics of cardiac arrhythmias," *Phys. Today*, pp. 40–45, Aug. 1996.
- [2] A. Garfinkel, M. L. Spano, W. L. Ditto, and J. N. Weiss, "Controlling cardiac chaos," *Sci.*, vol. 257, pp. 1230–1235, 1992.
- [3] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, pp. 1196–1199, 1990.
- [4] W. L. Ditto, "Applications of chaos in biology and medicine," in *Chaos and the Changing Nature of Science and Medicine: An Introduction*, D. E. Herbert, Ed. New York: AIP Press, 1996, pp. 175–201.
- [5] L. Glass and W. Zeng, "Bifurcations in flat-topped maps and the control of cardiac chaos," *Int. J. Bifurc. Chaos*, vol. 4, pp. 1061–1067, 1994.
- [6] M. E. Brandt and G. Chen, "Feedback control of a quadratic map model of cardiac chaos," *Int. J. Bifurc. Chaos*, vol. 6, pp. 715–723, 1996.
- [7] M. E. Brandt, A. Ademoglu, D. Lai, and G. Chen, "Autoregressive self-tuning feedback control of the Henon map," *Phys. Rev. E*, vol. 54, pp. 6201–6206, 1996.
- [8] H. K. Khalil, *Nonlinear Systems*. New York: Macmillan, 1992, p. 130.
- [9] M. E. Brandt and G. Chen, "Controlling the dynamical behavior of a circle map model of the human heart," *Biol. Cybern.*, vol. 74, pp. 1–8, 1996.
- [10] M. Courtemanche, L. Glass, J. Belair, D. Scaglioni, and D. Gordon, "A circle map in a human heart," *Physica D*, vol. 40, pp. 299–310, 1989.
- [11] W. Z. Zeng, M. Courtemanche, L. Sehn, A. Shrier, and L. Glass, "Theoretical computation of phase locking in embryonic atrial heart cell aggregates," *J. Theor. Biol.*, vol. 145, pp. 225–244, 1990.
- [12] Y. He, R. Chi, and N. He, "Lyapunov exponents of the circle map in human hearts," *Phys. Lett. A*, vol. 170, pp. 29–32, 1992.
- [13] K. Pyragas, "Continuous control of chaos by self-controlling feedback," *Phys. Lett. A*, vol. 170, pp. 421–428, 1992.

Targeting in Systems with Discontinuities, with Applications to Power Electronics

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Abstract—Targeting methods appropriate for systems with discontinuities are considered. The multivalued inverse function is used to generate multiple preimages of the target region which quickly cover the attractor. This method is applied to the current-controlled boost converter in order to jump between two controlled states. A significant reduction in the length of the target orbit is observed when compared with targeting methods for invertible maps.

Index Terms—Chaos, discontinuities, power electronics, targeting.

I. INTRODUCTION

Chaos is a common phenomenon in power converters when they are operated under feedback control. Because of their simplicity, dc–dc converters have been the most studied type of power converter, but other types are probably equally prone to chaos. The dynamics of a power converter can be described in terms of a mapping [7] which will be discontinuous when, in certain regions of phase space, adjacent points are mapped into widely separated points. This can happen in a system with one or more continuous state variables and an autonomous clock [14]. In the boost converter studied below, for instance, a switching event may occur just before or just after the arrival of a clock pulse. The state variables will evolve quite differently in the "before" and "after" cases, and a small change in initial conditions can make the difference between which actually occurs. Discontinuous maps are often noninvertible or equivalently, the inverse is a multivalued function. As will be shown, this property is very beneficial when it comes to targeting methods.

There has been much interest recently in methods for controlling chaos, following the pioneering paper of Ott, Grebogi, and Yorke (OGY) [8] in which small perturbations to a parameter are used to stabilize a saddle fixed point which is contained in a chaotic attractor. Alternative methods for controlling chaos which are particularly appropriate for switching circuits have also been developed [4], [11]. One drawback of these methods is that it is necessary to wait until an iterate falls near to the fixed point which is to be stabilized before the control method can be implemented, since it is based on a linear approximation to the dynamics near the fixed point. However, this chaotic transient can be very long in some cases.

