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Title: Modern Concrete Bridge Deck Analysis considering the Effects of Cracking

Authors: David Collings BSc CEng FICE, Researcher University of Surrey, Technical Director ARCADIS.

Juan Sagaseta PhD, DIC, CEng, MICE, MCICCP, Eur Ing, Senior Lecturer University of Surrey.

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Abstract

This paper outlines a refinement to current grillage and linear finite element analysis methods to better estimate the behaviour of reinforced concrete deck slabs on prestressed beam or steel girder bridges, suitable for modern codes of practice and computerised methods. The work is part of research aimed at unlocking the potential of compressive membrane action. The paper proposes a three-phase approach for the prediction of cracking, deflections, ductility and load capacity. The method increases the accuracy of current grillage and conventional linear finite element methods by taking into account flexural cracking extensions using an effective strain method. The method gives better estimates of deflections at serviceability and allows for better estimates of the ultimate load capacity of existing bridges. This results in more economic reinforcement designs with lower carbon footprint for new bridges.

Keywords

Bridges, Slabs and plates, Concrete structures, Design methods, Cracking, Compressive Membrane Action.

Notation

A	area of concrete
b	breadth of section
b _e	effective width
d	slab depth to reinforcement
E _c	elastic modulus of concrete
E _ε	elastic modulus of concrete based on stress-strain
EI _{eff}	effective EI of cracked section
f _{ck}	characteristic concrete strength
f _{cm}	cylinder strength
f _{ct}	tensile strength of concrete
f _y	reinforcement yield strength
h	depth of slab (soffit to top)
I _c	second moment of area of uncracked concrete
I _y	second moment of area of fully cracked concrete
k	normalised stiffness $\frac{EI_{eff}}{EI_c}$
L	length, span length
m	normalised moment $\frac{M}{M_y}$
M _{cr}	cracking moment
M _E	design moment
M _{EAV}	average design moment
M _{EC}	characteristic combination moment
M _y	yield moment
n	normalised axial force $\frac{N}{0.85 A f_{ck}}$
N	axial force
q	uniform load
Q	total load (q L)

s	dimension of wheel load
v	normalised punching shear $\frac{V}{V_p}$
V_p	punching shear capacity
W_z	Section modulus
x	depth to neutral axis
α	normalised strain $\frac{\epsilon_o}{\epsilon_y}$
β	Coefficient dependent on loading
β_p	Reinforcement coefficient
Δ	extension
ϵ_o	strain at slab mid-section (h/2)
ϵ_{od}	dilation strain
$\epsilon_{o\max}$	maximum strain at slab mid-section
ϵ_{C3}	concrete strain at end of elastic phase
ϵ_{U3}	ultimate concrete strain
ϵ_s	strain in reinforcement
ϵ_y	steel strain at yield
ξ	stress parameter
ρ	reinforcement ratio
σ_{sr}	reinforcement stress at first cracking
σ_s	reinforcement stress

Introduction

The purpose of structural analysis is to establish the distribution of either internal forces and moments, or stresses, strains and displacements, over the structure and the analyses shall be carried out using idealisations of both the geometry and the behaviour of the structure (BSI, 2002). The cracking of concrete is often ignored in the analysis of concrete bridge decks, it is usually considered in the analysis of composite steel-concrete bridges, for global analysis effects (Collings, 2013; BSI, 2005c). However, cracking is normally assumed for the section design of deck slabs analysed as uncracked. This paper outlines a more rational approach to the consideration of cracking in reinforced concrete bridge decks on steel girder or precast concrete beams for use with Eurocodes and other modern codes. The work is part of research aimed at unlocking the potential of compressive membrane action (Collings, 2017).

In design the authors perception is that bridge deck slab thickness and reinforcement content have been increasing with time. A snapshot of 15 UK, US and European concrete bridges from 1900 to the present and 15 composite bridges (all in UK) from 1950 is given in Figure 1. The increased concrete and reinforcement come at a cost and also increases the carbon footprint of a bridge (Collings, 2005). A more realistic analysis can help with refining the design and reducing both cost and carbon footprint. Part of the increased slab thickness is due to larger cover requirements for durability and part to changing loads. However, part is also due to a loss of knowledge across generations (Collings, 2008). Turner and Maillart (early pioneers of reinforced concrete) visualised slabs with a mixture of arching, flexure and shear in a way difficult for a modern computer-oriented engineer to comprehend. Their designs were often verified by testing. A brief historical perspective of bridge deck slab analysis is given to help explain this.

Figure 1 Span to thickness ratios of bridge deck slabs from 1900 to present day.

