THE CONCEPT OF PELLEVATION FOR SHAPING OF STRUCTURAL FORMS

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Abstract

Space structures have become increasingly important and many thousands of them have been build all over the world. However, the generation of architectural and engineering data for space structures is often difficult when the problem extends beyond standard forms such as planar grids, domes or barrel vaults. 'Configuration processing' using formex algebra is a conceptual tool which enables the designer to generate all forms of space structures with ease and elegance.

In this work a completely new concept for the shaping of structural forms is introduced which uses the concepts of formex algebra and which has been implemented in the environment of the interactive programming language Formian. The concept, referred to as 'pellevation', enables the alteration of any given configuration by changing the vertical coordinates of the nodes. Pellevation may be used in many different areas, however, it is of particular interest to the designers of grid domes and shell structures.

The present work provides a comprehensive treatment of the concept of pellevation, introducing an extensive range of pellevations together with their mathematical formulation. A major part of this work was the programming of the related pellevation functions. The structure of the program together with a small portion of the code is given in Appendix B. The Thesis further includes many practical examples of interest to structural engineers and architects.

Zusammenfassung

Räumliche Tragwerke haben in den vergangenen Jahren stark an Bedeutung gewonnen und zahlreiche Bauwerke dieser Art sind auf der ganzen Welt entstanden. Die Datengenerierung für räumliche Tragwerke ist jedoch oft schwierig, insbesondere wenn es sich um komplizierte Formen handelt, die nicht mehr als ebene Fachwerke, Kuppeln oder Tonnengewölbe erfaßt werden können. Die Datenverarbeitung von Konfigurationen (= 'Anordnung von Elementen innerhalb eines Tragwerks') auf der Grundlage der 'Formex Algebra' ermöglicht dem Architekten oder Ingenieur, Daten für räumliche Tragwerke auf einfache und elegante Weise zu generieren.

Diese Dissertation stellt eine völlig neuartige Methode vor, die es ermöglicht, bauliche Formen unter Verwendung der 'Formex Algebra' zu erzeugen. Das Konzept nennt sich 'Pellevation' (englisch *per elevation*, aus dem Lateinischen *elatus* = erhaben, erhöht) und wurde als Teil der interaktiven Programmiersprache 'Formian' umgesetzt. 'Pellevation' ermöglicht die Formänderung von gegebenen Konfigurationen, in dem die vertikalen Koordinaten der Knotenpunkte verändert werden. 'Pellevation' findet viele verschiedene Anwendungen, ist jedoch von besonderem Interesse für den Entwurf von Fachwerkkuppeln und Schalenträgwerken.

Die vorliegende Arbeit stellt eine umfassende Abhandlung des neuen Konzepts dar. Es werden zahlreiche Arten von 'Pellevationen' einschließlich deren mathematischer Herleitung vorgestellt. Das Programmieren der zugehörigen Funktionen war ein wesentlicher Teil dieser Arbeit. Die Programmstruktur sowie ein kurzer Auszug des eigentlichen Programms sind in Anhang B wiedergegeben. Weiterhin beinhaltet die Dissertation viele praktische Beispiele, die sowohl für den Ingenieur als auch für den Architekten von Interesse sind.

Gott gebe mir die Gelassenheit,
Dinge hinzunehmen,
die ich nicht ändern kann,
den Mut, Dinge zu ändern,
die ich ändern kann
und die Weisheit,
das eine vom anderen zu unterscheiden.

God grant me the serenity
to accept the things
that I cannot change.
Courage to change
the things I can
and the wisdom
to know the difference.

To Udo

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Table of Contents

Abstract	2
Acknowledgements	6
Table of Contents	7

CHAPTER 1: INTRODUCTION

1.0 What is a Space Structure?	12
1.1 Configuration Processing	15
1.2 Pellevation - A New Concept	17

CHAPTER 2: CAP PELLELEVATION

2.0 Introduction	19
2.1 Circular Cap Pellevation with Circular Base	20
2.1.1 Circular Cap Pellevation Function with Circular Base	22
2.1.2 Negative Pellevation	24
2.1.3 Mathematical Formulation	26
2.1.4 Applications	28
2.1.4.1 Superimposition of Pellevation	34
2.2 Elliptic Cap Pellevation with Circular Base	37
2.2.1 Elliptic Cap Pellevation Function with Circular Base	37
2.2.2 Mathematical Formulation	39
2.2.3 Applications	42
2.2.4 Change of Element Length, Distortion and Warping of Elements	46
2.3 Parabolic Cap Pellevation with Circular Base	50
2.3.1 Parabolic Cap Pellevation Function with Circular Base	50
2.3.2 Mathematical Formulation	52
2.3.3 Applications	53

2.4	Wedge Cap Pellevation with Circular Base	59
2.4.1	Wedge Cap Pellevation Function with Circular Base	59
2.4.2	Mathematical Formulation	60
2.4.3	Inclined Pellevation	62
2.4.4	Applications	64
2.5	Elliptic Cap Pellevation with Elliptic Base	67
2.5.1	Elliptic Cap Pellevation Function with Elliptic Base	67
2.5.2	Mathematical Formulation	69
2.5.3	Multi-layer Configurations	74
2.5.4	Applications	76
2.6	Circular Cap Pellevation with Elliptic Base	79
2.6.1	Circular Cap Pellevation Function with Elliptic Base	80
2.6.2	Mathematical Formulation	81
2.6.3	Selective Pellevation	81
2.6.4	Applications	84
2.7	Parabolic Cap Pellevation with Elliptic Base	86
2.7.1	Parabolic Cap Pellevation Function with Elliptic Base	86
2.7.2	Mathematical Formulation	88
2.7.3	Regional Pellevation	90
2.7.3.1	Rectangular Region	94
2.7.3.2	Ambit Region	95
2.7.3.3	Circular Region	96
2.7.3.4	Elliptic Region	96
2.7.3.5	Parabolic Region	97
2.8	Wedge Cap Pellevation with Elliptic Base	98
2.8.1	Wedge Cap Pellevation Function with Elliptic Base	99
2.8.2	Mathematical Formulation	100
2.8.3	Pellevation and Novation	101

C H A P T E R 3 : B A R R E L P E L L E V A T I O N

3.0	Introduction	104
3.1	Circular Barrel Pellevation with Rectangular Base	105
3.1.1	Circular Barrel Pellevation Function with Rectangular Base	106
3.1.2	Mathematical Formulation	107
3.2	Elliptic, Parabolic and Wedge Barrel Pellevations with Rectangular Base ...	110
3.2.1	Elliptic, Parabolic and Wedge Barrel Pellevation Functions with Rectangular Base	111
3.2.2	Mathematical Formulation	115

3.3	Circular Barrel Pellevation with Trapezoidal Base	120
3.3.1	Circular Barrel Pellevation Function with Trapezoidal Base	120
3.3.2	Mathematical Formulation	124
3.4	Elliptic, Parabolic and Wedge Barrel Pellevations with Trapezoidal Base ...	126
3.4.1	Elliptic, Parabolic and Wedge Barrel Pellevation Functions with Trapezoidal Base	127
3.4.2	Mathematical Formulation	135
3.5	Applications	139

CHAPTER 4: HALO PELLEATION

4.0	Introduction	153
4.1	Circular Halo Pellevation with Circular Base	154
4.1.1	Circular Halo Pellevation Function with Circular Base	155
4.1.2	Mathematical Formulation	158
4.2	Elliptic, Parabolic and Wedge Halo Pellevations with Circular Base	160
4.2.1	Elliptic, Parabolic and Wedge Halo Pellevation Functions with Circular Base	162
4.2.2	Mathematical Formulation	165
4.3	Circular Halo Pellevation with Elliptic Base	170
4.3.1	Circular Halo Pellevation Function with Elliptic Base	170
4.3.2	Definition of the Elliptic Boundary	172
4.3.3	Mathematical Formulation	174
4.4	Elliptic, Parabolic and Wedge Halo Pellevations with Elliptic Base	186
4.4.1	Elliptic, Parabolic and Wedge Halo Pellevation Functions with Elliptic Base	187
4.4.2	Mathematical Formulation	191
4.4.3	Difference between 'Stretched' Circular Base Halo and Elliptic Base Halo Pellevations	195
4.5	Applications	198

CHAPTER 5 : SADDLE PELLEVATION

5.0	Introduction	201
5.1	Circular Saddle Pellevation with Parabolic Base	202
5.1.1	Circular Saddle Pellevation Function with Parabolic Base	202
5.1.2	Mathematical Formulation	206
5.2	Elliptic, Parabolic and Wedge Saddle Pellevations with Parabolic Base	210
5.2.1	Elliptic, Parabolic and Wedge Saddle Pellevation Functions with Parabolic Base	211
5.2.2	Mathematical Formulation	215
5.3	Circular Saddle Pellevation with Hyperbolic Base	219
5.3.1	Circular Saddle Pellevation Function with Hyperbolic Base	219
5.3.2	Mathematical Formulation	222
5.4	Elliptic, Parabolic and Wedge Saddle Pellevations with Hyperbolic Base	225
5.4.1	Elliptic, Parabolic and Wedge Saddle Pellevation Functions with Hyperbolic Base	226
5.4.2	Mathematical Formulation	229
5.5	Applications	233
5.5.1	Data Export for Graphical Representation	235
5.5.2	Data Export for Analysis Purposes	238

CHAPTER 6 : RAFT PELLEVATION

6.0	Introduction	247
6.1	The Concept of Raft Pellevation	248
6.2	Raft Pellevation Function	251
6.3	Mathematical Formulation	252
6.4	Applications	255

CHAPTER 7 : MANDATE PELLEVATION

7.0 Introduction 258

7.1 The Concept of Mandate Pellevation 259

7.2 Mandate Pellevation Function 259

7.3 Mathematical Formulation 261

7.4 Applications 261

**CHAPTER 8 :
SELECTIVE AND REGIONAL PELLEVATIONS**

8.0 Introduction 266

8.1 Selective Pellevation 267

 8.1.1 Mathematical Formulation 267

8.2 Regional Pellevation 269

 8.2.1 Mathematical Formulation 271

 8.2.1.1 Ambit Region 272

 8.2.1.2 Rectangular Region 275

 8.2.1.3 Circular Region 277

 8.2.1.4 Elliptic Region 278

 8.2.1.5 Parabolic Region 279

**CHAPTER 9 : CONCLUSIONS AND
SUGGESTIONS FOR FUTURE WORK**

9.0 Summary 282

9.1 Summary and Conclusions 283

9.2 Suggestions for Future Work 285

Appendix A: Basic Pellevation Functions 286

Appendix B: Description of Pellevation Files 292

Appendix C: Analysis Output 304

References 309

Chapter 1

Introduction

1.0 What is a Space Structure?

The term 'space structure' refers to a structural system in which the load transfer mechanism involves three dimensions. This is in contrast with a 'plane structure', such as a plane truss, in which the load transfer mechanism involves no more than two dimensions. To elaborate, in the case of a plane structure, the external loads as well as the internal forces are in a single plane. This is the plane that also contains the (idealised) structure itself, both in its initial unloaded state and in its deformed loaded state. In the case of a space structure, the combination of the configuration, external loads, internal forces and displacements of the structure extends beyond a single plane.

The above definition is the 'formal' definition of a space structure. However, in practice, the term 'space structure' is simply used to refer to a number of families of structures that include grids, barrel vaults, domes, towers, cable nets, membrane systems, foldable assemblies and tensegrity forms. Space structures cover an enormous range of shapes and are constructed using different materials such as steel, aluminium, timber, concrete, fibre-reinforced composites, glass... or a combination of these. [1]

Examples of space structures include sports stadiums, gymnasiums, cultural centres, leisure centres, shopping malls, aircraft hangars and transmission towers amongst others, Figures 1.0.1 to 1.0.4.

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1.1 Configuration Processing

In the present context the term 'configuration' is used to mean an 'arrangement of parts'. The assembly of all the elements of a structure, for instance, is a configuration. The most common usage of the term configuration is in reference to a geometric composition consisting of points and / or lines and / or surfaces.

A configuration may be described using a numerical model (i.e. an arrangement of numbers). In particular, the internal computer representation of a configuration is bound to be in terms of a numerical model. The term 'configuration processing' is used to mean the 'creation and manipulation of numerical models that represent configurations'. In particular, the term 'formex configuration processing' is used to mean configuration processing with the aid of the concepts of 'formex algebra'.

Formex algebra is a mathematical system that provides simple and elegant conceptual tools for the presentation and processing of configurations. The basic ideas from which formex algebra has emerged were evolved in the early 1970's, Ref. [4], and the first textbook on the subject appeared in 1984, Ref. [5]. A concise description of the current state of the concepts of formex algebra is given in Ref. [6]. A convenient medium for using the concepts of formex configuration processing is the programming language 'Formian', Ref. [6]. [1]

Figure 1.1.1 gives a variety of configurations that have been obtained using the programming language Formian.

Figure 1.1.1: Examples of configurations obtained with Formian

1.2 Pellevation - A New Concept

The basic principle of the new concept which has been developed and established in this work can be seen in Figure 1.2.1. Here, the initial configuration represents a planar grid in the xy -plane, Figure 1.2.1(a). A geometric object, Figure 1.2.1(b), is then used to 'deform' this configuration by pushing the object up (along the z -axis). The result of this process can be seen in Figure 1.2.1(c).

This process could be repeated using a second geometric object, for example the top part of a sphere, Figure 1.2.1(d). The result of both processes can be seen in Figure 1.2.1(e).

Because the process actually involves the elevation of parts of the initial configuration it is referred to as 'pellevation'. 'Pellevation' comes from 'per elevation' which originates from the Latin word *elatus* (= raised, elevated).

The initial idea of the concept has first been introduced in Ref. [7]. The present work provides a comprehensive treatment of the concept of pellevation, introducing an extensive range of pellevations together with their mathematical formulation and software implementation. The concept of pellevation may be used in many different areas, however, it is of particular relevance to the designers of grid domes and shell structures. The Thesis includes many practical examples of interest to structural engineers and architects.

The material in this Thesis is organised as follows:

Chapters 2 to 7 are devoted to the description of various families of pellevations. Chapter 8 gives the background for some special features of the concept. Finally, Chapter 9 brings out the conclusions and suggestions for future work in this field of research.

Figure 1.2.1: (a) Typical configuration, (b) 1st geometric object, (c) deformed configuration, (d) 2nd geometric object, (e) deformed configuration

Chapter 2

Cap Pellevation

2.0 Introduction

This chapter introduces the first family of pellevations which is referred to as 'the family of cap pellevations'. The general features of the concept of pellevation will be explained in detail using different illustrative and practical examples involving cap pellevations.

2.1 Circular Cap Pellevation with Circular Base

There are several cap pellevations, the first of which is referred to as 'circular cap pellevation with circular base'. This cap pellevation can be imagined to describe the properties of a part of a sphere. This part of a sphere is used as a die to push up any part of a given configuration. Take, for example, the planar grid shown in Figure 2.1.1(a). This grid may represent a 30 by 30 lattice structure or it may represent a finite element mesh lying in the xy-plane.

The top part of the sphere which is given in Figure 2.1.1(b) is now used as a die. This die in the present context is called a 'pellevant' and its shape and position are defined by a number of parameters, namely, the coordinates of its centre, the span of its base and its height at the centre as shown in the figure.

If this pellevant is used to push up the central region of the configuration of Figure 2.1.1(a), the result will be as shown in Figure 2.1.1(c). This process is referred to as 'pellevation'. Here, it can be seen that outside the boundaries of the cap pellevant, the configuration remains unchanged while within the scope of the pellevant, the nodes of the configuration experience a 'push-up'. That is, the z-coordinates of the nodes are changed according to the shape of the pellevant. The above described circular cap pellevant is just one example out of a number of different pellevants which will be introduced in the sequel.

Typically, a cap pellevant is defined by

- a 'pellevation code' which characterises its basic shape in section and plan
- its position in the xy-plane with respect to a Cartesian coordinate system
- the size of the boundaries in plan
- the height of the pellevant at the centre

Figure 2.1.1: Typical pellevation process
(a) initial configuration, (b) pellevant, (c) pellevated configuration

Altogether, eight cap pellevation functions, referred to as capel functions, have been defined using pellevants with two different plan views in combination with four different cross-sections. These pellevants have four types of cross-sections, namely the

- circular cap
- elliptic cap
- parabolic cap
- wedge cap

where each one of these can be with either a circular or an elliptic base. Table A.1 in Appendix A shows the general characteristics of these cap pellevants.

2.1.1 Circular Cap Pellevation Function with Circular Base

The capel function, used to obtain the circular cap pellevation with circular base, is of the form

The function given above has actually been used to create the pellevated shape shown in Figure 2.1.1(c).

Figure 2.1.2: Details related to the parameters of the circular cap pellevation with circular base

As can be seen, the function requires a list of parameters, the first of which is called the 'pellevation code'. The circular cap pellevant with circular base has been allocated pellevation code 1. Details related to further parameters, in order of their appearance, are shown in Figure 2.1.2. They are the coordinates of the centre of the circular cap in plan $C_i (cx_i, cy_i)$, the span s in plan and the height h at the centre. The circular cap pellevation function with circular base can be written in its general form as:

Here, a convention is introduced that a cap pellevation function with pellevation code 1 may be referred to as '*capell*'. As will be seen later on, this convention is also used for other pellevation codes and other pellevation functions.

By definition of the currently described mode of the cap pellevation function, both the span and the height of *capell* have to be positive. Also, there is an important restriction which applies to the height of all circular cap pellevations. This restriction relates to the fact that the height of the pellevant cannot be greater than half the span, that is,

$$h \leq s/2$$

The restriction is introduced in order to have a unique solution to the problem. An example to clarify the reason for this restriction is shown in the sketch in Figure 2.1.3. It can be seen that the vertical line through a point may intersect with the surface of the pellevant twice. In this case, two solutions for the change in the z-coordinate occur.

Figure 2.1.3: Restriction of height for pellevants with circular cross-section

2.1.2 Negative Pellevation

The use of a negative pellevation code implies a 'negative pellevation', that is, the process will have a 'push-down' effect. Take the 30 by 30 grid shown in Figure 2.1.4. Here, the same initially flat configuration is used as in Figure 2.1.1. Also, the properties of the pellevant remain unchanged. Only the pellevation code has changed from 1 to -1. It can be seen that the z-coordinates of the points inside the boundary of the pellevant

have in this example been 'reduced' according to the shape of the pellevant. This convention of using positive pellevation codes to denote push-up pellevations and negative pellevation codes for push-down pellevations is used for all pellevation codes and all types of pellevations.

Figure 2.1.4: Example of a negative pellevation

A formex formulation for the configuration of Figure 2.1.4 is as follows:

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] # rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = capel(-1,15,15,24,5) | E;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,0,1);
draw F;
```

In the above formulation, formex E denotes the initial planar 30 by 30 configuration. Here, the configuration does not represent an actual structure but is used to show the basic properties of the pellevant and to illustrate a typical negative pellevation. In such a situation, the configuration is referred to as a 'tector'. Tectors will be used throughout this work whenever the intention is to demonstrate the properties of a pellevant or to bring out a special feature in an example.

Formex F, then, represents the tector after the pellevation where the parameters used are as explained before. The next line contains some use-items which set the view options, and the last line produces the actual picture shown in Figure 2.1.4. If the reader

is not familiar with Formian, Ref. [6] may be consulted where these basic concepts are explained in detail.

2.1.3 Mathematical Formulation

Both the section and the plan of a circular cap pellevant with circular base, outlining the variables required for the pellevation process, are given in Figure 2.1.5.

Figure 2.1.5: Geometric details for the circular cap pellevation with circular base

The procedure involved in the process is as follows.

Generally, all the points of the configuration to be pellevated are dealt with one by one and the procedure outlined below is repeated for each one of them. The process will be explained for a point $P(px, py)$ where point P stands for any point considered for pellevation.

In the first step the distance in plan between the points P and C_1 is obtained as

$$d = \sqrt{(px - cx_1)^2 + (py - cy_1)^2} \dots\dots\dots 2.1.1$$

where $C_1 (cx_1, cy_1)$ defines the centre of the pellevant, Figure 2.1.5. If the distance d is found to be greater than or equal to half the span, that is,

$$d \geq s/2$$

then point P lies outside the boundary of the pellevation cap and will not be affected by the pellevation process. If, however, d is found to be less than half the span,

$$d < s/2$$

then P will experience a push-up or push-down. Thus, the change in the z-coordinate needs to be determined from the properties of the pellevant. In order to do so, it is required to find the value of the radius r of the sphere. From the geometric properties of the pellevant's section as illustrated in Figure 2.1.5, the following relations may be derived:

$$\tan \alpha = \frac{s/2}{2r - h} \quad (\text{from triangle } \textcircled{1}\textcircled{2}\textcircled{3})$$

and

$$\tan \alpha = \frac{h}{s/2} \quad (\text{from triangle } \textcircled{2}\textcircled{3}\textcircled{4})$$

From these relations, it follows that

$$\frac{s/2}{2r - h} = \frac{h}{s/2}$$

or

$$(s/2)^2 = h(2r - h)$$

or

$$\frac{s^2}{4} = 2hr - h^2$$

or

$$2hr = \frac{s^2}{4} + h^2$$

and finally

$$r = \frac{s^2}{8h} + \frac{h}{2} = \frac{s^2 + 4h^2}{8h} \dots\dots\dots 2.1.2$$

Having obtained the radius r , it is possible to determine the value of e , Figure 2.1.5, as

$$e = \sqrt{r^2 - d^2} \dots\dots\dots 2.1.3$$

The amount of push-up or push-down at any location within the boundary of the pellevant is then given by

$$\Delta z = h - r + e$$

Hence, the new z-value of point P is obtained as

$$z_{new} = z_{old} + (h - r + e) \quad \quad \quad (for \textit{push-up})$$

$$z_{new} = z_{old} - (h - r + e) \quad \quad \quad (for \textit{push-down})$$

It has to be clearly understood that pellevation only operates on the z-coordinates and, therefore, the plan view of any pellevated configuration is the same before and after the pellevation.

2.1.4 Applications

The concept of pellevation serves to create many different types of structural shapes in a quick and efficient way. One of the main uses of the concept of pellevation is for obtaining all varieties of grid domes. A grid dome is a structural form that is commonly used throughout the world. A grid dome is created by starting from a planar grid which is then given a dome shape. Shallow cap pellevations are ideal for creating these structural forms.

Figure 2.1.6 shows four typical grids where for each one of them the initial configuration is shown together with the boundary and the section of the pellevant in dotted lines on the left. The corresponding pellevated configuration and the boundary of the pellevant are shown on the right together with the formex formulation.

Figure 2.1.6(a) shows a square grid which has been pellevated using a circular cap pellevant with circular base. In this first case, the four corner points of the configuration lie on the boundary of the pellevant. Therefore, these points remain in the same position before and after pellevation while all other nodes are pushed up in the process.

Figure 2.1.6(b) shows a grid dome with a diagonal pattern which has been created using a *capell* pellevation. This time, the boundaries of the pellevant in plan lie well outside the configuration and all points of the grid are affected by the process.

Figure 2.1.6(c) shows a triangular grid dome where the boundaries of the circular cap pellevant lie just outside the three corner points. While all points will be pushed up in the process as before, the absolute push-up of the corner points of this example will be much smaller than in the previous example. The reason being that the corner points are closer in plan to the boundary than the corner points of the configuration in Figure 2.1.6(b).

The grid shown in Figure 2.1.6(d) is of a hexagonal shape and has been pellevated using a *capell* pellevation. Similar to the previous examples, the outline of the pellevant in plan is larger than the actual configuration. Therefore, all nodes of the configuration obtain new z-coordinates in the process of pellevation.

There is one feature that the above given examples have in common: They have all been pellevated using a centrally placed pellevant. This, however, need not necessarily apply in all cases. The centre of the pellevant can be at any other location. For instance, consider the hexagonal grid of Figure 2.1.6(d). It is used again to obtain the grid dome given in Figure 2.1.7(a).

Figure 2.1.6: Examples of grid domes using circular cap pellevation with circular base

This time, the dome has been obtained using a *capell* with a different centre. The centre of the pellevant is, in this case, not coincident with the centre of the configuration. The centre of the pellevant has been chosen such that one of the corner nodes of the configuration lies on the boundary of the pellevant while the other nodes lie further inside. Thus, all the nodes except for the one on the boundary will experience a push-up. The result is a grid dome which is pushed up non-symmetrically.

Figure 2.1.7: Examples of grid domes using circular cap pellevation with circular base

As an other example, the triangular grid dome shown in Figure 2.1.7(b) is created from the same initial configuration as the example in Figure 2.1.6(c). Only this time, the centre of pellevation has been placed near one of the corner points of the configuration. Hence, this corner point is pellevated higher than the other two corner points which only experience small changes in z-coordinates.

The examples shown so far have all involved skeletal grids. In other words, the configurations have been composed of discrete bar elements forming lattice structures. However, each of the configurations shown in Figures 2.1.6 and 2.1.7 may also represent a finite element mesh of a shell structure. In this case, instead of using bar elements, the configuration is obtained using elements with three or more nodes. Figure 2.1.8 gives four shell structures starting with a hexagonal shape like the one already introduced in Figure 2.1.6(d). For all the examples, the finite element mesh and its formex formulation are given on the left while a rendered view of the shell is shown on the right.

Figure 2.1.8(b) shows a variation where the shell has a hole at the middle. The pellevation process itself is carried out irrespective of any discontinuity of the shell structure. The relevant part of the pellevant simply remains ineffective in these regions.

Figures 2.1.8(c) and (d) show shell structures similar in mesh to the grid dome of Figure 2.1.6(a) but with irregular boundaries and missing elements.

(a)

```

a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(8,8,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(7,7,2,1,1,-1) | b1;
c1 = bb(1,sqrt(3)) | (a2#b2); c2 = rosad(8,8*sqrt(3,6,60)) | c1;
S1 = capel(1,8,8*sqrt(3,46,12)) | c2;
use vm(2), vt(1), vh(-4,-5,3,0,0,0,0,0,1);
clear; draw S1;

```

(b)

```

a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(8,5,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(7,5,2,1,1,-1) | b1;
c1 = bb(1,sqrt(3)) | (a2#b2); c2 = rosad(8,8*sqrt(3,6,60)) | c1;
S2 = capel(1,8,8*sqrt(3,46,12)) | c2;
use vm(2), vt(1), vh(-4,-5,3,0,0,0,0,0,1);
clear; draw S2;

```

(c)

```

m=10;
a1 = tran(1,-m/2) | rinid(m,m,1,1) | [0,0,0; 1,0,0; 1,1,0; 0,1,0];
a2 = rosad(0,0) | a1;
S3 = capel(1,0,0,44,8) | a2;
use vm(2), vt(1), vh(-3,-5,3,0,0,0,0,0,1);
clear; draw S3;

```

(d)

```

m=10;
a1 = tran(1,-m/2) | rinid(m,m,1,1) | [0,0,0; 1,0,0; 1,1,0; 0,1,0];
a2 = rosad(0,0) | a1;
a3 = lux(rosad(0,0) | (rin(1,3,1) | [3,0,0])) | a2;
S4 = capel(1,0,0,44,8) | a3;
use vm(2), vt(1), vh(-3,-5,3,0,0,0,0,0,1);
clear; draw S4;

```

Figure 2.1.8: Examples of shells using circular cap pellevations with circular base

2.1.4.1 Superimposition of Pellevation

The examples shown so far, have all been the results of a single pellevation. However, the effects of pellevation are additive, that is, they superimpose. Consider, for instance, the configuration shown in Figure 2.1.9(a). This 25 by 20 mesh could represent a pellevated grid or the finite element mesh of a shell structure.

If this pellevated configuration is taken as the basis of another cap pellevation with smaller base and different centre, the configuration shown in Figure 2.1.9(b) can be obtained. Then, a third pellevation can be applied to this configuration and the configuration shown in Figure 2.1.9(c) is obtained. The ability of superimposing the effects of pellevation is one of the most important features of the pellevation concept and can be employed to achieve many interesting shapes like, for instance, the timber lattice shell in the Noborioji Area in Japan, Figure 2.1.10(a) or the Theme Pavilion of Aomori Expo '88, Figure 2.1.10(b).

Figure 2.1.9: Superimposition of pellevation

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Figure 2.1.10: (a) Timber lattice shell in the Noborioji Area in Japan, Ref. [8],
(b) Theme Pavilion of Aomori Expo '88, Ref. [9]

Figure 2.1.11 gives some more variations of the hexagonal configuration introduced in Figure 2.1.6(d), this time making use of the effects of superimposition. Figure 2.1.11(a) has been obtained through two subsequent cap pellevations with identical centres but different sizes. Figure 2.1.11(b) shows the result of adding a third cap pellevation with the same centre but smaller dimensions. The nodes in the vicinity of the edges of the configuration are still only subjected to a single pellevation since they do not fall into

the boundaries of the remaining two pellevations. The points in the central region, however, have acquired new z-coordinates due to three pellevations. The new z-coordinate for a point in the central part could be given as

$$z_{new} = z_{old} + \Delta z_1 + \Delta z_2 + \Delta z_3$$

where Δz_1 , Δz_2 and Δz_3 are the amounts of push-up resulting from the first, second and third pellevation, respectively.

(a)

```
a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(10,10,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(9,9,2,1,1,-1) | b1;
c1 = bb(1,sqrt(3)) | (a2#b2);
c2 = rosad(10,10*sqrt(3,6,60)) | c1;
c3 = capel(1,10,10*sqrt(3,30,4)) | c2;
S1 = capel(1,10,10*sqrt(3,50,10)) | c3;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,1);
clear; draw S1;
```

elevation

(b)

```
a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(10,10,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(9,9,2,1,1,-1) | b1;
c1 = bb(1,sqrt(3)) | (a2#b2);
c2 = rosad(10,10*sqrt(3,6,60)) | c1;
c3 = capel(1,10,10*sqrt(3,30,4)) | c2;
c4 = capel(1,10,10*sqrt(3,50,8)) | c3;
S2 = capel(1,10,10*sqrt(3,14,3)) | c4;
use vm(2), vt(1), vh(-4,-5,6,0,0,0,0,1);
clear; draw S2;
```

elevation

(c)

```
a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(10,10,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(9,9,2,1,1,-1) | b1;
c1 = bb(1,sqrt(3)) | (a2#b2); c2 = capel(1,10,0,18,2) | c1;
c3 = rosad(10,10*sqrt(3,6,60)) | c2;
c4 = capel(1,10,10*sqrt(3,30,4)) | c3;
S3 = capel(1,10,10*sqrt(3,50,8)) | c4;
use vm(2), vt(1), vh(-4,-5,6,0,0,0,0,1);
clear; draw S3;
```

elevation

Figure 2.1.11: Examples using superimposition of pellevation

Figure 2.1.11(c), gives a variation of centrally and eccentrically placed cap pellevations of different sizes. This idea can be utilised to obtain any type of 'bubbly' or 'hilly' lattice or continuous structure.

2.2 Elliptic Cap Pellevation with Circular Base

'Elliptic cap pellevation with circular base' is the second type of cap pellevation. It uses a part of an ellipsoid of revolution about the z-axis as pellevant. A typical tector is shown in Figure 2.2.1 with the formex formulation alongside. The purpose of this tector is to illustrate the basic shape of the pellevant as explained in Section 2.1.2.

Figure 2.2.1: Illustrative example of elliptic cap pellevation with circular base

2.2.1 Elliptic Cap Pellevation Function with Circular Base

This particular pellevation is obtained by the *capel* function with pellevation code ± 2 as the first parameter. The other parameters required include the coordinates of the centre $C_1(cx_1, cy_1)$, the span s of the base and the height h at the centre. These parameters have the same significance as those outlined for *capell* and are shown in Figure 2.2.2. However, the elliptic cap pellevation with circular base requires one additional

parameter which is simply added at the end of the list. This parameter is the height of the ellipsoid, h_e , and it denotes the length of the vertical semi-axis of the ellipsoid. Together with the height h of pellevation, h_e is a measure of how much of the ellipsoid serves as pellevant.

Figure 2.2.2: Details related to the parameters of the elliptic cap pellevation with circular base

The general form of the elliptic cap pellevation function with circular base is given as

By definition, the values of s , h and h_e have to be positive and the height h cannot be greater than h_e , that is,

$$h \leq h_e$$

This restriction is introduced because, otherwise, the solution for the pellevation would not be unique in a similar way as described for the *capell* in Section 2.1.1 and as shown in Figure 2.2.3.

Figure 2.2.3: Restriction of height for pellevants with elliptic cross-section

2.2.2 Mathematical Formulation

The initial part of the mathematical formulation follows a similar path as the one outlined for *capell*. To begin with, the distance d of point P from the centre of pellevation is determined (Equation 2.1.1) and only if it is less than half the span s of the pellevant, P will be affected by the pellevation process.

In order to compute the amount of push-up (positive pellevation code) or push-down (negative pellevation code) of point P , the geometric details of the ellipse in section are used, as shown in Figure 2.2.4.

Figure 2.2.4: Geometric details for the elliptic cap pellevation with circular base

The equation of this ellipse is of the general form

$$\frac{x^2}{a^2} + \frac{z^2}{h_e^2} = 1 \quad \dots\dots\dots 2.2.1$$

Here, the boundary point $B((s/2), (h_e - h))$, as shown in Figure 2.2.4, lies on the ellipse and, therefore, satisfies Equation 2.2.1. Substitution of its coordinates yields

$$\frac{(s/2)^2}{a^2} + \frac{(h_e - h)^2}{h_e^2} = 1$$

Solving this equation for the unknown parameter a gives

$$\frac{(s/2)^2}{a^2} = 1 - \frac{(h_e - h)^2}{h_e^2}$$

or

$$a^2 = \frac{(s/2)^2}{1 - \frac{(h_e - h)^2}{h_e^2}}$$

or

$$a = \frac{s/2}{\sqrt{1 - \frac{(h_e - h)^2}{h_e^2}}} \dots\dots\dots 2.2.2$$

The x-coordinate of point P relative to the elliptic cap is given by its distance d to the centre as shown Figure 2.2.4. Therefore, Equation 2.2.1 can be rewritten and solved for z .

$$\frac{d^2}{a^2} + \frac{z^2}{h_e^2} = 1$$

or

$$\frac{z^2}{h_e^2} = 1 - \frac{d^2}{a^2}$$

or

$$z^2 = h_e^2 \left(1 - \frac{d^2}{a^2} \right)$$

and finally

$$z = h_e \sqrt{1 - \frac{d^2}{a^2}} \dots\dots\dots 2.2.3$$

where a is known from Equation 2.2.2.

As can be seen from Figure 2.2.4, the new z -coordinate of point P is given by

$$z_{new} = z_{old} + (h - h_e + z) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - (h - h_e + z) \quad \text{(for push-down)}$$

2.2.3 Applications

The nature of the elliptic cap pellevation enables the creation of rather shallow grid domes or continuous shell structures as shown in Figure 2.2.5, where the change in z-coordinates is relatively small. The main difference with a shallow circular cap pellevation lies in the boundary regions. To elaborate, consider Figure 2.2.6(a). Here, the difference in section is shown between a circular and an elliptic cap pellevation of the same span and height. Notice that whereas the central region is almost identical, the elliptic cross-section becomes higher much faster. In other words, the slope at the boundary is comparatively steep which may be an important issue for design considerations.

Having said that an elliptic cap pellevation can give rise to rather shallow structures, it also has to be mentioned that a *capel2* pellevation may as well be used to create structures with a moderate increase in z-coordinates, that is, structures with a comparatively high rise-to-span ratio. This latter case will result in structural forms with a slope at the boundary which is more shallow than that of a corresponding *capel1* pellevation, as can be seen in Figure 2.2.6(b). The difference in obtaining either one of these results can be explained as follows:

- A shallow structural form, as illustrated in Figure 2.2.6(a), is obtained through the application of an elliptic cap pellevation where

$$s/2 \gg h$$

and

$$h \cong h_e$$

So, approximately the top half of a shallow ellipsoid is used as pellevant.

(a)