This leads to ideas of targeting, in which a parameter perturbation is used in order to direct an orbit to the neighborhood of the fixed point in a small number of iterations [12]. This method was first applied experimentally to the chaotic motion of a magnetoelastic ribbon in an oscillating external applied magnetic field [13]. A number of variations on the original method have been proposed. An optimal control approach has been used by Xu and Bishop [15] and Paskota, Mees, and Teo [9] to find small perturbations of a parameter at each iteration which will direct the orbit to the target in as few iterations as possible. Targeting methods often fail in the presence of small

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Fig. 1. Circuit diagram of the current-controlled boost converter.

amplitude noise and a number of methods have been proposed to overcome this problem [3], [10], [12].

We consider a new method of targeting which is applicable to systems with discontinuities and which are not uniquely invertible. The method of Shinbrot *et al.* [12] involves backward iteration of the target region. For an invertible map, each point has a unique inverse and so there is a unique region obtained at each back iteration of the target region. However, for noninvertible maps, each point may have several preiterates because the inverse map is multivalued. Thus, back iteration of the target region for noninvertible maps results in several new regions at each iteration. The attractor can therefore be covered with preimages of the target region much more quickly for a noninvertible map than for an invertible one. Hence, transitions between different controlled states can be achieved very quickly. These ideas are applied to the boost converter, which is described by a two-dimensional (2-D) mapping.

II. THE CURRENT-CONTROLLED BOOST CONVERTER

As an example we consider the current-controlled boost converter shown in Fig. 1 and derive a 2-D map which describes the behavior of this system, which is known to exhibit chaos in certain parameter regimes. Our derivation follows that in Deane [5] but we express the final equations in a slightly different form.

The circuit has two states, which occur when the switch S is closed or open. When S is closed, the circuit is described by two uncoupled first-order differential equations, one for the inductor current and one for the capacitor voltage. The current grows linearly during this phase and any clock pulses which occur are ignored. Once the current reaches the reference current I_{ref} , the switch is opened.

With S open, the circuit is described by a pair of coupled first-order differential equations and the current falls. When a clock pulse occurs, the switch is then closed again. The current and voltage are sampled at the clock pulse which closes the switch and the dynamics of the circuit can then be described in terms of a 2-D mapping involving i_n and v_n , the current and voltage at the sampling points. This behavior is illustrated in Fig. 2. Note that the number of clock pulses between successive iterates may vary, since clock pulses are ignored in the first phase when the switch is closed.

Let (i_n, v_n) be the current and voltage at a clock pulse at which the switch is closed. Then the two uncoupled first-order differential

equations have the solution

$$v(t) = v_n e^{-2kt}, \quad i(t) = \frac{V_I}{L} t + i_n$$

where $k = 1/(2RC)$. Thus, the current reaches the reference value I_{ref} at time t_n where

$$t_n = \frac{L(I_{\text{ref}} - i_n)}{V_I} \quad (1)$$

at which point the switch is opened. The general solution of the coupled first-order differential equations for i is then

$$i(t) = e^{-kt}(A_1 \sin \omega t + A_2 \cos \omega t) + \frac{V_I}{R} \quad (2)$$

where $\omega = \sqrt{1/LC - k^2}$, and the capacitor voltage v is

$$v(t) = V_I - L \frac{di(t)}{dt}. \quad (3)$$

The next clock pulse occurs at a time $t'_n = T[1 - (t_n/T \bmod 1)]$ after S last opened, where T is the time between clock pulses. Setting $t = 0$ immediately after S has opened, and using the fact that v and i must be continuous gives

$$\begin{aligned} i(0) &= A_2 + \frac{V_I}{R} = I_{\text{ref}} \\ v(0) &= V_I - L \frac{di(0)}{dt} = v_n e^{-2kt_n}. \end{aligned} \quad (4)$$