Historic Perspective

In the early part of the 20th century, during the initial development of reinforced concrete, Turner (**Gasparini, 2002**) and Maillart (**Zastavni, 2009**) visualised the behaviour of flat slab designs as a mixture of a flat dome and flexure. Both Turner and Maillart used empirical methods which were often validated by testing of the structures, the structures typically carried significantly more load than they were designed for. Maillart used low bending coefficients in his slab designs (**Furse and Marti, 1997**). In early versions of codes of practice, the increased test results (due to internal arching) was also acknowledged through the use of reduced bending coefficients. The UK code of practice for concrete, CP114 (**BSI, 1957**) for instance gave a total design moment of $QL/10$ for a building slab, 20% lower than the $QL/8$ derived from flexure-torsion only theories (where Q is the total uniform load on the span).

In the 1920's Westergaard and Slater (**1921**) defined a flexural theory for slabs, they acknowledged the effects of internal arching action, but did not incorporate it into their theory. Westergaard's work was highly influential and his paper dealing with the computation of the stresses in bridge slabs due to local wheel loads is still relevant to grillage analysis. A simplified expression was given by Westergaard (**1930**) on the basis of the effective width concept. Assuming that a local wheel load could be estimated as acting on an effective width (b_e) which is dependent on the size of the wheel (s) and the span (L) of the slab, as Equation 1. This effective width concept is used in modern codes such as AASHTO (**2007**). The effective width is the basis for modern grillage and finite element meshes which should be finer than this width to capture local effects.

$$b_e = 0.58L + 2s$$

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Various other influence curves by Morice and Little (**1954**), and Pucher (**1964**), among others, were developed. The Pusher influence surfaces are more comprehensive than Westergaard's and able to cover various load types and boundary conditions. However, following a discussion with Mr Little many years ago (noted in **Collings, 2002**) the influence surface has seemed a laborious way to achieve a conservative answer. The strip method, popularised in the UK by Wood and Armer (**1968**) but based on the Hillerborg (**1959**) theory, is preferred. It is interesting to note that in his tests Armer (**1968**) acknowledges the influence of arching action on the ultimate capacity, however, like Westergaard before he did not include it in the theory.

Elastic analysis by modern computers was another method of analysis noted by Wood and Armer and was being developed at that time. In a discussion of the strip method paper

McMillan (in **Wood and Armer, 1969**) noted that the strip method could be thought of as being similar to a torsion-less grillage method. The grillage method became more popular after the publication of Bridge Deck analysis by Hambly (**1976**), and by its second edition (Hambly, 1991, which interestingly reintroduced the torsion-less grillage) found grillage analysis the standard method in all bridge design offices. Typically, the grillage is a two-dimensional (2-D) analysis with all sections defined geometrically within a horizontal plane and with only vertical loads, shears, moments and torsions being considered (Figure 2a). The 2-D grillage, being constrained to the beam neutral axis, does not accurately represent in-plane effects in the deck slab like the tension, and subsequent cracking, over intermediate beam supports. The 2-D grillage is often improved by offsetting the beam or girder properties from the slab in a pseudo three-dimensional (3-D) analysis or a full 3-D frame analysis which can account for in-plane effects (Figure 2b).

Figure 2 a) 2-D and b) 3-D grillage or FEM node degrees of freedom

The finite element method (FEM) was developed at the same time as the grillage analysis method, with the advent of modern computers. The FEM uses similar stiffness-based methods, but rather than using beam elements with two nodes, it uses plate or shell elements with 3, 4 or more nodes. A key difference between grillage type and FEM is in the mesh size, grillages typically being near Westergaard's effective width (Equation 1), whereas FEM models tend to need to be an order of magnitude smaller to achieve accuracy. With similar properties, spans and restraints the linear elastic FEM gives moments very similar to those of Westergaard or Pucher.

Ingerslev (**1923**) first proposed the yield line method of analysis, Johansen (**1943**) extended this but it was not translated into English and more widely used until the 1960's. The destructive test on the Old Dental Hospital in Johannesburg, South Africa, reported by

Ockleston (1955), gave collapse loads of 3 to 4 times those predicted by yield line theory. McDowell, et al. (1956) outlined a theory for strength enhancement of masonry using internal arching. Ockleston then appreciated that the extent of strength enhancement of his tests was caused by internal arching action (Ockleston, 1958). Christiansen, Park, Eyre, Kirkpatrick and others (see Collings and Sagaseta, 2016) have developed this theory linking arching or compressive membrane action (CMA) with yield line theory. Yield line and CMA theories, which involve assumptions of plasticity are less favoured for the design of bridge deck slabs in modern codes (BSI, 2005b). These theories are however perhaps more akin to the visualisations of Turner and Maillart.