```
a = rinid(17,16,1,1) | [0,0,0; 0,1,0] #
  rinid(16,17,1,1) | [0,0,0; 1,0,0];
b = lux(rosad(8,8)[[6,6,0]] | a;
c = capel(2,8,8,16*sqrt(2)+1,3,3) | b;
use vm(2), vt(1), vh(-4,-5,3,0,0,0,0,1);
clear; draw c;
```

elliptic push-up

elevation

(b)

```
a = {[0,0,0; 6,0,0], [6,0,0; 3,4,0], [3,4,0; 0,0,0]};
b = genic(1,2,12,12,6,4,3,-1) | a;
c = capel(2,(12*6)/2,(12*4)/3+16,100,16,18) | b;
use vm(2),vt(1),vh(-2,-4,1.5,0,0,0,0,1);
clear;draw c;
```

elevation

(c)

```
a = [0,0,0; 2,0,0; 1,1,0]; b = genid(8,6,2,1,1,-1) | a;
c = [2,0,0; 1,1,0; 3,1,0]; d = genid(7,6,2,1,1,-1) | c;
e = bb(1,sqrt(3)) | (b#d);
f = rosad(8,8*sqrt(3),6,60) | e;
g = capel(2,8,8*sqrt(3),46,12,12) | f;
use vm(2), vt(1), vh(-4,-5,3,0,0,0,0,1);
clear; draw g;
```

elevation

(c)

```
a = [0,0,0; 2,0,0; 1,1,0]; b = genid(10,10,2,1,1,-1) | a;
c = [2,0,0; 1,1,0; 3,1,0]; d = genid(9,9,2,1,1,-1) | c;
e = bb(1,sqrt(3)) | (b#d);
f = rosad(10,10*sqrt(3),6,60) | e;
g = capel(2,10,10*sqrt(3),24,2,2) | f;
h = capel(2,10,10*sqrt(3),50,10,10) | g;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,1);
clear; draw h;
```

elevation

Figure 2.2.5: Examples of grids and shells using elliptic cap pellevation with circular base

- A structural form with an increased rise-to-span ratio, like the one sketched in Figure 2.2.6(b), is obtained by using an elliptic cap pellevant where,

$$s/2 \leq h$$

and

$$h \ll h_e$$

Furthermore, structural forms with a considerable change in z-coordinate, that is, a high rise-to-span ratio, can be obtained using a *capel2* pellevation with large values of h . Although the value of the height h is restricted to

$$h \leq h_e$$

(Section 2.2.1) it is possible to achieve high rise-to-span ratios by increasing the value of h_e at the same as increasing the value of h . Figure 2.2.7 shows an example of some tent structures with a high rise-to-span ratio.

(a)

(b)

Figure 2.2.6: Differences between circular and elliptic cross-sections

(a)

```

a1 = rinit(8,8,1,1) | [10,1,0; 10,1,1; 10,2,1; 10,2,0];
a2 = rin(2,8,1) | [10,0,8; 10,0,8; 10,0,11; 10,0,11];
a = a1 # a2;
b = bs(1,360/8,8) | a;
c = capel(2,0,0,22,20,20) | b;
use vm(2), vt(1), vh(-6,8,1.5,0,0,0,0,1);
clear; draw c;

```

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Figure 2.2.7: Tent structures
 (a) Configuration and formex formulation,
 (b) Rest area of Interior Pavilion at World Design Exhibition in Aichi 1989, Ref. [8]

2.2.4 Change of Element Length, Distortion and Warping of Elements

As has been mentioned before, pellevation by definition only operates on the z-coordinates of the points of a configuration, and the plan views before and after pellevation are identical. As a consequence of this characteristic of the pellevation process, the length of an element in a lattice structure with at least one point inside the scope of pellevation may change. To elaborate, Figures 2.2.8 to 2.2.10 show three typical elements from an initially planar grid structure that has been pellevated, similar to the one shown in Figure 2.2.2. Here,

- Figure 2.2.8 shows an element which lies completely outside the scope of the pellevant. Therefore, it remains unchanged before and after the pellevation process.
- Figure 2.2.9 shows an element that crosses the boundary of the pellevant. So, one of its nodes lies outside and the other one lies inside the scope of the pellevant. As a consequence, one of the points is pushed up (or pushed down) and while the projected length of the member remains unchanged, its actual length is changed in the process.
- Figure 2.2.10 shows an element which lies completely inside the boundary of the pellevant and both ends obtain new z-coordinates. Since, in this case, the changes in the z-coordinate of these nodes are of different numerical values, the actual member length is changed while, again, the projected length remains unchanged.

Figures 2.2.9 and 2.2.10 both show elements whose length has been increased. However, the element length may as well decrease if the element is inclined initially. In some special case, the element length could even remain unchanged although one or more nodes lie inside the boundary of the pellevant.

Figure 2.2.8: Bar element with both nodes outside the boundary of the pellevant

Figure 2.2.9: Bar element with one node outside the boundary of the pellevant

Figure 2.2.10: Bar element with both nodes inside the boundary of the pellevant

In a similar way as the element length of a discrete member can be changed, the shape of a finite element along the boundary or inside the scope of a pellevant may get distorted. A further consequence of this phenomenon is that the nodes of an initially planar finite element with more than three nodes may not remain in one plane. In other words, some of the elements may be warped after pellevation.

The phenomenon is explained visually in Figure 2.2.11 which is a close-up of some elements of Figure 2.2.2 situated along the boundary. Figure 2.2.11(a) shows a quadrilateral element before the pellevation. Figure 2.2.11(b) shows the same element after the pellevation when the element is warped. The corner points of this element are not in the same plane any more since two points of the originally square element have not moved ($\Delta 0$) while the other two have moved to different levels ($\Delta 1$ and $\Delta 2$).

Figure 2.2.11: Distortion of quadrilateral finite element

This fact has to be kept in mind when using pellevation to generate a mesh for finite element analysis since some programs may not cater for warped elements. If the finite element package used for further processing does not allow for this sort of element, it is recommended to use a mesh that is made up of triangular elements only.

Also, the pellevation process may result in irregular boundary lines when, for example, using a pellevant with circular boundaries on a square grid. Figure 2.2.12 shows a close-up of Figure 2.2.2 where the actual boundary of the pellevant is shown in dotted lines while the 'zigzag' boundary formed by the elements of the configuration is shown in thick lines.

Figure 2.2.12: 'Zigzag' boundary of configuration

2.3 Parabolic Cap Pellevation with Circular Base

The third type of cap pellevation is referred to as 'parabolic cap pellevation with circular base' and uses a part of a paraboloid of revolution as pellevant. The orientation of this paraboloid is such that its axis of revolution is parallel to or coincident with the global z-axis as can be seen from the tector shown in Figure 2.3.1.

Figure 2.3.1: Illustrative example of parabolic cap pellevation with circular base

2.3.1 Parabolic Cap Pellevation Function with Circular Base

The parabolic cap pellevation is applied through the capel function with pellevation code ± 3 as the first parameter. Figure 2.3.2 shows the other details related to the parameters which are the coordinates of the centre $C_l (cx_l, cy_l)$, the span s of the base and the height h at the centre.

Figure 2.3.2: Details related to the parameters of the parabolic cap pellevation with circular base

The general form of the parabolic cap pellevation function with circular base is given by

Here, the values of the span s and the height h may not be negative. However, there is no upper limit for the value of h in the way that there has been for *capel1* and *capel2*.

2.3.2 Mathematical Formulation

The way in which the *capel3* pellevation is set up in mathematical terms is as follows:

As for the previous cap pellevations, the first step is to determine the distance d of the point to be pellevated, P , from the centre (Equation 2.1.1). If d is found to be less than half the span s of the pellevant, point P will be subjected to the pellevation process.

Figure 2.3.3: Geometric details for the parabolic cap pellevation with circular base

The solution is graphically represented in Figure 2.3.3. In order to numerically determine the new z-coordinate of point P , the general equation of the cross-sectional parabola opening downwards is used.

$$\Delta z = -ax^2 + h \quad \dots\dots\dots 2.3.1$$

Point B((s/2),0), as shown in Figure 2.3.3, represents one of the boundary points and its coordinates can be used to solve Equation 2.3.1 for the unknown parameter a . Hence,

$$0 = -a(s/2)^2 + h$$

or

$$a = \frac{4h}{s^2} \dots\dots\dots 2.3.2$$

Substitution of Equation 2.3.2 into Equation 2.3.1 and replacing x by the distance d of point P from the centre gives the change in z-coordinate of point P :

$$\Delta z = h - \frac{4h}{s^2}d^2 \dots\dots\dots 2.3.3$$

The new z-coordinate can then be computed as

$$z_{new} = z_{old} + \left(h - \frac{4h}{s^2}d^2\right) \qquad (for\ push-up)$$

$$z_{new} = z_{old} - \left(h - \frac{4h}{s^2}d^2\right) \qquad (for\ push-down)$$

2.3.3 Applications

A parabolic cap pellevation can be used to create a grid dome or a shell structure in the way described for the previous cap pellevations. Examples of these types of structures are shown in Figure 2.3.4. The difference between a shallow structure obtained through a *capel3* as opposed to one obtained through a *capel1* or a *capel2* pellevation is of rather small visual impact. However, the difference in shape and curvature of the structure may be important for architectural purposes or for the different structural behaviour.

(a)

```

a = rosad(1,1) | (rosad(0.5,0.5) | [0,0,0; 1,0,0]
# [1,0,0; 0,1,0]);
b = rinid(6,6,2,2) | a;
c = capel(3,6,6,18,3,5) | b;
use vm(2), vt(1), vh(-5,-7,4,0,0,0,0,1);
clear; draw c;

```

parabolic push-up

elevation

(b)

```

a = rosad(1,1) | (rosad(0.5,0.5) | [0,0,0; 1,0,0]
# [1,0,0; 0,1,0]);
b = rinid(6,6,2,2) | a;
c = rosad(6,6) | rosad(4,4) | [3,3,0];
d = lux(c) | b;
e = capel(3,6,6,20,3,5) | d;
use vm(2), vt(1), vh(-5,-3,3,0,0,0,0,1);
clear; draw e;

```

elevation

(c)

```

a = rinid(16,16,1,1) | [0,0,0; 0,1,0; 1,1,0; 1,0,0];
b = rinid(2,2,8,8) | rosad(4,4) | [3,3,0];
c = lux(b) | a;
d = capel(3,8,8,23,4) | c;
use vm(2), vt(1), vh(-5,-4,4,0,0,0,0,1);
clear; draw d;

```

elevation

(d)

```

a = rinid(16,16,1,1) | [0,0,0; 0,1,0; 1,1,0; 1,0,0];
b = rosad(8,8) | rin(1,3,1) | [7,1,0];
c = lux(b) | a;
d = capel(3,8,8,23,4) | c;
use vm(2), vt(1), vh(-5,-4,4,0,0,0,0,1);
clear; draw d;

```

elevation

Figure 2.3.4: Examples of grid and shell structures using *capel3*

Furthermore, *capel3* and all other pellevations can be applied to dome-shape configurations which, for example, have been obtained using the basispherical retronorm in Formian, Ref. [6]. In this context, the concept of pellevation serves as 'tuning' for the basic dome shape. Consider the dome shown in Figure 2.3.5(a). This has been obtained with the basispherical retronorm in Formian. If a shallow parabolic cap pellevation with a span equal to the span of the dome is applied centrally to the dome, a structure of increased rise-to-span ratio will result, Figure 2.3.5(b). From the elevation of this dome, it can be observed that for any point P on the surface of the dome, the z -coordinate will be altered according to the parabolic shape of the pellevant. Therefore, the elevation of the dome is not circular any more after pellevation.

Figure 2.3.5(c) gives an example where the edge of a dome remains in its original shape, that is, circular in section, but the central region of the dome is pushed up using a parabolic cap pellevant. This principle of 'tuning' works both ways and can, therefore, also be used to lower the elevation of a dome as shown in Figure 2.3.5(d).

The pellevant need not always be placed centrally as in Figures 2.3.5(b), (c) and (d). The pellevant may be placed at any other location of the dome. Figure 2.3.6(a) gives an example where the pellevant has been located near the edge of the dome to push-up a region which could, for instance, be the entrance area of the dome. On the right, a sketch of the elevation shows that only some of the points are pushed up from their initial position. These are the ones that lie inside the scope of the pellevant.

The combination of the basic spherical dome shapes with pellevations along the edges is valuable for obtaining different variations of domes. Figure 2.3.6(b) shows a case with two adjacent push-ups near the edge which could, for example, be used as the entrance and the exit of a sports dome.

(a)

```

<> DOME 1 <>
m=36; (*) no. of triangles (*)
n=3; (*) no. of double-rows (*)
c=0.80; (*) scaling of central region (*)
d=5*c; (*) factor to match central piece (*)
e1 = rin(2,ric(m/4),8) | ([10,0,0; 10,0,1]
# lamit(0,2) | [10,0,1; 10,4,2] # [10,4,2; 10,4,3]);
e2 = e1#rin(2,ric(m/2),4) | (lamit(0,4) | [10,0,3;
10,2,4] # [10,2,4; 10,2,5]); (*) centre piece (*)
e3 = bt(1,1,c) | e2;
e4 = e3 # rinit(m,n,2,2) | {[10,1,d+1; 10,3,d+1],
[10,3,d+1; 10,2,d],[10,2,d; 10,1,d+1],
[10,2,d+2; 10,4,d+2], [10,4,d+2; 10,3,d+1],
[10,3,d+1; 10,2,d+2]};
dome1 = bs(1,360/(2*m),10/n) | e4;
use vm(2),vt(1),vh(20,4,10,0,0,0,0,0,1);
clear; draw dome1;

```

(b)

```

<> DOME 2 <>
m=36; (*) no. of triangles (*)
n=3; (*) no. of double-rows (*)
c=0.80; (*) scaling of central region (*)
d=5*c; (*) factor to match central piece (*)
e1 = rin(2,ric(m/4),8) | ([10,0,0; 10,0,1]
# lamit(0,2) | [10,0,1; 10,4,2] # [10,4,2; 10,4,3]);
e2 = e1#rin(2,ric(m/2),4) | (lamit(0,4) | [10,0,3;
10,2,4] # [10,2,4; 10,2,5]); (*) centre piece (*)
e3 = bt(1,1,c) | e2;
e4 = e3 # rinit(m,n,2,2) | {[10,1,d+1; 10,3,d+1],
[10,3,d+1; 10,2,d],[10,2,d; 10,1,d+1],
[10,2,d+2; 10,4,d+2], [10,4,d+2; 10,3,d+1],
[10,3,d+1; 10,2,d+2]};
dome2 = capel(3,0,0,12,1,4) | dome1;
use vm(2),vt(1),vh(20,4,10,0,0,0,0,0,1);
clear; draw dome2;

```

Figure 2.3.5: 'Tuning' of domes using pellevation

(c)

<> DOME 3 <>

```

m=40; (*) division along perimeter, factor of 4 (*)
n=2; (*) no. of replications (*)
c=1.10; (*) scaling of central region (*)
d=5*c; (*) factor to match central piece (*)
e1 = rin(2,ric(m/4),8) | ([10,0,0; 10,0,1]
    # lamit(0,2) | [10,0,1; 10,4,2] # [10,4,2; 10,4,3]);
e2 = e1 # rin(2,ric(m/2),4) | (lamit(0,4) | [10,0,3;
    10,2,4] # [10,2,4; 10,2,5]); (*) centre piece (*)
e3 = bt(1,1,c) | e2 # rinit(m,n,2,4) | (lamit(0,d+1.5) |
    [10,0,d; 10,1,d+1] #
    { [10,1,d+1; 10,1,d+2], [10,0,d+3; 10,0,d+4]});
e4 = e3#rin(2,m,2) | {[10,0,d+4*n; 10,1,d+4*n+1],
    [10,1,d+4*n+1; 10,2,d+4*n]};
dome = bs(1,180/m,5/n)e4;
dome3 = capel(3,0,0,7,0.7) | dome;
use vm(2),vt(1),vh(20,-13,8,0,0,0,0,0,1);
clear; draw dome3;

```

(d)

<> DOME 4 <>

```

m=40; (*) division along perimeter, factor of 4 (*)
n=2; (*) no. of replications (*)
c=1.10; (*) scaling of central region (*)
d=5*c; (*) factor to match central piece (*)
e1 = rin(2,ric(m/4),8) | ([10,0,0; 10,0,1] # lamit(0,2) |
    [10,0,1; 10,4,2] # [10,4,2; 10,4,3]);
e2 = e1 # rin(2,ric(m/2),4) | (lamit(0,4) | [10,0,3;
    10,2,4] # [10,2,4; 10,2,5]); (*) centre piece (*)
e3 = bt(1,1,c) | e2 # rinit(m,n,2,4) | (lamit(0,d+1.5) |
    [10,0,d; 10,1,d+1] #
    { [10,1,d+1; 10,1,d+2], [10,0,d+3; 10,0,d+4]});
e4 = e3#rin(2,m,2) | {[10,0,d+4*n; 10,1,d+4*n+1],
    [10,1,d+4*n+1; 10,2,d+4*n]};
dome = bs(1,180/m,5/n)e4;
dome4 = capel(3,0,0,12,-0.5) | dome;
use vm(2),vt(1),vh(20,-13,9,0,0,0,0,0,1);
clear; draw dome4;

```

Figure 2.3.5: 'Tuning' of domes using pellevation

Figure 2.3.6: 'Tuning' of domes using pellevation

Figure 2.3.6(c) extends the basic idea of altering the edges of the dome. Here, four identical cap pellevants have been used in different locations to give a rather new 'overall look' to the originally circular dome.

2.4 Wedge Cap Pellevation with Circular Base

'Wedge cap pellevation with circular base' is the fourth type of cap pellevation. The pellevant, in this case, has the appearance of a cone. However, the term 'wedge' is used because of the cross-section of the pellevant which reminds one of the shape of a wedge. The result of a typical wedge pellevation using the familiar 30 by 30 tector shows the general features of the pellevant in Figure 2.4.1. Also, the corresponding formex formulation for this example is given alongside.

Figure 2.4.1: Illustrative example of wedge cap pellevation with circular base

2.4.1 Wedge Cap Pellevation Function with Circular Base

Examples of the kind shown in Figure 2.4.1 can be formulated through the capel function with pellevation code ± 4 as the first parameter. The details related to all the other parameters are shown in Figure 2.4.2. These parameters are the coordinates of the centre of the pellevant $C_1(cx_1, cy_1)$, the span s of the base and the height h at the centre.

Figure 2.4.2: Details related to the parameters of the wedge cap pellevation with circular base

The general form of the function with its parameters is given as

The *capel4* pellevation does not have an upper limit for the height and may, therefore, be used for structures with low rise-to-span as well as high rise-to-span ratios.

2.4.2 Mathematical Formulation

The mathematical procedure involved in the *capel4* function will be described below where *P* represents a typical point to be pellevated.

The first step is the determination of the distance d of point P from the centre of the pellevant (Equation 2.1.1) in order to examine whether this distance is less than half the span. Only if this is the case, the pellevation will affect the z-coordinate of P .

Figure 2.4.3: Geometric details for the parabolic cap pellevation with circular base

To find an expression for the change in the z-coordinate, a linear relationship using similar triangles is used:

$$\frac{h}{s/2} = \frac{\Delta z}{s/2 - d} \quad \dots\dots\dots 2.4.1$$

as can be seen in Figure 2.4.3. Manipulation of this equation will give the value of the unknown parameter Δz as

$$\Delta z = \frac{h(s/2) - hd}{s/2}$$

or

$$\Delta z = h - \frac{2hd}{s} \quad \dots\dots\dots 2.4.2$$

Therefore, the new z-coordinate of P is given by

$$z_{new} = z_{old} + \left(h - \frac{2hd}{s} \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{2hd}{s} \right) \quad (\text{for push-down})$$

2.4.3 Inclined Pellevation

The height of any type of pellevation is always measured vertically since any push-up or push-down takes place in the z-direction only. However, if it is required to pellevate in a direction other than vertical, any pellevation function can be used together with the veritition function. Readers who are not familiar with the latter function are referred to Ref. [6].

Figure 2.4.4: Example of inclined pellevation

In the example shown in Figure 2.4.4, the same 30 by 30 tector as used in Figure 2.4.1 has been used. Also, the same wedge cap pellevation has been applied to it, as can be seen from the formex formulation given alongside. Only this time, the grid has been rotated prior to pellevation and then has been returned to its original position after pellevation. Figures 2.4.5(a) to 2.4.5(d) show the steps involved. Here, a perspective view of the tector together with the coordinate system is shown on the left, and the sketches of the elevation are given on the right.

Figure 2.4.5: Process of inclined pellevation

The steps shown in the figure are:

- (a) Creation of a square tector lying in the xy-plane with one edge along the x- and one edge along the y-axis.
- (b) Rotation of the tector about the global y-axis using the veritition function.
- (c) Application of a wedge cap pellevation to the inclined tector.
- (d) Rotation back of the pellevated tector into its original position - once again using the veritition function.

The entire process outlined above is referred to as 'inclined pellevation'.

2.4.4 Applications

Capel4 is an efficient tool to create linearly sloping grid domes and continuous structures. Figure 2.4.6 gives four typical examples of this kind.

The first example, Figure 2.4.6(a), shows a pentagonal shell where the wedge cap pellevant has been placed at the centre of the configuration. The span of the pellevant covers the entire grid.

The example in Figure 2.4.6(b) creates an octagonal shell with a hole in the middle. A centrally placed *capel4* pellevation has been used to pellevate the initially planar configuration. In this case, the span of the pellevant is less than the span of the shell and, therefore, the elements along the edges of the shell remain at their initial level.

Figure 2.4.6(c) shows a triangular lattice structure that has been pellevated using a *capel4* with its centre at one of the corner points of the configuration. Hence, that point is pushed up to the maximum height of the pellevant whereas the remaining structure slopes down according to the properties of the pellevant.

(a)

```
p1 = [0,0,0; 2,0,0; 1,1,0]; p2 = genid(8,8,2,1,1,-1) | p1;
p3 = [2,0,0; 1,1,0; 3,1,0]; p4 = genid(7,8,2,1,1,-1) | p3;
p5 = bb(1,tan|54) | (p2#p4);
p6 = rosad(8,8*tan|54,5,72) | p5;
p7 = capel(4,8,8*tan|54,30,6) | p6;
use vm(2), vt(1), vh(-4,-1,2.5,0,0,0,0,1);
clear; draw p7;
```

elevation

(b)

```
h1 = [0,0,0; 2,0,0; 1,1,0]; h3 = genid(8,7,2,1,1,-1) | h1;
h3 = [2,0,0; 1,1,0; 3,1,0]; h4 = genid(7,7,2,1,1,-1) | h3;
h5 = bb(1,tan|67.5)|(h2#h4);
h6 = rosad(8,8*tan|67.5,8,45) | h5;
h7 = capel(4,8,8*tan|67.5,30,5) | h6;
use vm(2), vt(1), vh(-4,-5,3,0,0,0,0,1);
clear; draw h7;
```

elevation

(c)

```
a1 = {[0,0,0; 6,0,0], [6,0,0; 3,4,0], [3,4,0; 0,0,0]};
a2 = genic(1,2,12,12,6,4,3,-1) | a1;
a3 = capel(4,(12*6)/2,(12*4)/3+32,126,36) | a2;
use vm(2),vt(1),vh(-2,-1,1,0,0,0,0,1);
clear; draw a3;
```

elevation

(d)

```
e1 = rinid(17,12,1,1) | [0,0,0; 0,1,0] #
rinid(16,13,1,1) | [0,0,0; 1,0,0];
e2 = rinid(8,6,2,2) | rosad(1,1) | [1,0,0; 0,1,0];
e3=veras(0,0,20) | (e1#e2);
e4=capel(4,10,6,24,5) | e3;
e5=veras(0,0,-20) | e4;
use vm(2), vt(1), vh(-4,-5,4,0,0,0,0,1);
clear; draw e5;
```

y (into
paper)

elevation

Figure 2.4.6: Examples of grid and shell structures using *capel4*

The last example, Figure 2.4.6(d), is the result of an inclined pellevation. In this case, the initially planar grid has been rotated about the global y-axis, then it has been pellevated using a *capel4* and finally it has been rotated back into its original position. This process has been explained in detail for Figure 2.4.5 in Section 2.4.3.

In addition to the previously outlined applications, wedge cap pellevations with circular base can be used to introduce a linear rise in roof-type structures, as shown in the example in Figure 2.4.7 where the configuration is actually composed of four separate parts.

(a)

```
a1 = bp(1,360/40) | rinid(5,40,1,1) | [1,0,0; 2,0,0; 2,1,0;
1,1,0]; a2 = capel(4,0,0,32,3) | a1;
b1 = bp(1,360/35) | rinid(4,35,1,1) | [1,0,0.1; 2,0,0.1;
2,1,0.1; 1,1,0.1]; b2 = capel(4,0,0,24,4) | b1;
c1 = bp(0.9,360/30) | rinid(3,30,1,1) | [1,0,0.2; 2,0,0.2;
2,1,0.2; 1,1,0.2]; c2 = capel(4,0,0,16,5) | c1;
d1 = bp(1,360/20) | rinid(2,20,1,1) | [0,0,0.6; 1,0,0.6;
1,1,0.6; 0,1,0.6]; d2 = capel(4,0,0,20,5) | d1;
f = d2 # c2 # b2 # a2; f = capel(4,0,0,50,6) | f;
use vm(2), vt(1), vh(-6,6,2,0,0,0,0,1); draw f;
```

MATERIAL REDACTED AT REQUEST OF UNIVERSITY

Figure 2.4.7: Roof structure
(a) Configuration and formex configuration,
(b) Exhibition Building in Kumamoto 1992, Ref. [3]

2.5 Elliptic Cap Pellevation with Elliptic Base

An elliptic cap pellevation with elliptic base uses a part of an ellipsoid as pellevant. It belongs to a group of cap pellevations with an elliptic boundary in plan. Basically, this group of pellevations works on the same principles as the cap pellevations with circular base.

Following the general pattern of grouping the cap pellevation functions as outlined in Table A.1 in Appendix A, this section should have related to the 'circular cap pellevation with elliptic base'. However, a circular cap pellevation with elliptic base does not really exist, since vertical sections through the centre of an ellipsoid are normally not circular. Therefore, it has been decided that this type of cap pellevation is defined as a special case of the elliptic cap pellevation with elliptic base and it will be dealt with in Section 2.6.

2.5.1 Elliptic Cap Pellevation Function with Elliptic Base

The elliptic cap pellevation with elliptic base has been allocated pellevation code ± 6 of the capel function. Further parameters are, first of all, the x- and y-coordinates of three points: $C_1 (cx_1, cy_1)$, $C_2 (cx_2, cy_2)$ and $C_3 (cx_3, cy_3)$. These three points are required to define the ellipse in plan as shown in Figure 2.5.1. By definition, the first point describes the centre of the ellipse and points C_2 and C_3 specify the ends of major and minor axes. The order in which the major and minor axes are defined is arbitrary. The next parameter is the height h at point C_1 , and the last parameter, h_e , denotes the length of the vertical semi-axis of the ellipsoid as explained for *capel2*.

Figure 2.5.1: Details related to the parameters of the elliptic cap pellevation with elliptic base

The general form of the *capel6* function is given by

In the same way as outlined for the *capel2* pellevation and as already shown in Figure 2.2.3, it is required that

$$h \leq h_e$$

in order to have a unique solution for each point to be pellevated.

A 40 by 30 tector has been used to illustrate the general shape of a typical elliptic cap pellevant with elliptic base. The result of the pellevation is shown in Figure 2.5.2.

Figure 2.5.2: Illustrative example of elliptic cap pellevation with elliptic base

2.5.2 Mathematical Formulation

For the mathematical formulation of the *capel6* function, the plan view of the pellevant is examined first. Here, for convenience, the entire configuration is subjected to a translational coordinate transformation in the xy -plane which leaves the centre point C_l as the new origin. In other words, the x - and y -coordinates of any point (u'_1, u'_2) are transferred to (u_1, u_2) according to the following relationships

$$u_1 = u'_1 - cx_l$$

$$u_2 = u'_2 - cy_l$$

Subsequently, a rotational coordinate transformation is carried out in order to make the major axis the new global x -axis. This relationship is described by the transformation matrix

$$\begin{bmatrix} px \\ py \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Here, (px, py) are the final coordinates of point P after both translational and rotational coordinate transformations have been carried out. Both transformations are shown graphically in Figure 2.5.3.

Figure 2.5.3: Translational and rotational coordinate transformations

In the case when $\overline{C_1C_2} \geq \overline{C_1C_3}$:

$$a_r = \sqrt{(cx_1 - cx_2)^2 + (cy_1 - cy_2)^2} \quad \dots\dots\dots 2.5.1$$

and

$$b_r = \sqrt{(cx_1 - cx_3)^2 + (cy_1 - cy_3)^2} \quad \dots\dots\dots 2.5.2$$

Also,

$$a = cx_2 - cx_1$$

and

$$b = cy_2 - cy_1$$

However, when $\overline{C_1C_2} < \overline{C_1C_3}$:

$$a_r = \sqrt{(cx_1 - cx_3)^2 + (cy_1 - cy_3)^2} \dots\dots\dots 2.5.3$$

and

$$b_r = \sqrt{(cx_1 - cx_2)^2 + (cy_1 - cy_2)^2} \dots\dots\dots 2.5.4$$

Also,

$$a = cx_3 - cx_1$$

and

$$b = cy_3 - cy_1$$

In the above equations, a_r relates to the length of the semimajor axis and b_r refers to the length of the semiminor axis while a and b serve to compute the elements of the rotation matrix. To elaborate,

$$\cos \alpha = \frac{a}{a_r}$$

and

$$\sin \alpha = \frac{b}{a_r}$$

Within the newly defined coordinate system, it is now possible to determine whether or not point P lies inside the elliptic boundaries in plan by utilising the general ellipse equation

$$\frac{px^2}{a_r^2} + \frac{py^2}{b_r^2} < 1.0 \dots\dots\dots 2.5.5$$

If Equation 2.5.5 is satisfied, point P lies within the scope of the pellevant and its distance to the centre is evaluated:

$$d = \sqrt{px^2 + py^2}$$

It is further required to determine the coordinates of a point $P_e(x_e, y_e)$ that lies where the straight line through C_1 (the new origin) and P intersects with the ellipse in plan as shown in Figure 2.5.4. For this purpose, the parametric formulation of the ellipse is considered:

$$x_e = a_r \cos \theta \quad \dots\dots\dots 2.5.6$$

and

$$y_e = b_r \sin \theta \quad \dots\dots\dots 2.5.7$$

where a_r and b_r are the length of the semimajor and semiminor axes and their numerical values are known from either Equations 2.5.1 and 2.5.2 or Equations 2.5.3 and 2.5.4. Therefore, angle θ is the only unknown parameter.

Figure 2.5.4: Geometric details for the elliptic cap pellevation with elliptic base

Considering Figure 2.5.4, the following mathematical approach is used to determine θ :

$$\tan \alpha = \frac{y_e}{x_e}$$

Substitution of Equations 2.5.6 and 2.5.7 into the above equation yields

$$\tan \alpha = \frac{b_r \sin \theta}{a_r \cos \theta}$$

Hence

$$\tan \alpha = \frac{b_r}{a_r} \tan \theta$$

or

$$\tan \theta = \frac{a_r}{b_r} \tan \alpha$$

or

$$\theta = \tan^{-1}\left(\frac{a_r}{b_r} \tan \alpha\right) \dots\dots\dots 2.5.8$$

The angle θ can then be evaluated and used to compute the coordinates of point $P_e(x_e, y_e)$ by back-substitution into Equations 2.5.6 and 2.5.7.

In order to determine the change in the z-coordinate of point $P(px, py)$ within the boundary of the pellevant, the section through the points C_l and P is further examined as shown in Figure 2.5.4. Here, the vertical distance d_e of point P_e from the centre is computed as

$$d_e = \sqrt{x_e^2 + y_e^2} \dots\dots\dots 2.5.9$$

In a manner similar to the case of *capel2*, the length of the horizontal axis of the ellipse s_e in Figure 2.5.4 can be obtained as

$$s_e = \frac{d_e}{\sqrt{1 - \frac{(h_e - h)^2}{h_e^2}}} \dots\dots\dots 2.5.10$$

and thus

$$z = h_e \sqrt{1 - \frac{d_e^2}{s_e^2}} \dots\dots\dots 2.5.11$$

Finally, the new z-coordinate of point P is given by

$$z_{new} = z_{old} + (h - h_e + z) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h - h_e + z) \quad (\text{for push-down})$$

2.5.3 Multi-layer Configurations

An important feature of all types of pellevations will be brought out at this stage. The idea will be explained in terms of an elliptic cap pellevation with elliptic base. For this purpose, the 20 by 15 single-layer tector as shown in Figure 2.5.5(a) is duplicated into the 20 by 15 double-layer tector shown in Figure 2.5.5(b).

Figure 2.5.5: (a) Typical single-layer configuration,
(b) typical double-layer configuration

Pellevation generally implies a change of the z-coordinates of all nodes within the boundaries of the pellevant. So if the same elliptic cap pellevation as applied in Figure 2.5.2 is now applied to the double-layer tector of Figure 2.5.5(b), the appropriate nodes

of both layers are affected. The result of the process can be seen in Figure 2.5.6. As with the single-layer examples, all x- and y-coordinates of the nodes remain unchanged, and also the node connectivity is not influenced by the pellevation process. At any point in plan, the distance between the two layers of the tector remains constant, Δ , as shown in the elevation in Figure 2.5.6.

The above outlined principles do not only apply to double-layer configurations. Indeed, the ideas can be extended to any number of layers. The elevation shown in Figure 2.5.6 includes two dashed lines to indicate the situation for any additional layers.