Using $i(t)$ from (2) and solving gives

$$A_1 = \frac{kL \left(I_{\text{ref}} - \frac{V_I}{R} \right) + V_I - v_n e^{-2kt_n}}{\omega L}$$

and

$$A_2 = I_{\text{ref}} - \frac{V_I}{R}.$$

Now, by definition $i_{n+1} = i(t'_n)$, so using (2) and (4) gives

$$\begin{aligned} i_{n+1} &= e^{-kt'_n} \left[\frac{kLI'_{\text{ref}} + V_I - v_n e^{-2kt_n}}{\omega L} \right. \\ &\quad \left. \cdot \sin \omega t'_n + I'_{\text{ref}} \cos \omega t'_n \right] + \frac{V_I}{R} \end{aligned}$$

where $I'_{\text{ref}} = I_{\text{ref}} - V_I/R$. Similarly, $v_{n+1} = v(t'_n)$ and so

$$\begin{aligned} v_{n+1} &= V_I - e^{-kt'_n} \left[\left(k v_n e^{-2kt_n} - k V_I - \frac{I'_{\text{ref}}}{C} \right) \right. \\ &\quad \left. \cdot \frac{\sin \omega t'_n}{\omega} + (V_I - v_n e^{-2kt_n}) \cos \omega t'_n \right]. \end{aligned}$$

Using the dimensionless variables

$$V_n = \frac{v_n}{V_I}, \quad I_n = \frac{L}{V_I T} \left(i_n - \frac{V_I}{R} \right), \quad x_n = \frac{t_n}{T}$$

and the dimensionless parameters

$$\alpha = \frac{I'_{\text{ref}} L}{V_I T}, \quad \beta = kT, \quad \gamma = \omega T, \quad \eta = \frac{T^2}{LC}$$

we can rewrite the mapping in the form

$$\begin{aligned} I_{n+1} &= \frac{e^{-\beta x'_n}}{\gamma} [(\alpha\beta + 1 - V_n e^{-2\beta x_n}) \sin \gamma x'_n \\ &\quad + \alpha \cos \gamma x'_n], \\ V_{n+1} &= 1 - \frac{e^{-\beta x'_n}}{\gamma} \{[\beta(V_n e^{-2\beta x_n} - 1) - \alpha\eta] \sin \gamma x'_n \\ &\quad - (V_n e^{-2\beta x_n} - 1) \cos \gamma x'_n\} \end{aligned} \quad (5)$$

Fig. 2. Voltage and current waveforms in the boost converter.

where

$$x_n = \alpha - I_n, \quad x'_n = 1 - (x_n \bmod 1). \quad (6)$$

Note that (6) is the nondimensional form of (1).

When $i = I_{\text{ref}}$, then $I = \alpha$ and so α is the reference value for the dimensionless current. Thus, $I_n \leq \alpha$ for all n .

Now the mapping (5) has discontinuities. Depending on whether the current $i(t)$ reaches I_{ref} just before or just after a clock pulse, the resulting value of i_{n+1} will be very different [5]. This is because the switch open phase will last for either a very short time or for a time of approximately T , respectively. Thus, lines of discontinuities can be defined in the phase space corresponding to $i = I_{\text{ref}}$ occurring precisely at a clock pulse. Since the equations for the current and voltage are decoupled when the switch is closed, the lines of discontinuity will be independent of the voltage and so will consist of straight lines of the form $i = \text{constant}$. Now the condition that $i = I_{\text{ref}}$ at a clock pulse requires $t_n = jT$ for some positive integer j . In terms of the dimensionless time, this corresponds to $x_n = j$. By (6), this occurs when $I_n = \alpha - j$ and so these values define the lines of discontinuities. If a line in phase space crosses a line of discontinuities, then under iteration it will be cut into two pieces which are mapped to different parts of the phase space.

Another important feature of these equations is that they are non-invertible or equivalently, that the inverse is a multivalued function. This is due to the clock pulses which are ignored when the switch is closed. This property is the basis for our targeting method in the next section.

