Performance Analysis of Conventional Detection in BFWA Systems

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Abstract—The systems under study are broadband wireless fixed access (BFWA) systems over multipath fading channels. Conventional detection methods like coherent and non-coherent detection are examined theoretically for both QPSK and 16-QAM modulated BFWA systems in this paper and shown to yield unsatisfactory performance. The theoretical analysis for different algorithms are validated by Monte-Carlo simulations and proved to be accurate. They give us an insight into the physical limitations of the BFWA channels and suggest solutions to improve the capacity and performance of future BFWA systems.

I. INTRODUCTION

It is becoming apparent that access to the Internet is of growing economic and political importance. It is also clear that low bandwidth dial-up Internet access is restricting the services and applications that can be offered. What is required is a quantum leap in access bandwidth to free up the Internet for innovative applications. One possible solution is to use the existing local-loop. This approach requires the installation of digital subscriber line (DSL) equipment at the exchange and customer premises. Unfortunately, the length and quality of the local-loop infrastructure will prevent this service being offered universally. Another option for providing broadband access is via cable TV networks. However, the availability of these services is far from universal and contention will seriously degrade throughput per user. An alternative approach is to deploy broadband fixed wireless access (BFWA) technology. The advantage of such an approach is that it enables operators in a competitive environment to roll-out broadband services in a rapid and cost efficient manner. BFWA networks generally employ a point-to-multi-point architecture [1], where a single based station (BS) communicates with many subscriber units (SU) placed at the user locations. Standardization of BFWA systems is currently undertaken by the IEEE 802.16 working group [2] and the ETSI HIPERMAN group [3].

One of the limiting factors in outdoor wireless transmission is the multipath channel between the transmitter and the receiver causing intersymbol interference (ISI), which degrades the system performance and limits the maximum achievable data rate. In this paper, we provide a theoretical approach to analyze the effect of ISI on the performance

of the BFWA systems, with an attempt to gain a deep insight into physical limitations of the BFWA channels with conventional detection techniques. The multipath channel can be modeled by an equivalent baseband system where the transmit filter, the channel and the receive filter, are represented by a discrete-time L -tap transversal filter with finite-length impulse response $h_n = \sum_{l=0}^{L-1} h_l \delta_{n-l}$ where h_l denotes the complex channel coefficients. Tailored for different terrain conditions, a set of 6 typical channel models called Stanford University Interim (SUI) Channel Models used for simulation, design, development and testing of technologies suitable for fixed broadband wireless applications were proposed in [4]. All of them are simulated using 3 taps, having either Rayleigh or Ricean amplitude distributions. For the purpose of this study, we select SUI-3 channel with tap spacing of 500ns, and maximum tap delay at 1000ns. To simplify the analysis, we assume the transmitted data rate is 4Mbps with QPSK modulation or 8Mbps WITH 16-QAM modulation so that the multipath fading is modelled as a tapped-delay line with adjacent taps equally spaced at symbol rate. The received signal is formed as

$$r_n = h_0 s_n + h_1 s_{n-1} + h_2 s_{n-2} + v_n$$

where the channel coefficients h_0, h_1, h_2 are complex Gaussian distributed and assumed to remain constant during the transmission of one block of data. They, however, vary from block to block. The amplitude of the first tap $|h_0|$ is characterized by a Ricean distribution due to the line of sight propagation. The amplitudes of the taps $|h_1|, |h_2|$ are Rayleigh distributed. The transmitted PSK/QAM symbol at time instant n is denoted as $s_n = x_n + jy_n$, and v_n is the complex additive white Gaussian noise with zero mean and variance N_0 .

II. PERFORMANCE OF COHERENT DETECTION

Denote $\hat{h}_0$ as an estimate of h_0 , and assume it is an accurate estimate of h_0 , i.e., $\hat{h}_0 \approx h_0$. To detect the transmitted symbols coherently, we multiply the received signal with the conjugate of $\hat{h}_0$, i.e.,

$$\begin{aligned} r'_n &= \hat{h}_0^* (h_0 s_n + h_1 s_{n-1} + h_2 s_{n-2} + v_n) \\ &= \hat{h}_0^* h_0 s_n + \underbrace{\hat{h}_0^* (h_1 s_{n-1} + h_2 s_{n-2} + v_n)}_{\text{combined ISI and noise}} \\ &= \hat{h}_0^* h_0 s_n + w_n \approx |h_0|^2 s_n + w_n \end{aligned}$$

Fig. 1. QPSK constellation and decision regions.

where the combined ISI and noise $w_n = w_I + jw_Q$ is a complex Gaussian random variable with PDF $\sim \mathcal{CN}(0, N_w)$ and variance $N_w = |h_0|^2(\mathbb{E}[|h_1|^2] + \mathbb{E}[|h_2|^2] + N_0) = |h_0|^2(P_1 + P_2 + N_0)$.