```
bot = rinid(21,15,2,2) | [0,0,0; 0,2,0] # rinid(20,16,2,2) | [0,0,0; 2,0,0];
top = rinid(21,15,2,2) | [0,0,0.9; 0,2,0.9] # rinid(20,16,2,2) | [0,0,0.9; 2,0,0.9];
grid = bot # top;
tec = capel(6,20,15,34,22,16,23,6,16) | grid;
use vm(2), vt(1), vh(-4,-5,7,0,0,0,0,1);
clear; draw tec;
```

Figure 2.5.6: Pellevation of multi-layer grids

For the sake of visual clarity, the double-layer tector used above did not have any web elements. However, the addition of web elements does not affect any of the principles. Since node connectivity remains unchanged, the web elements will simply be pushed up or down together with the nodes to which they are connected. And as with any other element, the length of the web elements may increase, decrease or remain constant as explained in Section 2.2.

2.5.4 Applications

Elliptic cap pellevations with elliptic base may be used for many types of lattice or continuous structures. The fact that the pellevants have an elliptic base makes them a useful tool for configurations which extend more in one direction in plan than they do in the other.

The examples in Figures 2.5.7(a) and 2.5.7(b) show two typical rectangular double-layer grids pellevated by a centrally placed *capel6*. In both cases the boundaries of the pellevant lie outside the grid such that all the nodes of the grid are affected in the course of the pellevation process. It can be seen in these examples that the web elements are pellevated in the same way as any members of the top or bottom layer. It may also be noted that the vertical web elements in Figure 2.5.7(b) remain constant in length because the distance between the two layers remains constant before and after the pellevation.

Having said that pellevation normally affects all the nodes within the boundaries of the pellevant, it has to be mentioned that there are ways of restricting the effects of pellevation to selected nodes only. These could, for example, be the nodes of one layer of a double-layer grid. This special feature of the pellevation functions is referred to as 'selective pellevation' and will be introduced in Section 2.6.

Figure 2.5.8 gives some more examples of configurations that have been obtained using *capel6* pellevations. Figure 2.5.8(a) shows a shell with an elongated hexagonal boundary. An elliptic cap pellevant has been used to pellevate the shell. The boundaries of the centrally placed pellevant lie completely outside the boundaries of the configuration.

Figure 2.5.7: Examples of pellevated double-layer grids

Figure 2.5.8(b) is based on an initially planar rectangular shell. An elliptic cap pellevant has been placed centrally with its boundary just outside the corners of the shell. The surface is subsequently subjected to a second elliptic cap pellevant. This second *capel6* has also been placed at the centre of the configuration but with its boundary inside the shell. The properties of the second pellevant have been chosen such that the elliptic base is rather narrow as can be seen in the plan. Therefore, the additional push-up effect on the shell will only affect a number of elements located near the central line of the configuration. The effect obtained by the second pellevation may be used to increase the stiffness of the structure or to cater for skylights as indicated by the white elements, Figure 2.5.8(b).

(a)

```

a1 = [0,0,0; 2,0,0; 1,1,0]; a2 = genid(8,8,2,1,1,-1) | a1;
b1 = [2,0,0; 1,1,0; 3,1,0]; b2 = genid(7,8,2,1,1,-1) | b1;
c = bb(1,sqrt(3)) | (a2 # b2); d = rosad(8,8*sqrt(3,6,60) | c;
e = bb(1.5,1) | d;
f = capel(6,12,8*sqrt(3,12,-5,36,8*sqrt(3,5,8) | e;
use vm(2), vt(1), vh(-3,-5,3,0,0,0,0,1);
clear; draw f;

```

elliptic push-up

elevation

plan

(b)

```

a = [0,0,0; 1,0,0; 1,1,0; 0,1,0]; b = rinid(14,12,1,1) | a;
c = capel(6,7,6,7,-7,-2,6,4,8) | b;
c1 = capel(6,7,6,7,3,0,6,0,7,3) | c;
use vm(2), vt(1), vh(-2,-5,2,0,0,0,0,1);
clear; draw c1;

```

elevation of long side

plan (not
to scale)

(c)

```

a = [0,0,0; 1,0,0; 1,1,0; 0,1,0];
b = rinid(14,12,1,1) | a;
c2 = capel(6,16,6,16,-7,-2,6,6,8) | b;
use vm(2), vt(1), vh(-2,-5,2,0,0,0,0,1);
clear; draw c2;

```

elevation

plan

Figure 2.5.8: Examples using *capel6*

Figure 2.5.8(c) has been obtained from the same initial configuration as the previous example. However, the centre of the pellevant is now near an edge of the shell. So, two of the corner points lie near the boundary of the pellevant and only experience a very small push-up. Shapes of this kind give rise to a variety of roof type structures as will also be seen in the subsequent sections.

2.6 Circular Cap Pellevation with Elliptic Base

Returning to the previously mentioned circular cap pellevation with elliptic base. This case, as mentioned before, is considered as a special case of the above described *capel6*. Here, the pellevant is of the shape of an ellipsoid of revolution with its axis of revolution in the xy-plane or parallel to it. Therefore, any vertical section at the minor axis or parallel to it is circular. An illustrative example of this kind of pellevation is shown in Figure 2.6.1.

Figure 2.6.1: Illustrative example of circular cap pellevation with elliptic base

2.6.1 Circular Cap Pellevation Function with Elliptic Base

the circular cap pellevation function with elliptic base has pellevation code ± 5 as its first parameter. Besides the pellevation code, the circular cap pellevation with elliptic base requires three points as parameters: $C_1(cx_1, cy_1)$, $C_2(cx_2, cy_2)$ and $C_3(cx_3, cy_3)$. The last parameter is the height h at point C_1 . It is then a convention that the length of the semiminor axis of the ellipse in plan is also used as the length of the vertical semi-axis of the ellipsoid, h_e . In mathematical terms this relationship is represented by

$$h_e = b_r$$

The general form of the function is given by

Figure 2.6.2: Geometric details for the elliptic cap pellevation with elliptic base

2.6.2 Mathematical Formulation

As a consequence of the above conventions some small alterations in the mathematical formulation apply, as compared with *capel6*. Because an ellipsoid of revolution serves as pellevant, that is,

$$h_e = b_r$$

Equation 2.5.10 can, for this special case, be rewritten as

$$s_e = \frac{d_e}{\sqrt{1 - \frac{(b_r - h)^2}{b_r^2}}} \dots\dots\dots 2.6.1$$

In the above equation, b_r can be obtained from Equation 2.5.2 or 2.5.4 and d_e can be evaluated using Equation 2.5.9. Also, Equation 2.5.11 can be rewritten as

$$z = b_r \sqrt{1 - \frac{d^2}{d_e^2}} \dots\dots\dots 2.6.2$$

Finally, the new z-coordinate of point P is obtained as

$$z_{new} = z_{old} + z \quad \quad \quad (for \textit{push-up})$$

$$z_{new} = z_{old} - z \quad \quad \quad (for \textit{push-down})$$

2.6.3 Selective Pellevation

It has been pointed out on several occasions that pellevation generally affects all nodes that lie within the boundaries of the pellevant. Consider now a double-layer tector similar to the one introduced in Figure 2.5.5(b). This tector is subjected to a circular cap pellevation with elliptic base which affects the nodes of the top layer only. The result is as shown in Figure 2.6.3. It can be seen that the nodes of the top layer which

lie within the boundaries of the pellevant have been pushed up while the corresponding nodes of the bottom layer are still in their original place. Any further layers introduced would also have remained in place as indicated by the dashed lines in the elevation of Figure 2.6.3. The above described special mode of the pellevation function is referred to as 'selective pellevation' since only a number of nodes are selected to be considered for the pellevation process.

```
bot = rinid(21,15,2,2) | [0,0,0; 0,2,0] # rinid(20,16,2,2) | [0,0,0; 2,0,0];
top = rinid(21,15,2,2) | [0,0,1.5; 0,2,1.5] # rinid(20,16,2,2) | [0,0,1.5; 2,0,1.5];
grid = top # bot;
a = capel(15,20,15,34,22,17,21,6,3,1.5) | grid;
use vm(2), vt(1), vh(-4,-5,9,0,0,0,0,0,1);
clear; draw a;
```

Figure 2.6.3: Selective pellevation of multi-layer grids

The complete formex formulation for the example in Figure 2.6.3 is given below the configuration where the actual function used to carry out the selective pellevation process is given by

capel(15,20,15,34,22,17,21,6,3,1.5)

Here, the first parameter is the pellevation code. In the case of the selective pellevation, a '1' is added in front of the 'basic' pellevation code. In other words, whereas the 'basic' circular cap pellevation with elliptic base has been allocated pellevation code 5, the corresponding selective pellevation has been allocated pellevation code 15. In the same

manner, a negative pellevation code -5 is changed to -15 to indicate selective push-down pellevation. In this context, the term 'basic' is used to refer to any pellevation with a one-digit pellevation code which implies a pellevation effective for all nodes situated inside the boundaries of the pellevant.

The seven parameters following the pellevation code denote the x- and y-coordinates of the centre and the ends of major and minor axes and the height as explained in Section 2.6.1. The properties of the pellevant used in this case are identical to the one used in Figure 2.6.1.

The last two parameters of the function are specific to the selective pellevation. In order of their appearance, they are referred to as 'position indicator' *un* (i.e. 'uniple number') and 'value indicator' *uv* (i.e. 'uniple value'). The role of *un* is to specify the position of the uniple that is consulted to see whether or not the node should be considered for pellevation. The value indicator *uv* is then used to select the nodes which will be considered for pellevation. A node is selected if the numeric value of its uniple in position *un* is equal to *uv*.

Therefore, the general form of the function is given by

In terms of the example in Figure 2.6.3, the position indicator *un* has been given the value 3, that is, the position of the z-coordinate. This implies that the z-coordinate of

each node of the grid is compared with the value of uv . In this case, uv is equal to 1.5. Therefore, only the nodes whose z-coordinate is equal to 1.5 will be selected and considered for pellevation. As can be seen from the formex formulation, this will then result in the selection of all the nodes of the top layer.

The above described concept of selective pellevation may be used in relation to all types of pellevation. As far as the pellevation parameters are concerned, a '1' is always placed in front of the basic pellevation code and the two additional parameters un and uv are added at the end of the list of parameters. The value of uv can take any numeric value. However, the value of un must be an integer number to refer to a uniple position. This position need not be the x-, y- or z-coordinate of a node (un equal to 1, 2 or 3). It can also be position four or five or higher (un equal to 4, 5, etc.). Actually it is recommended that the z-coordinate is not used for cases which incorporate superimposition of pellevation. Because once a pellevation has been applied, the z-coordinate of a node may have changed. The previously chosen value indicator will then not be useful anymore for any subsequent pellevations.

2.6.4 Applications

The selective pellevation can be used to obtain interesting structures. Consider the double-layer grid shown in Figure 2.6.4(a). This initially planar double-layer grid has been subjected to a selective pellevation of the top layer using a *capell15*. It can be noticed that the web elements become 'stretched' because their bottom nodes remain in place while their top nodes experience a push-up following the shape of the pellevant. For the sake of visual clarity, a reduced number of vertical web members has been chosen in this example, since a large number of sloping web elements would have resulted in a confusing graphic form.

The double-layer grid of Figure 2.6.4(a) is subjected to a second selective pellevation. The result can be seen in Figure 2.6.4(b). Here, the second pellevation is a *capell16*

which has been applied to the bottom layer only. So, the top layer of Figure 2.6.4(b) is identical to the top layer of Figure 2.6.4(a). But the bottom layer of Figure 2.6.4(b) has been given a different shape to the top layer. The new position and length of a web element is governed by the movement of its start and end nodes. So the web elements may become 'stretched' in some places and 'compressed' in others.

This basic idea can easily be extended to any multi-layer grid where the different layers require to be pellevated in different ways.

(a)

```
m=12; n=6;
top = rinid(m+1,n,1,1) | [0,0,0.6; 0,1,0.6] #
      rinid(m,n+1,1,1) | [0,0,0.6; 1,0,0.6];
bot = rinid(m+1,n,1,1) | [0,0,0; 0,1,0] #
      rinid(m,n+1,1,1) | [0,0,0; 1,0,0];
web = rinid(m/2+1,n/2+1,2,2) | [0,0,0; 0,0,0.6];
grid = top # bot # web;
a = capel(15,6,3,18,3,6,-2,3,0.6) | grid;
use vm(2), vt(1), vh(-2,-5,4,0,0,0,0,1);
clear; draw a;
```

(b)

```
m=12; n=6;
top = rinid(m+1,n,1,1) | [0,0,0.6; 0,1,0.6] #
      rinid(m,n+1,1,1) | [0,0,0.6; 1,0,0.6];
bot = rinid(m+1,n,1,1) | [0,0,0; 0,1,0] #
      rinid(m,n+1,1,1) | [0,0,0; 1,0,0];
web = rinid(m/2+1,n/2+1,2,2) | [0,0,0; 0,0,0.6];
grid = top # bot # web;
a1 = capel(15,6,3,18,3,6,-2,3,0.6) | grid;
a2 = capel(16,6,3,16,3,6,-8,4,6,3,0) | a1;
use vm(2), vt(1), vh(-2,-5,4,0,0,0,0,1);
clear; draw a2;
```

Figure 2.6.4: Examples of pellevated double-layer grids using selective pellevation

The idea of selective pellevation can easily be extended to any part of any structure. In other words, selective pellevation may be used to apply pellevation to only one group of nodes which need not lie on different layers but may well be in different places of a single-layer grid or continuous structure. The grouping of these nodes may be done by introducing a fourth uniple to the nodes of a three-dimensional structure.

2.7 Parabolic Cap Pellevation with Elliptic Base

The third type of elliptic cap pellevation is referred to as 'parabolic cap pellevation with elliptic base'. The base of this type of pellevant is elliptic, similar to the previous two types, whereas the cross-section of the pellevant is parabolic. A rectangular tector has been used to show the general appearance of the pellevant in Figure 2.7.1. The formex formulation for the example is given alongside the plot of the tector.

Figure 2.7.1: Illustrative example of parabolic cap pellevation with elliptic base

2.7.1 Parabolic Cap Pellevation Function with Elliptic Base

In order to apply the parabolic cap pellevation with elliptic base, the capel function with pellevation code ± 7 as the first parameter is used. The remaining parameters are

similar to the ones introduced for *capel6*: the x- and y-coordinates of points $C_1(cx_1, cy_1)$, $C_2(cx_2, cy_2)$ and $C_3(cx_3, cy_3)$ to define the elliptic shape of the base and the height h at the centre, Figure 2.7.2. Definition of the three points is as outlined for the *capel6* pellevation.

Figure 2.7.2: Details related to the parameters of the parabolic cap pellevation with elliptic base

The general form of the function is given as

2.7.2 Mathematical Formulation

For the mathematical formulation of the parabolic cap pellevation with elliptic base, the procedure outlined below has been used.

First of all, a translational and subsequent rotational coordinate transformation serve to move the origin to point C_1 and leave the major axis - either $\overline{C_1C_2}$ or $\overline{C_1C_3}$ - as the new global x-axis. This process has been explained in detail in Section 2.5, Figure 2.5.3. In the next step, it is determined whether a point $P(px, py)$ to be pellevated lies within the boundaries of the pellevant in plan. That is, Equation 2.5.5 has to be evaluated.

If Equation 2.5.5 is satisfied, point P lies within the scope of the pellevant and further steps are taken. Here, the coordinates of the corresponding point $P_e(x_e, y_e)$ on the ellipse in plan are found in order to examine the situation in the cross-section as shown in Figure 2.7.3. For this purpose, Equations 2.5.6 to 2.5.8 are used.

Figure 2.7.3: Geometric details for the parabolic cap pellevation with elliptic base

It is known that

$$d = \sqrt{px^2 + py^2}$$

and also

$$d_e = \sqrt{x_e^2 + y_e^2}$$

with the expressions for x_e and y_e as derived in Section 2.5.

The cross-section of the pellevant is considered next with the general equation of the parabola as given by Equation 2.3.1 in Section 2.3.

This enables to derive the specific equation of the parabola at the section through C_l and P . In this section, the coordinates of point P_e may be represented by $P_e(d_e, 0)$. Substitution of these values into Equation 2.3.1 yields

$$0 = -ad_e^2 + h$$

or

$$a = \frac{h}{d_e^2}$$

The above equation gives an expression for the unknown parameter a of the parabola in the section examined. Substitution of this expression into Equation 2.3.1 yields

$$\Delta z = h - \frac{h}{d_e^2} d^2 \quad \dots\dots\dots 2.7.1$$

Finally, the new z -value of P is given by

$$z_{new} = z_{old} + \left(h - \frac{h}{d_e^2} d^2 \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{h}{d_e^2} d^2 \right) \quad (\text{for push-down})$$

2.7.3 Regional Pellevation

It has been shown so far that the concept of pellevation can be applied universally. It has also been explained how pellevation can be restricted to selected nodes, for example, the nodes of one layer of a double-layer grid. It is further possible to restrict the effects of the pellevation to a specified boundary in plan rather than the boundary of the pellevant.

Consider, for example, the rectangular tector shown in Figure 2.7.4(a). The illustrated effect has been obtained using a shallow *capel7* pellevation. Figure 2.7.4(b) has been obtained using the same pellevant as in Figure 2.7.4(a). However, this time a square 'region' is specified which has been placed centrally inside the elliptic boundary of the pellevant. From the plan of Figure 2.7.4(b) it can be seen that the square region lies completely inside the elliptic boundary of the pellevant. This specified region 'overrides' the boundary of the pellevant. In other words, the boundary of the pellevant is ignored and the effects of the pellevation are restricted to the square region only.

Figure 2.7.4(c) gives another example of this kind using the elliptic base pellevant of Figure 2.7.4(a). In this case, a circular region which is near one end of the major axis of the elliptic boundary of the pellevant is used as indicated in Figure 2.7.4(c). Here, four different areas can be distinguished.

1. Areas which lie outside the elliptic boundary of the pellevant as well as outside the circular boundary of the region. These areas are not considered for pellevation.
2. Areas which lie inside the elliptic boundary of the pellevant but outside the circular region. These areas are also not considered for pellevation.
3. Areas which lie inside the circular boundary of the region but outside the elliptic boundary of the pellevant. Points in these areas are not pellevated.
4. Areas which lie inside both boundaries. Here, the pellevant is effective and pellevation takes place.

Figure 2.7.4: Illustrative examples of regional pellevation using *capel7*
 (a) initial tector, (b) rectangular region, (c) circular region

Thus, in the above described mode of the pellevation, the pellevant is only effective where both the regional boundary and the boundary of the pellevant overlap. This mode is referred to as 'regional pellevation' and is applicable for all pellevation functions.

Returning to the example shown in Figure 2.7.4(b), with its formex formulation given alongside. The regional pellevation is obtained through

Here, the first parameter is the pellevation code which has a '10' added before the basic pellevation code to indicate the regional pellevation. In this case, pellevation code 107 is used to denote regional parabolic cap pellevation with elliptic base (similarly, a negative pellevation code -7 is changed to -107 to denote regional push-down pellevation). The following six parameters give the x- and y-coordinates of the centre and the ends of major and minor axes of the elliptic boundary of the pellevant. Following these coordinates is the height of the pellevant.

The very last parameter is the 'region plenix'. Plenices are generally used as a medium for the organisation and correlations of information. The 'opening plenix bracket', `~{` and `}~`, are used to indicate the start and the end of the plenix. Each of the entities within a plenix, separated by a comma, is referred to as a

'panel'. Readers who are not familiar with this concept are referred to Ref. [6] for further details.

In the case of the region plenix, the plenix consists of two panels. The first panel is an integer number referred to as the 'region specifier', rs , and the second panel is an ingot referred to as the 'region ingot'. The region specifier is used to specify the general shape of the region. The region ingot consists of a number of x- and y- coordinates of points that specify both the position and the dimensions of the region.

In the example of Figure 2.7.4(b), rs is equal to 2 which is the region specifier for a 'rectangular region', and the region ingot consists of two points specifying two diametrically opposite corners of the rectangle in plan. These points will be further explained in Section 2.7.3.2. In all cases where a regional pellevation is used, the region plenix, containing both rs and the region ingot, is added at the very end of the list of parameters.

Altogether, five different types of regions have been established which are specified by the region specifier. These regions are as follows:

- 1 - ambit region
- 2 - rectangular region
- 3 - circular region
- 4 - elliptic region
- 5 - parabolic region

The regions and their corresponding region ingots will be discussed below.

2.7.3.1 Ambit Region

The first type of region has been allocated region code 1 and it is referred to as the ambit region. Figure 2.7.5 shows a typical ambit region consisting of four corner points on the left and the corresponding region plenix on the right. An ambit region is defined by the x- and y-coordinates of a number of corner points where

- at least three corner points have to be given which have to be non-collinear;
- the points have to be given in a clockwise order whereby the starting point can be chosen anywhere;
- the boundary of an ambit region has to be convex, that is, the boundary lines may not 'cut the region'.

Figure 2.7.5: Ambit region

In this context, the above described corner points are referred to as 'vertex points'. The reasons for the given conventions will be discussed in detail in Chapter 8 together with the mathematical formulation of the regional pellevation.

In general terms, a typical region plenix for an ambit region is of the form:

$$\sim\{1,\{[x_1,y_1],[x_2,y_2],[x_3,y_3],\dots[x_n,y_n]\}\}\sim$$

where $P_1(x_1,y_1)$ to $P_n(x_n,y_n)$ are the coordinates of the vertices.

2.7.3.2 Rectangular Region

Region specifier 2 has been allocated to a rectangular region. A typical rectangular region is shown in Figure 2.7.6 together with the corresponding region plenix. All regions of this kind are defined by the x- and y-coordinates of two diametrically opposite corners of the rectangle.

Figure 2.7.6: Rectangular region

The general form of the region plenix of a rectangular region is, therefore, given by

$$\sim\{2,\{[x_1,y_1],[x_2,y_2]\}\}\sim$$

2.7.3.3 Circular Region

Region specifier 3 is used for a circular region. A typical circular region as well as its region plenix are shown in Figure 2.7.7. A circular region is defined by the x- and y-coordinates of two points. The first point, P_1 , is the centre of the circular region and the second point, P_2 , is any point on the circumference of the circle. Therefore, the radius of the circle is given by the distance between the two points.

Figure 2.7.7: Circular region

The general form of the circular region plenix is:

$$\sim\{3, \{[x_1, y_1], [x_2, y_2]\}\} \sim$$

2.7.3.4 Elliptic Region

Elliptic regions have been given region code 4. Figure 2.7.8 shows a typical elliptic region with the region plenix alongside. In this case, the region ingot requires three points. The first of these points, P_1 , is the centre of the ellipse while the remaining two points, P_2 and P_3 , specify the end points of the major and minor axes.

Figure 2.7.8: Elliptic region

The general form of this type of region is given by

$$\sim\{4, \{[x_1, y_1], [x_2, y_2], [x_3, y_3]\}\} \sim$$

2.7.3.5 Parabolic Region

The last type of region is referred to as parabolic region and it has been allocated region specifier 5. A typical parabolic region and the corresponding region plenix are given in Figure 2.7.9. Here, the x- and y-coordinates of three points are used in the region plenix. The first two points, P_1 and P_2 are two points which lie at equal distance from the axis of the parabola. The third point P_3 is an arbitrary point on the parabola which defines the side to which the parabola opens and the slope of the curve. The parabolic region is the only region that does not define a closed area. Further details will be outlined together with the mathematical formulation in Chapter 8.

Figure 2.7.9: Parabolic region

The general form of the region plenix for parabolic regions is given by

$$\sim\{5,\{[x_1,y_1],[x_2,y_2],[x_3,y_3]\}\}\sim$$

2.8 Wedge Cap Pellevation with Elliptic Base

The last type of cap pellevation is the 'wedge cap pellevation with elliptic base'. The shape of the pellevant used to obtain this type of pellevation is conical at any section through the centre with an elliptic base as can be seen from the tector shown in Figure 2.8.1.

Figure 2.8.1: Illustrative example of wedge cap pellevation with elliptic base

2.8.1 Wedge Cap Pellevation Function with Elliptic Base

Pellevations of the kind shown in Figure 2.8.1 can be obtained using the capel function together with pellevation code ± 8 as the first parameter. The list of function parameters further includes the x- and y-coordinates of the three points $C_1(cx_1, cy_1)$, $C_2(cx_2, cy_2)$ and $C_3(cx_3, cy_3)$ to define the elliptic plan as outlined for *capel6*, and the height h of the pellevant at the centre point C_1 as shown in Figure 2.8.2.

Figure 2.8.2: Details related to the parameters of the wedge cap pellevation with elliptic base

The general form of the function is given as

2.8.2 Mathematical Formulation

The formulation of the present function is similar to that of the other elliptic base cap pellevations where at the beginning a translational and rotational coordinate transformation are carried out as explained in detail for the *capel6* pellevation.

Figure 2.8.3: Geometric details for the wedge cap pellevation with elliptic base

From the equation of the ellipse in plan, as shown in Figure 2.8.3, it is possible to determine whether or not point $P(px, py)$ lies inside the scope of the pellevant (Equation 2.5.5). If this is the case, both d and d_e as shown in Figure 2.8.3 are obtained in the same way as outlined in Section 2.5.2.

In the next step, the linear cross-section of the pellevant, as shown in the Figure 2.8.3, is used to arrive at the relationship:

$$\frac{h}{d_e} = \frac{\Delta z}{d_e - d} \quad \dots\dots\dots 2.8.1$$

Manipulation of this equation yields:

$$\Delta z = \frac{h}{d_e}(d_e - d)$$

or

$$\Delta z = h - \frac{hd}{d_e} \dots\dots\dots 2.8.2$$

The new z-value of point *P* is finally obtained as:

$$z_{new} = z_{old} + \left(h - \frac{hd}{d_e} \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{hd}{d_e} \right) \quad (\text{for push-down})$$

2.8.3 Pellevation and Novation

The concept of pellevation can be combined with any other function as has been shown in the previous sections. However, the combination of the concepts of pellevation and that of the novation function is a very valuable one and it will, therefore, be elaborated upon in this section.

Novation is a concept which allows specified translocations of a number of nodes of a configuration. The remaining nodes either remain in place ('sharp novation') or experience conformity translocation following the rule of exponential decay ('conformity novation'). Conformity novation is of special interest for the combination of pellevation and novation. For further details on the concept of novation, readers are referred to Ref. [10].

Figure 2.8.4(a) shows a 25 by 25 tector which has been pellevated using a *capel8* pellevation. The end of the major axis of the pellevant lies on the edge of the tector as can be seen in the elevation in Figure 2.8.4(b). The result shown in Figure 2.8.4(c) has

been obtained by a conformity novation applied to the pellevated tector of Figure 2.8.4(a). The process of this novation moves the four corner points of the tector outwards and down. A sketch of the elevation of the tector after both geometric manipulations can be seen in Figure 2.8.4(d).

On the right of Figure 2.8.4, the corresponding configurations for the same manipulations as given on the left are shown but, this time, the novation process is carried out prior to the pellevation process. Figure 2.8.4(e) shows the tector after the novation with its four corner points moved outwards and down. The corresponding elevation is given in Figure 2.8.4(f). The result of the subsequently applied pellevation is shown in Figure 2.8.4(g) together with its elevation, Figure 2.8.4(h).

Comparison of the two results shows that, in the case of the pellevation and the novation, it makes a difference in which order the two concepts are applied. The two results, Figure 2.8.4(c) and 2.8.4(g) are not identical. The differences are as follows:

- If the pellevation is carried out first, the pellevant leaves an 'imprint' of certain properties which have been defined through the function parameters. In the case of the *capel8*, the 'imprint' has the properties of a certain ellipse in plan, defined by the major and minor axes, and the height at the centre. If a conformity novational transformation follows this pellevation, the original properties of the 'imprint' get distorted according to the novation parameters. It can be seen from the final elevation shown in Figure 2.8.4(d) that the original wedge push-up has increased in span compared to the initial elevation while the height has decreased, Figure 2.8.4(b).
- If the novational transformation is carried out first, Figure 2.8.4(e), the original size of the tector in plan is changed at the beginning by the outwards and down translocation of the four corner points. This is indicated by the dashed and full lines of the elevation shown in Figure 2.8.4(f). The subsequently applied wedge cap pellevation, then, operates on the already transformed tector. Therefore, the

'imprint' of the pellevant shows the original properties in the end which have been defined by the pellevation parameters, Figures 2.8.4(g) and 2.8.4(h).

Figure 2.8.4: Combination of pellevation and novation

Chapter 3

Barrel Pellevation

3.0 Introduction

In this chapter, the second main family of pellevations is introduced. This is referred as the family of 'barrel pellevations'. Altogether, there are eight basic barrel pellevations. Similar to case of the cap pellevations introduced in the previous chapter, the barrel pellevation can be divided into two groups according to the shape of the base of the pellevants. The two types of bases are referred to as 'rectangular' and 'trapezoidal' with either

- circular,
- elliptic,
- parabolic or
- wedge

cross-section as shown in Table A.2 in Appendix A. In this context, 'rectangular' always implies a barrel of infinite length and constant cross-section while 'trapezoidal' implies a barrel with changing cross-section that terminates at one or both ends. Further details will be discussed in the appropriate sections in the sequel.

3.1 Circular Barrel Pellevation with Rectangular Base

The first barrel pellevation to be introduced is the 'circular barrel pellevation with rectangular base'. The pellevant used has the shape of a top part of a cylindrical barrel as can be seen from the example shown in Figure 3.1.1. In this case, the cross-section is circular while the boundaries in plan are defined by two parallel lines of infinite length.

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(1,15,15,15,30,20,6) | E;
use vm(2), vt(1), vh(-3,-5,5,0,0,0,0,1);
clear; draw F;
```

Figure 3.1.1: Illustrative example of circular barrel pellevation with rectangular base

3.1.1 Circular Barrel Pellevation Function with Rectangular Base

The circular barrel pellevation with rectangular base is obtained by the bapel function with pellevation code ± 1 as the first parameter. Further parameters, as shown in Figure 3.1.2, are the x- and y-coordinates of two points $C_1 (cx_1, cy_1)$ and $C_2 (cx_2, cy_2)$ to define the axis of the barrel pellevant in plan, the span s , that is, the width of the barrel at base level, and the height h of the pellevant along the axis.

Figure 3.1.2: Details related to the parameters of the circular barrel pellevation with rectangular base

The general form of this type of barrel pellevation function is given by

As for all circular pellevations, the circular cross-section is restricted to half a circle. That is, the height cannot be greater than half the span:

$$h \leq s/2$$

The reason for this restriction has been explained in detail for the *capell* in Chapter 2.

3.1.2 Mathematical Formulation

For the formulation of the *bapell* (that is, the barrel pellevation function with pellevation code ± 1) it is required to go through the process of a translational coordinate transformation in the xy-plane which makes point C_1 the new origin. Mathematically speaking, the initially given x- and y-coordinates of any point (u'_1, u'_2) are changed into (u_1, u_2) following the relationship, Figure 3.1.3,

$$\begin{aligned} u_1 &= u'_1 - cx_1 \\ u_2 &= u'_2 - cy_1 \end{aligned}$$

This translational coordinate transformation is followed by a rotational coordinate transformation as shown in Figure 3.1.3. The rotational transformation is carried out with the intention of making $\overline{C_1C_2}$ the new global x-axis. The final x- and y-coordinates of any point $P(px, py)$ are computed from the previous ones (u_1, u_2) using the transformation

$$\begin{bmatrix} px \\ py \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Here,

$$\begin{aligned} \cos \alpha &= a/c \\ \sin \alpha &= b/c \end{aligned}$$

with

$$a = cx_2 - cx_1$$

$$b = cy_2 - cy_1$$

as shown in Figure 3.1.3. The distance c between the points C_1 and C_2 is given by

$$c = \sqrt{a^2 + b^2}$$

Figure 3.1.3: Translational and rotational coordinate transformations

In order to determine whether point P lies inside the boundaries of the pellevant, it is required to check if the absolute value of its new y -coordinate, py , is less than half the span. That is,

$$d = |py| < s/2$$

Figure 3.1.4: Geometric details for the circular barrel pellevation with rectangular base

If this condition is satisfied, point P will be affected by the pellevation process as shown in Figure 3.1.4 and the change in the z -coordinate is obtained as follows:

$$\tan \alpha = \frac{s/2}{2r - h} \quad (\text{from triangle } ①②③)$$

or

$$\tan \alpha = \frac{h}{s/2} \quad (\text{from triangle } ②③④)$$

and from these relations,

$$\frac{s/2}{2r - h} = \frac{h}{s/2}$$

or

$$r = \frac{s^2}{8h} + \frac{h}{2} = \frac{s^2 + 4h^2}{8h} \quad \dots\dots\dots 3.1.1$$

This will give the radius r of the barrel in terms of the initially given parameters s and h . The change in z is then given by

$$\Delta z = h - r + e \quad \dots\dots\dots 3.1.2$$

where

$$e = \sqrt{r^2 - d^2} \quad \dots\dots\dots 3.1.3$$

Therefore, the new z -value of point P under consideration is found as

$$z_{new} = z_{old} + (h - r + e) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - (h - r + e) \quad \text{(for push-down)}$$

3.2 Elliptic, Parabolic and Wedge Barrel Pellevations with Rectangular Base

The remaining barrel pellevations with rectangular base, as outlined in the previous section, are referred to as the 'elliptic, parabolic and wedge barrel pellevations with rectangular base'. As can be seen from the examples given in Figure 3.2.1, the cross-sections of the pellevants in these cases are elliptic, parabolic or wedge-shaped.