To derive the inverse of the map, we consider first the switch open phase. By reversing the sign of time t (or equivalently x) in the nondimensionalized form of (2) and (3), we can derive new functions $I^-(x)$ and $V^-(x)$ which describe the motion during this phase in backward time, when combined with the initial conditions $(I^-(0), V^-(0)) = (I_{n+1}, V_{n+1})$. The value of x'_n can then be found by solving $I^-(x'_n) = \alpha$, since α is the dimensionless value of the

reference current, which is given by

$$\frac{e^{\beta x'_n}}{\gamma} [(V_{n+1} - 1 - \beta I_{n+1}) \sin \gamma x'_n + I_{n+1} \cos \gamma x'_n] = \alpha.$$

This equation cannot be solved analytically and so the inverse can only be evaluated numerically. The corresponding value of the dimensionless voltage when $x = x'_n$ is

$$V' = 1 + \alpha\beta + e^{\beta x'_n} [(V_{n+1} - 1 - \beta I_{n+1}) \cdot \cos \gamma x'_n - \gamma I_{n+1} \sin \gamma x'_n].$$

The two decoupled first-order equations for the switch closed phase can then be solved with time reversed and with initial conditions $I^-(0) = \alpha$ and $V^-(0) = V'$. The multiple values of the inverse function arise by considering solutions at different clock pulses, which are given by

$$\begin{aligned} I_n^m &= I^-(x_n + m), \\ V_n^m &= V^-(x_n + m), \quad m = 0, 1, 2, \dots \end{aligned}$$

where $x_n = 1 - x'_n$. The maximum possible value of m is determined by the condition that $I_n^m > 0$. Due to the simple relationship (6) between x_n and I_n , there is one value of the inverse value of the current I in each of the intervals $(\alpha - m - 1, \alpha - m)$. This process is shown in Fig. 3. In general, not all of these back iterates will be contained in the chaotic attractor but there are often several which are.

III. THE TARGETING METHOD

We first describe the targeting method proposed by Shinbrot *et al.* [12] and then show how this approach can be adapted to deal with maps which have a multivalued inverse.

Consider the iteration

$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n, p)$$

where $\mathbf{F}: \mathbf{R}^2 \times \mathbf{R} \rightarrow \mathbf{R}^2$ and p is a system parameter which can be varied about a nominal value p^* . Assuming that only small parameter

Fig. 3. An illustration of the multiple inverses of the mapping: $I_n^1, \dots, I_n^4$ are all inverse values of I_{n+1} .

perturbations are allowed, we require that $|p - p^*| \leq \delta p_{\max}$ for some specified $\delta p_{\max}$.

Suppose that we want to target a region R when starting at an initial point $\mathbf{x}_s$. This is achieved by specifying a perturbation to the parameter p at the first iteration only. Due to the sensitive dependence on initial conditions of chaotic systems, this small change made at the first step can result in a large change in the orbit and so, if the parameter perturbation is chosen appropriately, the orbit can be directed to the target region in a few iterations.

In order to find the appropriate parameter perturbation, the first iterate of the map with $\mathbf{x}_0 = \mathbf{x}_s$ and for $p \in [p^* - \delta p_{\max}, p^* + \delta p_{\max}]$ is determined, giving a short line segment, which we denote by ℓ . The line segment ℓ is then iterated n_1 times until it approximately spans the attractor. The target region R is also backward iterated n_2 times until it intersects the n_1 th iterate of ℓ . A point of intersection between the backward iterated region and the forward iterated line can be iterated back to a point on the original line segment ℓ which is associated with a particular value of δp . By applying this perturbation at the first iterate, the target region will then be reached in $n_1 + n_2$ iterations.

Suppose now that the map $\mathbf{F}$ has discontinuities and a multivalued inverse. The discontinuities affect the forward iterations while the multivalued inverse has implications for the backward iterations. When iterating the line segment ℓ forward, the n th iterate of the line may fall across a line of discontinuities. The $(n+1)$ th iterate of the line will then be cut into two pieces which lie on different parts of the attractor. Thus, under repeated forward iteration, the original line segment ℓ may be cut into many pieces owing to the presence of the discontinuities. There is no benefit for the targeting method in this effect but neither is it detrimental.