A. Coherence detection for QPSK modulation

QPSK signal constellation is plotted in Fig. 1. Due to the symmetry of the QPSK constellation, the symbol error probability P_e equals the conditional error probabilities, i.e., $P_e = P(e|s_n = s_0) = P(e|s_n = s_1) = P(e|s_n = s_2) = P(e|s_n = s_3)$. With maximum likelihood detection, the optimum decision regions are simply four quadrants R_0, R_1, R_2, R_3 . Suppose s_0 is transmitted, the probability of making correct decision $P(c|s_n = s_0)$ is the probability of r'_n falling in the correct decision region R_0 (the first quadrant), i.e.,

$$\begin{aligned} P(c|s_n = s_0) &= P_r\{r'_n = |h_0|^2 s_0 + w_n \in R_0\} \\ &= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I + j\left(\frac{|h_0|^2}{\sqrt{2}} + w_Q\right) \in R_0\right\} \\ &= P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_I > 0\right\} \cdot P_r\left\{\frac{|h_0|^2}{\sqrt{2}} + w_Q > 0\right\} \\ &= P_r\left\{w_I > -\frac{|h_0|^2}{\sqrt{2}}\right\} \cdot P_r\left\{w_Q > -\frac{|h_0|^2}{\sqrt{2}}\right\} \end{aligned} \quad (1)$$

The third equality in (1) holds since the Gaussian random variables $w_I \sim \mathcal{N}(0, N_w/2)$ and $w_Q \sim \mathcal{N}(0, N_w/2)$ are statistically independent. Normalizing w_I, w_Q to unit vari-

ance, yield

$$\begin{aligned} P(c|s_0) &= P_r\left\{\frac{w_I}{\sqrt{N_w/2}} > -\frac{|h_0|^2}{\sqrt{2}\sqrt{N_w/2}}\right\} \\ &\quad \cdot P_r\left\{\frac{w_Q}{\sqrt{N_w/2}} > -\frac{|h_0|^2}{\sqrt{2}\sqrt{N_w/2}}\right\} \\ &= \left[1 - Q\left(\frac{|h_0|^2}{\sqrt{|h_0|^2(P_1 + P_2 + N_0)}}\right)\right]^2 \\ &= \left[1 - Q\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right)\right]^2 \end{aligned}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt$ is the complementary Gaussian cumulative distribution function. The relationship between P_e and $P(c|s_0)$ is

$$\begin{aligned} P_e &= P(e|s_0) = 1 - P(c|s_0) \\ &= 2Q\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right) - Q^2\left(\frac{|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right) \end{aligned}$$

We know that $r = |h_0|$ is Ricean distributed with PDF

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0 \quad (2)$$

where $I_0(x)$ is the 0th order modified Bessel function of the first kind [5, p. 44]. For gray coded QPSK, $P_b \approx P_e/2$, therefore

$$P_b(r) \approx P_e(r)/2 \approx Q\left(\frac{r}{\sqrt{P_1 + P_2 + N_0}}\right) \quad (3)$$

To obtain the error probability when r is random, we must average $P_b(r)$ given in (3) over distribution of r , i.e.,

$$\begin{aligned} \bar{P}_b &= \int_0^\infty P_b(r) p(r) dr \\ &= \int_0^\infty Q\left(\frac{r}{\sqrt{P_1 + P_2 + N_0}}\right) \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{rs}{\sigma^2}\right) dr \\ &= \frac{1}{2} \exp\left(-\frac{s^2}{2\sigma^2}\right) \sum_{n=1}^\infty g(n) (1 - \beta)^n \sum_{k=0}^{n-1} 2^{-k} \binom{n-1+k}{k} (1 + \beta)^k \end{aligned} \quad (4)$$

where

$$\begin{aligned} g(n) &= \frac{(\sigma s)^{2n-2}}{4^{n-1} \sigma^{4n-4} (n-1)!} \\ \alpha &= \sqrt{P_1 + P_2 + N_0}/\sigma, \quad \beta = (\alpha^2 + 1)^{-1/2} \end{aligned}$$

In (4), the derivation of the closed form expression is omitted to conserve space.