(a)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(2,15,15,15,30,20,3,5) | E;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,1);
clear; draw F;
```

(b)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(3,15,15,15,30,20,7) | E;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,1);
clear; draw F;
```

(c)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(4,15,15,15,30,20,7) | E;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,1);
clear; draw F;
```

Figure 3.2.1: Illustrative example of
 (a) elliptic barrel pellevation with rectangular base,
 (b) parabolic barrel pellevation with rectangular base,
 (c) wedge barrel pellevation with rectangular base

3.2.1 Elliptic Barrel Pellevation Functions with Rectangular Base

The barrel pellevation function as introduced in Section 3.1 can also be used to obtain the presently discussed types of pellevation. In this context, the first parameter, that is, the pellevation code is different. For the elliptic cross-section, the pellevation code is ± 2 . The other parameters are shown in Figure 3.2.2 and their significance is as described for the *bapel* function. The last parameter, h_e , denotes the length of the

vertical semi-axis of the ellipse in a similar manner as explained for *capel2* where the height cannot be greater than h_e :

$$h \leq h_e$$

Figure 3.2.2: Details related to the parameters of the elliptic barrel pellevation with rectangular base

The general form of this mode of the barrel pellevation function can be written as

The general form of the function for the parabolic cross-section is given by

The parameters used are shown graphically in Figure 3.2.3. Finally, the function in its general form for the linear cross-section can be written as

where the significance of the parameters is shown in Figure 3.2.4, and where the value of the height h for both bapel3 and bapel4 has no upper limit.

Figure 3.2.3: Details related to the parameters of the parabolic barrel pellevation with rectangular base

Figure 3.2.4: Details related to the parameters of the wedge barrel pellevation with rectangular base

3.2.2 Mathematical Formulation

The mathematical formulation for *bapel2*, *bapel3* and *bapel4* is initially the same as that for the circular barrel pellevation with rectangular base discussed in Section 3.1. If for a point P the distance from the axis is found to be less than half the width of the barrel, the different geometric forms of the cross-sections of the pellevant are used to obtain the change in the z -coordinate.

Figure 3.2.5: Geometric details for the elliptic barrel pellevation with rectangular base

For *bapel2*, the following relationship can be obtained from Figure 3.2.5:

$$\frac{y^2}{a^2} + \frac{z^2}{h_e^2} = 1 \quad \dots\dots\dots 3.2.1$$

Since the boundary point $B((s/2), (h_e - h))$, as shown in the figure, lies on the ellipse, its coordinates satisfy Equation 3.2.1 and, therefore,

$$\frac{(s/2)^2}{a^2} + \frac{(h_e - h)^2}{h_e^2} = 1$$

Algebraic manipulation of the above equation, in a manner similar to that shown in Section 2.2.2, will yield an expression for the unknown parameter a :

$$a = \frac{s/2}{\sqrt{1 - \frac{(h_e - h)^2}{h_e^2}}} \quad \dots\dots\dots 3.2.2$$

Having found this parameter of the ellipse, it is possible to evaluate the value of z from Equation 3.2.1:

$$\frac{z^2}{h_e^2} = 1 - \frac{y^2}{a^2}$$

or

$$z = h_e \sqrt{1 - \frac{y^2}{a^2}}$$

The y-coordinate of point P after the coordinate transformations is equal to the distance d of the point to the axis of the barrel, hence

$$z = h_e \sqrt{1 - \frac{d^2}{a^2}} \quad \dots\dots\dots 3.2.3$$

The new z-coordinate of a point P that lies inside the boundary of the *bapel2* pellevant is then given by

$$z_{new} = z_{old} + (h - h_e + z) \quad (for \textit{push-up})$$

$$z_{new} = z_{old} - (h - h_e + z) \quad (for \textit{push-down})$$

Figure 3.2.6: Geometric details for the parabolic barrel pellevation with rectangular base

In the case of *bapel3*, the general equation of the parabola in the *yz*-plane, as shown in Figure 3.2.6, is used as the starting point:

$$\Delta z = -ay^2 + h \quad \dots\dots\dots 3.2.4$$

Here, the boundary point $B((s/2),0)$ lies on the parabola and, therefore, satisfies Equation 3.2.6, that is,

$$0 = -a(s/2)^2 + h$$

Manipulation of this equation will yield an expression for the unknown parameter *a*.

$$a = \frac{4h}{s^2} \quad \dots\dots\dots 3.2.5$$

Substitution of Equation 3.2.5 into Equation 3.2.4 gives

$$\Delta z = h - \frac{4h}{s^2}y^2$$

or

$$\Delta z = h - \frac{4h}{s^2}d^2 \quad \dots\dots\dots 3.2.6$$

where the y-coordinate has been replaced by the distance *d* of point *P* from the axis of the pellevant, Figure 3.2.6. The new z-coordinate of point *P* for *bapel3* can, therefore, be obtained as

$$z_{new} = z_{old} + \left(h - \frac{4h}{s^2}d^2 \right) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - \left(h - \frac{4h}{s^2}d^2 \right) \quad \text{(for push-down)}$$

Figure 3.2.7: Geometric details for the wedge barrel pellevation with rectangular base

Finally, for the wedge-shape cross-section of the *bapel4* pellevation, the following relation is obtained from Figure 3.2.7:

$$\frac{h}{s/2} = \frac{\Delta z}{s/2 - d} \dots\dots\dots 3.2.7$$

Solving this equation for the unknown parameter Δz yields:

$$\Delta z = \frac{h(s/2) - hd}{s/2}$$

or

$$\Delta z = h - \frac{2hd}{s} \dots\dots\dots 3.2.8$$

Therefore, the new z-coordinate can be computed as

$$z_{new} = z_{old} + \left(h - \frac{2hd}{s} \right) \quad (for \textit{push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{2hd}{s} \right) \quad (for \textit{push-down})$$

3.3 Circular Barrel Pellevation with Trapezoidal Base

The 'circular barrel pellevation with trapezoidal base' is a barrel pellevation with varying cross-section characterised by a 'trapezoidal' boundary in plan. This boundary of the pellevant in plan is defined by two straight lines which are normally not parallel to each other but sloping at an angle symmetric with respect to the axis of the pellevant. The cross-section of the pellevant is circular but varies in size along the axis. An example showing the features of the pellevant can be seen in Figure 3.3.1. In the case of the trapezoidal base barrel pellevant, the continuation of the base is not infinite at both ends (as it has been for the barrel pellevations with rectangular base) but terminates at one or both ends. The details of this will be discussed in the following section.

Figure 3.3.1: Illustrative example of circular barrel pellevation with trapezoidal base

3.3.1 Circular Barrel Pellevation Function with Trapezoidal Base

The circular barrel pellevation function with trapezoidal base is the fifth mode of the *bapel* function with its first parameter being the pellevation code ± 5 . Therefore, this function may be referred to as *bapel5*. The other parameters required are, firstly, the coordinates of the points $C_1(cx_1, cy_1)$ and $C_2(cx_2, cy_2)$ which define the position of the axis of the barrel. Also required are the spans s_1 and s_2 and the heights h_1 and h_2 at points C_1 and C_2 as shown in Figure 3.3.2.

Figure 3.3.2: Details related to the parameters of the circular barrel pellevation with trapezoidal base

The general form of the function is given by

The following conventions are used:

- The values of both spans s_1 and s_2 and both heights h_1 and h_2 have to be greater than or equal to zero.
- Due to the circular form of the cross-section, the following relationships must be satisfied: $h_1 \leq s_1/2$ and $h_2 \leq s_2/2$.
- For any point n along the axis of the barrel the span s_n and the height h_n are determined proportionally. To elaborate, the value of the span s_n is found by linear interpolation / extrapolation of the given spans s_1 and s_2 , based on the distance of point n from points C_1 and C_2 . The value of the height h_n is found by linear interpolation / extrapolation of the given heights h_1 and h_2 , also based on the distance of point n from points C_1 and C_2 .
- The trapezoidal barrel ends wherever one of the following three criteria is reached:
 - (a) The value of the height h_n along the axis becomes zero, Figure 3.3.3(a).
 - (b) The value of the span s_n along the axis becomes zero, Figure 3.3.3(b).
 - (c) The value of the height h_n at any point becomes greater than half the corresponding span s_n , Figure 3.3.3(c).

These termination points may lie outside the boundaries of the actual configuration as shown for the tector in Figure 3.3.3(d).

Figure 3.3.3: Termination of circular barrel pellevation with trapezoidal base

Figure 3.3.3: Termination of circular barrel pellevation with trapezoidal base

3.3.2 Mathematical Formulation

The mathematical formulation for the *bapel5* function is initially the same as for *bapel1* to *bapel4* and begins with a translational and subsequent rotational coordinate transformation in order to make C_1 the new origin and the axis of the pellevant the new x-axis. The procedure is shown in Section 3.1 for the *bapel1* pellevation.

As far as all barrel pellevations with trapezoidal base are concerned, it is then possible to find the span s_n and the height h_n for any point n along the axis of the pellevant, Figure 3.3.4:

$$s_n = s_1 + \frac{(s_2 - s_1)}{c} x_n \dots\dots\dots 3.3.1$$

and

$$h_n = h_1 + \frac{(h_2 - h_1)}{c} x_n \dots\dots\dots 3.3.2$$

where c is the distance between the points C_1 and C_2 .

Figure 3.3.4: Geometric details for the circular barrel pellevation with trapezoidal base

Examining the situation in plan, Figure 3.3.4, it is necessary to check for any point $P(x,y)$ if the following conditions are satisfied:

- $s_n > 0$,
- $h_n > 0$,
- $h_n \leq \frac{s_n}{2}$ and
- $|y| = d < \frac{s_n}{2}$

If all the above conditions are satisfied, point P is within the scope of the pellevant and is subjected to the pellevation. The geometric details of the cross-section, as shown in Figure 3.3.4, are then used to evaluate the change in the z-coordinate:

$$\Delta z = h_n - r + e$$

with

$$r = \frac{s_n^2 + 4h_n^2}{8h_n} \dots\dots\dots 3.3.3$$

and

$$e = \sqrt{r^2 - d^2} \dots\dots\dots 3.3.4$$

The complete derivation of the above expressions is given in Section 3.1. The new z-value of point P can then be obtained as

$$z_{new} = z_{old} + (h_n - r + e) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - (h_n - r + e) \quad \text{(for push-down)}$$

3.4 Elliptic, Parabolic and Wedge Barrel Pellevations with Trapezoidal Base

There are three further barrel pellevations with trapezoidal base, these are the 'elliptic, parabolic and wedge barrel pellevations with trapezoidal base'. The cross-section of the pellevant is elliptic, parabolic or linear while the boundary in plan is trapezoidal as discussed for the *bapel5* in the previous section. The particulars can be seen in the examples shown in Figure 3.4.1. Both the span and the height of the pellevants vary linearly but independently from each other along the axis.

(a)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(6,15,5,15,40,6,30,1,6,7) | E;
use vm(2), vt(1), vh(4,-2,4,0,0,0,0,0,1);
clear; draw F;
```

(b)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(7,15,5,15,40,6,30,2,6) | E;
use vm(2), vt(1), vh(4,-2,4,0,0,0,0,0,1);
clear; draw F;
```

(c)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = bapel(8,15,4,15,40,6,30,3,12) | E;
use vm(2), vt(1), vh(4,-2,4,0,0,0,0,0,1);
clear; draw F;
```

Figure 3.4.1: Illustrative example of
 (a) elliptic barrel pellevation with trapezoidal base,
 (b) parabolic barrel pellevation with trapezoidal base,
 (c) wedge barrel pellevation with trapezoidal base

3.4.1 Elliptic, Parabolic and Wedge Barrel Pellevation Functions with Trapezoidal Base

The *bapel* function in combination with pellevation code ± 6 , ± 7 or ± 8 as the first parameter is used to obtain the elliptic, parabolic or wedge barrel pellevation with trapezoidal base, respectively. Generally, the other parameters of the function are as discussed for the *bapel5*.

In the case of *bapel6*, the details related to the parameters are shown in Figure 3.4.2.

Figure 3.4.2: Details related to the parameters of the elliptic barrel pellevation with trapezoidal base

The general form of the function can be written as

The additional parameter h_{el} , which is at the end of the list, denotes the length of the vertical semi-axis of the ellipse at point C_l . The value of h_{el} has to be positive and greater than or equal to the corresponding height h_l , that is,

$$h_{el} \geq h_l$$

for the same reasons as discussed in Section 2.2 for *capel2*. In the case of *bapel7*, the function is given in its general form by

where the significance of the parameters is shown in Figure 3.4.3. Finally, the general form of the function in mode eight, that is, the wedge barrel pellevation with trapezoidal base, is given by

with the details related to the parameters as shown in Figure 3.4.4.

Figure 3.4.3: Details related to the parameters of the parabolic barrel pellevation with trapezoidal base

Figure 3.4.4: Details related to the parameters of the wedge barrel pellevation with trapezoidal base

The following conventions are used:

- The values of both spans s_1 , s_2 and the heights h_1 , h_2 have to be greater than or equal to zero.
- The span s_n and the height h_n at any point n along the axis of the pellevant are determined proportionally, that is, through linear interpolation / extrapolation of the given values s_1 , s_2 and h_1 , h_2 . Additionally, in the case of the *bapel6* pellevation, h_{en} at any point n along the axis can be evaluated by keeping a constant ratio h_{en}/h_n .
- The trapezoidal barrel pellevant with elliptic, parabolic or wedge-shaped cross-section ends where one of the following two criteria is reached:
 - (a) The value of the height h_n along the axis becomes zero, Figure 3.4.5(a).
 - (b) The value of the span s_n at any point along the axis becomes zero, Figure 3.4.5(b).

Note that these termination points may lie beyond the outline of the configuration as shown in Figure 3.4.5(c).

Figure 3.4.5: Termination of circular barrel pellevation with trapezoidal base

However, if the termination point of a trapezoidal pellevant lies within the configuration, it is possible to 'extend' the pellevant beyond this point by applying a second 'corresponding' pellevant. For instance, the pellevant used for the example in Figure 3.4.5(a) can be 'extended' to give a result like the one shown in Figure 3.4.6(a). In this case, the 'extension' has been obtained through the application of a second barrel pellevation with the same properties and the same axis but opposite direction. Therefore, the second pellevant ends in the same location as the first pellevant but opens in the opposite direction.

Figure 3.4.6(b) shows an 'extension' of the example in Figure 3.4.5(b). In a similar way as explained in the above paragraph, a second pellevation is applied with the same properties and the same axis but opening the opposite way. In order to avoid superimposition of the pellevation effects, the second pellevant is this time defined in a way that it terminates just next to the termination point of the first pellevant.

Whereas the previous two examples showed the effects using positive pellevations only, it is possible to obtain results of this kind incorporating negative pellevation. Figure 3.4.6(c) uses two negative pellevations while Figure 3.4.6(d) 'extends' a positive trapezoidal barrel pellevant with its corresponding negative one. In the latter case, the result appears as if the barrel 'intersects' with the initial plane of the tector.

(b)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F1 = bapel(6,15,15,15,40,0,30,0,6,12) | E;
F2 = bapel(6,15,15,1,15,-10,0,30,0,6,12) | F1;
use vm(2), vt(1), vh(-4,-5,7,0,0,0,0,1);
clear; draw F2;

```

plan

elevation

(c)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F1 = bapel(-6,15,15,15,40,4,30,0,6,10) | E;
F2 = bapel(-6,15,15,15,-10,4,30,0,6,10) | F1;
use vm(2), vt(1), vh(-4,-5,7,0,0,0,0,1);
clear; draw F2;

```

plan

elevation

(d)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F1 = bapel(6,15,15,15,40,6,30,0,8,10) | E;
F2 = bapel(-6,15,15,15,-10,6,30,0,8,10) | F1;
use vm(2), vt(1), vh(-4,-5,7,0,0,0,0,1);
clear; draw F2;

```

plan

elevation

Figure 3.4.6: 'Extension' of trapezoidal base barrel pellevations

3.4.2 Mathematical Formulation

The mathematical formulation of the *bapel6* to *bapel8* pellevations is initially the same as discussed for *bapel5* and begins with a translational and subsequent rotational coordinate transformation in order to make point C_1 the new origin and the axis of the barrel the new x-axis. Details of the procedure are given in Section 3.1.

Following these transformations, the span s_n and the height h_n for any point n along the axis of the pellevant can be obtained by interpolation / extrapolation as

$$s_n = s_1 + \frac{(s_2 - s_1)}{c} x_n \dots\dots\dots 3.4.1$$

and

$$h_n = h_1 + \frac{(h_2 - h_1)}{c} x_n \dots\dots\dots 3.4.2$$

where c denotes the distance between the two points C_1 and C_2 , Figures 3.4.7 to 3.4.9. In the case of *bapel6*, the length h_{en} of the vertical semi-axis of the ellipse at point n can be obtained from

$$h_{en} = \frac{h_{e1}}{h_1} h_n \dots\dots\dots 3.4.3$$

In order to obtain the new z-value of a point $P(x,y)$ the following conditions have to be satisfied:

- $s_n > 0$,
- $h_n > 0$ and
- $|y| = d < \frac{s_n}{2}$

If the conditions are satisfied, the cross-section of the pellevant is considered to obtain the new z-coordinate of point P .

Figure 3.4.7: Geometric details for the elliptic barrel pellevation with trapezoidal base

In the case of the elliptic cross-section, the value of z is obtained as follows, Figure 3.4.7:

$$z = h_{en} \sqrt{1 - \frac{d^2}{a^2}} \quad \dots\dots\dots 3.4.4$$

where

$$a = \frac{s_n/2}{\sqrt{1 - \frac{(h_{en} - h_n)^2}{h_{en}^2}}} \quad \dots\dots\dots 3.4.5$$

The derivation of the above equations is similar to that given in Section 3.2 for *bapel2*. Finally, the new z -value of point P can be obtained as

$$z_{new} = z_{old} + (h_n - h_{en} + z) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h_n - h_{en} + z) \quad (\text{for push-down})$$

Figure 3.4.8: Geometric details for the parabolic barrel pellevation with trapezoidal base

In the case of *bapel7*, the parabolic cross-section as shown in Figure 3.4.8 is used to find the change in the z -coordinate:

$$\Delta z = h_n - \frac{4h_n}{s_n^2} d^2 \quad \dots\dots\dots 3.4.6$$

where the derivation of this equation is similar to that given in Section 3.2. The new z -coordinate can then be found as

$$z_{new} = z_{old} + \left(h_n - \frac{4h_n}{s_n^2} d^2 \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h_n - \frac{4h_n}{s_n^2} d^2 \right) \quad (\text{for push-down})$$

Figure 3.4.9: Geometric details for the wedge barrel pellevation with trapezoidal base

Finally, in the case of the *hapel8*, the cross-section of the pellevant as shown in Figure 3.4.9 is used and from the linear relationship the change in the z-coordinate is given as:

$$\Delta z = h_n - \frac{2h_n d}{s_n} \dots\dots\dots 3.4.7$$

Finally, the new z-coordinate is obtained from

$$z_{new} = z_{old} + \left(h_n - \frac{2h_n d}{s_n} \right) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - \left(h_n - \frac{2h_n d}{s_n} \right) \quad \text{(for push-down)}$$

3.5 Applications

Circular barrel pellevations can be used to obtain many types of barrel vaults. Figure 3.5.1(a) shows a barrel vault obtained from a square grid with three 'holes' along one diagonal. In this case, a *bapell* has been used with its axis along the diagonal of the configuration and a span equal to the length of the diagonal. As a consequence, two of the corner points of the configuration lie on the boundary of the pellevant and remain at their original level while all other nodes are pushed up according to the circular cross-section of the barrel.

Figure 3.5.1(b) shows a similar barrel vault obtained with the use of cylindrical coordinates. The perspective view of this example looks rather similar to the one shown in Figure 3.5.1(a). However, due to the differences between the definition of the barrel pellevation and that of the cylindrical retronorm, the two barrel vaults are not identical.

The difference can be seen from the plan views in Figure 3.5.1(c) where the pellevated vault is shown on the left and the vault obtained from the cylindrical retronorm is shown on the right. In the first case, using the barrel pellevation, the length of some elements is changed depending on their position relative to the barrel pellevant. The plan of the configuration, however, remains unchanged. In the latter case, using the cylindrical retronorm, the length of the elements with the same orientation is identical. So, the circular barrel pellevation and the cylindrical retronorm are not to be confused with one another. Typically, the barrel pellevation would be used for comparatively shallow barrel vaults while the cylindrical retronorm is useful for higher barrel vaults.

(a)

```

a = rosad(0.5,0.5) | {[0,0,0; 0.5,0.5,0], [0,0,0; 1,0,0]};
b = rinid(12,12,1,1) | a;
c = { [3,3,0],[6,6,0],[9,9,0]};
d = lux(c) | b;
e = bapel(1,0,0,12,12,12*sqrt(2,4) | d;
use vm(2), vt(1), vh(-1,-5,4,0,0,0,0,1);
clear; draw e;

```

(b)

```

a = rosat(0.5,0.5) | {[11,0,0; 11,0.5,0.5], [11,0,0; 11,1,0]};
b = rinid(12,12,1,1) | a;
c = { [11,3,3],[11,6,6],[11,9,9]};
d = lux(c) | b; d = verat(0,0,45) | d;
e = bc(5,360/60,6) | d;
use vm(2), vt(1), vh(2,-2,3,0,0,0,1,0,0);
clear; draw e;

```

(c)

Figure 3.5.1: (a) Circular barrel pellevation, (b) Barrel using cylindrical coordinates, (c) Plan views of (a) on the left and (b) on the right

In order to demonstrate the effects of superimposed barrel pellevations, consider the grid shown in Figure 3.5.2(a) with a single elliptic barrel pellevation. If a second barrel pellevation of exactly the same properties is applied at 90° to it, a 'hump' will form in the centre of the grid as can be seen in Figure 3.5.2(b). This effect may, for example, be desired in a structure to raise certain areas of the roof within the building.

Figure 3.5.2: Superimposition of intersecting barrel pellevants
(a) single *bapel2* pellevation, (b) two *bapel2* pellevations at 90°

If for any reason this effect of the superimposition is not wanted and the result of the two intersecting barrel pellevations is required without the 'hump' in the middle, Figure 3.5.3(a), the shape can be obtained in the following manner :

- one quarter of the tector to be pellevated is generated,
- this part is pellevated using the elliptic barrel pellevation function and
- the resulting configuration is rotated four times by 90° and composed using the *rosad* function, Figure 3.2.5(b).

Figure 3.5.3: Intersection of barrel pellevants without 'hump'
 (a) configuration and formex formulation, (b) process

Multiple use and superimposition of barrel pellevations give rise to further interesting examples. Figure 3.5.4(a) gives an example where a number of small barrel pellevants are used and eventually superimposed with a much bigger *bapel2* pellevation which is located centrally to the configuration at 90° to the other barrel pellevants. One way of obtaining this configuration is to generate the entire mesh first and then apply five *bapel2* pellevations at equal intervals before applying the big *bapel2* pellevation. Another approach which has been chosen for the generation of Figure 3.5.4(a) is, however, quicker. Here, only the first 'unit' of the mesh is generated and pushed up using the small *bapel2* pellevation. Then, the basic 'unit' is replicated and finally subjected to the big *bapel2* pellevation. As a matter of fact, the order in which the small and big barrel pellevations are applied does not affect the final result since the effects simply add up as mentioned in Section 2.1.4.

Making use of the same ideas, the structure shown in Figure 3.5.4(b) has been obtained. This time, the configuration has been composed of the initial mesh and its mirror image about the xy-plane after the small *bapel2* pellevations have been applied and prior to the application of the big *bapel2*. The result is a barrel vault which is composed of tube-like units. This approach may be used to create structures similar to the ones shown in the picture of Figure 3.5.5.

Figure 3.5.4: Multiple use and superimposition of barrel pellevations

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Figure 3.5.5: (a) Exhibition spaces at Expo '85 in Tsukuba, Ref. [11],
(b) Fuji Group Pavilion at Japan World Exposition 1970 in Osaka, Ref. [9]

The difference between the cylindrical retronorm and the barrel pellevation has been discussed in Section 3.5. However, similar examples to the ones obtained with the superimposition of barrel pellevations may result from the combination of the cylindrical retronorm and the barrel pellevation. Figure 3.5.6 shows such a combination used to obtain a structure similar to the vault of Figure 3.5.4(a). This time, the overall barrel curvature is obtained with the retronorm while the small push-ups at 90° to the axis of the barrel vault result from five *bapel2* pellevations. Note that this configuration

as well as any one of the other configurations may represent a skeletal structure or the finite element mesh of a continuous shell.

Figure 3.5.6: Combination of cylindrical coordinates and barrel pellevations

Figure 3.5.7 gives a new variety of an idea which has been introduced previously, that is, roofs with skylights. In this example, a basic unit composed of square finite elements has been created. A rectangular region is then defined which restricts the regional *bapel102* to one half of the barrel pellevant only. Both the region and the axis of the barrel pellevant are indicated in the plan of Figure 3.5.7. The resulting wave-like unit is finally replicated to obtain a structure which may be used as a roof catering for skylights along the steep sides of the 'cut-off' elliptic barrels. The skylight openings are indicated by the white elements in the perspective view.

Figure 3.5.7: Example of roof structure using *bapel102* pellevation

Barrel pellevations can further be used in combination with cap pellevations to obtain different types of dome and roof structures. Figure 3.5.8(a) shows an example where a *bapel3* has been applied to a triangular mesh 'unit' as can be seen in plan and elevation. The axis of the barrel pellevant coincides with the axis of symmetry of the 'unit' and the span of the pellevant is identical to the base length of the triangular mesh.

Rotational replication of this basic 'unit' about vertex *A* of the triangle by 60° creates a dome-like structure which is further modified through a centrally placed cap pellevant with parabolic cross-section. The result in both the perspective view and the elevation shows the overall appearance of the structure.

Figure 3.5.8(b) shows a similar but nevertheless different structure. This time, the basic 'unit' is not composed of finite elements but of discrete line elements. The same *bapel3* pellevation as in the previous example is used on this 'unit' and also the same rotational replication about point *A* is applied. However, this time, a wedge-shape

capel4 pellevation has been applied centrally. The result is a 'pointed' skeletal dome with its peak being where the six triangular 'units' meet.

Figure 3.5.8: Examples of domes using cap and barrel pellevations
(a) *bapel3* and *capel3*, (b) *bapel3* and *capel4*

With the same approach, the octagonal configuration shown in Figure 3.5.9 has been obtained starting from a single 'unit'. A *bapel4* pellevation is applied to this 'unit' which is then replicated about point *A* to obtain a linearly undulating roof. This 'folded' effect can be used for roof structures similar to the one shown in the picture of Figure 3.5.10.

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Figure 3.5.10: Wooden Pavilions at Yokohama Exotic Showcase 1989, Ref. [9]

The trapezoidal base barrel pellevations can be used for configurations which combine an increasing height with decreasing span. In other words, the pellevants used are narrow but high on one side and wide but shallow on the other side. Figure 3.5.11 gives a configuration which has been obtained with this sort of pellevant.

Configurations similar to the one in Figure 3.5.11 have already been shown in Figure 3.5.8. Only then, the rise of the central region was achieved by superimposing a cap pellevation to the effects of the rectangular base barrel pellevation. In the present example, this rise is achieved by the trapezoidal barrel pellevation only and no additional cap pellevation is required to obtain the effect.

Figure 3.5.11: Example using a trapezoidal base barrel pellevant

Figure 3.5.12 shows another application using a pellevant with a trapezoidal base. The hexagonal structure with a star-like push-up of the centre has been obtained from a basic triangular 'unit' which has been subjected to a *bapel6* pellevation as indicated in

the figure. The pointed end of the pellevant lies on the base line of the triangular 'unit' whereas the opening side of the pellevant is towards vertex *A* of the triangle. Finally, the hexagonal shape is obtained through the rotational replication of the 'unit' about point *A*.

Figure 3.5.12: Example using a trapezoidal base barrel pellevant

Figure 3.5.13 gives an example using a *bapel7* pellevation where the boundary lines as well as the termination point of the *bapel7* pellevant lie inside the triangular 'unit' of the configuration. After rotational replication of the 'unit' about point *A*, the configuration is subjected to a centrally placed wedge cap pellevation. The result is a shallow grid dome or shell structure with parabolic rises along the sides of the hexagon and a small linear rise of the overall structure.

Figure 3.5.13: Example using a trapezoidal base barrel pellevant

Examples of a totally different nature can be obtained using inclined barrel pellevation. Figure 3.5.14 shows an example obtained through an inclined *bapel3* pellevation. The process has four distinct steps as indicated in the elevation on the right in Figure 3.5.14 and as given in the formex formulation below the configuration. These steps are as follows:

1. Generation of the finite element mesh in plan,
2. rotation of the mesh about the y-axis by an angle of 50° ,
3. application of a *bapel3* pellevation, and
4. rotation of the pellevated configuration about the y-axis by an angle of -40° .

This procedure may be used to obtain structures like parts of the roof of the new Kansai Airport in Japan, Figure 3.5.15.

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(b)

Figure 3.5.15: Kansai International Airport Passenger Terminal Building, Ref. [12]

(a) Picture of part of roof

(b) Technical detail of roof structure

Chapter 4

Halo Pellevation

4.0 Introduction

The present chapter introduces the third main family of pellevations which is referred to the family of 'halo pellevations'. Eight halo pellevations with two different bases and four different cross-sections have been defined. These are referred to as

- circular halo,
- elliptic halo,
- parabolic halo and
- wedge halo

each of which have either a 'circular' or an 'elliptic' base. In this context, 'circular' base stands for a boundary in plan which has the shape of a circular ring and 'elliptic' base refers to a base with an elliptic ring outline.

4.1 Circular Halo Pellevation with Circular Base

The first halo pellevation is the 'circular halo pellevation with circular base'. A 30 by 30 tector is used in Figure 4.1.1 to illustrate the general features of this first type of halo pellevation which is characterised by the annular shape of the pellevant and a circular cross-section.

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = hapel(1,15,15,22,22,6,1.5) | E;
use vm(2), vt(1), vh(-3,-5,5,0,0,0,0,1);
clear; draw F;
```

Figure 4.1.1: Illustrative example of circular halo pellevation with circular base

4.1.1 Circular Halo Pellevation Function with Circular Base

The halo pellevation function can be used to obtain examples of the kind shown in Figure 4.1.1. The function requires several parameters. In the case of the circular halo pellevation with circular base, the list of parameters begins with the pellevation code ± 1 . This is followed by six parameters which define the size and the position of the annular halo pellevant. These include the x- and y-coordinates of the centre point $C_1(cx_1, cy_1)$ and a point $C_2(cx_2, cy_2)$ which lies on the central line of the ring and specifies the radius of the ring, Figure 4.1.2. The remaining parameters are the span s (width of the ring) and the height h of the ring at the position of its central line.

Figure 4.1.2: Details related to the parameters of the circular halo pellevation with circular base

The general form of the circular halo pellevation function with circular base is given by

As in the case of the *capell* pellevation, the height h of the circular section may not be greater than half the span, that is,

$$h \leq s/2$$

For all halo pellevations with circular base, the values of the height h and the span s are restricted to positive values only. However note that increasing values of s have different effects on the appearance of the pellevant. To elaborate, it is possible to distinguish between three cases:

- $s < 2\overline{C_1C_2}$, that is, the span is less than twice the distance between the points C_1 and C_2 , Figure 4.1.3(a). In this case, the pellevant has an annular shape.
- $s = 2\overline{C_1C_2}$, that is, the span is exactly equal to twice the distance between the two points C_1 and C_2 , Figure 4.1.3(b). In this case, the central circle of the pellevant reduces to a point.
- $s > 2\overline{C_1C_2}$, that is, the span is greater than twice the distance between the two points C_1 and C_2 , Figure 4.1.3(c). In this case, parts of the pellevant 'overlap' in the central region.

A special case of the halo pellevation occurs if the two points C_1 and C_2 coincide. In this case, the effect obtained is identical to that obtained with a cap pellevation using corresponding parameters, Figure 4.1.4.

Figure 4.1.3: Different effects with increasing values of span s (a) span $<$ radius of central line of pellevant,(b) span $=$ radius of central line of pellevant,(c) span $>$ radius of central line of pellevant

Figure 4.1.4: Special case of circular base halo pellevation (C_1 is coincident with C_2)

4.1.2 Mathematical Formulation

The procedure used for the mathematical formulation of the *hapel1* pellevation begins with the determination of r_0 which defines the distance between the two points C_1 and C_2 as shown in Figure 4.1.5.

Figure 4.1.5: Geometric details for the circular halo pellevation with circular base

$$r_0 = \sqrt{(cx_2 - cx_1)^2 + (cy_2 - cy_1)^2} \dots\dots\dots 4.1.1$$

In the next step, the inner and outer radii of the ring in plan, s_1 and s_2 , are obtained from

$$s_1 = r_0 - \frac{s}{2} \dots\dots\dots 4.1.2$$

and

$$s_2 = r_0 + \frac{s}{2} \dots\dots\dots 4.1.3$$

Also, the distance r_1 of point $P(px,py)$ from the centre C_1 as shown in Figure 4.1.5 is obtained as

$$r_1 = \sqrt{(px - cx_1)^2 + (py - cy_1)^2} \dots\dots\dots 4.1.4$$

where P is a typical point considered for pellevation. From Equations 4.1.1 and 4.1.4, the distance d of point P from the central line of the halo pellevant can be found as

$$d = |r_1 - r_0| \dots\dots\dots 4.1.5$$

It is now possible to determine whether or not point P lies within the boundaries of the pellevant by comparing the value of r_1 with the values of the inner and outer radii, s_1 and s_2 . In other words, if the conditions

$$r_1 < s_2 \text{ and } r_1 > s_1$$

are satisfied, then point P lies inside the scope of the halo pellevant and is subjected to pellevation.

In the case of the *hapell* pellevation, the new z-value will be obtained from the properties of the circular cross-section of the pellevant as shown in Figure 4.1.5. Here,

$$\tan \alpha = \frac{s/2}{2r - h} \quad (\text{from triangle ①②③})$$

but also

$$\tan \alpha = \frac{h}{s/2} \quad (\text{from triangle ②③④})$$

From the above relations,

$$\frac{s/2}{2r-h} = \frac{h}{s/2}$$

or

$$r = \frac{s^2}{8h} + \frac{h}{2} = \frac{s^2 + 4h^2}{8h} \quad \dots\dots\dots 4.1.6$$

The change in z-value is then given by

$$\Delta z = h - r + e$$

where e can be obtained as

$$e = \sqrt{r^2 - d^2} \quad \dots\dots\dots 4.1.7$$

Finally, the new z-value of point P is found as

$$z_{new} = z_{old} + (h - r + e) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h - r + e) \quad (\text{for push-down})$$

4.2 Elliptic, Parabolic and Wedge Halo Pellevations with Circular Base

The 'elliptic, parabolic and wedge halo pellevations with circular base' use pellevants with the same circular ring base as introduced in Section 4.1 while the cross-section of the ring is elliptic, parabolic or wedge-shaped, as can be seen in the examples shown in Figures 4.2.1(a) to 4.2.1(c).

(a)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = hapel(2,15,15,21,21,10,2,2) | E;
use vm(2), vt(1), vh(-4,-5,5,0,0,0,0,0,1);
clear; draw F;

```

(b)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = hapel(3,15,15,21,21,9,2,5) | E;
use vm(2), vt(1), vh(-4,-5,6,0,0,0,0,0,1);
clear; draw F;

```

(c)

```

E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
  rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = hapel(4,15,15,21,21,9,3) | E;
use vm(2), vt(1), vh(-4,-5,7,0,0,0,0,0,1);
clear; draw F;

```

Figure 4.2.1: Illustrative example of
 (a) elliptic halo pellevation with circular base,
 (b) parabolic halo pellevation with circular base,
 (c) wedge halo pellevation with circular base

4.2.1 Elliptic, Parabolic and Wedge Halo Pellevation Functions with Circular Base

The halo pellevation function can be used for the elliptic, parabolic and wedge halo pellevations with circular base in a similar way as described for the *hapell* pellevation in Section 4.1. As far as the parameters of the function are concerned, only the first parameter, that is, the pellevation code, changes according to the cross-section of the pellevant. Figure 4.2.2 shows the details related to the remaining parameters for the second mode of the function.