In this paper; to establish the distribution of internal forces, moments, stresses and strains; the elastic methods of analysis favoured by bridge designers is extended to consider cracking, extensions (or dilations) and restraint using an effective strain method. This 3-phase analysis allowing a more realistic simulation of behaviour and the consideration of some internal arching or CMA.

Three-Phase Analysis

A 3-phase analysis considers the cracking and non-linear behaviour of concrete deck slabs as three discrete linear phases. The method considers; an initial un-cracked phase up to the tensile strength of the concrete (f_{ct}); a second cracked but elastic phase generally at the serviceability limit state (SLS); and a third yielding or plastic post-yield phase at the ultimate limit state (ULS) (see Figure 3). A flow chart outlining the 3-phase analysis is given in Figure 4. It should be noted that the proposed 3-phase method is a step-by-step linear elastic analysis aiming to more accurately represent concrete cracking under multiple loading. It is not intended as a non-linear analysis; post-yielding redistribution of moments and local non-linearity is not taken into account.

Figure 3 Typical slab strains and stresses during the three phase analysis proposed, a) un-cracked, b) cracked but elastic and, c) plastic post yield.

Figure 4 Flow chart of the proposed 3-phase analysis of concrete bridge deck slabs.

The initial phase is characterised by the simple elastic bending of the un-cracked concrete section with linear stress and strain assumptions. In this first phase no dilation or extension takes place as the centroid strain (ϵ_0) is zero. The section reaches the end of this phase when the applied moment is larger than the cracking resistance moment (M_{cr}) or, the tensile stress at the extreme fibre exceeds the tensile strength of the concrete (f_{ct}) and the section

cracks, as Equation 2. The analysis during this phase is the conventional linear elastic grillage or FEM using full section properties.

$$M_{cr} = f_{ct}/W_z = 0.167 f_{ct} b h^2 \quad 2$$

Where W_z is the elastic section modulus, b is the width of slab and h the overall depth. To be compatible with Eurocode 4 (**BSI, 2005c**) where cracking of concrete is considered to derive cracked or uncracked analysis for composite sections, the slab section is considered to be fully cracked over a significant part of the span when the characteristic load combination (M_{EC}) causes cracking that exceeds twice the tensile strength of concrete or twice the cracking moment as Equation 3:

$$2M_{cr} \leq M_{EC} \quad 3$$

The linear elastic analysis of a number of bridges with deck slab spans of 1m to 4m (between beam web centres) were carried out using grillage models. The characteristic moments resulting from LM1, LM2 and LM3 load combinations, as defined in Eurocode 1 (**BSI, 2003a; BSI, 2003b**) were derived. The characteristic combination is defined in Eurocode 0 (**BSI, 2002**), it is a combination with all primary partial load factors set to 1.0. The moments were used to estimate the cracking limits for the concrete deck slabs using Equation 3. Figure 5 outlines the likely range of bridge deck slab thickness that would be cracked or uncracked for various spans using concrete tensile strengths of 2.5 to 3.5MPa representing reasonable values of the concrete tensile strength. The bridges noted in Figure 1 are also superimposed on figure 5, most of the existing structures are in the cracked region.

Figure 5 Span to thickness plot for various concrete strengths (MPa) outlining likely cracked and uncracked regions, with bridges of Figure 1 superimposed.

Where the bridge deck is uncracked at the characteristic combination then the conventional elastic grillage or FEM analysis with full cross section stiffness may be used to derive forces, moments and deflections at both the serviceability limit state (SLS) and the ultimate limit state (ULS). It however follows that crack widths and deflections will be small, stresses very likely to be well within allowable limits for the slab at SLS, and that the ULS will likely govern the design of reinforcement requirements for flexure or punching shear. Where the bridge deck is cracked at the characteristic combination then a second-phase analysis should be carried out.

In the second phase the concrete is considered cracked, but the section still behaves essentially elastically, governed by the composite action of the concrete and reinforcement. These are the typical assumptions for sections at the SLS when crack widths and deflections are to be estimated. The stiffness of this elastic cracked section is less than the un-cracked section and deflections are larger. Hambly (1991) notes that for reinforced concrete structures where there is cracking in both the main beams and transverse elements the distribution of moments and shears may be similar to the uncracked section. However, when the main beams remain uncracked, but the deck slab is cracked there will be a different distribution of moments and shears. Deflections and rotations will be increased in this case. Concrete Society (2007) recommends a reduction in flexural stiffness ($E_c I_c$) by half for the slab to account for this. Eurocode 2 (BSI, 2005a) outlines the flexural stiffness of the cracked section in a number of clauses and outlines reductions of 0.3 to 0.5 of the full stiffness. It also notes that for serviceability a gradual evolution of cracking should be considered and that at the ULS a reduced stiffness corresponding to cracked section may be assumed. The change in stiffness is proportional to a stress parameter ξ as Equations 4 to 7:

$$EI_{\text{eff}} = (1-\xi)E_c I_c + \xi E_\epsilon I_y \quad 4$$

$$\xi = 1 - \beta (\sigma_{\text{sr}}/\sigma_s)^2 \quad 5$$

Where E_c is the initial concrete elastic modulus, I_c is the full uncracked second moment of area, E_ϵ is the effective elastic modulus at higher strains (Equation 6). I_y is the cracked second moment of area at yield.

$$E_\epsilon = f_{\text{ck}}/\epsilon_{\text{c3}} \quad 6$$

Assuming a constant lever arm and using moments rather than stresses:

$$\xi = 1 - \beta (M_{\text{cr}}/M_E)^2 \quad 7$$

The coefficient β depends on the duration of the load, for a single loading $\beta = 1$, for multiple loading, such as wheels on a bridge deck, $\beta = 0.5$ (**BSI, 2005a**). The effects of multiple loads are to significantly reduce the effective stiffness of the slab at lower moments and stresses. Figure 6 shows the reduction in normalised stiffness (k) for slab sections 150-450mm thick with normalised moments and various reinforcement ratios (ρ) calculated assuming elastic section properties, with tension stiffening, different loading assumptions and that the stiffness changes as Equation 4.

$$k = \frac{E_{eff}}{E_{lc}}$$

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The torsional and transverse section properties should be reduced proportionally (a torsion-less slab analysis for this second phase may also be used to simplify data post-processing). The sectional area should also be reduced as Eyre and Kemp (**1994**) showed by testing that the axial stiffness of a cracked section is reduced.

Figure 6 M-EI curve for concrete slabs of reinforcement content and loading.

The strain at the centroid of the section during this phase is usually not zero but is a tensile strain (Figure 3b), indicating that the section increases in length. For an unrestrained beam or slab this change in length, or dilation, is often ignored, although Beeby and Faithbitaraf (**2001**) and Mathias et al. (**2019**) note that this can be an unsafe assumption. For a restrained or part restrained structure the restraint of this dilation causes some internal arching or CMA which may stiffen the structure, reduce crack widths and increase capacity (**Collings and Sagaseta, 2016**) under localised wheel axle or vehicle loading; bridge deck slabs offer this restraint. It is proposed that the local dilation strains may be added into the

phase-2 analysis as a load case using an effective strain approach described in the next section. The concrete section reaches the end of this second phase typically when the reinforcement approaches yield, or concrete strains approach the elastic limit (ϵ_{c3}).

The third phase again assumes a cracked section, but this time the reinforcement is at yield and the concrete approaching its compression limit (Figure 3c). The neutral axis is derived from strength and equilibrium adopting a plastic stress block rather than elastic assumptions. The distribution of strains are linear, but stresses in the concrete are non-linear. There is likely to be some localised rotations at regions of plasticity. These are typical assumptions for sections at the ultimate limit state. Again, the strain at the centroid (ϵ_o) is non-zero and a significant increase in length occurs leading to increased internal arching or CMA in restrained slabs. This increase is primarily at areas where plastic hinges will form. The section reaches the end of this phase typically when the concrete strain reaches its limiting value (ϵ_{u3}), the rotational limits are reached, the reinforcement fractures or de-bonds, or a shear failure occurs. For the design of bridges, it is not usual to assume significant yielding, redistribution of moments or plasticity so this third phase is typically not used to further consider flexural effects. For the assessment of existing structures, provided ductility requirements are complied with, then this phase could be considered.

Concrete Dilation and Effective Strain

The extension of an unrestrained reinforced concrete slab can be obtained from tests (**Vecchio and Tang, 1990**), derived from NL-FEA, or estimated as the sum of the centroid strains from the assumptions used to design conventional reinforced concrete, as Equation 9. Where Δ is the total extension of the slab (typically half of this total extension occurs at each end of the slab, although this may vary if the end restraints are different), L is the distance along the slab and ϵ_o is the strain at the mid-section of slab.

$$\Delta = \int_0^L \epsilon_o (i) di \quad 9$$

For lower moments at SLS or at ULS away from maximum moments, the centroid strain may be estimated based on the strain in the reinforcement using assumptions of steel stress to limit crack widths as Equation 10.