The lines of discontinuities divide the phase space into a number of disjoint parts each of which will contain precisely one back iterate of the map. Thus, a single back iterate of the target region will give rise to several regions owing to the multivalued inverse of the map. Each of these regions can also be back iterated and so the number of regions increases rapidly with each back iteration. If the maximum number of preiterates of any point is m , then the maximum number of preiterates of the target region after n back iterations is $\sum_{k=1}^n m^k \sim m^n$ for large n . Clearly this property is very advantageous for the targeting method since the attractor will be filled very quickly with back iterates of the target region R . In contrast, for an invertible map $m = 1$ and so there is only ever one region obtained from back iterating the target region and one back iterate of this region, and so on. Thus, after n backward iterations, there will be only n preiterates of this region.

We now consider targeting when it is coupled with controlling an unstable fixed or periodic point using the OGY method [8]. For the sake of simplicity, we will discuss switching between two fixed

Fig. 4. Chaotic attractor and two fixed points. — quadratic approximation to the stable manifold through $\mathbf{x}_2^*$, --- lines of discontinuities of the map.

points, although of course this can easily be generalized to periodic points.

Suppose that a saddle fixed point $\mathbf{x}_1^*$ with $p = p^*$ has been controlled using the OGY method and that we want to jump to another saddle fixed point $\mathbf{x}_2^*$ also with $p = p^*$. We first generate a line segment which consists of all possible first iterates with $\mathbf{x}_0 = \mathbf{x}_1^*$ and with $p^* - \delta p_{\max} \leq p \leq p^* + \delta p_{\max}$. Clearly this line segment passes through the fixed point $\mathbf{x}_1^*$ since this is the first iterate when $p = p^*$. In contrast to the original method of targeting, we do not forward iterate this line segment since there is no advantage in doing this. Now the line segment is given by

$$\begin{aligned} \mathbf{F}(\mathbf{x}_1^*, p) &= \mathbf{F}(\mathbf{x}_1^*, p^*) + \delta p \mathbf{F}_p(\mathbf{x}_1^*, p^*) + O(\delta p^2) \\ &= \mathbf{x}_1^* + \delta p \mathbf{w} + O(\delta p^2) \end{aligned}$$

where $\mathbf{w} = \mathbf{F}_p(\mathbf{x}_1^*, p^*)$, $\delta p = p - p^*$ and the restriction $|\delta p| \leq \delta p_{\max}$ applies. Thus, a first-order approximation to the line segment ℓ is given by

$$\mathbf{x}_1^* + \delta p \mathbf{w}, \quad -\delta p_{\max} \leq \delta p \leq \delta p_{\max}. \quad (7)$$

The target saddle fixed point $\mathbf{x}_2^*$ will also have a one-dimensional stable manifold. Control of this fixed point can be achieved once an iterate falls within a strip around the stable manifold [2], [8]. The width of this strip is determined by $\delta p_{\max}$ and so typically is small. Suppose that the eigenvalues of the Jacobian matrix $\mathbf{F}_x(\mathbf{x}_2^*, p^*)$ are given by λ_s and λ_u where $|\lambda_s| < 1 < |\lambda_u|$. Then along the stable manifold near to the fixed point, the iterates contract at a rate of $|\lambda_s|$ and along the unstable manifold, iterates expand at a rate of $|\lambda_u|$. However, under backward iteration, the roles of the stable and unstable manifolds are reversed. Thus, under backward iteration, the strip around the stable manifold in which control can be applied expands in the direction of the stable manifold and contracts in the direction of the unstable manifold. Now the width of the strip is small to start with, and so under backward iteration, it will shrink even further. Thus, rather than considering the strip around the stable manifold, a short length of the manifold itself is taken as the target. It is also easier to back iterate this line segment rather than a region.

Now the stable manifold is invariant under the map and so, under back iteration, a small section of the stable manifold will map to a longer section and will always include the fixed point. For an invertible map, back iterates of this short section will simply continue to expand along the manifold which may cross the attractor in a

Fig. 5. Preiterates of the stable manifold through x_2^* . — first preiterates, — — second preiterates, . . . third preiterates.

Fig. 6. The target trajectory from x_1^* to x_2^* .

complicated way but of course each point on the manifold has only *one* preiterate. (For example, the stable manifold for a fixed point of the Hénon map is shown in [9, Fig. 7].)