B. Coherent detection for 16-QAM

The 16-QAM constellation and maximum likelihood decision regions are depicted in Fig. 2. The decision regions can be squares (type x), squares with one open side (type

Fig. 2. 16-QAM constellation and decision regions.

y) or squares with two open sides (type z), see Fig. 2. To compute the bit error probability, we first need to compute the conditional error probability for these three types of regions. Conditioned on that we send a symbol that has a decision region of type x, y, z , and the probability of making a wrong bit decision is denoted by P_x, P_y, P_z .

To simplify the calculation, we assume that a symbol error results in maximum two bit errors. If the constellation is gray coded, 2 bit error occurs when the symbol is erroneously decoded to the symbol in diagonal neighbouring regions; 1 bit error occurs when the symbol is erroneously decoded to the symbol in non-diagonal neighbouring regions. The probability of these two events are denoted as P_{x2}, P_{y2}, P_{z2} , and P_{x1}, P_{y1}, P_{z1} for symbol with decision region of type x, y, z . Due to the symmetry of the constellation, $P_2 = P_{x2} = P_{y2} = P_{z2}$, and $P_1 = P_{x1} = P_{y1} = P_{z1}$. Given s_n is of type x symbol (e.g., the one pointed in Fig. 2), the received signal after phase correction is $r'_n \approx |h_0|^2 s_n + w_n = |h_0|^2 (A + jA) + w_n$. The probabilities P_1, P_2 can be computed as

$$\begin{aligned} P_1 &= P_r\{w_I > A|h_0|^2\} = P_r\{w_I < -A|h_0|^2\} \\ &= P_r\{w_Q > A|h_0|^2\} = P_r\{w_Q < -A|h_0|^2\} \\ &= Q\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \\ P_2 &= P_r\{w_I > A|h_0|^2, w_Q > A|h_0|^2\} \\ &= P_r\{w_I < -A|h_0|^2, w_Q < -A|h_0|^2\} \\ &= P_r\{w_I > A|h_0|^2, w_Q < -A|h_0|^2\} \\ &= P_r\{w_I < -A|h_0|^2, w_Q > A|h_0|^2\} \\ &= Q^2\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \end{aligned}$$

The conditional bit error probability P_x is computed as

$$\begin{aligned} P_x &= \frac{1}{k}[n_{x1} \cdot 1 \cdot P_1 + n_{x2} \cdot 2 \cdot P_2] = \frac{1}{4}[4P_1 + 8P_2] \\ &= Q\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) + 2Q^2\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \end{aligned}$$

where k stands for the number of bits per symbol (For 16-QAM, $k = 4$), and n_{x1}, n_{x2} are the number of neighbouring regions that differ in 1 and 2 bits, respectively, from the transmitted symbol having a decision region of type x .

Similarly,

$$\begin{aligned} P_y &= \frac{1}{k}[n_{y1} \cdot 1 \cdot P_1 + n_{y2} \cdot 2 \cdot P_2] = \frac{1}{4}[3P_1 + 4P_2] \\ &= \frac{3}{4}Q\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) + Q^2\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \\ P_z &= \frac{1}{k}[n_{z1} \cdot 1 \cdot P_1 + n_{z2} \cdot 2 \cdot P_2] = \frac{1}{4}[2P_1 + 2P_2] \\ &= \frac{1}{2}Q\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) + \frac{1}{2}Q^2\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \end{aligned}$$

where n_{y1}, n_{y2} (n_{z1}, n_{z2}) are the number of neighbouring regions that differ in 1 and 2 bits, respectively, from the transmitted symbol having a decision region of type y (z). The bit error probability is thus

$$\begin{aligned} P_b &= \frac{1}{16}(n_x P_x + n_y P_y + n_z P_z) = \frac{1}{16}(4P_x + 8P_y + 4P_z) \\ &= \frac{3}{4}Q\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) + \frac{9}{8}Q^2\left(\frac{A|h_0|^2}{\sqrt{N_w/2}}\right) \\ &= \frac{3}{4}Q\left(\frac{\sqrt{2}A|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right) + \frac{9}{8}Q^2\left(\frac{\sqrt{2}A|h_0|}{\sqrt{P_1 + P_2 + N_0}}\right) \end{aligned} \quad (5)$$

where n_x, n_y, n_z is the number of x, y, z type regions in the constellation. For 16-QAM, $n_x = 4, n_y = 8, n_z = 4$.