$$\epsilon_o = \epsilon_s \frac{0.5h - x}{d - x} \quad 10$$

Where x is the depth to the neutral axis. When the neutral axis tends towards $0.5h$ then the centroid strain reduces to zero, hence for high reinforcement ratios or low concrete strengths,

which increase x , the extensions will be smaller. The centroid strain is as proposed for the modified compression field theory (**Vecchio and Collins, 1986**) and the critical shear crack theory (**Muttoni and Ruiz, 2008**) used to estimate shear capacity. Figure 7 shows the normalised centroid strain (α) as a function of the reinforcement content (ρ) based on elastic properties and reinforcement characteristic strength of 500MPa. The larger the reinforcement content the lower the dilation. The strain is normalised against the steel strain at yield (ϵ_y) so at SLS the strains will be proportionally smaller and at ULS may be greater.

$$\alpha = \frac{\epsilon_o}{\epsilon_y}$$

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Figure 7 Normalised centroid strain with reinforcement content and β_ρ of Equation 16.

At the ULS in areas near the maximum moment where potential plastic hinges would form, or moment redistribution occurs the central strain may be estimated based on the strain distribution of the section using the maximum concrete strain (ϵ_{u3}) as Equation 12. Cracking due to shear can cause also cause some extension of the slab at lower span to depth ratios (**Sigrist, et al. 2013**) but is neglected in this current method.

$$\epsilon_o = \epsilon_{u3} \frac{0.5h - x}{x}$$

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The concrete extension may be calculated using Equations 9 and 10 based on the moments on the section. To gauge the accuracy of the method, the calculated strains and extensions are compared with measured values. Vecchio and Tang (**1990**) tested an un-restrained (TV1) and a restrained slab strip (TV2). The slab strip being 6m long, 1.5m wide, with a 3m span and 100mm slab thickness. For the slab strip the extensions and strains were measured. Theoretical centroid strains can be calculated for the test TV1 using Equation 10. Figure 8

shows the measured and calculated centroid or mid height strains (ϵ_o) along the length of the 3m span with the calculated strains peaking at mid span and with smaller peaks at supports. The measured strains are more variable at the support, this is likely to be due to local column head thickening. The test load is at 55kN which is slightly beyond the yield load at mid span (46kN) and so the additional centroid strain was determined from Equation 12 with an ultimate strain of 0.0035. The ultimate strain is confined to the hinge region and largest at mid span and reducing to the yield value at the end of the hinge region (0.6d from this maximum using BSI, 2005a). The theoretical and measured mid-height strains shown in Figure 8 are in good agreement at mid span.

Figure 8 Measured mid-height strains from test TV1 and theoretical centroid strains for a test load of 55kN.

The total extension calculated from the strains using Equation 9 are 1.2mm. The extensions are in good agreement with the 1.0mm value measured at the end of the slab strip. The extension at various loads can also be calculated. Figure 9 shows the measured and theoretical extensions of the test slab over the range of the test, the theoretical values following a similar line to the measured values.

The equivalent or effective strain (ϵ_{od}) is the extension per unit length of the slab as Equation 13. For most slab elements with almost linear change in moment then ϵ_o can be approximated by Equation 14. Where $\epsilon_{o\max}$ is the largest value of ϵ_o along the span of the slab.

$$\epsilon_{od} = \Delta/L \quad 13$$

$$\epsilon_{od} = 0.5 \epsilon_{o\max} \quad 14$$

For conventional concrete strength ($f_{ck} = 25$ to 60MPa) slabs with high yield reinforcement ($f_y = 500\text{MPa}$ and $\epsilon_y = 0.0025$) the dilation strain can be estimated with respect to the average moment (M_{EAV}) on the grillage beam or FEM element and a reinforcement modification factor β_p . The reinforcement factor is zero at 2.5% reinforcement ratio and increases as the reinforcement ratio reduces. β_p is derived from Figure 7 and given by Equation 16 it tends to zero at high reinforcement ratios and 0.6 at low reinforcement content.

$$\epsilon_{od} = 0.0025 \beta_p M_{EAV}/M_y \quad 15$$

$$\beta_p = 0.625 - 0.25 (100\rho) \quad 16$$

Proposed Effective Strain Method

This section outlines an effective strain method to incorporate the dilation strains into the phase 2 analysis, it gives a lower bound solution at first yield and can be used for analysis at the serviceability and ultimate limit state. The method is based on the forces generated by the restraint to the longitudinal dilation of the concrete. The dilation strains are applied to the 2-D or 3-D grillage members or FEM elements as an effective strain calculated using Equation 15. The following analysis steps are proposed (see also Figure 4):