For a map with a multivalued inverse, one back iterate of a short section of the stable manifold will consist of a longer section of the manifold, as is the case with invertible maps. However, there will also be back iterates which occur in other places on the attractor, well away from the fixed point. These lines can themselves be back iterated giving multiple preimages which quickly cover the attractor. After a few back iterations, one such line will lie close to the fixed point x_1^* . The parameter perturbation required for targeting can then be found from an intersection between one of the preiterates of the stable manifold of x_2^* and the line segment ℓ through x_1^* . If the intersection of an n_1 th preiterate of the stable manifold and the line segment ℓ occurs at x_I say, then using the linear approximation to the line segment ℓ given in (7), the magnitude of the required parameter perturbation is given by

$$|\delta p| = \frac{\|x_I - x_1^*\|}{\|w\|}$$

and the sign of δp can be easily found as the direction of the vector w is known. Applying this perturbation at the first iterate will result

Fig. 7. The target trajectory from x_2^* to x_1^* .

in the orbit lying on (or close to) the stable manifold of x_2^* after n_1 iterations, at which point the control algorithm can be applied.

If the map has a maximum of m back iterates for any point, then there can be at most $m - 1$ preiterates of the stable manifold other than itself. Each of these can then have a maximum of m preiterates. Thus, after n iterations, the maximum number of preiterates of the stable manifold is $(m - 1) \sum_{k=1}^n m^{k-1}$.

All of the calculations required for this method can be performed off-line and the perturbation parameter δp is all that is required in order to use the method. Thus, it is a very simple procedure to implement in practice.

In the original targeting method, it is advantageous to use forward iteration of the line segment ℓ and backward iteration of the region R since both are expanding (in different directions) and so using a few iterations of each prevents too great a loss of accuracy. In this method we do not use forward iterations but since the attractor is quickly filled with back iterates owing to the multivalued inverse, only a small number of back iterates is required and so there is not too great a loss of accuracy.

IV. EXAMPLE

We now apply the new targeting method to the boost converter. We use the parameter values $R = 20 \Omega$, $L = 1 \text{ mH}$, $C = 12 \mu\text{F}$, $T = 100 \mu\text{s}$, $V_I = 10 \text{ V}$, and $I_{\text{ref}} = 5.5 \text{ A}$. These give values of the dimensionless parameters as $\alpha = 5$, $\beta = \frac{5}{24}$, $\gamma = \frac{\sqrt{455}}{24}$, and $\eta = \frac{5}{6}$. The values of C , R , and T are chosen to ensure chaotic operation. The resulting output voltage ripple is larger than would be expected in a practical converter, where as a rule $CR \gg T$. However, the present purpose is not to design an optimum converter, but to provide a test bed for targeting techniques. The chaotic attractor with these parameter values is shown in Fig. 4. Also shown are the lines of discontinuities which have I constant and occur at integer values from the dimensionless reference current $\alpha = 5$. It can be seen that the smallest value of I on the attractor is between 2 and 3. Any point with I in this range will miss two clock pulses with the switch closed resulting in the next iterate occurring after three clock pulses. This also implies that there is no need to go back more than three clock pulses when evaluating the inverse of any point.

The nondimensional equation (5) have fixed points at $I_1^* = 4.309184$, $V_1^* = 3.668363$, and $I_2^* = 3.626192$, $V_2^* = 3.958709$ for which there are one and two clock pulses between the sampled points, respectively. (Note that if the period is determined by the

number of clock pulses, then (I_1^*, V_1^*) is a fixed point and (I_2^*, V_2^*) is a subharmonic of period 2.) The reference current I_{ref} is taken as the control parameter which is equivalent to using the nondimensional parameter α in (5). For the given parameter values, $\alpha = I_{\text{ref}} - 0.5$ and so the magnitude of perturbations in α is the same as for perturbations in I_{ref} . We first suppose that the fixed point $\mathbf{x}_1^* = (I_1^*, V_1^*)$ has been stabilized using the OGY method and we will target the fixed point $\mathbf{x}_2^* = (I_2^*, V_2^*)$. Thus, we need to back iterate the stable manifold of $\mathbf{x}_2^*$. Linearizing (5) about $\mathbf{x}_2^*$ gives