Figure 4.2.2: Details related to the parameters of the elliptic halo pellevation with circular base

The general form of the hapel function for the elliptic cross-section is given by

In this case, one additional parameter is added at the end of the list. This parameter is denoted by h_e and defines the length of the vertical semi-axis of the ellipse in section, as shown in Figure 4.2.2, in a similar manner as described for *capel2* in Section 2.2. As explained then, the height h of the *hapel2* is restricted to

$$h \leq h_e$$

The general form of the hapel function with the parabolic cross-section is given by

where the significance of the parameters is as shown in Figure 4.2.3. Also, the general form of the hapel function with the linear cross-section is given by

where the details related to the parameters are as indicated in Figure 4.2.4. Note, that neither *hapel3* nor *hapel4* have an upper limit for the height.

Figure 4.2.3: Details related to the parameters of the parabolic halo pellevation with circular base

Figure 4.2.4: Details related to the parameters of the wedge halo pellevation with circular base

4.2.2 Mathematical Formulation

The mathematical formulation involved in the *hapel2* to *hapel4* pellevations begins as described in Section 4.1.2. If then, point *P* lies inside the boundaries of the pellevant, its new *z*-value will be obtained. For this purpose, it is necessary to distinguish between the different cross-sections.

In the case of the *hapel2* pellevation, the elliptic cross-section shown in Figure 4.2.5 is represented by the equation

$$\frac{x^2}{a^2} + \frac{z^2}{h_e^2} = 1 \quad \dots\dots\dots 4.2.1$$

Here, the boundary point $B((s/2), (h_e-h))$ lies on the ellipse and, therefore, satisfies Equation 4.2.1:

$$\frac{(s/2)^2}{a^2} + \frac{(h_e - h)^2}{h_e^2} = 1$$

or

$$a = \frac{s/2}{\sqrt{1 - \frac{(h_e - h)^2}{h_e^2}}} \dots\dots\dots 4.2.2$$

Figure 4.2.5: Geometric details for the elliptic halo pellevation with circular base

Manipulation of Equation 4.2.1 will then yield an expression for the unknown parameter z :

$$\frac{z^2}{h_e^2} = 1 - \frac{x^2}{a^2}$$

or

$$z = h_e \sqrt{1 - \frac{x^2}{a^2}}$$

In the above equation, x can be replaced by the distance d of point P from the centre of the ellipse, Figure 4.2.5. That is,

$$z = h_e \sqrt{1 - \frac{d^2}{a^2}} \quad \dots\dots\dots 4.2.3$$

The new z -value of point P in the case of *hapel2* can finally be obtained as

$$z_{new} = z_{old} + (h - h_e + z) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h - h_e + z) \quad (\text{for push-down})$$

As far as the *hapel3* pellevation is concerned, the parabolic shape of the cross-section as shown in Figure 4.2.6 is used to obtain the change in the z -coordinate.

Figure 4.2.6: Geometric details for the parabolic halo pellevation with circular base

Here, the general equation of the parabola is given by:

$$\Delta z = -ax^2 + h \quad \dots\dots\dots 4.2.4$$

From the above equation, the unknown parameter a is found by substituting the coordinates of the boundary point $B((s/2), 0)$, as shown in Figure 4.2.6. That is,

$$0 = -a(s/2)^2 + h$$

or

$$a = \frac{4h}{s^2} \quad \dots\dots\dots 4.2.5$$

Substitution of Equation 4.2.4 into Equation 4.2.5 yields an expression for the change in the z-coordinate:

$$\Delta z = h - \frac{4h}{s^2}d^2 \quad \dots\dots\dots 4.2.6$$

Hence, the new z-coordinate of point P within the boundary of the parabolic halo pellevant can be obtained as

$$z_{new} = z_{old} + \left(h - \frac{4h}{s^2}d^2 \right) \quad (for \text{ push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{4h}{s^2}d^2 \right) \quad (for \text{ push-down})$$

Finally, in the case of the wedge halo pellevation with circular base, the following relationship is established using Figure 4.2.7:

$$\frac{h}{2/s} = \frac{\Delta z}{s/2 - d} \quad \dots\dots\dots 4.2.7$$

or

$$\Delta z = \frac{h(s/2) - hd}{s/2}$$

or

$$\Delta z = h - \frac{2hd}{s} \quad \dots\dots\dots 4.2.8$$

Figure 4.2.7: Geometric details for the wedge halo pellevation with circular base

The new z -value of point P within the boundaries of the *hapel5* pellevant is, therefore, obtained as

$$z_{new} = z_{old} + \left(h - \frac{2hd}{s} \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{2hd}{s} \right) \quad (\text{for push-down})$$

4.3 Circular Halo Pellevation with Elliptic Base

The 'circular halo pellevation with elliptic base' is the first halo pellevation with an elliptic ring base as can be seen from the example shown in Figure 4.3.1. The cross-section of this ring is circular.

Figure 4.3.1: Illustrative example of circular halo pellevation with elliptic base

4.3.1 Circular Halo Pellevation Function with Elliptic Base

The hapel function in combination with pellevation code ± 5 as the first parameter is used to apply the circular halo pellevation with elliptic base. Further parameters required for the specification of the size and the position of the pellevant are, first of all, the x- and y-coordinates of three points: $C_1 (cx_1, cy_1)$, $C_2 (cx_2, cy_2)$ and $C_3 (cx_3, cy_3)$, as shown in Figure 4.3.2. Here, point C_1 defines the centre of the pellevant and points C_2 and C_3 define the ends of the major and the minor axes of the elliptic central line of the ring as shown in the figure. The order in which the major and minor axes are defined is arbitrary. The last two parameters of the function are the span s of the ring and the height h at the location of the central line of the halo pellevant.

Figure 4.3.2: Details related to the parameters of the circular halo pellevation with elliptic base

The function in general terms is given by

As for any other pellevation with circular cross-section, the height of the pellevant is restricted to

$$h \leq s/2$$

The details on this restriction are discussed in Section 2.1 for the *capell* pellevation. A further restriction which applies to all halo pellevations with elliptic base is given by

$$s < 2b_r$$

where b_r is the length of the semiminor axis. The reasons for this restriction are related to the mathematical formulation and will be discussed in the sequel. A typical example where the span s is just less than twice the length of the semiminor axis is shown in Figure 4.3.3.

Figure 4.3.3: Example where $s \cong 2b_r$

4.3.2 Definition of the Elliptic Boundary

In order to define the elliptic halo boundary, a number of conventions have been introduced which apply to all halo pellevations with an elliptic ring base.

- The central line of the pellevant is an ellipse described by the equation $\frac{x^2}{a_r^2} + \frac{y^2}{b_r^2} = 1$ where a_r and b_r (the semimajor and semiminor axes) are defined by the coordinates of the three points C_1 , C_2 and C_3 , as shown in Figure 4.3.4.
- The span s of the pellevant (the width of the ring), is constant throughout the ring. That is, to define the boundaries of the pellevant the perpendicular

to any point on the elliptic central line is considered and the points on the inner and outer boundary lines are defined at distances $\pm \frac{s}{2}$ from the central line on this perpendicular.

- As a consequence of this definition, the inner and outer boundary lines of the base of the pellevant are not ellipses. Figure 4.3.5 shows the difference in plan between an elliptic halo base as defined here, Figure 4.3.5(a), and a ring where the inner and outer boundary lines are ellipses, Figure 4.3.5(b). In the latter case, the span of the ring is not constant.

Figure 4.3.4: Elliptic ring base

Figure 4.3.5: Difference in plan between
(a) elliptic ring base, (b) boundary defined by two ellipses

4.3.3 Mathematical Formulation

As far as the formulation of the elliptic base halo pellevations is concerned, it is divided into two distinct parts. The first part serves to determine whether or not a point P of the configuration lies inside the elliptic ring boundary. This part of the formulation is the same for all the halo pellevations with elliptic ring boundary. The second part of the formulation is used to obtain the change in the z -coordinate of point P . This part will be dealt with for each of the cross-sections in the appropriate sections in the sequel.

The first part of the procedure begins with a translational and subsequent rotational coordinate transformation as shown in Figure 4.3.6. These transformations make point C_1 the new origin and the greater one of $\overline{C_1C_2}$ and $\overline{C_1C_3}$, that is, the major axis, the new global x -axis. Thus, the coordinates of any point (u'_1, u'_2) are firstly transferred to (u_1, u_2) according to the following relationship

$$\begin{aligned} u_1 &= u'_1 - cx_1 \\ u_2 &= u'_2 - cy_1 \end{aligned}$$

The final x - and y -coordinates of the point are then obtained from

$$\begin{bmatrix} px \\ py \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where

$$\begin{aligned} \cos \alpha &= \frac{a}{a_r} \\ \sin \alpha &= \frac{b}{a_r} \end{aligned}$$

as shown in Figure 4.3.6 where a_r denotes the length of the semimajor axis.

Figure 4.3.6: Translational and rotational coordinate transformations

To proceed further, it is necessary to distinguish between two different cases. Namely where $\overline{C_1C_2} \geq \overline{C_1C_3}$ and where $\overline{C_1C_2} < \overline{C_1C_3}$.

For $\overline{C_1C_2} \geq \overline{C_1C_3}$:

$$a_r = \sqrt{(cx_1 - cx_2)^2 + (cy_1 - cy_2)^2} \dots\dots\dots 4.3.1$$

and

$$b_r = \sqrt{(cx_1 - cx_3)^2 + (cy_1 - cy_3)^2} \dots\dots\dots 4.3.2$$

Also,

$$a = cx_2 - cx_1$$

$$b = cy_2 - cy_1$$

For $\overline{C_1C_2} < \overline{C_1C_3}$:

$$a_r = \sqrt{(cx_1 - cx_3)^2 + (cy_1 - cy_3)^2} \dots\dots\dots 4.3.3$$

and

$$b_r = \sqrt{(cx_1 - cx_2)^2 + (cy_1 - cy_2)^2} \dots\dots\dots 4.3.4$$

Also,

$$a = cx_3 - cx_1$$

$$b = cy_3 - cy_1$$

In the subsequent course of formulation, only points that lie within the rectangle $(-a_r - s/2)$ to $(+a_r + s/2)$ in the x-direction and $(-b_r - s/2)$ to $(+b_r + s/2)$ in the y-direction are considered, Figure 4.3.7. This is done to reduce the computation time and effort since all the points that lie outside the specified rectangle are definitely outside the boundaries of the elliptic ring of the pellevant in plan.

In the next step, it is required to determine the perpendicular distance d_e of point P from the central line of the ellipse which is, in fact, the shortest distance of point P from the central line. There are two cases where this distance d_e can be obtained from a straightforward relationship. These are,

- when point $P(x_p, y_p)$ lies on the minor axis (if $x_p = 0$), that is, when P lies on the line $\overline{A_1A_1}$ in Figure 4.3.7, then

$$d_e = |b_r - |y_p|| \quad \dots\dots\dots 4.3.5$$

- when point P lies along the major axis (if $y_p = 0$), but also $|x_p| \geq (a_r - s/2)$, then

$$d_e = |a_r - |x_p|| \quad \dots\dots\dots 4.3.6$$

In Figure 4.3.7, the points that belong to this second group are the ones that lie on the lines $\overline{A_2A_2}$ or $\overline{A_3A_3}$. In this case, only points that lie to the right of line $\overline{B_1B_1}$ or to the left of line $\overline{B_2B_2}$ are considered because points along the major axis that lie between the two lines $\overline{B_1B_1}$ and $\overline{B_2B_2}$ may be closer to the end of the minor axis than they are to the end of the major axis.

Figure 4.3.7: Categories of points considered for pellevation

Where P does not lie on any of the lines indicated in Figure 4.3.7, a more complicated procedure is required for determining the value of d_e . Here, it is necessary to obtain the coordinates of a point on the central line of the ellipse, $P_e(x_e, y_e)$, where $\overline{PP_e}$ is the shortest perpendicular from P to the central line of the ellipse, Figure 4.3.8. That is to say, it is perpendicular to the nearest tangent to the ellipse. The reason for specifying the shortest distance d_e is that any point has more than one perpendicular to tangents of the ellipse, for instance, as shown for point E in Figure 4.3.8.

In any one of the cases outlined above, the distance d_e is used to check whether point P lies inside the boundary of the elliptic ring. The mathematical approach chosen for the last case, where d_e has to be obtained through the operation shown in Figure 4.3.8, is given below.

Figure 4.3.8: Situation in plan for determination of d_e

First of all, point P_e , as shown in Figure 4.3.8, lies on the central line of the ellipse in plan and, therefore, satisfies the general equation of the ellipse:

$$\frac{x_e^2}{a_r^2} + \frac{y_e^2}{b_r^2} = 1 \quad \dots\dots\dots 4.3.7$$

Equation 4.3.7 can be solved for y_e :

$$y_e^2 = b_r^2 \left(1 - \frac{x_e^2}{a_r^2} \right)$$

or

$$y_e = b_r \sqrt{1 - \frac{x_e^2}{a_r^2}}$$

or

$$y_e = b_r \left(1 - \frac{x_e^2}{a_r^2} \right)^{1/2} \quad \dots\dots\dots 4.3.8$$

Differentiation of Equation 4.3.8 with respect to x_e serves to find an expression for the slope y'_e of the tangent at point P_e :

$$y_e' = \frac{1}{2}b_r\left(1 - \frac{x_e^2}{a_r^2}\right)^{-1/2}\left(\frac{-2x_e}{a_r^2}\right)$$

or

$$y_e' = -\frac{b_rx_e}{a_r^2}\left(1 - \frac{x_e^2}{a_r^2}\right)^{-1/2}$$

or

$$y_e' = \frac{-b_rx_e}{a_r^2\sqrt{1 - \frac{x_e^2}{a_r^2}}} \dots\dots\dots 4.3.9$$

From this equation, slope m of the line $\overline{PP_e}$, Figure 4.3.8, is given by

$$m = -\frac{1}{y_e'}$$

or

$$m = \frac{a_r^2\sqrt{1 - \frac{x_e^2}{a_r^2}}}{b_rx_e} \dots\dots\dots 4.3.10$$

Therefore, the equation of the line through points P and P_e is given by

$$y_e = y_P + m(x_e - x_P) \dots\dots\dots 4.3.11$$

Here, an expression for the slope m is given by Equation 4.3.10 and the coordinates of point $P(x_P, y_P)$ are known (where P is the point considered for pellevation). On the other hand, the coordinates of point $P_e(x_e, y_e)$ are the unknowns to be determined.

Substitution of Equation 4.3.8 into Equation 4.3.11 yields

$$b_r\sqrt{1 - \frac{x_e^2}{a_r^2}} = y_P + m(x_e - x_P) \dots\dots\dots 4.3.12$$

Also, substituting Equation 4.3.10 into Equation 4.3.12 gives

$$b_r \sqrt{1 - \frac{x_e^2}{a_r^2}} = y_P + \frac{a_r^2 \sqrt{1 - \frac{x_e^2}{a_r^2}}}{b_r x_e} (x_e - x_P) \quad \dots\dots\dots 4.3.13$$

In this equation, x_e is the only unknown entity. However, obtaining the value of x_e from this equation requires the following operations:

First of all, division of the Equation 4.3.13 by the expression

$$\sqrt{1 - \frac{x_e^2}{a_r^2}}$$

yields

$$b_r = \frac{y_P}{\sqrt{1 - \frac{x_e^2}{a_r^2}}} + \frac{a_r^2}{b_r x_e} (x_e - x_P)$$

or

$$b_r - \frac{a_r^2}{b_r x_e} (x_e - x_P) = \frac{y_P}{\sqrt{1 - \frac{x_e^2}{a_r^2}}}$$

Then raising both sides of the equation to the power of two gives

$$b_r^2 - 2b_r \frac{a_r^2}{b_r x_e} (x_e - x_P) + \frac{a_r^4}{b_r^2 x_e^2} (x_e^2 - 2x_e x_P + x_P^2) = \frac{y_P^2}{1 - \frac{x_e^2}{a_r^2}}$$

or

$$b_r^2 - 2a_r^2 + \frac{2a_r^2 x_P}{x_e} + \frac{a_r^4}{b_r^2} - \frac{2a_r^4 x_P}{b_r^2 x_e} + \frac{a_r^4 x_P^2}{b_r^2 x_e^2} = \frac{y_P^2}{1 - \frac{x_e^2}{a_r^2}}$$

or

$$b_r^2 - 2a_r^2 + \frac{2a_r^2 x_P}{x_e} + \frac{a_r^4}{b_r^2} - \frac{2a_r^4 x_P}{b_r^2 x_e} + \frac{a_r^4 x_P^2}{b_r^2 x_e^2} - \frac{y_P^2}{1 - \frac{x_e^2}{a_r^2}} = 0$$

Multiplication of both sides of the equation by the factor $(b_r^2 x_e^2)$ gives

$$b_r^4 x_e^2 - 2a_r^2 b_r^2 x_e^2 + 2a_r^2 b_r^2 x_P x_e + a_r^4 x_e^2 - 2a_r^4 x_P x_e + a_r^4 x_P^2 - \frac{y_P^2 b_r^2 x_e^2}{1 - \frac{x_e^2}{a_r^2}} = 0$$

Multiplication of the above equation by the factor $\left(1 - \frac{x_e^2}{a_r^2}\right)$ gives

$$\left(1 - \frac{x_e^2}{a_r^2}\right) \left(b_r^4 x_e^2 - 2a_r^2 b_r^2 x_e^2 + 2a_r^2 b_r^2 x_P x_e + a_r^4 x_e^2 - 2a_r^4 x_P x_e + a_r^4 x_P^2\right) - y_P^2 b_r^2 x_e^2 = 0$$

or

$$b_r^4 x_e^2 - \frac{b_r^4 x_e^4}{a_r^2} - 2a_r^2 b_r^2 x_e^2 + 2b_r^2 x_e^4 + 2a_r^2 b_r^2 x_P x_e - 2b_r^2 x_P x_e^3 + a_r^4 x_e^2 - a_r^2 x_e^4 - 2a_r^4 x_P x_e + 2a_r^2 x_P x_e^3 + a_r^4 x_P^2 - a_r^2 x_P^2 x_e^2 - y_P^2 b_r^2 x_e^2 = 0$$

and finally separating the expressions containing x_e :

$$\left(2b_r^2 - \frac{b_r^4}{a_r^2} - a_r^2\right)x_e^4 + (2a_r^2 x_P - 2b_r^2 x_P)x_e^3 + (b_r^4 - 2a_r^2 b_r^2 + a_r^4 - a_r^2 x_P^2 - b_r^2 y_P^2)x_e^2 + (2a_r^2 b_r^2 x_P - 2a_r^4 x_P)x_e + a_r^4 x_P^2 = 0 \quad \dots\dots\dots 4.3.14$$

For convenience in computation, the following notation is introduced:

$$r_0 = a_r^4 x_P^2 \quad \dots\dots\dots 4.3.15$$

$$r_1 = 2a_r^2 b_r^2 x_P - 2a_r^4 x_P \quad \dots\dots\dots 4.3.16$$

$$r_2 = b_r^4 - 2a_r^2 b_r^2 + a_r^4 - a_r^2 x_P^2 - b_r^2 y_P^2 \quad \dots\dots\dots 4.3.17$$

$$r_3 = 2a_r^2 x_P - 2b_r^2 x_P \quad \dots\dots\dots 4.3.18$$

$$r_4 = 2b_r^2 - \frac{b_r^4}{a_r^2} - a_r^2 \quad \dots\dots\dots 4.3.19$$

Thus, Equation 4.3.14 can be re-written in the form:

$$r_4 x_e^4 + r_3 x_e^3 + r_2 x_e^2 + r_1 x_e + r_0 = 0 \quad \dots\dots\dots 4.3.20$$

Because this quadric equation is non-symmetric, that is, $r_4 \neq r_0$ and $r_3 \neq r_1$, no close solutions for the equation exist. Several approaches are possible for the solution of Equation 4.3.20, Ref. [13]. These include,

- Graphical solution:

The curve representing the polynomial is drawn and the location of $f(x_e) = 0$ is found graphically where the curve intersects with the x-axis.

- Regula falsi:

Regula falsi is a linear interpolation method where the curve between two starting points $P_1(x_1, f(x_1))$ and $P_2(x_2, f(x_2))$ is replaced by the chord. The next iteration cycle uses the obtained point $P_3(x_3, f(x_3))$ and the one of the starting point with opposite sign to $f(x_3)$. The process is repeated until a close enough solution is found, Figure 4.3.9.

- Newton-Raphson's iteration method:

The curve is replaced by its tangent to the starting point P_1 . The value of x where this tangent intersects with the x-axis serves as the new x in the next cycle of iteration, Figure 4.3.9. The method is a second order iteration method with quadratic convergence, that is, the number of correct digits of the result doubles in each cycle of the iteration.

Regula falsi

Newton-Raphson

Figure 4.3.9: Regular falsi and Newton-Raphson's methods of iteration

In the course of this research, it has been found that the Newton-Raphson's method of iteration is most suitable for the present problem since only one starting value is required and because of the fast convergence of the method. The general equation of Newton-Raphson is given by

$$x_{e,i+1} = x_{e,i} - \frac{f(x_{e,i})}{f'(x_{e,i})} \dots\dots\dots 4.3.21$$

where $x_{e,i}$ denotes a value for the required parameter x_e , and $x_{e,i+1}$ denotes the value of this parameter in the next cycle of iteration. Also, $f(x_{e,i})$ is the function in terms of the parameter x_e and, $f'(x_{e,i})$ is the first derivative of this function. In the present case the function is represented by

$$f(x_e) = r_4 x_e^4 + r_3 x_e^3 + r_2 x_e^2 + r_1 x_e + r_0 \dots\dots\dots 4.3.22$$

and its derivative is

$$f'(x_e) = 4r_4 x_e^3 + 3r_3 x_e^2 + 2r_2 x_e + r_1 \dots\dots\dots 4.3.23$$

The author's investigation in the context of the present problem has shown that the best starting value $x_{e,1}$ for the iteration is given by

$$x_{e,1} = b_r$$

where b_r is the length of the semiminor axis of the central line of the base of the pellevant. From the geometry of the ellipse, Figure 4.3.8, the lower and upper limit of the value of x_e are known to be in the range of:

$$0 \leq x_e \leq a_r$$

For a narrow ellipse, a starting value near the end of the major axis ($x_{e,1} \approx a_r$) leads to problems with the Newton-Raphson's iteration method. In this case, the process shows either no convergence or a convergence towards a wrong solution for x_e . The Newton-Raphson's method of iteration shows this behaviour for any function where the tangent to the curve (Equation 4.3.20) is almost horizontal in the vicinity of $f(x_e)=0$. This

situation occurs for ellipses where the span s of the ring is greater than or equal to twice the minor axis in plan, that is,

$$s \geq 2b_r$$

For this reason, the previously mentioned restriction of $s < 2b_r$ has been introduced. Hapel functions with an elliptic base whose parameters fall within this restriction are excluded from further computation. In any other case, the Newton-Raphson's method of iteration is entered with the starting value of

$$x_{e,1} = b_r$$

and repeated until

$$|x_{e,i+1} - x_{e,i}| \leq 10^{-5}$$

The Newton-Raphson's method converges very fast for the present problem because the starting value is rather well-defined from the geometry of the ellipse. Therefore, the chosen accuracy of 10^{-5} is normally reached with less than five iteration cycles. Using a standard personal computer with a Pentium processor, the configurations presented in this work are obtained within less than one second computation time.

The resulting value of x_e from the iteration process represents the required x-coordinate of point P_e . Finally, the distance d_e between the points P_e and P is found as

$$d_e = \sqrt{(x_P - x_e)^2 + (y_P - y_e)^2}$$

where y_e is obtained by substitution of the value of x_e into Equation 4.3.8. If then

$$d_e < s/2$$

point P lies inside the elliptic ring and is, therefore, subjected to the pellevation process.

Figure 4.3.10: Geometric details for the circular halo pellevation with elliptic base

At this point, the second part of the procedure starts. This part is different for the different types of cross-sections of the ring. In the present case, the circular form of the cross-section, as shown in Figure 4.3.10, is used to obtain the following relationships:

$$\tan \alpha = \frac{s/2}{2r-h} \quad (\text{from triangle } \textcircled{1}\textcircled{2}\textcircled{3})$$

and

$$\tan \alpha = \frac{h}{s/2} \quad (\text{from triangle } \textcircled{2}\textcircled{3}\textcircled{4})$$

From these relations,

$$\frac{s/2}{2r-h} = \frac{h}{s/2}$$

or

$$r = \frac{s^2}{8h} + \frac{h}{2} = \frac{s^2 + 4h^2}{8h} \quad \dots\dots\dots 4.3.24$$

Also,

$$\Delta z = h - r + e$$

where

$$e = \sqrt{r^2 - d_e^2} \quad \dots\dots\dots 4.3.25$$

The new z-coordinate is then given by

$$z_{new} = z_{old} + (h - h_e + z) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - (h - h_e + z) \quad \text{(for push-down)}$$

4.4 Elliptic, Parabolic and Wedge Halo Pellevations with Elliptic Base

There are three further types of halo pellevations with elliptic base. They are the 'elliptic, parabolic and wedge halo pellevations with elliptic base'. In all these cases, the definition of the elliptic ring boundaries in plan is exactly as described for *hapel5* in the previous section. However, in the present cases, the cross-section of the ring is elliptic, parabolic or wedge-shaped. An example of each type to illustrate the general features of the pellevant is given in Figure 4.4.1.

(b)

```

E = rinid(41,30,1,1) | [0,0,0; 0,1,0] #
  rinid(40,31,1,1) | [0,0,0; 1,0,0];
F = hapel(7,20,15,32,15,20,22,10,2.5) | E;
use vm(2), vt(1), vh(-3,-5,5,0,0,0,0,0,1);
clear; draw F;

```

(c)

```

E = rinid(41,30,1,1) | [0,0,0; 0,1,0] #
  rinid(40,31,1,1) | [0,0,0; 1,0,0];
F = hapel(8,20,15,32,15,20,22,10,3) | E;
use vm(2), vt(1), vh(-3,-5,5,0,0,0,0,0,1);
clear; draw F;

```

Figure 4.4.2: Illustrative example of
 (a) elliptic halo pellevation with elliptic base,
 (b) parabolic halo pellevation with elliptic base,
 (c) wedge halo pellevation with elliptic base,

4.4.1 Elliptic, Parabolic and Wedge Halo Pellevation Functions with Elliptic Base

The hapel function together with pellevation codes ± 6 , ± 7 or ± 8 as the first parameter is used to obtain the elliptic, parabolic and wedge halo pellevations with elliptic base, respectively. Figure 4.4.2 shows the details related to the parameters of the elliptic case.

Figure 4.4.2: Details related to the parameters of the elliptic halo pellevation with elliptic base

The hapel function in this mode is given by

Here, the parameters of the function are as outlined for *hapel5* in Section 4.3 except for the last parameter, h_e , that defines the length of the vertical semi-axis of the ellipse as explained for *capel2*. In a similar manner as discussed then, the restriction applies that

$$h \leq h_e$$

The general form of the function in seventh mode is given by

where the details related to the parameters of the function are as shown in Figure 4.4.3 and as discussed for *hapel5*. Also, the general form of the wedge halo pellevation function with elliptic base can be represented as

with the details related to the parameters as shown in Figure 4.4.4.

Figure 4.4.3: Details related to the parameters of the parabolic halo pellevation with elliptic base

Figure 4.4.4: Details related to the parameters of the wedge halo pellevation with elliptic base

4.4.2 Mathematical Formulation

As far as the formulations for *hapel6* to *hapel8* are concerned, the initial part is identical to that discussed for *hapel5* in Section 4.3.2. If the distance d_e between the two points P and P_e is found to be less than half the span of the pellevant, point P is subjected to pellevation. For this purpose, the actual cross-section of the pellevant is considered to obtain the new z -coordinate.

Figure 4.4.5: Geometric details for the elliptic halo pellevation with elliptic base

In the case of *hapel6*, Figure 4.4.5 shows the details that are used to obtain the ellipse equation:

$$\frac{d_e^2}{a^2} + \frac{z^2}{h_e^2} = 1 \quad \dots\dots\dots 4.4.1$$

Substitution of the coordinates of the boundary point $B((s/2), (h_e-h))$ into this equation gives an expression for the unknown parameter a :

$$a = \frac{s/2}{\sqrt{1 - \frac{(h_e - h)^2}{h_e^2}}} \dots\dots\dots 4.4.2$$

Manipulation of Equation 4.4.1 yields an expression for the required value of z :

$$z = h_e \sqrt{1 - \frac{d_e^2}{a^2}} \dots\dots\dots 4.4.3$$

The new z -value of point P inside the scope of the *hapel6* pellevant can finally be obtained as

$$z_{new} = z_{old} + (h - h_e + z) \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - (h - h_e + z) \quad \text{(for push-down)}$$

For mode 7 of the *hapel* function, the parabolic section shown in Figure 4.4.6 serves to obtain an expression for the change in the z -coordinate where the equation of the parabola is given by

$$\Delta z = -ax^2 + h \dots\dots\dots 4.4.4$$

In order to obtain an expression for the unknown parameter a , the coordinates of the boundary point $B((s/2), 0)$ are substituted into Equation 4.4.4:

$$0 = -a(s/2)^2 + h$$

or

$$a = \frac{4h}{s^2} \dots\dots\dots 4.4.5$$

Substitution of the above expression into Equation 4.4.4 yields:

$$\Delta z = h - \frac{4h}{s^2}x^2$$

where x should, in this case, be replaced by d_e :

$$\Delta z = h - \frac{4h}{s^2}d_e^2 \quad \dots\dots\dots 4.4.6$$

The new z -coordinate of point P within the boundaries of the *hapel7* pellevant can then be found as

$$z_{new} = z_{old} + \left(h - \frac{h}{s^2}d_e^2\right) \quad (for\ push-up)$$

$$z_{new} = z_{old} - \left(h - \frac{h}{s^2}d_e^2\right) \quad (for\ push-down)$$

Figure 4.4.6: Geometric details for the parabolic halo pellevation with elliptic base

Finally, in the case of the *hapel8* pellevation, the form of the cross-section as shown in Figure 4.4.7 gives rise to the relationship

$$\frac{h}{s/2} = \frac{\Delta z}{s/2 - d_e} \dots\dots\dots 4.4.7$$

or

$$\Delta z = \frac{h(s/2) - hd_e}{s/2}$$

or

$$\Delta z = h - \frac{2hd_e}{s} \dots\dots\dots 4.4.8$$

Therefore, the new z-coordinate is found by

$$z_{new} = z_{old} + \left(h - \frac{2hd_e}{s} \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h - \frac{2hd_e}{s} \right) \quad (\text{for push-down})$$

Figure 4.4.7: Geometric details for the wedge halo pellevation with elliptic base

4.4.3 Difference between 'Stretched' Circular Base Halo and Elliptic Base Halo Pellevations

It has been mentioned in the previous sections that the central line of the ring of an elliptic base halo pellevant is an ellipse. Note, however, that the *hapel5* to *hapel8* pellevations are **not** the same as 'squeezed' or 'stretched' circular base halo pellevations. To illustrate this fact, consider Figure 4.4.8. Figure 4.4.8(a) shows a 30 by 30 tector after pellevation using an elliptic halo pellevation with **circular base**. On the right, the following features are shown in plan:

- the centre point C_r ,
- the central line of the base as dashed line,
- the inner and outer boundaries of the base as full lines,
- the dimensions a_r and b_r , where $a_r = b_r$ = radius of the circle.

Figure 4.4.8(b) shows the result of a pellevation obtained with the same initial tector using the same *hapel2* pellevation as in the previous example. This time, the pellevated tector has subsequently been 'stretched' to twice its initial length by using a basibifect retronorm. On the right, the corresponding details of the base are given:

- the centre point C_r ,
- the central line of the elliptic base (that is, the 'stretched' circle) as dashed line,
- the inner and outer boundaries of the base as full lines,
- the semi-axes a_r and b_r of the central ellipse where $a_r = 2b_r$.

(a)

```

<> circle <>
a1 = rinid(31,30,1,1) | [-15,-15,0; -15,-14,0] #
    rinid(30,31,1,1) | [-15,-15,0; -14,-15,0];
circle = hapel(2,0,0,9,0,8,1.5,2) | a1;
use vm(2), vt(1), vh(-3,-5,8,0,0,0,0,1);
clear; draw circle;

```

(b)

```

<> Stretched Circle <>
a1 = rinid(31,30,1,1) | [-15,-15,0; -15,-14,0] #
    rinid(30,31,1,1) | [-15,-15,0; -14,-15,0];
circle = hapel(2,0,0,9,0,8,1.5,2) | a1;
ellipse1 = bb(2,1) | circle;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,1);
clear; draw ellipse1;

```

(c)

```

<> Ellipse <>
a2 = rinid(31,30,2,1) | [-30,-15,0; -30,-14,0] #
    rinid(30,31,2,1) | [-30,-15,0; -28,-15,0];
ellipse2 = hapel(6,0,0,18,0,0,9,8,1.5,2) | a2;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,1);
clear; draw ellipse2;

```

Figure 4.4.8: Difference between 'stretched' circular and elliptic base halo pellevation
(a) circular base, (b) 'stretched' circular base, (c) elliptic base

Finally, Figure 4.4.8(c) shows the result of an elliptic halo pellevation with **elliptic base** using a 30 by 30 tector. The parameters defining the *hapel6* pellevant are basically the same as in the previous case: the same centre point (compared to the previous example after using the basibifect retronorm), the same span, the same height, the same vertical semi-axis of the ellipse in section. Also, the points that define the major and minor axes, C_2 and C_3 , have been chosen such that $a_r = 2b_r$ (in harmony with the 'stretching' of the previous example to twice its initial length). Again, the important parameters are shown in plan on the right of the figure.

The common features and the difference between the example in Figure 4.4.8(b) and the one in Figure 4.4.8(c) now become clear and can be described as follows:

- The centre C_r , the length of the semimajor and semiminor axes of the central line of the base (a_r and b_r) are the same.
- The height h of the pellevant at the location of the central line is the same in both cases.
- The span s is **not the same**. It is constant for the entire ring in Figure 4.4.8(c) due to the definition of the elliptic base halo pellevation. This is not the case for the example in Figure 4.4.8(b) since the whole configuration has been 'stretched' to twice its initial length using the basibifect retronorm. This 'stretching' has also affected the span in the x-direction which is now twice as long as the span of the ring in the y-direction. Anywhere in between these two points the span varies from s to $2s$ and does not remain constant as it is for the '*real*' elliptic base halo pellevation.