- a) Carry out a linear elastic analysis of the un-cracked phase-1 structure, estimate the load that causes cracking using Equation 3. If the section is un-cracked use the conventional linear elastic results for design without considering cracking or dilation effects. If the section is cracked estimate the effective strains for the load cases to be considered using Equation 15.
- b) Carry out a conventional linear elastic analysis of the cracked phase-2 structure with modifications to flexural, torsional and axial properties to account for cracking and applying the effective strains as an additional load case to the loaded part of the structure where the peak moments occur in the phase-1 analysis. This analysis gives a revised estimate of the flexural capacity of the structure, the strength and cracking capacity of the structure will typically be increased due to the induced axial force and moment-axial interaction effects (see Appendix A).
- c) A further ultimate flexural analysis of the phase-3 structure with reduced flexural stiffness, increased dilations and increased axial forces could be carried out in some cases for a higher level of approximation or in cases where a more refined assessment is needed. However, the use of the phase-2 results is usually sufficient for design. Calculation of the shear capacity of the structure should be carried out, the presence of

the axial compression in the slab will result in an increase the shear capacity (see Appendix A).

Examples of the Effective strain Method

The first example is to reconsider the 6m long 3m span slab strip tests TV1 and TV2 of Vecchio and Tang. At 55kN the extension of TV1 was found to be approximately 1mm from test and 1.2mm by calculation (see Figure 9). The reinforcement ratio (ρ) is 0.66% and so the flexural stiffness will be about 0.4 of the full value (see Figure 5). Test TV2 is similar to TV1 but fully restrained in-plane at the ends, hence the full restraint to extension will induce an axial force (N):

$$N = EA_{\text{eff}} \Delta / L \quad 17$$

The full axial stiffness (EA/L) is approximately 720kN/mm, hence assuming the axial stiffness reduces in a similar proportion to the flexural stiffness (as noted above) then the axial force is estimated using equation 17 as 290kN. The measured axial force from test TV2 was 270kN and at similar deflection was carrying a load of 65kN compared with 55kN for TV1. Confirming that the restraint to dilation will induce compressive membrane action and increase the slabs load capacity.

Figure 9 Measured and theoretical extensions of the TV1 test slab.

In real bridges there is not a full restraint and there is an interaction of local and global flexural effects. A number of tests on full size or scale bridges have been carried out. One of the best instrumented tests were those by Jackson (1990) where a model bridge was tested with point loads and patch loads representing an abnormal vehicle. Jackson recorded the

behaviour of the uncracked and cracked structure. The Jackson test bridge is a half-scale test of four precast prestressed beams with a reinforced concrete deck slab. Key details of the test are outlined in Table 1.

Table 1 Jackson test bridge details.

Element	Characteristic
Main beams	4 number, 8m long half scale precast-prestressed M beams on elastomeric bearings.
Deck slab	80mm thick half scale concrete deck spanning 1m across precast beams.
Concrete	For beams $f_{cm}=62.4\text{MPa}$. For slab $f_{cm}=42.4\text{MPa}$.
Reinforcement	$f_y=460\text{MPa}$, reinforcement 4 orthogonal layers $\rho=0.33\%$.

An uncracked phase-1 pseudo 3-D grillage model of the bridge was made. A point load case and an abnormal load case were run to determine the flexural moments on the structure. The results of the phase-1 analysis for a single wheel load and an HB type abnormal vehicle (**Highways England, 2001**) indicate the slab cracks at a moment (M_{cr}) of 3.5kNm/m which is equivalent to a load of 35kN . It was confirmed that the slab cracked under both wheel and HB vehicle loads. The conventional flexure only capacity from the uncracked grillage is 64kN and the punching shear resistance 68kN based on Eurocode 2 (**BSI, 2005a**) with all material partial factors set to 1.0.

For the phase-2 analysis the dilation strains at first yield were calculated and added as a load case as applied strains (similar to temperature loading). This causes in-plane deformation of the slab and adjacent beams as shown in Figure 10a together with in plane axial compressions and tensions. The pattern of axial forces is compression immediately adjacent to the applied loads, where the largest moments occur, with a tensile ring around the load. In the analysis it is important that the transverse properties of the beams and slabs are correctly modelled. The phase-2 analysis indicates a single wheel load capacity of 120kN considering flexure-axial interaction. The shear capacity, taking into account the axial forces induced, is 92kN , this being the critical criteria. The moment-axial and shear-axial interaction diagrams for the slab are outlined in Appendix A.

Figure 10 In-plane grillage extensions and in plane forces for a) single wheel load and b) HB vehicle.