$$\mathbf{F}_{\mathbf{x}}(\mathbf{x}_2^*, \alpha^*) = \begin{bmatrix} -3.444 & 152 & -0.294 & 303 & 9 \\ 2.381 & 383 & 0.359 & 118 & 2 \end{bmatrix}$$

where $\alpha^* = 5$. The eigenvalues of this matrix are $\lambda_s = 0.1649273$ and $\lambda_u = -3.249961$. The eigenvector associated with the eigenvalue λ_s is $\mathbf{e}_s = [0.1032530, -1.266202]^T$ from which the slope of the tangent to the stable manifold at the fixed point can be derived. We derive a quadratic approximation to the manifold in order to maintain accuracy of the back iterates. To do this, we set

$$\delta I = h(\delta V) = c_1 \delta V + c_2 (\delta V)^2,$$

where $\delta I = I - I_2^*$ and $\delta V = V - V_2^*$. The coefficients c_1 and c_2 can be found from an equation derived from the invariance of the stable manifold [1] giving $c_1 = -0.0815454$ and $c_2 = 0.0089097$. The two fixed points and the quadratic approximation to the stable manifold of $\mathbf{x}_2^*$ are also shown in Fig. 4. An alternative approach to obtaining an accurate approximation to the stable manifold which spans the attractor is simply to back iterate a much shorter section which is based on a linear approximation.

The next stage is to back iterate the approximation to the manifold a sufficient number of times until there is a preiterate sufficiently close to the fixed point $\mathbf{x}_1^*$. It can be seen from Fig. 5 that a third preiterate is very close to $\mathbf{x}_1^*$ and so no further preiterates are required. Out of a maximum of 18 possible preiterates of the stable manifold, there are 16 which intersect the attractor.

We now need to determine the line ℓ through $\mathbf{x}_1^*$ of points which are accessible in one iteration with a small perturbation in the parameter α . We take $\delta\alpha_{\text{max}} = 0.02$ where $\delta\alpha = \alpha - 5$. The line ℓ is defined in terms of the vector $\mathbf{w} = \mathbf{F}_{\alpha}(\mathbf{x}_1^*, \alpha^*) = [3.958546, -3.206621]^T$.

The intersection between the line ℓ and the third preiterate of the stable manifold is found to be at $(I, V) = (3.643805, 4.339500)$ and the corresponding value of the perturbation is found to be $\delta\alpha = 0.0076584$. Starting at the fixed point $\mathbf{x}_1^*$ and using this perturbation at the first iteration gives an orbit which is on (or close to) the stable manifold of $\mathbf{x}_2^*$ after only four iterations and which is at a distance of 0.0498774 from the fixed point after only five iterations. At this stage, the OGY control method could be employed to stabilize the orbit on the fixed point $\mathbf{x}_2^*$. The targeted orbit is shown in Fig. 6.

A similar procedure can be performed to jump back from the fixed point $\mathbf{x}_2^*$ to $\mathbf{x}_1^*$. In this case a perturbation of $\delta\alpha = -0.0051709$ results in an orbit which is on (or close to) the stable manifold of $\mathbf{x}_1^*$ after 4 iterations as before, and which is a distance of 0.0188128 from $\mathbf{x}_1^*$ after 7 iterations. The targeted orbit in this case is shown in Fig. 7. To compare the orbit length with and without targeting, a perturbation of $\delta\alpha = +0.0051709$ was made at the first iteration (note the change of sign). In this case, it took 1929 iterations to come within a distance of 0.02 of the fixed point $\mathbf{x}_1^*$.

We note that the number of clock pulses in the orbit will be more than the number of iterations since in some cases, there is a missed clock pulse during the switch closed phase. However, the two fixed points $\mathbf{x}_1^*$ and $\mathbf{x}_2^*$ are in the regions in which there are none or one missed clock pulses per iteration of the map respectively and so the

number of clock pulses in the target orbits will only be slightly more than the number of iterations of the map.

The number of iterations in the target orbit for invertible maps is often around 12 [3], [12] and so we see that for this noninvertible map, the target orbit is much shorter so that very rapid jumping between states is possible.