4.5 Applications

Halo pellevations can be used to obtain curved surfaces by utilising, for instance, major parts of a halo pellevant. Figure 4.5.1 shows an example of this kind where a square finite element mesh has been subjected to a centrally placed *hapell* pellevation. In this case, the span of the ring is equal to the outer radius of the halo pellevant and, therefore, leaves a curved structure with lowered edges and a depression in the central region.

Figure 4.5.1: Example using a major part of a *hapell* pellevant

On the other hand, halo pellevations can also be used to introduce curvature to structures by utilising a small portion of the halo pellevant only. Figure 4.5.2 gives such an example where the top layer of an initially flat double-layer grid acquires its shape through a selective *hapell7* pellevation using a small part of the pellevant as shown in plan on the right.

Figure 4.5.2: Example using a small portion of a *hapel17* pellevant

Another group of practical examples involves the combination of halo pellevation and cap pellevation. Figure 4.5.3(a) gives an example of a round structure which consists of a cylindrical wall with a curved roof on top. The curvature of the roof is obtained through two centrally placed *capell1* and a *hapel2* pellevants. The first cap pellevant has a span that is equal to the inner diameter of the subsequently applied halo pellevant, Figure 4.5.3(a), while the second cap has a span that is equal to the diameter of the entire structure. In this example, *hapel2* is used to obtain a push-up along the outer region of the roof.

Figure 4.5.4(b) gives another example of the combination of halo and cap pellevations. This time, three *hapel2* pellevants of different sizes have been combined with two cap pellevants. The halo pellevants are all placed centrally and arranged such that the outer radius of the smaller halo is equal to the inner radius of the next bigger halo. The boundaries of the halo pellevants are shown in full lines while the central lines of the rings are given in dashed lines, Figure 4.5.4(b). Finally, a small *capell1* pellevation elevates the central region of the structure while a big *capell3* with its boundary well outside the outline of the structure (not shown in the figure) is used to obtain an overall curvature.

(a)

```

a = rinid(18,30,1,1) | [2,0,6; 2,1,6; 3,1,6; 3,0,6];
b = bp(1,360/30) | a;
c = rinit(30,4,1,1) | [1,0,0; 1,0,1; 1,1,1; 1,1,0];
d = bc(20,360/30,1.5) | c;
e = b#d;
f = capel(1,0,0,24,6) | e;
g = capel(1,0,0,40,6) | f;
h = hapel(2,0,0,16,0,8,3,5,5) | g;
use vm(2), vt(1), vh(5,6,8,0,0,0,0,0,1);
clear; draw h;

```

(b)

```

a = rinid(24,30,1,1) | [0,0,6; 0,1,6; 1,1,6; 1,0,6];
b = bp(1.5,360/30) | a;
c = hapel(2,0,0,22,22,8,3,3) | b;
d = hapel(2,0,0,16,16,8,3,3) | c;
e = hapel(2,0,0,10,10,8,3,3) | d;
f = capel(1,0,0,20,6) | e;
g = capel(3,0,0,120,50) | f;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,0,1);
clear; draw g;

```

Figure 4.5.3: Example combining halo and cap pellevations

Chapter 5

Saddle Pellevation

5.0 Introduction

This chapter is concerned with the family of 'saddle pellevations'. Saddle pellevations can be divided into two groups according to the shape of their base with four different cross-sections in each group, Table A.4 in Appendix A. The pellevations are referred to as

- circular saddle,
- elliptic saddle,
- parabolic saddle and
- wedge saddle

pellevations with either a parabolic or a hyperbolic base.

5.1 Circular Saddle Pellevation with Parabolic Base

The first type of saddle pellevation is the 'circular saddle pellevation with parabolic base'. Figure 5.1.1 illustrates the typical shape of the pellevant using a 30 by 30 tector. The cross-section of this first saddle pellevant is circular and its boundaries in plan are described by two parabolic curves that are symmetric with respect to the axis of the saddle.

Figure 5.1.1: Illustrative example of circular saddle pellevation with parabolic base

5.1.1 Circular Saddle Pellevation Function with Parabolic Base

The saddle pellevation function with pellevation code ± 1 as the first parameter is used for the application of the circular saddle pellevation with parabolic base. Further parameters are the coordinates of two points $C_1(cx_1, cy_1)$ and $C_2(cx_2, cy_2)$ which define

the axis of the saddle in plan as shown in Figure 5.1.2. The first point given is taken as the 'waist' of the saddle, that is, the point where the planes of symmetry of the saddle meet in plan. The other parameters are the spans s_1 and s_2 at points C_1 and C_2 and the height h_1 at point C_1 .

Figure 5.1.2: Details related to the parameters of the circular saddle pellevation with parabolic base

The general form of the function is given by

The height at point C_2 as well as the height at any other point n along the axis is obtained by keeping the ratio of s_n/h_n constant. Both spans s_1 and s_2 and the height h_1 have to be positive. The value of the height is further restricted to

$$h_1 \leq s_1/2$$

The reason for this restriction has been explained in detail in Section 2.1 for the *capell* pellevation.

By definition, the boundary of the pellevant in plan is formed by a parabolic curve and its mirror image about the axis of the saddle, Figure 5.1.2. As a result of this definition, some special cases can be derived from the general example shown in Figure 5.5.1 where the span s_2 was greater than the span at the waist s_1 . In this case, the result is a typical saddle where the pellevant is narrow at the waist and of infinite length. However, the following special are possible:

- A first special case is when both spans have the same value, that is, $s_1=s_2$ as shown in Figure 5.1.3(a). The result obtained is exactly the same as for a *bapell* pellevation using the appropriate parameters. The *bapell* solution would, however, be the quicker way for obtaining the same result because only one value for the span is required as parameter.
- Figure 5.1.3(b) shows a second special case where $s_1>s_2$. This case is possible due to the definition of the parabolic base of the pellevant, that is, a parabolic curve and its mirror image. The result is a pellevant which terminates at two points along the axis. These two points are at equal distance from point C_1 and are located where the parabolic curve intersects its mirror image. The termination points may lie outside the configuration, Figure 5.1.3(b), or as shown in Figure 5.1.3(c), inside the configuration. In the latter case, the shape of the pellevant is similar to a lentil.

Figure 5.1.3: Special cases of parabolic base saddle pellevations

5.1.2 Mathematical Formulation

As far as the formulation of the *sapell* pellevation is concerned, an initial translational coordinate transformation is carried out which makes the point C_1 the new origin of the coordinate system, Figure 5.1.4. Here, the coordinates of any point (u'_1, u'_2) are changed into (u_1, u_2) according to the following relationship

$$u_1 = u'_1 - cx_1$$

$$u_2 = u'_2 - cy_1$$

Figure 5.1.4: Translational and rotational coordinate transformations

Subsequently, a rotational coordinate transformation which leaves $\overline{C_1C_2}$ as new global x-axis transforms the coordinates of this point to $P(px, py)$ using the following transformation

$$\begin{bmatrix} px \\ py \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The components of the transformation matrix can be obtained as

$$\begin{aligned} \cos \alpha &= a/d \\ \sin \alpha &= b/d \end{aligned}$$

where

$$\begin{aligned} a &= cx_2 - cx_1 \\ b &= cy_2 - cy_1 \end{aligned}$$

and d , the distance between the points C_1 and C_2 :

$$d = \sqrt{a^2 + b^2}$$

In order to find out whether a point P considered for pellevation lies inside the boundary of the pellevant, the equation of the parabola in plan is used, Figure 5.1.5. That is,

$$y_{Pb} = a_{Pb} x^2 + \frac{s_1}{2} \dots\dots\dots 5.1.1$$

In this equation, a_{pb} is the only unknown parameter. However, point $(d, (s_2/2))$ lies on the parabola and, therefore, satisfies Equation 5.1.1:

$$\frac{s_2}{2} = a_{pb} d^2 + \frac{s_1}{2}$$

or

$$a_{pb} d^2 = \frac{s_2}{2} - \frac{s_1}{2}$$

or

$$a_{pb} = \frac{s_2 - s_1}{2d^2} \dots\dots\dots 5.1.2$$

Figure 5.1.5: Geometric details for the circular saddle pellevation with parabolic base

Having obtained an expression for the parameter a_{pb} enables the use of Equation 5.1.1 to determine whether or not point $P(px,py)$ lies inside the boundary of the pellevant, Figure 5.1.5. For this purpose, the x-coordinate of point P is substituted into Equation 5.1.1. If

$$|py| < y_{Pb}$$

then P lies inside the boundary and its z -value is subjected to change. In order to find the change in the z -coordinate, the cross-section at point P is used as shown in Figure 5.1.5.

Here, the span s is known as $2y_{pb}$. The height at the location of the axis in this section is obtained from the height h_i at point C_i and by keeping the ratio of s_n/h_n constant. Hence,

$$h_{Pb} = h_1 \frac{2|y_{Pb}|}{s_1} \dots\dots\dots 5.1.3$$

The change in the z-coordinate of point P is then obtained in the same manner as shown for *capell* and as given in Figure 5.1.5 where

$$\tan \alpha = \frac{y_{Pb}}{2r - h_{Pb}} \quad (\text{from triangle ①②③})$$

but also

$$\tan \alpha = \frac{h_{Pb}}{y_{Pb}} \quad (\text{from triangle ②③④})$$

From the above relations,

$$\frac{y_{Pb}}{2r - h_{Pb}} = \frac{h_{Pb}}{y_{Pb}}$$

Algebraic manipulation yields

$$r = \frac{y_{Pb}^2}{2h_{Pb}} + \frac{h_{Pb}}{2} = \frac{y_{Pb}^2 + h_{Pb}^2}{2h_{Pb}} \dots\dots\dots 5.1.4$$

Finally, the change in the z-coordinate is given as

$$\Delta z = h_{Pb} - r + e$$

where

$$e = \sqrt{r^2 - py^2} \dots\dots\dots 5.1.5$$

The new z-value of point P is then obtained as

$$z_{new} = z_{old} + (h_{Pb} - r + e) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h_{Pb} - r + e) \quad (\text{for push-down})$$

5.2 Elliptic, Parabolic and Wedge Saddle Pellevations with Parabolic Base

The 'elliptic, parabolic and wedge saddle pellevations with parabolic base' are further types of saddle pellevations with a parabolic base as described in Section 5.1. In these cases, the cross-section of the saddle is elliptic, parabolic or wedge-shaped as shown in Figure 5.2.1.

(a)

```
E = rinid(31,30,1,1) | [0,0,0;0,1,0] #
    rinid(30,31,1,1) | [0,0,0;1,0,0];
F = sapel(2,15,15,30,15,12,30,2,3) | E;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,1);
clear; draw F;
```

(b)

```
E = rinid(31,30,1,1) | [0,0,0;0,1,0] #
    rinid(30,31,1,1) | [0,0,0;1,0,0];
F = sapel(3,15,15,30,15,12,28,4) | E;
use vm(2), vt(1), vh(-3,-5,8,0,0,0,0,1);
clear; draw F;
```

(c)

```

E = rinid(30,31,1,1) | [0,0,0; 1,0,0] #
  rinid(31,30,1,1) | [0,0,0; 0,1,0];
F = sapel(4,15,15,30,15,7,30,2) | E;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,1);
clear; draw F;

```

Figure 5.2.1: Illustrative example of
 (a) elliptic saddle pellevation with parabolic base,
 (b) parabolic saddle pellevation with parabolic base,
 (c) wedge saddle pellevation with parabolic base

5.2.1 Elliptic, Parabolic and Wedge Saddle Pellevation Functions with Parabolic Base

In order to use the elliptic, parabolic or wedge saddle pellevations with parabolic base, the *sapel* function together with pellevation code ± 2 , ± 3 or ± 4 as the first parameter can be used. The remaining parameters of the *sapel* function in these modes are as discussed in Section 5.1 for the *sapel1* pellevation.

Figure 5.2.2: Details related to the parameters of the elliptic saddle pellevation with parabolic base

The general form of the function in the second mode is given by

where the significance of the parameters is shown in Figure 5.2.2. Here, the last parameter h_{e1} denotes the length of the vertical semi-axis of the ellipse at point C_1 . By definition, the ratio of h_{en}/h_n is kept constant for any point n along the axis of the saddle. Also, as for all pellevations with an elliptic cross-section,

$$h_1 \leq h_{e1}$$

in order to limit the push-up or the push-down to half an ellipse.

In the case of the parabolic saddle pellevation with parabolic base, the function in the general form is given by

The parameters are shown in Figure 5.2.3. Finally, for the *sapel4*, the general form of the function is

where the significance of the parameters is given in Figure 5.2.4

Figure 5.2.3: Details related to the parameters of the parabolic saddle pellevation with parabolic base

Figure 5.2.4: Details related to the parameters of the wedge saddle pellevation with parabolic base

5.2.2 Mathematical Formulation

The formulation of the *sapel2*, *sapel3* and *sapel4* pellevations is initially the same as described for *sapel1* in Section 5.1. If any point $P(px,py)$ considered for pellevation is found to lie inside the parabolic boundary in plan, that is,

$$|py| < y_{pb}$$

then the cross-section of the pellevant has to be considered to obtain the change in the z-coordinate. In the case of the elliptic cross-section, Figure 5.2.5, the height of the vertical semi-axis of the ellipse at point P can be found by linear proportion, in a similar way as h_{pb} can be obtained using Equation 5.1.3:

$$h_{e,pb} = h_{e1} \frac{2|y_{pb}|}{s_1} \dots\dots\dots 5.2.1$$

Figure 5.2.5: Geometric details for the elliptic saddle pellevation with parabolic base

In the next step, the equation of the ellipse in section is obtained as

$$\frac{y^2}{a_{Pb}^2} + \frac{z^2}{h_{e,Pb}^2} = 1 \quad \dots\dots\dots 5.2.2$$

where the boundary point $B(y_{Pb}, (h_{e,Pb} - h_{Pb}))$ lies on the ellipse and, therefore, satisfies Equation 5.2.2. That is,

$$\frac{(y_{Pb})^2}{a_{Pb}^2} + \frac{(h_{e,Pb} - h_{Pb})^2}{h_{e,Pb}^2} = 1$$

or

$$a_{Pb} = \frac{y_{Pb}}{\sqrt{1 - \frac{(h_{e,Pb} - h_{Pb})^2}{h_{e,Pb}^2}}} \quad \dots\dots\dots 5.2.3$$

in the same way as explained for *capel2* in Section 2.2. With the expression derived for a_{Pb} , Equation 5.2.2 can be used to obtain an expression for the unknown parameter z :

$$\frac{z^2}{h_{e,Pb}^2} = 1 - \frac{py^2}{a_{Pb}^2}$$

or

$$z = h_{e,Pb} \sqrt{1 - \frac{py^2}{a_{Pb}^2}} \quad \dots\dots\dots 5.2.4$$

The new z -coordinate of point P is given by

$$z_{new} = z_{old} + (z - h_{e,Pb} + h_{Pb}) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (z - h_{e,Pb} + h_{Pb}) \quad (\text{for push-down})$$

In the case of *sapel3*, the cross-section shown in Figure 5.2.6 is used to obtain the parabolic relationship

$$\Delta z = -a py^2 + h_{Pb} \quad \dots\dots\dots 5.2.5$$

Here, the boundary point $B(y_{pb}, 0)$ lies on the parabola and can, therefore, be used to obtain an expression for the unknown parameter a :

$$0 = -a y_{pb}^2 + h_{pb}$$

or

$$a = \frac{h_{pb}}{y_{pb}^2} \dots\dots\dots 5.2.6$$

Substitution of Equation 5.2.6 into Equation 5.2.5 gives an expression for the change in the z-coordinate:

$$\Delta z = h_{pb} - \frac{h_{pb}}{y_{pb}^2} py^2 \dots\dots\dots 5.2.7$$

The new z-value of point P is then given by

$$z_{new} = z_{old} + \left(h_{pb} - \frac{h_{pb}}{y_{pb}^2} py^2 \right) \quad (for \textit{push-up})$$

$$z_{new} = z_{old} - \left(h_{pb} - \frac{h_{pb}}{y_{pb}^2} py^2 \right) \quad (for \textit{push-down})$$

Figure 5.2.6: Geometric details for the parabolic saddle pellevation with parabolic base

Finally, for the wedge saddle pellevation with parabolic base, the following linear relationship is formulated using Figure 5.2.7 :

$$\frac{h_{Pb}}{y_{Pb}} = \frac{\Delta z}{y_{Pb} - py} \dots\dots\dots 5.2.8$$

or

$$\Delta z = \frac{h_{Pb}(y_{Pb} - py)}{y_{Pb}}$$

or

$$\Delta z = h_{Pb} - h_{Pb} \frac{py}{y_{Pb}} \dots\dots\dots 5.2.9$$

The new z-value is then obtained as

$$z_{new} = z_{old} + \left(h_{Pb} - h_{Pb} \frac{py}{y_{Pb}} \right) \quad (for \text{ push-up})$$

$$z_{new} = z_{old} - \left(h_{Pb} - h_{Pb} \frac{py}{y_{Pb}} \right) \quad (for \text{ push-down})$$

Figure 5.2.7: Geometric details for the wedge saddle pellevation with parabolic base

5.3 Circular Saddle Pellevation with Hyperbolic Base

The 'circular saddle pellevation with hyperbolic base', is the first saddle pellevation which is characterised by a hyperbolic boundary in plan. Any hyperbolic boundary consists of two curves, therefore, no mirror image of any curve is required as it was in the case of the parabolic boundary, Section 5.1. The cross-section of the pellevant of the present type of saddle pellevation is circular. A typical tector used to illustrate the general features is shown in Figure 5.3.1.

Figure 5.3.1: Illustrative example of circular saddle pellevation with hyperbolic base

5.3.1 Circular Saddle Pellevation Function with Hyperbolic Base

The *sapel* function in combination with pellevation code ± 5 as the first parameter can be used for the application of the circular saddle pellevation with hyperbolic base. The required details for the parameters of the function in mode 5 are shown in Figure 5.3.2. As for the *sapell* pellevation, they are the x- and y-coordinates of the points $C_1(cx_1, cy_1)$ and $C_2(cx_2, cy_2)$ that define the position and orientation of the axis of the pellevant, spans s_1 and s_2 at points C_1 and C_2 as well as the height h_1 at point C_1 .

Figure 5.3.2: Details related to the parameters of the circular saddle pellevation with hyperbolic base

The general form of the function is given by

As usual, the spans s_1 and s_2 as well as the height h_1 have to be positive. However, a further restriction applies for the spans of all hyperbolic base saddles: the span at the waist of the pellevant s_1 has to be less than or equal to s_2 , that is,

$$s_1 \leq s_2$$

This restriction originates from the hyperbolic nature of the boundary. Also, the value of the height h_i at point C_i is limited to

$$h_1 \leq s_1/2$$

as for all other pellevations with a circular cross-section. The height at any point n along the axis of the pellevant is obtained by keeping the ratio s_n/h_n constant, in the same way as described for *sapel1*.

Since the span s_1 at the waist cannot be greater than the span s_2 , only one special case of the saddle pellevation with hyperbolic base exists, Figure 5.3.3. Here, the values of the spans s_1 and s_2 are identical such that the boundary in plan is represented by two straight lines. Note that the same result could have been obtained using a *sapel1* or, in a more elegant and quicker way, using a *bapel1* pellevation.

Figure 5.3.3: Special case of hyperbolic base saddle pellevations

Generally speaking, the hyperbolic base appears rather similar to the parabolic base. However, one or the other way may be preferred in various contexts.

5.3.2 Mathematical Formulation

For the formulation of the *sapel5* pellevation, a translational and a rotational coordinate transformation are carried out as explained in detail for *sapel1*. In this process, any point considered for the pellevation receives its new coordinates $P(px,py)$. It is further necessary to write the equation of the hyperbola in plan with respect to the newly established coordinate system where C_1 is the new origin and the axis through C_1 and C_2 is the new x-axis. That is,

$$\frac{y_{Hy}^2}{a_{Hy}^2} - \frac{x_{Hy}^2}{b_{Hy}^2} = 1 \quad \dots\dots\dots 5.3.1$$

as shown in Figure 5.3.4. Also, the distance d between the points C_1 and C_2 can be evaluated as

$$d = \sqrt{(cx_2 - cx_1)^2 + (cy_2 - cy_1)^2}$$

Figure 5.3.4: Geometric details for the circular saddle pellevation with hyperbolic base

In order to find an expression for the two unknown parameters a_{Hy} and b_{Hy} in Equation 5.3.1, the coordinates of two points that lie on the hyperbola at the sections through points C_1 and C_2 are used. The coordinates of these points, $(0, (s_1/2))$ and $(d, (s_2/2))$, are substituted into Equation 5.3.1. From the coordinates of the first point, a_{Hy} is found as

$$a_{Hy} = s_1/2 \quad \dots\dots\dots 5.3.2$$

Substitution of the coordinates of the second point yields an expression that can be solved for b_{Hy} :

$$\frac{(s_2/2)^2}{(s_1/2)^2} - \frac{d^2}{b_{Hy}^2} = 1$$

or

$$\frac{s_2^2}{s_1^2} - 1 = \frac{d^2}{b_{Hy}^2}$$

or

$$b_{Hy}^2 = \frac{d^2}{\frac{s_2^2}{s_1^2} - 1}$$

or

$$b_{Hy} = \frac{d}{\sqrt{\frac{s_2^2}{s_1^2} - 1}} \quad \dots\dots\dots 5.3.3$$

Once expressions for the parameters a_{Hy} and b_{Hy} have been obtained, Equation 5.3.1 can be solved for y_{Hy} . That is,

$$\frac{y_{Hy}^2}{a_{Hy}^2} = 1 + \frac{x_{Hy}^2}{b_{Hy}^2}$$

or

$$y_{Hy} = a_{Hy} \sqrt{1 + \frac{x_{Hy}^2}{b_{Hy}^2}} \quad \dots\dots\dots 5.3.4$$

This y-coordinate of the hyperbola, y_{Hy} , can be compared with the y-coordinate of any point $P(px,py)$ considered for pellevation. If

$$|py| < y_{Hy}$$

point P lies inside the boundaries of the pellevant, Figure 5.3.4, and its z-coordinate will be changed using the pellevation process. In this case, the circular cross-section as shown in Figure 5.3.3 is examined. Here, the span at any location along the axis is equal to the distance of the two arms of the hyperbola, that is, $2y_{Hy}$, Figure 5.3.4. Also, the corresponding height h_{Hy} at the location of the axis is determined by keeping a constant ratio of span to height, hence

$$h_{Hy} = h_1 \frac{2y_{Hy}}{s_1} \dots\dots\dots 5.3.5$$

In a manner similar to the case of the *sapell* pellevation, the change in the z-coordinate for the circular section is given by

$$\Delta z = h_{pb} - r + e$$

where

$$r = \frac{y_{Hy}^2 + h_{Hy}^2}{2h_{Hy}} \dots\dots\dots 5.3.6$$

and

$$e = \sqrt{r^2 - py^2} \dots\dots\dots 5.3.7$$

The new z-coordinate of point P can then be obtained as

$$z_{new} = z_{old} + (h_{Hy} - r + e) \qquad (for\ push-up)$$

$$z_{new} = z_{old} - (h_{Hy} - r + e) \qquad (for\ push-down)$$

5.4 Elliptic, Parabolic and Wedge Saddle Pellevations with Hyperbolic Base

Another three saddle pellevations with hyperbolic base have been defined. They are referred to as 'elliptic, parabolic and wedge saddle pellevations with hyperbolic base'. The cross-sections of these pellevants are elliptic, parabolic and wedge-shaped, respectively. The general features of these pellevants can be seen from the illustrative examples shown in Figure 5.4.1.

(a)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = sapel(6,15,15,30,15,14,28,2,2) | E;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,0,1);
clear; draw F;
```

(b)

```
E = rinid(31,30,1,1) | [0,0,0; 0,1,0] #
    rinid(30,31,1,1) | [0,0,0; 1,0,0];
F = sapel(7,15,15,30,15,9,28,2.5) | E;
use vm(2), vt(1), vh(-3,-5,6,0,0,0,0,0,1);
clear; draw F;
```

Figure 5.4.1: Illustrative example of
(a) elliptic saddle pellevation with hyperbolic base,
(b) parabolic saddle pellevation with hyperbolic base,
(c) wedge saddle pellevation with hyperbolic base

5.4.1 Elliptic, Parabolic and Wedge Saddle Pellevation Functions with Hyperbolic Base

The *sapel* function with pellevation code ± 6 , ± 7 or ± 8 as the first parameter is used for the elliptic, parabolic and wedge saddle pellevation with hyperbolic base. In the case of *sapel6*, the general form of the function is

with the significance of the parameters as shown in Figure 5.4.2 and as described for *sapel5* in Section 5.3. The last parameter, h_{e1} , denotes the length of the vertical semi-axis of the ellipse at point C_1 . As for all elliptic cross-sections,

$$h_1 \leq h_{e1}$$

Figure 5.4.2: Details related to the parameters of the elliptic saddle pellevation with hyperbolic base

For the parabolic cross-section, the parameters are shown in Figure 5.4.3 and the general form of the function is given by

Figure 5.4.3: Details related to the parameters of the parabolic saddle pellevation with hyperbolic base

In a similar manner, the parameters for *sapel8* are shown in Figure 5.4.4 and the general form of the function in mode 8 can be written as

Figure 5.4.4: Details related to the parameters of the wedge saddle pellevation with hyperbolic base

5.4.2 Mathematical Formulation

The initial procedure for the mathematical formulation of the elliptic, parabolic and wedge saddle pellevation with hyperbolic base is as outlined in Section 5.3 for the *sapel5* pellevation. If point $P(px,py)$ is found to lie inside the hyperbolic boundaries in plan, that is,

$$|py| < y_{Hy}$$

the cross-section of the pellevant is to be considered to obtain the change in the z-coordinate. In the case of the elliptic cross-section, Figure 5.4.5, the height h_{Hy} and the value of $h_{e,Hy}$ at the section through point P are obtained by proportion:

$$h_{Hy} = h_1 \frac{2y_{Hy}}{s_1} \dots\dots\dots 5.4.1$$

and

$$h_{e,Hy} = h_{e1} \frac{2y_{Hy}}{s_1} \dots\dots\dots 5.4.2$$

Figure 5.4.5: Geometric details for the elliptic saddle pellevation with hyperbolic base

From the elliptic form of the pellevant in section, an expression for the parameter z is obtained:

$$z = h_{e,Hy} \sqrt{1 - \frac{py^2}{a_{Hy}^2}} \dots\dots\dots 5.4.3$$

where

$$a_{Hy} = \frac{y_{Pb}}{\sqrt{1 - \frac{(h_{e,Pb} - h_{Pb})^2}{h_{e,Pb}^2}}} \dots\dots\dots 5.4.4$$

The derivation of the above equation is similar to the one shown for *capel2* in Section 2.2.

The new z -coordinate of point P within the boundaries of the *sapel6* pellevant can finally be obtained as

$$z_{new} = z_{old} + (h_{Hy} - h_{e,Hy} + z) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - (h_{Hy} - h_{e,Hy} + z) \quad (\text{for push-down})$$

In the case of the parabolic cross-section, Figure 5.4.6, the parameter h_{Hy} can be obtained from Equation 5.4.1. The change in the z-coordinate of a point P is then found from the equation of the parabola in section:

$$\Delta z = h_{Hy} - \frac{h_{Hy}}{y_{Hy}^2} py^2 \quad \dots\dots\dots 5.4.5$$

where the derivation of the above expression is similar to the one shown for *capel3* in Section 2.3. The new z-value of point P for the *sapel7* pellevation is given by

$$z_{new} = z_{old} + \left(h_{Hy} - \frac{h_{Hy}}{y_{Hy}^2} py^2 \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h_{Hy} - \frac{h_{Hy}}{y_{Hy}^2} py^2 \right) \quad (\text{for push-down})$$

Finally, in the case of the wedge saddle pellevation, the linear section shown in Figure 5.4.7 is used to obtain the change in the z-coordinate:

$$\Delta z = h_{Hy} - h_{Hy} \frac{|py|}{y_{Pb}} \quad \dots\dots\dots 5.4.6$$

The new z-coordinate of point P within the boundaries of the *sapel8* pellevant is given by

$$z_{new} = z_{old} + \left(h_{Hy} - h_{Hy} \frac{|py|}{y_{Hy}} \right) \quad (\text{for push-up})$$

$$z_{new} = z_{old} - \left(h_{Hy} - h_{Hy} \frac{|py|}{y_{Hy}} \right) \quad (\text{for push-down})$$

Figure 5.4.6: Geometric details for the parabolic saddle pellevation with hyperbolic base

Figure 5.4.7: Geometric details for the wedge saddle pellevation with hyperbolic base

5.5 Applications

Saddle pellevations can generally be used to obtain many types of saddles in a quick and convenient way. Figure 5.5.1(a) shows a hyperbolic base saddle with a circular cross-section. The structure shown has been obtained through a single *sapel5* pellevation where the boundary of the pellevant lies completely outside the configuration as indicated in the plan.

In a similar manner, Figure 5.5.1(b) represents the finite element mesh of a square shell. The configuration has been subjected to a single *sapel6* pellevation with its axis along one diagonal of the structure as shown in the sketch on the right.

The example shown in Figure 5.5.1(c) is based on the same grid as the one used in Figure 5.5.1(a). This time, the grid has been subjected to two *sapel7* pellevations which intersect in the middle at 90°. Due to the 'overlap' of the two saddles in the middle, a 'hump' forms in this region while along the edges of the structure only one saddle pellevant is effective. If the effect of the superimposition is not desired, the method discussed in Section 3.5 (where one quarter of the structure is generated and then replicated) can be used, Figure 5.5.1(d).

Figure 5.5.1: Examples using saddle pellevations

Figure 5.5.2 gives three examples that show some possibilities of combining the saddle pellevation with the concept of novation. To show the effect, the examples all use a simple 30 by 30 tector which has initially been pellevated using the same *sapel2* pellevant as applied in Figure 5.2.1(a).

In the first example, Figure 5.5.2(a), the four corner points of the pellevated configuration have been moved away from the configuration along the diagonals. In the configuration of Figure 5.5.2(b), the same four corner points have been moved inside the configuration, also along the diagonals of the configuration. In the third example, Figure 5.5.2(c), the two sides of the configuration that are parallel to the axis of the saddle pellevant have been translocated to a parabolic curve in plan which follows the same equation as the parabolic boundary of the pellevant.

Figure 5.5.3 'repeats' the examples shown in Figure 5.5.2. However, this time the novation is applied prior to the pellevation. It can be seen that the results are not identical. Due to the way in which both the pellevation and the novation are defined, it does matter in which order the two concepts are applied, Section 2.8.3.

5.5.1 Data Export for Graphical Representation

Whereas Formian is mainly used to generate the geometric information of a structure, there are a number of programs which offer advanced graphical features such as CorelDRAW!™ or AutoCAD®. What can be done with those programs nicely complements the features of Formian. In order to combine the advantages of Formian and the graphical presentation programs, Formian provides a 'Reformat Formex File' option which includes a range of different formats to select from.

Figure 5.5.2: Combination of 1. saddle pellevation, 2. novation

Figure 5.5.3: Combination of 1. novation, 2. saddle pellevation

A frequently used format is the plt-format (HPGL plot format) which has, in this dissertation, been used as the transfer medium between Formian and CorelDRAW!™. For further details on CorelDRAW!™, the reader is referred Ref. [15].

Another very popular format is the universally compatible dxf-format (AutoCAD® format). This format enables, for example, the export of data for a shell structure from Formian to AutoCAD® where it can be rendered photo-realistically. Some rendered pictures have already been presented in Section 2.1.4.

Figure 5.5.5 shows another rendered example which uses the shell structure from Figure 5.5.1(b). For further details on the rendering process within AutoCAD®, the reader is referred to Ref. [16].

Figure 5.5.5: Rendered view of shell structure obtained with AutoCAD®

5.5.2 Data Export for Analysis Purposes

Design often involves a tedious procedure of finding the right configuration with suitable proportions. If a designer has finally come to the point where he or she has found the 'right' configuration, it is in most cases required that the geometric information obtained, for example using Formian, is sent to a suitable analysis package.

The 'Submission Form' option in Formian is available for the analysis packages ABAQUS®, Lusas® and Sap 2000®. If it is necessary to send the information to any other package, the dxf-format in the 'Reformat Formex File' option of Formian can again be used. As far as the data for structural analysis is concerned, the information to be transferred from Formian to an analysis package generally includes

- the nodal coordinates,
- the element connectivity,
- the support conditions, and
- loading

In order to illustrate the application of the data export feature in Formian the pellevated finite element mesh introduced in Figure 5.5.1(b) is used. The geometric information obtained from Formian has been exported as dxf-file (AutoCAD® format) and then analysed using the analysis package SOFiSTiK, Ref. [14]. The individual steps of the entire process are briefly outlined below:

1. Generation of the configuration in Formian where 1 unit of the configuration represents 1 metre of the structure to be analysed:

```
a = {[0,0,0; 0,1,0; 1,0,0], [1,0,0; 0,1,0; 1,1,0]};
b = rinid(8,8,1,1) | a;
saddle = sapel(-6,4,4,8,8,12,20,3,5) | b;
```

2. Reformatting of the formex 'saddle' (formex file *saddle.fmx*) as AutoCAD® file and saving the information under the new format and the new file name *saddle.dxf*.
3. Import of the information into the analysis package SOFiSTiK.
4. Definition of the nodes, elements and their properties within the analysis package.

```
C:\statik\hofmann\test\saddle4.kno 17.01.1999 16:19
$ PROG GENF
$ MONET VERSION 2297 17.01.99
SYST raum ODIV 1000
KNOT NR x y z
1 .000000E+00 -.800000E+01 .517000E+00
2 .100000E+01 -.800000E+01 .134900E+01
```



```

3          .200000E+01      -.800000E+01      .204800E+01
4          .300000E+01      -.800000E+01      .265900E+01
...
78         .300000E+01      .000000E+00      .370000E+01
79         .200000E+01      .000000E+00      .416000E+01
80         .100000E+01      .000000E+00      .459100E+01
81         .000000E+00      .000000E+00      .500000E+01

$ PROG GENF
$ MONET VERSION 2497 28.04.99
$ Material
BETO 1 B 25.0 17.5 2.1 EC 30000. .20 25.0 .000010
STAH 2 BST 500.0 500.0 550.0 ES 210000. .30 78.5 .000012
GRUP NRG 0
QUAD NR K1 K2 K3 K4 MNR MBW D LAGE C CQ
1 11 2 1 - 1 2 .1000 - - -
2 11 1 10 - 1 2 .1000 - - -
3 12 3 2 - 1 2 .1000 - - -
4 12 2 11 - 1 2 .1000 - - -
...
125 78 70 79 - 1 2 .1000 - - -
126 80 71 72 - 1 2 .1000 - - -
127 79 71 80 - 1 2 .1000 - - -
128 80 72 81 - 1 2 .1000 - - -

```

5. Definition of the support conditions.