To obtain the error probability when $r = |h_0|$ is random, we must average $P_b(r)$ given in (5) over distribution of r , i.e.,

$$\begin{aligned} \bar{P}_b &= \int_0^\infty P_b(r)p(r)dr \\ &= \int_0^\infty \left[\frac{3}{4}Q\left(\frac{\sqrt{2}Ar}{\sqrt{P_1 + P_2 + N_0}}\right) + \frac{9}{8}Q^2\left(\frac{\sqrt{2}Ar}{\sqrt{P_1 + P_2 + N_0}}\right) \right] \\ &\quad \cdot \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{rs}{\sigma^2}\right) dr \end{aligned} \quad (6)$$

The closed form of (7) is difficult to derive due to the quadratic term of the Q-function. One way of tackling this problem is to use the method proposed by Holtzman in [6]. It gives a simple and accurate method to evaluate the expectation without carrying out the integration. With Holtzman approximation, the solution to our problem becomes

$$\begin{aligned} P_b(r) &= \frac{3}{4}Q\left(\frac{\sqrt{2}Ar}{\sqrt{P_1 + P_2 + N_0}}\right) + \frac{9}{8}Q^2\left(\frac{\sqrt{2}Ar}{\sqrt{P_1 + P_2 + N_0}}\right) \\ \bar{P}_b &\approx \frac{2}{3}P_b(\mu_r) + \frac{1}{6}P_b(\mu_r + \sqrt{3}\sigma_r) + \frac{1}{6}P_b(\mu_r - \sqrt{3}\sigma_r) \end{aligned} \quad (7)$$

where $\sigma_r = \sqrt{E[r^2] - \mu_r^2} = \sqrt{E[|h_0|^2] - \mu_r^2}$, and μ_r can be derived as

$$\begin{aligned}\mu_r &= \frac{\exp(-s^2/2\sigma^2)}{\sigma^2} \int_0^\infty \exp\left(-\frac{r^2}{2\sigma^2}\right) \sum_{k=0}^\infty \frac{r^{2k+2} s^{2k}}{(2\sigma^2)^{2k} (k!)^2} dr \\ &= \frac{\exp(-s^2/2\sigma^2)}{\sigma^2} \int_0^\infty \exp\left(-\frac{r^2}{2\sigma^2}\right) \sum_{n=1}^\infty \frac{r^{2n} s^{2n-2}}{(2\sigma^2)^{2n-2} [(n-1)!]^2} dr\end{aligned}$$

According to [7, p. 255]

$$\int_0^\infty x^{2n} \exp(-px^2) dx = \frac{(2n-1)!!}{2(2p)^n} \sqrt{\frac{\pi}{p}}$$

$p > 0, n = 0, 1, 2, \dots \quad (2n-1)!! = 1 \cdot 3 \cdot 5 \cdots (2n-1)$

Denote $p = \frac{1}{2\sigma^2}$, we have

$$\begin{aligned}\mu_r &= 2p \exp(-ps^2) \int_0^\infty \exp(-pr^2) \sum_{n=1}^\infty \frac{r^{2n} s^{2n-2} p^{2n-2}}{[(n-1)!]^2} dr \\ &= \sqrt{\pi p} \exp(-ps^2) \sum_{n=1}^\infty \frac{s^{2n-2} p^{n-2} (2n-1)!!}{2^n [(n-1)!]^2}\end{aligned}\quad (8)$$

III. PERFORMANCE OF NON-COHERENT DETECTION

Non-coherent detection is performed on the original received signal without correcting the phase shift. Let us reform the received signal as

$$r_n = h_0 s_n + \underbrace{h_1 s_{n-1} + h_2 s_{n-2} + v_n}_{\text{combined ISI and noise}} = h_0 s_n + w_n$$

where $w_n = h_1 s_{n-1} + h_2 s_{n-2} + v_n \sim \mathcal{N}(0, N_w)$ stands for combined ISI and noise, which is Gaussian distributed, and $N_w = E[|h_1|^2] + E[|h_2|^2] + N_0 = P_1 + P_2 + N_0$.