Normal operation of power converters aims to avoid subharmonics and chaos, mainly because they are poorly understood and ripple currents and voltages are greater. However, it may be advantageous in some cases to jump rapidly among different periodic behaviors in order to improve electromagnetic compatibility by spreading the interference spectrum [6]. There may also be other physical systems with discontinuities in which rapid jumping from one periodic point to another could have practical advantages.

V. CONCLUSION

A new method for targeting in systems with discontinuities is presented and applied to the current-controlled boost converter. The multivalued inverse function of such systems is used to obtain multiple back iterates of the target region which soon cover the attractor. This method results in target orbits which are considerably shorter than those obtained for invertible maps.

This brief could be extended by considering the case when the mapping $\mathbf{F}$ is not known explicitly. Targeting methods are known to be sensitive to noise and this should be addressed for this method since power converters are renowned generators of electromagnetic interference. It may also be useful in applications to track a fixed point with a slowly varying parameter.

REFERENCES

- [1] P. J. Aston, "An investigation of a mode interaction involving period-doubling and symmetry-breaking bifurcations," in *Dynamics, Bifurcation and Symmetry, New Trends and New Tools*, P. Chossat, Ed. Boston, MA: Kluwer, 1994, pp. 49–58.
- [2] P. J. Aston and C. M. Bird, "Analysis of the control of chaos—extending the basin of attraction," *Chaos, Sol. Fract.*, vol. 8, pp. 1413–1429, 1997.
- [3] C. M. Bird and P. J. Aston, "Targeting in the presence of noise," Tech. Rep. 96/13/Ma, Dept. Math. Comput. Sci., Univ. Surrey, U.K., 1996.
- [4] K. Chakrabarty and S. Banerjee, "Control of chaos in piecewise-linear systems with switching nonlinearity," *Phys. Lett. A*, vol. 200, pp. 115–120, 1995.
- [5] J. H. B. Deane, "Chaos in a current-mode controlled boost dc–dc converter," *IEEE Trans. Circuits Syst. I*, vol. 39, pp. 680–683, 1992.
- [6] D. C. Hamill, J. H. B. Deane, and P. J. Aston, "Some applications of chaos in power converters," *Inst. Elect. Eng. Colloquium: Update on New Power Electronic Techniques*, London, England, Ref. 1997/091, May 1–5, 1997.
- [7] D. C. Hamill, J. H. B. Deane, and D. J. Jefferies, "Modeling of chaotic dc–dc converters by iterated nonlinear mappings," *IEEE Trans. Power Electron.*, vol. 7, pp. 25–36, 1992.
- [8] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, pp. 1196–1199, 1990.
- [9] M. Paskota, A. I. Mees, and K. L. Teo, "Directing orbits of chaotic dynamical systems," *Int. J. Bifurc. Chaos*, vol. 5, pp. 573–583, 1995.
- [10] —, "Directing orbits of chaotic systems in the presence of noise: Feedback correction," *Dynam. Contr.*, vol. 7, pp. 25–47, 1997.
- [11] G. Poddar, K. Chakrabarty, and S. Banerjee, "Control of chaos in the boost converter," *Electron. Lett.*, vol. 31, pp. 841–842, 1995.
- [12] T. Shinbrot, E. Ott, C. Grebogi, and J. A. Yorke, "Using chaos to direct trajectories to targets," *Phys. Rev. Lett.*, vol. 65, pp. 3215–3218, 1990.
- [13] T. Shinbrot, W. Ditto, C. Grebogi, E. Ott, M. Spano, and J. A. Yorke, "Using sensitive dependence of chaos (the 'butterfly effect') to direct trajectories in an experimental chaotic system," *Phys. Rev. Lett.*, vol. 68, pp. 2863–2866, 1992.
- [14] Y. S. Tang, A. I. Mees, and L. O. Chua, "Synchronization and chaos," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 620–626, 1983.
- [15] D. Xu and S. R. Bishop, "Steering dynamical trajectories to target a desired state," *Chaos, Sol. Fract.*, vol. 4, pp. 1931–1942, 1994.