The load deflection diagram of the test is given in Figure 11. The phase-1 and phase-2 analysis results are superimposed on this. It is noted that the conventional uncracked grillage analysis gives a significantly lower load capacity than the 3-phase analysis method and the test.

Figure 11 Load deflection of deck slab for single wheel test and analysis methods.

A similar analysis was carried out for the multi-wheel HB vehicle and an additional dilation load case applied to the grillage. The in-plane deformed shape and axial forces are shown in Figure 10b. Again, the broad pattern of compression local to wheels and tensions around the perimeter of the vehicle can be seen. The load deflection diagram of the test is shown in Figure 12 with the phase-1, phase-2 and punching shear results superimposed. The difference in the initial uncracked test and the test after further wheel loads in this and other locations have been applied can be seen. Again, the phase-2 results show an increase compared to the phase-1 analysis. For this simulation the flexural and punching shear capacity are similar.

Figure 12 Load deflection of deck slab for HB vehicle test and analysis methods.

Summary & Conclusions

This paper outlines a simple 3-phase method to more accurately analyse bridge decks in accordance with modern codes. An advantage of the method is that it uses conventional linear elastic methods common in design.

The conventional linear elastic phase-1 analysis derives the predicted cracking moment and determines if cracking is likely to occur using criteria consistent with other Eurocodes.

Results indicate that slabs with a span to thickness (L/h) ratio greater than approximately 10 to 12 are likely to be cracked.

The phase-2 analysis using cracked section properties more accurately estimates moments and deflections at higher stresses. With cracking comes some dilation and extension of the concrete local to the loaded area, which can be included in the phase-2 analysis as an applied effective strain load case to more realistically model in plane effects and consider Compressive Membrane Action.

A phase 3 analysis with further cracking and dilation may be appropriate for the assessment of some existing structures. However, for the design of new structures the authors recommend the use of the phase-2 analysis with the shear capacity only calculated at phase-3.

The 3-phase method outlined in this paper shows significant improvement in accuracy of design and assessment compared with current practice which adopts only the phase 1 analysis.

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References

AASHTO (2007) LRFD Bridge Design Specifications, Washington DC, 7th Edition, Chapter 9, Bridge deck systems.

Armer GST (1968) Ultimate load tests of slabs designed by the strip method, ICE Proceedings, Volume 41, Issue 2, 313 –331.

Beeby A W Faithibitaraf F (2001) Membrane Effects in Reinforced Concrete Frames – A Proposal for Change in the Design of Frame structures, Engineering Structures, 23, 82-93.

BSI (British Standards Institution) (1957) The Structural Use of Reinforced Concrete in Buildings, CP 114.

BSI (2002) Eurocode 0: BS EN 1990: Basis of Structural Design.

BSI (2003a) Eurocode 1: BS EN 1991-2: Actions on structures - Part 2: Traffic loads on bridges

BSI (2003b) Eurocode 1:BS NA EN 1991-2: UK National Annex to Eurocode 1: Actions on structures. Traffic loads on bridges

BSI (2005a) Eurocode 2: BS EN 1992-2: Design of concrete structures Part 2: Concrete bridges — Design and detailing rules.

BSI (2005b) Eurocode 2: BS EN 1992-2: UK National Annex to Eurocode 2: Design of concrete structures Part 2: Concrete bridges — Design and detailing rules.

BSI (2005c) Eurocode 4: EN 1994-2: Design of composite steel and concrete structures – Part 2, General rules and rules for bridges.

Collings D (2002) The design of bridge decks utilising arching effects, Proceedings ICE, Structures & buildings 152, August 2002, 277 – 282.

Collings D (2005) An Environmental Comparison of Bridge Forms, Proceedings of ICE, Bridge Engineering 159, Issue BE4, 163–168.

Collings D (2008) Lessons from historical bridge failures, Proceedings of ICE, Civil Engineering 161, November 2008, 20–27.

Collings D (2013) Steel-concrete composite bridges, designing for Eurocodes, ICE Publishing, London.

Collings D (2017) Unlocking the potential of compressive membrane action in concrete, Civil Engineering, Volume 170 Issue CE1, 12.

Collings D and Sagaseta J (2016) A Review of Arching and Compressive Membrane Action, Proceedings of the Institution of Civil Engineers - Bridge Engineering, 169(4), 271-284.

Concrete Society (2007) Guide to the design & construction of reinforced concrete flat slabs, TR64, Crowthorn, UK.

Eyre JR and Kemp KO (1994) In-plane stiffness of reinforced concrete slabs under compressive membrane action, Magazine of Concrete Research, Volume 46, Issue 166, 67-77.

Furse A and Marti P (1997) Robert Maillart's Design Approach for flat Slabs, Journal of Structural Engineering, Vol. 123. No.8, 1102-1110.