A. Non-coherent detection for QPSK

The probability of making a symbol error, e.g., $\hat{s}_n = s_1$ given s_0 is transmitted can be computed as

$$\begin{aligned}P(\hat{s}_n = s_1 | s_n = s_0) &= P_r\{h_0 s_0 + w_n \in R_1\} \\ &= P_r\left\{(h_I + j h_Q) \left(\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}\right) + w_n \in R_1\right\} \\ &= P_r\left\{\frac{h_I - h_Q}{\sqrt{2}} + w_I < 0\right\} \cdot P_r\left\{\frac{h_I + h_Q}{\sqrt{2}} + w_Q > 0\right\} \\ &= Q\left(\frac{h_I - h_Q}{\sqrt{P_1 + P_2 + N_0}}\right) Q\left(-\frac{h_I + h_Q}{\sqrt{P_1 + P_2 + N_0}}\right)\end{aligned}\quad (9)$$

Similarly, other conditional error probabilities can be obtained as

$$\begin{aligned}P(\hat{s}_n = s_2 | s_n = s_0) &= Q\left(\frac{h_I - h_Q}{\sqrt{P_1 + P_2 + N_0}}\right) Q\left(\frac{h_I + h_Q}{\sqrt{P_1 + P_2 + N_0}}\right) \\ P(\hat{s}_n = s_3 | s_n = s_0) &= Q\left(-\frac{h_I - h_Q}{\sqrt{P_1 + P_2 + N_0}}\right) Q\left(\frac{h_I + h_Q}{\sqrt{P_1 + P_2 + N_0}}\right)\end{aligned}\quad (10)$$

Denote $x = h_I - h_Q$ and $y = h_I + h_Q$. Both of them are Gaussian distributed random variables with PDFs

$$\begin{aligned}p(x) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{x^2}{2\sigma^2}\right], \quad -\infty \leq x \leq \infty \\ p(y) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y - m_y)^2}{2\sigma^2}\right], \quad -\infty \leq y \leq \infty\end{aligned}\quad (11)$$

According to the signal constellation plotted in Fig. 1, the error event ($\hat{s}_n = s_2 | s_n = s_0$) results in 2 bits error, while both ($\hat{s}_n = s_1 | s_n = s_0$) and ($\hat{s}_n = s_3 | s_n = s_0$) result in 1 bit error. The relationship between bit error probability and the conditional symbol error probabilities is therefore

$$\begin{aligned}P_b &= \frac{1}{k} [2 \times P(\hat{s}_n = s_2 | s_n = s_0) + 1 \times P(\hat{s}_n = s_1 | s_n = s_0) \\ &\quad + 1 \times P(\hat{s}_n = s_3 | s_n = s_0)]\end{aligned}\quad (12)$$

where k stands for the number of bits per symbol. For QPSK modulation, $k = 2$. Based on (9), (10) and (12), the bit error probability is expressed as

$$\begin{aligned}P_b(x, y) &= \frac{1}{2} Q\left(\frac{x}{\sqrt{N_w}}\right) Q\left(\frac{-y}{\sqrt{N_w}}\right) + \\ &\quad \frac{1}{2} Q\left(\frac{-x}{\sqrt{N_w}}\right) Q\left(\frac{y}{\sqrt{N_w}}\right) + Q\left(\frac{x}{\sqrt{N_w}}\right) Q\left(\frac{y}{\sqrt{N_w}}\right)\end{aligned}\quad (13)$$

where $N_w = P_1 + P_2 + N_0$. The average bit error probability $\bar{P}_b$ is derived by taking expectation of $P_b(x, y)$ expressed in (13) with respect to x and y . Since x is a zero-mean normal random variable, $E[Q(\lambda x)] = Q(0) = 0.5$ [8, p. 102]. Therefore, expectation of (13) with respect to x is equivalent to

$$P_b(y) = \frac{1}{4} Q\left(\frac{-y}{\sqrt{N_w}}\right) + \frac{3}{4} Q\left(\frac{y}{\sqrt{N_w}}\right)\quad (14)$$