```

KNOT NR      FIX      DX      DY      DZ
1          YP
2          YP
3          YP
4          YP
...
45         XP
48         XP
55         XP
64         XP

```

6. Definition of the loading (UDL of $q = 20 \times \text{dead load} = 20 \times 2.5 \text{ kN/m}^2 = 50 \text{ kN/m}^2$)

```

$ PROG ASE
$ MONET VERSION 2297      17.01.99
LF 1 FAKT 1.0000 EGZ 20.0000
    BEZ 'Vollast'

```

7. Run the analysis.

8. Output of the analysis (the numerical output is given in Appendix C).

Chapter 6

Raft Pellevation

6.0 Introduction

This chapter introduces the concept of 'raft pellevation'. Raft pellevation differs from the previously introduced types of pellevation in that it does not use any predefined shapes of pellevants. Raft pellevation enables the user to define any required surface as a pellevant where the surface is specified through a number of points.

6.1 The Concept of Raft Pellevation

The concept of raft pellevation is based on the idea of creating a user-defined, tailor-made surface as the pellevant. Consider, for example, the group of 150 points shown in Figure 6.1.1.(b). These points represent a surface which is, in this case, used as the pellevant of a raft pellevation and is applied to an initially planar 30 by 20 tector, Figure 6.1.1(a). The result of this raft pellevation can be seen in Figure 6.1.1(c). In the present context, the surface used as the pellevant is referred to as a 'raft pellevant'.

Theoretically speaking, a raft pellevant is defined by the x-, y- and z-coordinates of at least three points. However, practically speaking, the result of a raft pellevation is greatly influenced by the number of points used. To elaborate, the more points are used, the better defined the surface and, therefore, the better the result. In other words, increasing the number of points that define the raft pellevant improves the description of the surface and results in a more accurate pellevated configuration.

Consider the group of points shown in Figure 6.1.2(a). The surface described here is the same as the one described in Figure 6.1.1(b) but, this time, a group of 300 points is used. In this case, the result of the raft pellevation, Figure 6.1.2(b), is 'smoother' and shows a better 'imprint' of the intended surface.

In order to further improve the result, the process is repeated once more using a raft pellevant with 600 points, Figure 6.1.3(a). The result, Figure 6.1.3(b), has further improved although the differences between Figures 6.1.2(b) and 6.1.3(b) are less obvious than those between Figures 6.1.1(c) and 6.1.2(b). It is, therefore, completely up to the user to decide what degree of accuracy is required. However, computation time should be included in these considerations since it is a significant factor. In the first case, using 150 points, the computation time lies between 1 and 2 seconds on a personal computer with a Pentium processor. In the second case, using 300 points, this time increases to approximately 5 to 6 seconds. In the last case, using 600 points, computation time is about 4 times as long as in the case with the 300 points.

Comparison between the above three examples shows that the differences occur especially along the edges and any regions where the slope changes abruptly. In the present case, the result along the edges is less accurate because the outline of the pellevant is approximately the same as the outline of the tector and, therefore, points for the interpolation / extrapolation of the z-coordinate are only defined to one side of the edge.

(a)

(b)

(c)

Figure 6.1.1: Typical raft pellevation
(a) initial configuration, (b) raft pellevant - 150 points, (c) pellevated configuration

Figure 6.1.2: (a) Pellevant - 300 points, (b) pellevated configuration

Figure 6.1.3 (a) Pellevant - 600 points, (b) pellevated configuration

Figure 6.1.4 shows a comparison between the sections of the pellevated configurations. Figure 6.1.4(a) gives the sections along the far edge where the different examples are shown in different line styles. The second diagram, Figure 6.1.4(b), shows the sections along the middle of the pellevated configurations. In both cases it can be seen that the improvement from the first to the second example (150 points to 300 points) is more significant than the one from the second to the third example (300 points to 600 points).

Figure 6.1.4: Section through pellevated configurations
(a) along far edge, (b) along middle of configuration

6.2 Raft Pellevation Function

The raft pellevation function requires two parameters. The first parameter is the pellevation code. Since there is only one basic raft pellevation, the pellevation code is either 1 to denote push-up or -1 to denote push-down pellevation (for the selective and regional pellevations the conventions as introduced in Sections 2.6.3 and 2.7.3 apply). The second parameter is a formex which contains the points defining the raft surface. These points have to be at least of grade three, that is, x -, y - and z -coordinate, and a minimum of three points has to be given. The general form of the raft pellevation function is given by

In the case of the examples given in Figures 6.1.1 to 6.1.3, the raft pellevation function is of the form

$$F = \text{rapel}(1, \text{raft}) \mid E$$

where E represents the initially planar 30 by 20 tector, F represents the tector after the raft pellevation and raft is a formex representing the group of 150, 300 or 600 nodes for each of the examples. The manner in which the coordinates of these nodes are obtained is completely up to the user. They may result from different functions, they may be the output of an analysis or a test, etc.

6.3 Mathematical Formulation

At the beginning of the mathematical formulation of the raft pellevation, the distance d_n of a point $P(x,y)$ considered for pellevation and any point $P_n(x_n,y_n)$ of the raft pellevant is obtained:

$$d_n = \sqrt{(x - x_n)^2 + (y - y_n)^2} \dots\dots\dots 6.3.1$$

For the evaluation of the change in the z-coordinate a value of z needs to be obtained. In this context, z represents the interpolation / extrapolation of the z-coordinate of the raft surface at the location of P . For this purpose, two cases are distinguished:

Figure 6.3.1: Situation when P coincides with a point of the raft pellevant in plan

Figure 6.3.2: Situation when P does not coincide with a point of the raft pellevant in plan

Firstly, there is the case where the x- and y-coordinates of point P coincide with those of a point P_i defining the raft pellevant, Figure 6.3.1. That is, the distance between the points P and P_i in plan is less than the tolerance:

$$d_1 \leq tol$$

In this case,

$$z = z_1 \quad \dots\dots\dots 6.3.2$$

where z_1 is the z-coordinate of point P_i . In any other case, Figure 6.3.2,

$$z = \frac{z_1/d_1 + z_2/d_2 + z_3/d_3}{1/d_1 + 1/d_2 + 1/d_3} \quad \dots\dots\dots 6.3.3$$

where z_1 is the z-coordinate of the point of the raft pellevant that is closest to P in plan with d_1 denoting the distance between these two points. Correspondingly, z_2 and d_2 are entities similar to z_1 and d_1 but relate to the point of the raft pellevant which is second closest to P , and z_3 and d_3 are those relating to the point of the raft pellevant which is third closest to P .

If the value of z obtained from Equation 6.3.2 or 6.3.3 is greater than zero, that is,

$$z > 0$$

Then the new z-coordinate of point P is obtained as

$$z_{new} = z_{old} + z \quad \text{(for push-up)}$$

$$z_{new} = z_{old} - z \quad \text{(for push-down)}$$

In the case when

$$z \leq 0$$

the pellevant is ineffective and

$$z_{new} = z_{old}$$

6.4 Applications

The applications of the raft pellevation are basically without limits since every shape of pellevant can be created as long as the shape is described by a number of points. It is up to the user to obtain the points with any method and also to decide on the number of points used to describe the surface - always keeping in mind that a number too low might give results that are not accurate enough and a number too high might waste computation time unnecessarily.

Figure 6.4.1 gives an example where a number of points have been used to imitate the effects of a *capell* pellevation with a span of $s=18$ and a height of $h=2$ on an initially planar 20 by 20 tector. The first case uses 100 points, the second case uses 200 points and the final one uses 400 points, Figures 6.4.1(a) to (c). It can be seen from the configuration on the right how the result becomes 'smoother' with the increasing number of points. However, computation time also increases approximately in proportion to the number of points. So, whereas the first examples is obtained in about 1.5 seconds, the second and third raft pellevations take around three and six seconds, respectively.

Figure 6.4.2 shows two examples of lattice grids obtained through different raft pellevations. In both cases, the points defining the raft pellevant are shown on the left while the pellevated configurations which have been obtained from originally planar configurations are given on the right. The formex formulation of the structure is also given below the figure where the formex *raft* has been obtained through a separate formulation which is not shown here.

(a)

(b)

(c)

(d)

```

pointy = '[b,s,h,n:
r = (s*s/(8*h)+h/2); t = asin(s/(2*r));
m = ric | ((sqrt(9+12*n)-3)/6);
e1 = [r,0,0] # lib(i=1,m) | lib(j=1,i*6) | tranit(j*60/i,i*t/m) | [r,0,0]; e2 = bs(1,1,1) | e1;
a1 = rinid(b,b+1,1,1) | [0,0,0; 1,0,0]#rinid(b+1,b,1,1) | [0,0,0; 0,1,0];
a1 = tranid(-b/2,-b/2) | a1; a2 = rapel(1,e2) | a1; a2 = tranid(b/2,b/2) | a2;
use vt(1), vm(2), vh(-4,-3,4,0,0,0,0,10); clear; draw a2];
<> pointy(20,18,2,100); <> pointy(20,18,2,200); <> pointy(20,18,2,400); <>

```

Figure 6.4.1: Raft pellevation examples using
(a) 100 points, (b) 200 points, (c) 400 points, (d) formex formulation

Figure 6.4.2: Further raft pellevation examples

In the latter case, Figure 6.4.2, the outline of the points defining the raft pellevant is smaller than the outline of the tector as indicated by the dashed lines in the picture on the left. In this case, the part of the tector that lies outside the scope of the pellevant is obtained through extrapolation using the points of the raft pellevant. As a result of this, the nodes in the areas outside the raft pellevant (that is, the ones that obtain their z-coordinate through extrapolation) have similar z-coordinates to the nodes along the corresponding edges of the configuration.

Chapter 7

Mandate Pellevation

7.0 Introduction

The present chapter introduces the concept of 'mandate pellevation'. Mandate pellevation allows the creation of a user-defined pellevant by specifying the shape of its surface through an equation.

7.1 The Concept of Mandate Pellevation

The concept of a mandate pellevation is different to the ones described in Chapters 2 to 5 since it does not use any pre-defined pellevants like, for example, the caps for the cap pellevations and the saddles for the saddle pellevations.

Consider the example shown in Figure 7.1.1. The finite element mesh shown has been obtained through a basic mandate pellevation. In this case, the shape of the pellevant is described by the following equation:

$$z = 1.5\sqrt{x}$$

The x-y-z-coordinate axes are indicated in Figure 7.1.1 and the initial mesh before pellevation is shown in dashed lines.

Figure 7.1.1: Typical mandate pellevation

7.2 Mandate Pellevation Function

The formex formulation of the example shown in Figure 7.1.1 is given alongside the perspective view of the configuration where the actual pellevation is obtained through the mandate pellevation function,

`mapel(1,[':x,y,z:z=1.5*(sqrt|x|)'])`

Here, the first parameter is the pellevation code where $pc=1$ denotes the basic push-up and $pc=-1$ denotes the basic push-down pellevation. The second parameter is a string constant of the form

`[':x,y,z:z=1.5*(sqrt|x|)']`

whose value is a 'mandate'. The mandate consists of two parts, that is,

`:x,y,z:`

and

`z=1.5*(sqrt|x|)`

The first part is referred to as the 'register', enclosed between two colons, and the second part is an equation. The entity given on the right-hand side of the equality sign is the expression of the equation. In the present example, the function is of the form

where x , y and z are the variables of the function. In this particular case, x and y are the independent variables and z is the dependent variable.

7.3 Mathematical Formulation

The formulation of the mandate pellevation is based on the consideration of each point of the configuration individually. Here, the equation of the mandate is evaluated and the value of z is computed for any point P considered using a routine which has been implemented in Formian by Yoshihiko Kuroiwa. Then, if

$$z > 0$$

the new z -coordinate of point P is obtained as

$$z_{new} = z_{old} + z \quad (\text{for push-up})$$

$$z_{new} = z_{old} - z \quad (\text{for push-down})$$

otherwise the z -coordinate remains unchanged.

7.4 Applications

The applications of the mandate pellevation are wide-ranging. They include, first of all, any kind of sloping grid or shell structures incorporating a linear relationship between the dependent and the independent variables, Figure 7.4.1.

Furthermore, mandate pellevation is a tool which enables the definition of curved surfaces of many different kinds which cannot be obtained through *capel*, *bapel*, *hapel* and *sapel*. Figure 7.4.2(c) gives such an example. The initially planar surface, Figure 7.4.2(a), has been subjected to a mandate pellevation using a sinusoidal pellevant. The extend of this pellevant in the x - y -direction is actually infinite but a part of it is shown in Figure 7.4.2(b). Figure 7.4.2(d) shows a variation of the pellevated configuration where the result has been subjected to a subsequent *bapel3* pellevation placed at right angle to the sinus curves.

(a)

```

b1 = [0,0,0,0; 2,0,0,0]; b2 = genid(8,8,2,1,1,-1) | b1;
b3 = [0,0,0,0; 1,1,0,0] # [1,1,0,0; 2,0,0,0];
b4 = genid(8,8,2,1,1,-1) | b3;
grid = b2 # b4;
a = mapel(1,[':x,y,z:z=0.2*y']) | grid;
use vm(2), vt(1), vh(-1,0,0,0,0,0,0,1);
clear; draw a;

```

(b)

Figure 7.4.1: Example of linear mandate pellevation
(a) pellevated configuration, (b) part of pellevant used

Figure 7.4.3(d) shows a tector which has been subjected to two mandate pellevations. The first one uses a pellevant which is sinusoidal in the x-direction, Figure 7.4.3(b), and the second one uses a pellevant which is also sinusoidal but in the y-direction, Figure 7.4.3(c). The result has 'dips' where both pellevants have a minimum and 'humps' in the regions of local maximums of both pellevants.

Consider the second mandate pellevation of the present example. The function is of the form

$$\text{mapel}(1,[':x,y,z: z=0.8*\sin|(30*y)+1|'])$$

where the x- and y-coordinates are the dependent variables and the z-coordinate is the dependent variable.

(a)

```
a = rinid(40,20,1,1) | [0,0,0; 0,1,0; 1,1,0;
1,0,0];
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw a;
```

(b)

(c)

```
a = rinid(40,20,1,1) | [0,0,0; 0,1,0; 1,1,0; 1,0,0];
b = mapel(1,['x,y,z:z=sin(40*x)+2']) | a;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw b;
```

(d)

```
a = rinid(40,20,1,1) | [0,0,0; 0,1,0; 1,1,0; 1,0,0];
b = mapel(1,['x,y,z:z=sin(40*x)+2']) | a;
c = bapel(3,0,20,25,20,50,20) | b;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw c;
```

Figure 7.4.2: Example of sinusoidal mandate pellevation
 (a) initial configuration, (b) mandate pellevant,
 (c) pellevated configuration, (d) variation of pellevated configuration

Figure 7.4.3: Example of superimposition of two sinusoidal mandate pellevations
 (a) initial configuration, (b) first mandate pellevant,
 (c) second mandate pellevant, (d) pellevated configuration

However, the variables need not be referred to as x , y and z . In the case of the mandate pellevation, the first two variables of the register are always taken as the independent x - and y -coordinates while the third variable is always taken as the dependent z -coordinate. As a result of this definition,

```
mapel(1,['x,y,z: z=0.8*sin|(30*y)+1|'])
```

and

```
mapel(1,['a,b,c: c=0.8*sin|(30*b)+1|'])
```

give identical results.

Chapter 8

Selective and Regional Pellevations

8.0 Introduction

Any type of pellevation may be applied in its general basic mode or in combination with the special effects referred to as 'selective pellevation' and 'regional pellevation'. These effects have been discussed in detail in Chapter 2 and their applications have been included throughout this work. This chapter is devoted to the explanation of the theory of these effects.

8.1 Selective Pellevation

The term 'selective pellevation' is used to describe a special form of a pellevation function which allows the restriction of the effects of the pellevation to selected nodes only. These selected nodes may, for example, be the nodes of one layer of a multi-layer grid. Details on the application of this concept have been given in Section 2.6.3. Figure 8.1.1(b) shows a double-layer grid that has been subjected to a basic mandate pellevation and Figure 8.1.1(c) shows the same grid after the application of a selective mandate pellevation. The grid before the pellevations is shown in Figure 8.1.1(a).

The way in which the concept of selective pellevation has been defined is the same for all the pellevation functions and can be described as follows: The one-digit pellevation code used for any basic pellevation is extended into a two-digit pellevation code by adding a '1' in front of the code. To elaborate, in the example given in Figure 8.1.1, the pellevation code for the basic mandate pellevation is $pc=1$ whereas the corresponding selective mandate pellevation is obtained through the pellevation code $pc=11$. (Similarly, a negative selective pellevation code must be preceded by a negative sign to indicate push-down pellevation.) Here, the pellevation code denoting the selective pellevation requires the definition of two further parameters which are added at the end of the list. These are, the 'position indicator', un , and the 'value indicator', uv , as discussed in detail in Section 2.6.3.

8.1.1 Mathematical Formulation

For the formulation of the selective pellevation, the following approach has been chosen: Prior to the actual pellevation process, a condition is introduced which checks whether any point P considered for pellevation is a 'selected node'. In this check, the value given by uv is compared with the value of the uniple at position un of point P .

(a)

<> Initial Double-layer Grid <>

```

m=30; n=16; h=-2;
top = rinid(m+1,n,1,1) | [0,0,0; 0,1,0] #
      rinid(m,n+1,1,1) | [0,0,0; 1,0,0];
top = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,0]) | top;
bot = rinid(m+1,n,1,1) | [0,0,h; 0,1,h] #
      rinid(m,n+1,1,1) | [0,0,h; 1,0,h];
bot = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,h]) | bot;
web = rinid(ric(m/2+1),ric(n/2+1),2,2) | [0,0,0; 0,0,h];
grid = top # bot # web;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw grid;

```

(b)

<> Basic Mandate Pellevation <>

```

m=30; n=16; h=-2;
top = rinid(m+1,n,1,1) | [0,0,0; 0,1,0] #
      rinid(m,n+1,1,1) | [0,0,0; 1,0,0];
top = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,0]) | top;
bot = rinid(m+1,n,1,1) | [0,0,h; 0,1,h] #
      rinid(m,n+1,1,1) | [0,0,h; 1,0,h];
bot = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,h]) | bot;
web = rinid(ric(m/2+1),ric(n/2+1),2,2) | [0,0,0; 0,0,h];
grid = top # bot # web;
a = mapel(1,['x,y,z:z=sin(30*x)+1']) | grid;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw a;

```

(c)

<> Selective Mandate Pellevation <>

```

m=30; n=16; h=-2;
top = rinid(m+1,n,1,1) | [0,0,0; 0,1,0] #
      rinid(m,n+1,1,1) | [0,0,0; 1,0,0];
top = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,0]) | top;
bot = rinid(m+1,n,1,1) | [0,0,h; 0,1,h] #
      rinid(m,n+1,1,1) | [0,0,h; 1,0,h];
bot = lux(rinid(ric(m/2),ric(n/2),2,2)|[1,1,h]) | bot;
web = rinid(ric(m/2+1),ric(n/2+1),2,2) | [0,0,0; 0,0,h];
grid = top # bot # web;
b = mapel(1,['x,y,z:z=sin(30*x)+1'],3,0) | grid;
use vm(2), vt(1), vh(-3,-5,4,0,0,0,0,1);
clear; draw b;

```

Figure 8.1.1: Concept of selective pellevation: double-layer grid
(a) before pellevation, (b) after basic pellevation, (c) after selective pellevation

To elaborate, consider the coordinates of point $P(x_p, y_p, z_p, \dots, m_p)$. If un and uv are given as

$$un = 3$$

and

$$uv = 0$$

then point P is considered for pellevation only if

$$|z_p - uv| < tol \quad \dots\dots\dots 8.1.1$$

where tol is the currently set tolerance, which is normally given by

$$tol = 10^{-5}$$

For details on the concept of tolerances the reader is referred to Ref. [6]. Only if Equation 8.1.1 is satisfied, it is checked whether the considered node lies within the boundaries of the pellevant.

8.2 Regional Pellevation

The term 'regional pellevation' is used to describe the form of a pellevation function where the effect of the pellevation is not only restricted by the boundaries of the currently applied pellevant, but is also restricted by the boundaries of a 'region' in plan. This concept has been discussed in detail in Section 2.7.3. Figure 8.2.1(a) shows a grid which has been subjected to a basic *capel4* pellevation. Figure 8.2.1(b) shows the result of a corresponding regional *capel104* pellevation using the same initial grid. This time, the effect of the wedge cap pellevation is restricted to a square region located centrally with respect to the grid.

(a)

```

E = rinid(14,15,1,1) | [0,0,0; 1,0,0] #
  rinid(15,14,1,1) | [0,0,0; 0,1,0];
E = E # rinid(14,14,1,1) | {[0,0,0; 1,1,0], [1,0,0; 0,1,0]};
F = capel(4,7,7,13,3) | E;
use vt(1), vm(2), vh(-3,-5,6,0,0,0,0,1);
clear; draw F;

```

(b)

```

E = rinid(14,15,1,1) | [0,0,0; 1,0,0] #
  rinid(15,14,1,1) | [0,0,0; 0,1,0];
E = E # rinid(14,14,1,1) | {[0,0,0; 1,1,0], [1,0,0; 0,1,0]};
rect = {[3,3], [11,11]};
F = capel(104,7,7,13,3,~{2,rect}~) | E;
use vt(1), vm(2), vh(-3,-5,6,0,0,0,0,1);
clear; draw F;

```

Figure 8.2.1: Concept of regional pellevation: grid subjected to
(a) basic cap pellevation, (b) regional cap pellevation

The way in which the regional pellevation works is as follows: The basic one-digit pellevation code is extended by placing '10' before it. In other word, the basic pellevation code $pc=4$ used in Figure 8.2.1(a) is changed into $pc=104$ to denote a regional wedge cap pellevation in Figure 8.2.1(b). Correspondingly, a negative

pellevation code $pc=-4$ is changed into $pc=-104$. Also, one parameter is added at the end of the list of parameters. This last parameter is referred to as a 'region plenix'.

Furthermore, if it is required to combine the concepts of selective and regional pellevation, the pellevation code is extended by placing a '11' in front of it. For example, a basic pellevation code $pc=6$ is changed into $pc=116$ and $pc=-6$ is changed into $pc=-116$. In this case, un , uv and the region plenix are added at the end of the list of parameters.

As explained in Section 2.7.3, there are five types of regions available each of which is defined by a region specifier, rs , and a number of points. For the full details on these regions and the definition of the points, the reader is referred to Section 2.7.3. The region specifiers and the corresponding regions are,

- 1 - ambit region
- 2 - rectangular region
- 3 - circular region
- 4 - elliptic region
- 5 - parabolic region

The combination of the region specifier and the list of nodes defining the region is referred to as the 'region plenix'. In the case of the example in Figure 8.2.1(b), the region specifier is $rs=2$ to specify a rectangular region and the two points that define the region in plan are $P_1(3,3)$ and $P_2(11,11)$. The region plenix in this case is given by

$$\sim\{2,\{[3,3],[11,11]\}\}\sim$$

8.2.1 Mathematical Formulation

In order to apply the regional pellevation, prior to the actual pellevation process it is checked whether point $P(x_p, y_p)$ considered for pellevation lies inside the regional

boundaries in plan. Only if this is the case, point P is considered further to check whether it also lies inside the boundaries of the pellevant. The manner in which this check is carried out depends on the type of the region.

8.2.1.1 Ambit Region

The ambit region is a region that is denoted by the region specifier $rs=1$ and is defined by a number of vertex points, Figure 8.2.2. These points have to be given in a clockwise order and have to define a convex shape, Section 2.7.3.1. The following procedure is used to check whether a given point P lies inside the ambit region:

Figure 8.2.2: Typical ambit region

First of all, a so-called 'parameter array', pa , is set up. This two-dimensional array has as many rows as there are points defining the ambit region plus one. The number of columns of this array is equal to two in catering for the x- and y-coordinates of the vertex points. For the example of Figure 8.2.2, the parameter array consists of five rows (four points plus one) and two columns (x- and y-coordinates). Here, each row relates to a vertex point, in the order of their appearance in the region plenix. In the last row of the array the information given in the first row is repeated. This is done in order

to 'close' the ambit region for computation purposes. In general terms, the parameter array pa is given by

$$pa = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{bmatrix} \dots\dots\dots 8.2.1$$

All the points of the configuration to be pellevated are then 'taken one by one' and entered into the next step of the procedure. Here, the equation of the straight line through the first two points of the boundary of the region, that is $\overline{P_1P_2}$, is set up using the first two lines of the parameter array.

Using determinants, it is checked if any point $P(x_p, y_p)$ of the configuration lies to the **right** of the line $\overline{P_1P_2}$ in plan (looking from point P_1 towards point P_2), Figure 8.2.2. That is, it is checked whether

$$\begin{vmatrix} x_p & y_p & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \leq 0 \dots\dots\dots 8.2.2$$

If Equation 8.2.2 is satisfied, point P lies to the right of line $\overline{P_1P_2}$ in plan and the procedure is repeated for the remaining points of the parameter array. In other words, the equations of the lines $\overline{P_2P_3}$, $\overline{P_3P_4}$, ... $\overline{P_{n-1}P_n}$ and $\overline{P_nP_1}$ are considered and the checks are carried out. The procedure is interrupted if the condition is not satisfied for any one of these lines because point P is then found to lie outside the boundaries of the ambit region. However, if the condition is satisfied for all the lines, P is inside the region in plan.

The last one of these checks, considering the line $\overline{P_nP_1}$, will check the straight line from the last vertex point P_n back to the first point P_1 . It is for this purpose that the first row of the parameter array has been repeated as the last row as mentioned earlier.

At this stage, the restriction that the outline of the ambit region has to have a convex shape can be considered and discussed. As a start, consider the region shown in Figure 8.2.3. The outline of this region is said to have a non-convex shape and, therefore, is not accepted as an ambit region.

Figure 8.2.3: Typical non-convex region

To elaborate, consider Figure 8.2.4(a). In carrying out the above described check using the two lines $\overline{P_2P_3}$ and $\overline{P_3P_4}$ wrong results may be obtained by cutting the actually wanted ambit region to a portion denoted by the shaded area. The reason for this is, that a number of points of the non-convex region lie to the left of the two lines $\overline{P_2P_3}$ and $\overline{P_3P_4}$ and are, therefore, considered to lie outside the region. If, however, a region of the shape shown in Figure 8.2.3 is wanted, it can be obtained by defining two convex ambit regions, for example, $P_1P_2P_3$ and $P_1P_4P_5$, Figure 8.2.4(b).

Figure 8.2.4: Definition of non-convex regions for regional pellevation

Finally, if point P considered is found to lie inside the ambit region, that is, if the entire process of checking the position of P relative to the boundary lines has been completed successfully, the point is forwarded for the actual pellevation process.

8.2.1.2 Rectangular Region

The rectangular region is denoted by the region specifier $rs=2$. The region is defined by the x - and y -coordinates of two diametrically opposite corners of the rectangle as shown in Figure 8.2.5.

Figure 8.2.5: Typical rectangular region

The formulation of the rectangular region is treated as a special case of the ambit region. Here, the two corner points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are given and the remaining two vertex points can be deduced accordingly. In order to set up the parameter array pa , two cases are distinguished.

When $(x_1 \leq x_2 \text{ and } y_1 \leq y_2)$ or $(x_1 \geq x_2 \text{ and } y_1 \geq y_2)$, Figures 8.2.6(a) and (b), then:

$$pa = \begin{bmatrix} x_1 & y_1 \\ x_1 & y_2 \\ x_2 & y_2 \\ x_2 & y_1 \\ x_1 & y_1 \end{bmatrix} \dots\dots\dots 8.2.3$$

otherwise, Figures 8.2.6(c) and (d):

$$pa = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_1 \\ x_2 & y_2 \\ x_1 & y_2 \\ x_1 & y_1 \end{bmatrix} \dots\dots\dots 8.2.4$$

The parameter array pa and the remainder of the procedure are carried out in the same way as described for the ambit region in Section 8.2.1.1

Figure 8.2.6: Diametrically opposite corner points of rectangular region

Alternatively, the formulation could be done in terms of the vertical and horizontal lines through the corner points where, for example, in the case of Figure 8.2.6(a) a point $P(x_P, y_P)$ is considered to lie inside the rectangular region if

$$x_1 \leq x_P \leq x_2 \dots\dots\dots 8.2.5$$

and also

$$y_1 \leq y_P \leq y_2 \dots\dots\dots 8.2.6$$

In the other cases (Figures 8.2.6(b) to 8.2.6(d)), Equations 8.2.5 and 8.2.6 are adjusted accordingly.

8.2.1.3 Circular Region

The circular region is denoted by the region specifier $rs=3$ and is defined by two points, P_1 and P_2 , Figure 8.2.7. The first point, P_1 , is the centre and the second point, P_2 , is a point on the circumference of the circular region. From the coordinates of these points, the radius r of the region can be obtained as

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \dots\dots\dots 8.2.7$$

For each point $P(x_p, y_p)$ of the configuration it is checked whether

$$d \leq r$$

where

$$d = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2} \quad \dots\dots\dots 8.2.8$$

denotes the distance between point P and the centre. If d is less than the radius, point P lies inside the circular boundary and is further considered for pellevation.

Figure 8.2.7: Typical circular region

8.2.1.4 Elliptic Region

The elliptic region is denoted by the region specifier $rs=4$. The position and shape of the region are defined by three points P_1 , P_2 and P_3 , Figure 8.2.8. The first point P_1 defines the centre of the elliptic region and the points P_2 and P_3 define the semimajor and semiminor axes of the region. The order in which these axes are given is arbitrary.

In the case of the elliptic region, a translational and a rotational coordinate transformation are carried out. These transformations will result in point P_1 to become the new origin of the global coordinate system and the major axis to become the new x-axis. Details of the transformations are similar to the ones shown in Section 2.5 for *capel5*.

For any point P of the configuration, the following relationship is examined:

$$\frac{x_P^2}{a^2} + \frac{y_P^2}{b^2} \leq 1 \quad \dots\dots\dots 8.2.9$$

where a and b denote the length of the semimajor and semiminor axes, respectively. If Equation 8.2.9 is satisfied, point P lies inside the elliptic boundary and is considered for the pellevation process.

Figure 8.2.8: Typical elliptic region

8.2.1.5 Parabolic Region

The parabolic region is denoted by the region specifier $rs=5$. A parabolic region is defined by the x- and y-coordinates of three points P_1 , P_2 and P_3 , Figure 8.2.9. Here, the first two points P_1 and P_2 define two points on the parabola which lie at equal distances from the axis of the parabola ($\overline{P_1P_2}$ is perpendicular to the axis of the parabola). Point P_3 is any arbitrary point on the parabola (other than P_1 and P_2). The parabolic region is the only region that is open on one side and does not define a closed area as did the previously discussed regions.

In order to find out whether any point P of the configuration lies inside the parabolic region, a translational and a rotational coordinate transformation are carried out which make the midpoint M between the points P_1 and P_2 the new origin and the line $\overline{MP_1}$ the new x-axis, Figure 8.2.10, where the coordinates of point $M(x_M, y_M)$ are given by

$$x_M = \frac{x_2 + x_3}{2} \dots\dots\dots 8.2.10$$

and

$$y_M = \frac{y_2 + y_3}{2} \dots\dots\dots 8.2.11$$

Details of the coordinate transformations are similar to the ones shown in Figure 3.1.3 in Section 3.1, where C_1 corresponds with M and C_2 corresponds with P_1 in the present problem.

Figure 8.2.9: Typical parabolic region

Figure 8.2.10: Details for formulation of parabolic region

The general equation of the boundary of the parabolic region with respect to the new coordinate system is given by

$$y = a_P x^2 + b_P \quad \dots\dots\dots 8.2.12$$

In this equation, a_P and b_P are the unknown parameters. An expression for b_P is found by substituting the new coordinates of point $P_1(x_{1,new}, y_{1,new})$ into Equation 8.2.12. That is,

$$y_{1,new} = a_P x_{1,new}^2 + b_P$$

where, the new coordinates of point P_1 (after the transformations) are

$$\begin{aligned} x_{1,new} &= \frac{d}{2} \\ y_{1,new} &= 0 \end{aligned}$$

Therefore,

$$0 = a_P \left(\frac{d}{2}\right)^2 + b_P$$

or

$$b_P = -a_P \left(\frac{d}{2}\right)^2 \quad \dots\dots\dots 8.2.13$$

Substitution of Equation 8.2.13 into Equation 8.2.12 and replacement of x and y by the new coordinates of the point P_3 yields

$$y_{3,new} = a_P x_{3,new}^2 - a_P \left(\frac{d}{2}\right)^2$$

or

$$y_{3,new} = a_P \left(x_{3,new}^2 - \left(\frac{d}{2}\right)^2 \right)$$

or

$$a_P = \frac{y_{3,new}}{\left(x_{3,new}^2 - \left(\frac{d}{2}\right)^2 \right)} \dots\dots\dots 8.2.14$$

Finally, substitution of the above expression back into Equation 8.2.12 yields the value of b_P .

Once all the parameter of the equation describing the boundary of the parabolic region are found, the coordinates of any point $P(x_P, y_P)$ are used in Equation 8.2.12 to determine whether point P lies inside the parabolic region. For this purpose, two cases have to be distinguished:

When $a_P \geq 0$ then

$$x_P \geq a_P y_P^2 + b_P \dots\dots\dots 8.2.15$$

and when $a_P < 0$ then

$$x_P \leq a_P y_P^2 + b_P \dots\dots\dots 8.2.16$$

In either of the above cases point P lies inside the parabolic region and is forwarded for pellevation.

Chapter 9

Conclusions and Suggestions for Future Work

9.0 Summary

In this work, a new concept for the shaping of grid and continuous shell structures has been introduced and presented through a number of illustrative examples. The new concept, referred to as 'pellevation', has been formulated in mathematical terms and the related 'pellevation functions' have been implemented in the environment of the interactive programming language Formian. In the following sections the conclusions together with the ideas for possible future work are discussed.

9.1 Summary and Conclusions

The new concept of pellevation operates on the principle of changing the z-coordinates of points of a configuration while preserving any other feature such as member connectivity. The z-coordinates are changed in the process according to the shape of a pellevant, that is, a geometric object used to push up or push down a part of the initial configuration. A pellevation superimposes with the effects of any other type of pellevation as well as those of any other functions.

The present work has shown that pellevation is a powerful conceptual tool which offers convenience of application and allows the creation of many space structure forms with ease and elegance using concise formulations.

Pellevations have been established in a basic mode which covers a vast number of practical examples. However, the additional selective and regional features further enrich the basic concept and broaden the field of applications.

Chapter 1 sets the environment for this area of research and briefly outlines the concept of pellevation. Chapter 2 establishes the first type of pellevation referred to as cap pellevation. This first family of pellevations is used to derive and explain the new concept in detail and to bring out many special features and particulars of the pellevations. These are, for instance, the change of element lengths, and the distortion and possible warping of finite elements.

Also, the possibilities of inclined pellevation, multi-layer applications and the concept of selective and regional pellevations are introduced through a number of illustrative examples. The combination of the concept of pellevation with that of the novation function is presented at the end of the chapter.

Chapter 3 is devoted to the family of barrel pellevations which operate on the same principles as the cap pellevations but may be used to obtain various cylindrical forms. Many practical examples can be obtained by combining the cap and barrel pellevations.

Chapter 4 establishes the family of halo pellevations. Whereas the first four halo pellevations, namely the ones with a circular base, can be formulated using a close mathematical solution, the halo pellevations with an elliptic base do not have a close solution. In this latter case, an iterative approach has been adopted. Particular attention is paid to the elliptic base halo pellevations and the conventions and special cases are fully discussed using different illustrative examples.

In Chapter 5, the family of saddle pellevations is introduced and explained in detail. Saddle pellevations serve to create many commonly used saddle shapes in a quick and convenient way. It is further demonstrated how data export for graphical and analysis purposes works and the export of data and the analysis of an elliptic saddle concrete shell are presented.

Chapters 6 and 7 are devoted to the raft and mandate pellevations which are two types of pellevations that enable the users to define their own surfaces by either describing a geometric object through a number of points or by giving the mathematical equation of the object's surface.

Finally, Chapter 8 gives the details of the selective and regional pellevations which have been introduced in Chapter 2 and used throughout the work. Chapter 8 discusses the mathematical background of these special features and their computer implementation.

9.2 Suggestions for Future Work

A wide range of pellevations have been introduced in this work. However, there are of course many other possible types of pellevations. One type of pellevation referred to as 'facet pellevation' has already been initiated on the basis of what has been done in Ref. [17]. However, the facet pellevation has not been established as a general pellevation function in this work.

Another point of interest and possible field of future work is given in the solution of the raft pellevation function. The proposed method of interpolation / extrapolation is not the only possible solution and different approaches for the interpolation / extrapolation of the z-coordinates of the points could be established. The different approaches may, for example, be implemented as new modes of the raft pellevation function. Additionally, the inclusion of randomly generated raft points for the generation of a raft pellevant would add an interesting feature to the general concept.

Also, the regional pellevation using ambit regions can only be solved for convex regions. However, there might be the possibility of accepting non-convex ambit regions as a single region. A mathematical formulation could possibly be established for this case in order to drop the currently imposed restriction.

Another presently imposed restriction is related to the elliptic base halo pellevations where the span of the ring has to be less than twice the minor axis. Some investigation suggests that genetic algorithm may be used to solve the so far disallowed cases. Further work in this field might reveal a universally valid solution to the problem discussed in Chapter 4.

Furthermore, genetic algorithm could be used as a kind of post-processor of the pellevation to regularise member lengths and the normally distorted elements of a finite element mesh which result from the pellevation process, Ref. [19].

Appendix A

Basic Pellevation Functions

Cross-Section of Pellevant	Base of Pellevant	General Form of Function (Basic Mode)
circular	circular	$\text{capel}(\pm 1, cx_1, cy_1, s, h)$
elliptic	circular	$\text{capel}(\pm 2, cx_1, cy_1, s, h, h_e)$
parabolic	circular	$\text{capel}(\pm 3, cx_1, cy_1, s, h)$
wedge	circular	$\text{capel}(\pm 4, cx_1, cy_1, s, h)$
circular	elliptic	$\text{capel}(\pm 5, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, h)$
elliptic	elliptic	$\text{capel}(\pm 6, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h, h_e)$
parabolic	elliptic	$\text{capel}(\pm 7, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h)$
wedge	elliptic	$\text{capel}(\pm 8, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h)$

Table A.1: Cap pellevation functions

Cross-Section of Pellevant	Base of Pellevant	General Form of Function (Basic Mode)
circular	rectangular	$\text{bapel}(\pm 1, cx_1, cy_1, cx_2, cy_2, s, h)$
elliptic	rectangular	$\text{bapel}(\pm 2, cx_1, cy_1, cx_2, cy_2, s, h, h_e)$
parabolic	rectangular	$\text{bapel}(\pm 3, cx_1, cy_1, cx_2, cy_2, s, h)$
wedge	rectangular	$\text{bapel}(\pm 4, cx_1, cy_1, cx_2, cy_2, s, h)$
circular	trapezoidal	$\text{bapel}(\pm 5, cx_1, cy_1, cx_2, cy_2, s_1, s_2, h_1, h_2, h_{e1})$
elliptic	trapezoidal	$\text{bapel}(\pm 6, cx_1, cy_1, cx_2, cy_2, s_1, s_2, h_1, h_2)$
parabolic	trapezoidal	$\text{bapel}(\pm 7, cx_1, cy_1, cx_2, cy_2, s_1, s_2, h_1, h_2)$
wedge	trapezoidal	$\text{bapel}(\pm 8, cx_1, cy_1, cx_2, cy_2, s_1, s_2, h_1, h_2)$

Table A.2: Barrel pellevation functions

Cross-Section of Pellevant	Base of Pellevant	General Form of Function (Basic Mode)
circular	circular	$\text{hapel}(\pm 1, cx_1, cy_1, cx_2, cy_2, s, h)$
elliptic	circular	$\text{hapel}(\pm 2, cx_1, cy_1, cx_2, cy_2, s, h, h_c)$
parabolic	circular	$\text{hapel}(\pm 3, cx_1, cy_1, cx_2, cy_2, s, h)$
wedge	circular	$\text{hapel}(\pm 4, cx_1, cy_1, cx_2, cy_2, s, h)$
circular	elliptic	$\text{hapel}(\pm 5, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h)$
elliptic	elliptic	$\text{hapel}(\pm 6, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h, h_c)$
parabolic	elliptic	$\text{hapel}(\pm 7, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h)$
wedge	elliptic	$\text{hapel}(\pm 8, cx_1, cy_1, cx_2, cy_2, cx_3, cy_3, s, h)$

Table A.3: Halo pellevation functions

Cross-Section of Pellevant	Base of Pellevant	General Form of Function (Basic Mode)
circular	parabolic	$\text{sapel}(\pm 1, cx_1, cy_1, cx_2, cy_2, s, h)$
elliptic	parabolic	$\text{sapel}(\pm 2, cx_1, cy_1, cx_2, cy_2, s, h, h_c)$
parabolic	parabolic	$\text{sapel}(\pm 3, cx_1, cy_1, cx_2, cy_2, s, h)$
wedge	parabolic	$\text{sapel}(\pm 4, cx_1, cy_1, cx_2, cy_2, s, h)$
circular	hyperbolic	$\text{sapel}(\pm 5, cx_1, cy_1, cx_2, cy_2, s, h)$
elliptic	hyperbolic	$\text{sapel}(\pm 6, cx_1, cy_1, cx_2, cy_2, s, h, h_c)$
parabolic	hyperbolic	$\text{sapel}(\pm 7, cx_1, cy_1, cx_2, cy_2, s, h)$
wedge	hyperbolic	$\text{sapel}(\pm 8, cx_1, cy_1, cx_2, cy_2, s, h)$

Table A.4: Saddle pellevation functions

Definition of Pellevant	General Form of Function (Basic Mode)
<div> <p>points</p></div>	$\text{rapel}(\pm 1, \{[x_1, y_1, z_1], [x_2, y_2, z_2], [x_3, y_3, z_3], \dots [x_n, y_n, z_n]\})$
<div> <p>equation</p></div>	$\text{mapel}(\pm 1, '[:x, y, z:z=f(x, y)]')$

Table A.5: Raft and mandate pellevation functions

Appendix B

Description of Peilevation Files

Formian is a program that is continually improved and updated through the addition of new functions. The pellevation functions introduced in this work have also been made available as standard functions within Formian. A general description of the method for the addition of new standard functions to the Formian software is given in Ref. [18].

In the present case, all pellevation functions have been defined in the file *pellevat.c* (code given at the end of this appendix). This c-file includes the functions *capel*, *bapel*, *hapel*, *sapel*, *rapel* and *mapel*. Within these functions, several other functions are called depending on the pellevation code.

In the case of the cap pellevation, the related functions have been saved in the file *p_capel.c*. These functions are:

<i>c1_4</i> and <i>c5_8</i>	for pellevation codes ± 1 to ± 4 and ± 5 to ± 8 referring to the basic cap pellevations
<i>c11_14</i> and <i>c15_18</i>	for pellevation codes ± 11 to ± 14 and ± 15 to ± 18 referring to the selective cap pellevations
<i>c101_104</i> and <i>c105_108</i>	for pellevation codes ± 101 to ± 104 and ± 105 to ± 108 referring to the regional cap pellevations
<i>c111_114</i> and <i>c115_118</i>	for pellevation codes ± 111 to ± 114 and ± 115 to ± 118 referring to the selective and regional cap pellevations

In the same way, the barrel, halo and saddle pellevations have been organised where the functions have been stored in the files *p_bapel.c*, *p_hapel.c* and *p_sapel.c*, respectively. The name of the functions corresponds to the pattern outlined for the cap pellevation where *b1_4* and *b5_8* execute the basic barrel pellevations, *h1_4* and *h5_8* execute the basic halo pellevations, *s1_4* and *s5_8* execute the basic saddle pellevations, etc.

Similarly, the raft pellevation function calls further functions which are stored in the file *p_rapel.c*. These are,

<i>rlu11</i>	for pellevation codes ± 1 and ± 11 referring to the basic and selective raft pellevations
<i>rl01u111</i>	for pellevation codes ± 101 and ± 111 referring to the regional and the selective and regional raft pellevations

In the same manner, the functions *mlu11* and *ml01u111* are stored in the file *p_mapel.c* for the mandate pellevations, that is,

<i>mlu11</i>	for pellevation codes ± 1 and ± 11 referring to the basic and selective mandate pellevations
<i>ml01u111</i>	for pellevation codes ± 101 and ± 111 referring to the regional and the selective and regional mandate pellevations

The overall tree-like organisation of the files and functions is illustrated in Figure B.1.

In addition to the above described files, there is a file *region.c* which contains all the region checks for the regional pellevations. The functions in this file are named

<i>region1</i>	for the ambit region
<i>region2</i>	for the rectangular region
<i>region3</i>	for the circular region
<i>region4</i>	for the elliptic region
<i>region5</i>	for the parabolic region

The formulations of the equations in the implementation of the pellevation functions use the same notation as introduced in the sections on the mathematical formulation for each type of pellevation. For instance, the height of the pellevant is always referred to as h (or h_1 or h_2) and the span is always referred to as s (or s_1 or s_2) etc. Where appropriate, comments are added to the code.

The error messages that are related to the pellevation functions are stored in file *fferror.c* along with the error messages for all the other standard functions. The error messages for the pellevation functions have been allocated case 2000.

All the pellevation functions have been included in the *ffunc.c* file in two places where the integer functions are declared as follows:

```
...
int tran(), rin(), ref(), lam(), ver(), ros(), proj(), dil(),
  tranid(), rinid(), refid(), lamid(), projid(), dilid(),
  tranis(), rinis(), refis(), lamis(), projis(), dilis(),
  tranit(), rinit(), refit(), lamit(), projit(), dilit(),
  tranix(), rinix(), refix(), lamix(), projix(), dilix(),
  tranad(), rinad(), refad(), lamad(), verad(), rosad(), projad(), dilad(),
  tranas(), rinas(), refas(), lamas(), veras(), rosas(), projas(), dilas(),
  tranat(), rinat(), refat(), amat(), verat(), rosat(), projat(), dilat(),
  tranax(), rinax(), refax(), lamax(), verax(), rosax(), projax(), dilax(),
  tranic(), rinic(), refic(), lamic(), projic(), dilic(),
  genis(), genic(), genid(), genit(),
  pan(), dep(), vin(), lat(),
  lux(), col(), nex(), con(), pac(), cop(), rel(), dic(),
  red(), nov(), nnov(), tnov(), ras(), rav(), tec(), ram(),
  rf_bb(), rf_mb(), rf_bt(), rf_mt(), rf_bp(), rf_mp(), rf_bc(), rf_mc(),
  rf_bs(), rf_ms(), rf_ba(), rf_ma(), rf_bu(), rf_mu(),
  rf_db(), rf_dt(), rf_dp(), rf_dc(), rf_ds(), rf_da(), rf_bhp(), rf_bh(),
  glisic(), glisid(), glisis(), glisit(),
  rf_bd(), sbc(), sbd(), sbs(),
  br(), pol(), rhom(), apol(), pos(), pod(), mep(),
  size(), alpha(), swap(), trac(), dxf(), igs(), basp(), axi(),
  capel(), bapel(), hapel(), sapel(), rapel(), mapel(),
  lev(), pel(), fred(),
  f_met();
...
```

and where the formex function table is declared as follows:

```
...
struct FORMEX_FUNCTION_TABLE
{
  char      key[7];
  int       (*func)();          /*number,option,value */
  unsigned int m[12];          /* 99 : no check in this n_funcc() */
}
  tbl2[] = {
```



```

/*cardin*/  "TRAN"  ,tran,   2,0,1,2,0,0,0,0,0,0,0,0,
            "RIN"   ,rin,   3,0,1,1,2,0,0,0,0,0,0,0,
            "REF"   ,ref,   2,0,1,2,0,0,0,0,0,0,0,0,
            "LAM"   ,lam,   2,0,1,2,0,0,0,0,0,0,0,0,
            "PROJ"  ,proj,  2,0,1,2,0,0,0,0,0,0,0,0,
            "DIL"   ,dil,   2,0,1,2,0,0,0,0,0,0,0,0,
            "VER"   ,ver,   4,1,1,1,2,2,2,0,0,0,0,0,
            "ROS"   ,ros,   4,2,1,1,2,2,1,2,0,0,0,0,
            ...
            "CAPEL" ,capel, 99,0,0,0,0,0,0,0,0,0,0,0,
            "BAPEL" ,bapel, 99,0,0,0,0,0,0,0,0,0,0,0,
            "HAPEL" ,hapel, 99,0,0,0,0,0,0,0,0,0,0,0,
            "SAPEL" ,sapel, 99,0,0,0,0,0,0,0,0,0,0,0,
            "RAPEL" ,rapel,  2,2,1,3,1,2,0,0,0,0,0,0,
            "MAPEL" ,mapel,  2,2,1,4,1,2,0,0,0,0,0,0,
            "LEV"   ,lev,   1,0,3,0,0,0,0,0,0,0,0,0,
            "$"     ,NULL,   0,0,0,0,0,0,0,0,0,0,0,0 /*possibility of supplement*/
};
...

```

Figure B.1: Organisation of functions and files for pellevation
(*italic* indicates a function name)

```

/*****
/* Pellevat.c
/* (Isabell - last change 08.02.99)
*****/

#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "formian.h"

extern VALUE_TABLE *value_table;
extern float pi2;
extern USE_ITEM u;
extern int error_status[10];

void c1_4(int n,int *v); // capel
void c5_8(int n,int *v);
void c11_14(int n,int *v);
void c15_18(int n,int *v);
void c101_104(int n,int *v);
void c105_108(int n,int *v);
void c111_114(int n,int *v);
void c115_118(int n,int *v);
void b1_4(int n,int *v); // bapel
void b5_8(int n,int *v);
void b11_14(int n,int *v);
void b15_18(int n,int *v);
void b101_104(int n,int *v);
void b105_108(int n,int *v);
void b111_114(int n,int *v);
void b115_118(int n,int *v);
void h1_4(int n,int *v); // hapel
void h5_8(int n,int *v);
void h11_14(int n,int *v);
void h15_18(int n,int *v);
void h101_104(int n,int *v);
void h105_108(int n,int *v);
void h111_114(int n,int *v);
void h115_118(int n,int *v);
void s1_4(int n,int *v); // sapel
void s5_8(int n,int *v);
void s11_14(int n,int *v);
void s15_18(int n,int *v);
void s101_104(int n,int *v);
void s105_108(int n,int *v);
void s111_114(int n,int *v);
void s115_118(int n,int *v);
void r1u11(int n,int *v); // rapel
void r10u111(int n,int *v);
void m1u11(int n,int *v); // mapel
void m10u111(int n,int *v);

/* *****/

capel(int n,int *v)
{
    int pc;
    static int nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

    // Global Cap Pellevation w/o Layer Indication (circular base)

    if (abs(pc)==1||abs(pc)==2||abs(pc)==3||abs(pc)==4) {
        c1_4(n, v);
        return(0);
    }
}

```

```

// Global Cap Pellevation w/o Layer Indication (elliptic base)
    if (abs(pc)==5||abs(pc)==6||abs(pc)==7||abs(pc)==8) {
        c5_8(n, v);
        return(0);
    }

// Global Cap Pellevation with Layer Indication (circular base)
    if (abs(pc)==11||abs(pc)==12||abs(pc)==13||abs(pc)==14) {
        c11_14(n, v);
        return(0);
    }

// Global Cap Pellevation with Layer Indication (elliptic base)
    if (abs(pc)==15||abs(pc)==16||abs(pc)==17||abs(pc)==18) {
        c15_18(n, v);
        return(0);
    }

// Regional Cap Pellevation w/o Layer Indication (circular base)
    if (abs(pc)==101||abs(pc)==102||abs(pc)==103||abs(pc)==104) {
        c101_104(n, v);
        return(0);
    }

// Regional Cap Pellevation w/o Layer Indication (elliptic base)
    if (abs(pc)==105||abs(pc)==106||abs(pc)==107||abs(pc)==108) {
        c105_108(n, v);
        return(0);
    }

// Regional Cap Pellevation with Layer Indication (circular base)
    if (abs(pc)==111||abs(pc)==112||abs(pc)==113||abs(pc)==114) {
        c111_114(n, v);
        return(0);
    }

// Regional Cap Pellevation with Layer Indication (elliptic base)
    if (abs(pc)==115||abs(pc)==116||abs(pc)==117||abs(pc)==118) {
        c115_118(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */
bapel(int n,int *v)
{
    int          pc;
    static int    nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

// Global Barrel Pellevation w/o Layer Indication (rectangular base)
    if (abs(pc)==1||abs(pc)==2||abs(pc)==3||abs(pc)==4) {
        b1_4(n, v);
        return(0);
    }

```

```

// Global Barrel Pellevation w/o Layer Indication (trapezoidal base)
    if (abs(pc)==5||abs(pc)==6||abs(pc)==7||abs(pc)==8) {
        b5_8(n, v);
        return(0);
    }

// Global Barrel Pellevation with Layer Indication (rectangular base)
    if (abs(pc)==11||abs(pc)==12||abs(pc)==13||abs(pc)==14) {
        b11_14(n, v);
        return(0);
    }

// Global Barrel Pellevation with Layer Indication (trapezoidal base)
    if (abs(pc)==15||abs(pc)==16||abs(pc)==17||abs(pc)==18) {
        b15_18(n, v);
        return(0);
    }

// Regional Barrel Pellevation w/o Layer Indication (rectangular base)
    if (abs(pc)==101||abs(pc)==102||abs(pc)==103||abs(pc)==104) {
        b101_104(n, v);
        return(0);
    }

// Regional Barrel Pellevation w/o Layer Indication (trapezoidal base)
    if (abs(pc)==105||abs(pc)==106||abs(pc)==107||abs(pc)==108) {
        b105_108(n, v);
        return(0);
    }

// Regional Barrel Pellevation with Layer Indication (rectangular base)
    if (abs(pc)==111||abs(pc)==112||abs(pc)==113||abs(pc)==114) {
        b111_114(n, v);
        return(0);
    }

// Regional Barrel Pellevation with Layer Indication (trapezoidal base)
    if (abs(pc)==115||abs(pc)==116||abs(pc)==117||abs(pc)==118) {
        b115_118(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */

hapel(int n,int *v)
{
    int pc;
    static int nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

// Global Halo Pellevation w/o Layer Indication (circular base)
    if (abs(pc)==1||abs(pc)==2||abs(pc)==3||abs(pc)==4) {
        h1_4(n, v);
        return(0);
    }

```

```

// Global Halo Pellevation w/o Layer Indication (elliptic base)
    if (abs(pc)==5||abs(pc)==6||abs(pc)==7||abs(pc)==8) {
        h5_8(n, v);
        return(0);
    }

// Global Halo Pellevation with Layer Indication (circular base)
    if (abs(pc)==11||abs(pc)==12||abs(pc)==13||abs(pc)==14) {
        h11_14(n, v);
        return(0);
    }

// Global Halo Pellevation with Layer Indication (elliptic base)
    if (abs(pc)==15||abs(pc)==16||abs(pc)==17||abs(pc)==18) {
        h15_18(n, v);
        return(0);
    }

// Regional Halo Pellevation w/o Layer Indication (circular base)
    if (abs(pc)==101||abs(pc)==102||abs(pc)==103||abs(pc)==104) {
        h101_104(n, v);
        return(0);
    }

// Regional Halo Pellevation w/o Layer Indication (elliptic base)
    if (abs(pc)==105||abs(pc)==106||abs(pc)==107||abs(pc)==108) {
        h105_108(n, v);
        return(0);
    }

// Regional Halo Pellevation with Layer Indication (circular base)
    if (abs(pc)==111||abs(pc)==112||abs(pc)==113||abs(pc)==114) {
        h111_114(n, v);
        return(0);
    }

// Regional Halo Pellevation with Layer Indication (elliptic line base)
    if (abs(pc)==115||abs(pc)==116||abs(pc)==117||abs(pc)==118) {
        h115_118(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */
sapel(int n,int *v)
{
    int          pc;
    static int    nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

// Global Saddle Pellevation w/o Layer Indication (parabolic base)
    if (abs(pc)==1||abs(pc)==2||abs(pc)==3||abs(pc)==4) {
        s1_4(n, v);
        return(0);
    }

```

```

// Global Saddle Pellevation w/o Layer Indication (hyperbolic base)
    if (abs(pc)==5||abs(pc)==6||abs(pc)==7||abs(pc)==8) {
        s5_8(n, v);
        return(0);
    }

// Global Saddle Pellevation with Layer Indication (parabolic base)
    if (abs(pc)==11||abs(pc)==12||abs(pc)==13||abs(pc)==14) {
        s11_14(n, v);
        return(0);
    }

// Global Saddle Pellevation with Layer Indication (hyperbolic base)
    if (abs(pc)==15||abs(pc)==16||abs(pc)==17||abs(pc)==18) {
        s15_18(n, v);
        return(0);
    }

// Regional Saddle Pellevation w/o Layer Indication (parabolic base)
    if (abs(pc)==101||abs(pc)==102||abs(pc)==103||abs(pc)==104) {
        s101_104(n, v);
        return(0);
    }

// Regional Saddle Pellevation w/o Layer Indication (hyperbolic base)
    if (abs(pc)==105||abs(pc)==106||abs(pc)==107||abs(pc)==108) {
        s105_108(n, v);
        return(0);
    }

// Regional Saddle Pellevation with Layer Indication (parabolic base)
    if (abs(pc)==111||abs(pc)==112||abs(pc)==113||abs(pc)==114) {
        s111_114(n, v);
        return(0);
    }

// Regional Saddle Pellevation with Layer Indication (hyperbolic base)
    if (abs(pc)==115||abs(pc)==116||abs(pc)==117||abs(pc)==118) {
        s115_118(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */
rapel(int n,int *v)
{
    int          pc;
    static int    nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

// Global Raft Pellevation with and w/o Layer Indication
    if (abs(pc)==1||abs(pc)==11) {
        rl11(n, v);
        return(0);
    }

```

```

// Regional Raft Pellevation with and w/o Layer Indication

    if (abs(pc)==101||abs(pc)==111) {
        r101u111(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */

mapel(int n,int *v)
{
    int pc;
    static int nn;

    nn=n;

    if (value_table[v[1]].type==1) pc=value_table[v[1]].u.i;
    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code

// Global Mandate Pellevation with and w/o Layer Indication

    if (abs(pc)==1||abs(pc)==11) {
        m1u11(n, v);
        return(0);
    }

    if (abs(pc)==101||abs(pc)==111) {
        m101u111(n, v);
        return(0);
    }

    else {error_status[0]=20; error_rtn(2000);} // invalid pel. code
}

/* ***** */

```

Part of the code for the basic circular cap pellevation with circular base (*capell*):

```

... file_name_make(f1.f_name,nn);
file_name_make(f2.f_name,value_table[v[v[0]+1]].u.i);
f1.a_mode=2; f2.a_mode=1; f1.cn_size=f2.cn_size=s_cantle;
f1.cantle=f2.cantle=cantle;
if ((r=open_fmx(&f2)) != 0) error_rtn(r);
f1.fd.type=2;
f1.fd.grade=f2.fd.grade; f1.fd.homog=f2.fd.homog;
if ((r=open_fmx(&f1)) != 0) { close_fmx(&f2); error_rtn(r); }
if (f2.fd.grade < 3) {error_status[0]=10; error_rtn(2000);} // grade < 3
if (s<u.tol || h<u.tol) {error_status[0]=30; error_rtn(2000);} // neg. s or h
if (abs(pc)==1 && s>u.tol && (h-u.tol)>s/2)
{error_status[0]=40; error_rtn(2000);} // cap > hemisphere
if (pc>0) q= 1.0; // up- / downwards
else q=-1.0;
for (i=1;i<=f2.fd.c_size;i++){ // cantle by cantle
    if ((r=read_fmx(&f2)) !=0) error_rtn(r);
    for (k=1;k<=f2.fd.grade*f2.cantle[0];k+=f2.fd.grade){ // signet by signet
        a=f2.cantle[k]-cx1;
        b=f2.cantle[k+1]-cy1;
        d=sqrt(a*a+b*b); // distance point / centre
        if (d<s/2) { // point inside boundary
            if (abs(pc)==1) { // circ. cap w. circ. base
                r1=(s*s+4*h*h)/(8*h); // radius of sphere
                e=sqrt(r1*r1-d*d); // parameter e
                aa=e-r1+h; // aa = z = change in z
            }
        }
        f1.cantle[k+2]=f1.cantle[k+2]+q*aa;
    }
}
if ((r=write_fmx(&f1)) != 0) error_rtn(r);
}
close_fmx(&f1); close_fmx(&f2);
value_table[nn].type=3; value_table[nn].u.i=nn; return;

```

Appendix C

Analysis Output

Braun & Partner GbR - Beratende Ingenieure * 75179 Pforzheim
 Datei: C:\STATIK\HOFMANN\TEST\SADDLE4.NDB 17.01.99

GENF - EINGABE FINITER ELEMENTE (V 2.2-98) 28.04.99

M A T E R I A L I E N

Nr. 1 B 25 (DIN 1045) (mod)

E-Modul	30000	[MPa]	Sicherheit	1.00	[-]
Querdehnzahl	.20	[-]	Festigkeit fc	17.50	[MPa]
G-Modul	12500	[MPa]	Nennfestigkeit	25.00	[MPa]
K-Modul	16667	[MPa]	Zugfestig. fctm	2.10	[MPa]
Wichte	25.0	[kN/m3]	Zugfestig. fctk	2.14	[MPa]
Wichte Auftrieb	.0	[kN/m3]	Zugfestig. fctk	3.08	[MPa]
Temperaturkoeff.	1.00E-05	[-]	Druckbruchenerg.	20.00	[kN/m]
			Zugbruchenergie	.05	[kN/m]
			Reibung im Riß	.20	[-]

Nr. 2 BST 500 (DIN 1045) (mod)

E-Modul	210000	[MPa]	Sicherheit	1.00	[-]
Querdehnzahl	.30	[-]	Fließgrenze fy	500.00	[MPa]
G-Modul	80769	[MPa]	Zugfestigk. ft	550.00	[MPa]
K-Modul	175000	[MPa]	Bruchdehnung	10.00	[o/o]
Wichte	78.5	[kN/m3]	Relaxation .70ft	.00	[-]
Wichte Auftrieb	.0	[kN/m3]	Relaxation .55ft	.00	[-]
Temperaturkoeff.	1.20E-05	[-]	Verbund national	200.00	[-]
			Verbund K1 (EC2)	.80	[-]
			Verfestig. Modul	.00	[MPa]

ASE - ALLGEMEINE STATIK (V 2.2-98) 28.04.99

L A S T F A L L 1 Vollast

Eigengewicht in Z-Richtung 20.000

S U M M E D E R L A S T E N

LF	PXX [kN]	PYY [kN]	PZZ [kN]	MXX [kNm]	MYX [kNm]	MZZ [kNm]
1	.0	.0	3514.5	.00	.00	.00

A U F L A G E R K R A E F T E U N D R E S T K R A E F T E

Lastfall 1 Vollast

Knoten	P-XX	P-YY	P-ZZ	M-XX	M-YY	M-ZZ
Nr	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
1	-47.5		-13.7			
2	-146.1		-101.2			
3	-170.4		-111.7			
4	-195.0		-115.0			
5	-233.3		-124.7			
6	-251.5		-130.6			
7	-233.7		-128.2			
8	-225.0		-120.0			
9	-46.0		-22.3			
10		-190.6	-138.3			
18		271.0	-138.8			

19		-173.7	-114.1
27		233.6	-128.2
28		-195.1	-115.1
36		251.3	-130.5
44		-233.3	-124.7
45		233.3	-124.7
48		195.1	-115.1
54		-251.3	-130.5
55		173.7	-114.1
63		-233.6	-128.2
64		190.6	-138.3
72		-271.0	-138.8
73	47.5		-13.7
74	146.1		-101.2
75	170.4		-111.7
76	195.0		-115.0
77	233.3		-124.7
78	251.5		-130.6
79	233.7		-128.2
80	225.0		-120.0
81	46.0		-22.3

S U M M E DER A U F L A G E R K R A E F T E U N D L A S T E N						
Last	P-X	P-Y	P-Z	M-X	M-Y	M-Z
Fall	[kN]	[kN]	[kN]	[kNm]	[kNm]	[kNm]
1	.0	.0	-3514.5			
	.0	.0	3514.5	.00	.00	.00

MAXIMA - LASTFALLÜBERLAGERUNG (V 1.9-97) 28.04.99

```

1 PROG MAXIMA
2 $ Dat : C:\statik\hofmann\phd_calc\saddle4.dat (.#04) 28.04.1999
3 $ Job : phd_calc/PENTIUM-200:000434 17:05
4 KOPF DATEI: C:\STATIK\HOFMANN\TEST\SADDLE2.NDB 17.01.99
5 KOPF UEBERLAGERUNG SCHNITTKRAEFTE
6 ECHO VOLL NEIN ; ECHO TABU JA
7 QUAD ; QUAK
8 UEBE MAMI MX LF1 801 802
9 UEBE MAMI MY LF1 803 804
10 UEBE MAMI MXY LF1 805 806
11 UEBE MAMI SX LF1 811 812 $ NORMALKRAFT NX
12 UEBE MAMI SY LF1 813 814 $ NORMALKRAFT NY
13 UEBE MAMI TAU LF1 815 816 $ NORMALKRAFT NXY
14 KNOT
15 UEBE MAMI PX LF1 901 902
16 UEBE MAMI PY LF1 903 904
17 UEBE MAMI PZ LF1 905 906
18 $ SOLLTE NACH ANWENDERWUENSCHEN ERGAENZT WERDEN!
19 $ ACHTUNG : AUF WECHSELWIRKUNGEN MIT BEMESS ACHTEN
20 ENDE

```

UEBERLAGERUNG SCHNITTKRAEFTE

L A S T F A L L A U S W A H L

Nummer	Faktor	Typ	Bezeichnung
1	1.00	Ständige Last	Vollast

E R Z E U G T E L A S T F A E L L E

Nummer Bezeichnung

801 MAX-MX
802 MIN-MX
803 MAX-MY
804 MIN-MY
805 MAX-MXY
806 MIN-MXY
811 MAX-SX
812 MIN-SX
813 MAX-SY
814 MIN-SY
815 MAX-TAU
816 MIN-TAU
901 MAX-PX
902 MIN-PX
903 MAX-PY
904 MIN-PY
905 MAX-PZ
906 MIN-PZ

BEMESS - STAHLBETONBEMESSUNG FLÄCHENTRAGWERKE (V 1.1-98)

1 PROG BEMESS
2 \$ Dat : C:\statik\hofmann\phd_calc\saddle4.dat (.#05) 28.04.1999
3 \$ Job : phd_calc/PENTIUM-200:000434 17:05
4 KOPF DATEI: C:\STATIK\HOFMANN\TEST\SADDLE2.NDB 17.01.99
5 ECHO VOLL NEIN ; ECHO PARA,DUST VOLL
6 DUST NEIN
7 LF 801,802,803,804,805,806 \$ 905,906
8 LF 811,812,813,814,815,816 \$ 905,906
9 GEO - 3 1 3 1
10 \$RICH 0 0
11 DREI 0 90 - 0 90 -
12 \$ SOLLTE NACH ANWENDERWUENSCHEN ERGAENZT WERDEN!
13 \$ ACHTUNG : AUF WECHSELWIRKUNGEN MIT BEMESS ACHTEN
14 ENDE

BEMESS - STAHLBETONBEMESSUNG FLÄCHENTRAGWERKE (V 1.1-98)

L A S T F A E L L E mit Faktoren der ständig wirkenden Last in Prozent

LfNr	Faktor	LfNr	Faktor	LfNr	Faktor	LfNr	Faktor	LfNr	Faktor
801	.0	802	.0	803	.0	804	.0	805	.0
806	.0	811	.0	812	.0	813	.0	814	.0
815	.0	816	.0						

M A T E R I A L (DIN 1045)

Mat	Beta-R	Beta-S	Tau-011	Tau-02	Param.	Beta-Z	N	minQ	Art
	[MPa]	[MPa]	[MPa]	[MPa]		[MPa]	[-]	[-]	
1	17.5	.350	1.800	K1S	2.100	10.0	.20	vorw.	ruhend

Mindestbew. nach DIN 1045, 25.2.2.1: .80 Proz. des stat. erf. Quer.

2	500.0								
---	-------	--	--	--	--	--	--	--	--

Schnittgrößen und Lastfälle enthalten Ergebnisse auf Gebrauchslastniveau

Material-Sicherheitsbeiwerte:

Mat	Beton	SC1	SC2	Stahl	SS1	SS2
1		1.75	2.10			
2				1.75	2.10	

Schubbemessung nach DIN 1045

Bedeutung der Werte von Param.:

K1: Beiwert nach DIN 1045, 17.5.5, Gl.14

S: Bewehrung ist teilweise im Zugbereich verankert, es werden
die Werte der Zeile 1a in Tabelle 13 DIN 1045 verwendet

G E O M E T R I E (Achismaß der Betondeckung)

Nr	ha-oben	hm-oben	hi-oben	ha-unten	hm-unten	hi-unten	Elementdicke
	[cm]	[cm]	[cm]	[cm]	[cm]	[cm]	[cm]
1	3.0	4.0		3.0	4.0		10.0

Bewehrung wird in der Datenbasis gespeichert

Nummer der gespeicherten Bewehrungsverteilung: 1

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