Gasparini D A (2002) Contributions of C A P Turner to Development of Reinforced Concrete Flat Slabs 1905-1909, Journal of Struct Eng, 1243-1252.

Hambly, EC (1976) Bridge deck behaviour, E & FN Spon. London, UK.

Hambly, EC (1991) Bridge deck behaviour, 2nd Edition, E & FN Spon. London, UK.

Highways England (2001) Loads for highway bridges, Design manual for roads and bridges, Vol 1, section 3, Part 14, BD37/01.

Hillerboarg A (1959) *Strimlernetoden*. Svenska Riksbyggen, Stockholm, 1959.

Ingerslev A (1923) The strength of rectangular slabs, J. of Institution of Structural Engineering, London, Vol. 1, No.1, 1923, 3-14.

Johansen KW (1943) Brudlinieteorier, Jul. Gjellerups Forlag, Copenhagen, Yieldline theory, translated by Cement & Concrete Association, 1963, 191.

Jackson PA (1990) The global and local behaviour of bridge deck slabs, Structural Engineer, Vol 68 No 6, 112-116.

Mathias N, Li Z, Keileh J and Sarkisian M (2019) Flexural extension considerations in reinforced concrete, SEI, 1/19, 40-47.

McDowell EL, McKee KE and Sevin E (1956), Arching Action Theory of Restrained Masonry Walls, ASCE Journal of Structural Engineering, Vol 82, No ST2, 915 1- 18.

Morice P B and Little G (1954) Load Distribution in Prestressed Concrete Bridge Systems, The Structural Engineer, Issue 3, March 1954, pp. 83-112.

Muttoni A and Fernandez-Ruiz M (2008) Shear Strength of members without Transverse Reinforcement as function of critical shear crack width, ACI Structural Journal, 163-172.

Ockleston AJ (1955) Load tests on a three-storey building in Johannesburg, The Structural Engineer, Vol. 33, 304 – 322.

Ockleston AJ (1958) Arching action in reinforced concrete slabs, The Structural Engineer, Vol. 36, No.6, 197 – 201.

Pucher A (1964) Influence surfaces of elastic plates, Wien, New York, 1964.

Sigrist V, Bentz E, Ruiz M F, Foster S and Muttoni A (2013) Background to the fib Model Code 2010 shear provisions – part I: Beams and slabs, Structural Concrete, Volume 14, Issue 3, 195–203.

Vecchio FL and Collins MP (1986) The modified compression field theory for reinforced concrete elements subject to shear, ACI Structural journal, 83 (2), 219-231.

Vecchio FL and Tang K (1990) Membrane action in reinforced concrete slabs, Canadian Journal of Civil Engineering, Vol 17, 1990, 686-697.

Westergaard HM (1930) Computation of stresses in bridge slabs due to wheel loads. Public Roads, Vol 11.

Westergaard HM and Slater WA (1921) Moments and stresses in slabs, Proceedings of the American Concrete Institute, 1921, Vol. 17, pp 415 - 538.

Wood R H and Armer GST (1968) The theory of the strip method for design of slabs (includes Appendix), ICE Proceedings, Volume 41, Issue 2, 285–311.

Wood R H and Armer GST (1969) Discussion, The theory of the strip method for design of slabs, ICE Proceedings, Volume 43, Issue 2, 291–306.

Zastavni D (2009) What was truly innovative about Maillart's designs using reinforced concrete, Proceedings of third international congress on construction history, Cottbus.

Appendix A: Moment and shear with axial interaction

Both moments and punching shear are increased with axial loads based on Eurocode 2 methods (**BSI, 2005a**). For moments the basic flexural capacity is increased by an additional moment of resistance from the axial force at an eccentricity. Figure A1 outlines a normalised interaction diagram for the slab of the Jackson test, in which the normalised flexural capacity (m) increases up to a normalised axial load (n) of 0.45 then reduces after $n = 0.5$. The normalised capacity at $n = 0$ is $m = 1.0$. The capacity at $n = 0.1$ increases to $m = 1.87$. For normalised punching shear (v) the capacity is increased by an axial component directly proportional to the average axial stress. Figure A2 outlines a normalised interaction diagram for the slab of the Jackson test, in which the punching capacity increases to $n = 0.2$ and is then constant. The normalised capacity at $n = 0$ is $v = 1.0$. The capacity at $n = 0.1$ increases to $v = 1.34$.

Figure A1 Normalised moment-axial interaction with phase-1 and phase-2 results.

Figure A2 Normalised shear-axial interaction with phase-1 and phase-2 results.