According to [8, p. 102], if z is a zero-mean, unit-variance, normal random variable, then $E[Q(\mu + \lambda z)] = Q\left(\frac{\mu}{\sqrt{1+\lambda^2}}\right)$. Assign $z = (y - m_y)/\sigma$ such that z is a zero-mean, unit-variance, normal random variable. By taking the expectation of (14), we derive the average bit error probability as

$$\begin{aligned}\bar{P}_b &= E\left[\frac{1}{4} Q\left(-\frac{\sigma z + m_y}{\sqrt{N_w}}\right) + \frac{3}{4} Q\left(\frac{\sigma z + m_y}{\sqrt{N_w}}\right)\right] \\ &= \frac{1}{4} Q\left(\frac{\mu_1}{\sqrt{1+\lambda_1^2}}\right) + \frac{3}{4} Q\left(\frac{\mu_2}{\sqrt{1+\lambda_2^2}}\right)\end{aligned}\quad (15)$$

where $\lambda_1 = -\frac{\sigma}{\sqrt{N_w}}$, $\mu_1 = -\frac{m_y}{\sqrt{N_w}}$, $\lambda_2 = \frac{\sigma}{\sqrt{N_w}}$, $\mu_2 = \frac{m_y}{\sqrt{N_w}}$.

B. Non-coherent detection for 16-QAM

Given s_n is a type a symbol, the received signal

$$\begin{aligned}r_n &= h_0 s_n + w_n = (h_I + j h_Q)(A + j A) + w_n \\ &= A(h_I - h_Q) + w_I + j A(h_I + h_Q) + j w_Q\end{aligned}$$

In this case, the probability of 1 error event along I axis is different from the one along Q axis. They are denoted by

P_{1I}, P_{1Q} , respectively. The probability of 2 errors event is denoted as P_2 . These probabilities are computed as

$$\begin{aligned}
P_{1I} &= P_r\{A(h_I - h_Q) + w_I < 0\} = P_r\{w_I < -A(h_I - h_Q)\} \\
&= P_r\left\{\frac{w_I}{\sqrt{N_w/2}} < -\frac{\sqrt{2}A(h_I - h_Q)}{\sqrt{N_w}}\right\} \\
&= Q\left[\frac{\sqrt{2}A(h_I - h_Q)}{\sqrt{N_w}}\right] \\
P_{1Q} &= P_r\{A(h_I + h_Q) + w_Q < 0\} = P_r\{w_Q < -A(h_I + h_Q)\} \\
&= P_r\left\{\frac{w_Q}{\sqrt{N_w/2}} < -\frac{\sqrt{2}A(h_I + h_Q)}{\sqrt{N_w}}\right\} \\
&= Q\left[\frac{\sqrt{2}A(h_I + h_Q)}{\sqrt{N_w}}\right] \\
P_2 &= P_r\{A(h_I - h_Q) + w_I < 0\}P_r\{A(h_I + h_Q) + w_Q < 0\} \\
&= Q\left[\frac{\sqrt{2}A(h_I - h_Q)}{\sqrt{N_w}}\right]Q\left[\frac{\sqrt{2}A(h_I + h_Q)}{\sqrt{N_w}}\right]
\end{aligned}$$

The conditional bit error probabilities P_a, P_b, P_c are computed as

$$\begin{aligned}
P_a &= \frac{1}{4}[n_{a1I} \cdot P_{1I} + n_{a1Q} \cdot P_{1Q} + n_{a2} \cdot 2 \cdot P_2] \\
&= \frac{1}{4}[2P_{1I} + 2P_{1Q} + 8P_2] \\
P_b &= \frac{1}{4}[n_{b1I} \cdot P_{1I} + n_{b1Q} \cdot P_{1Q} + n_{b2} \cdot 2 \cdot P_2] \\
&= \frac{1}{4}[P_{1I} + 2P_{1Q} + 4P_2] \\
P_c &= \frac{1}{4}[n_{c1I} \cdot P_{1I} + n_{c1Q} \cdot P_{1Q} + n_{c2} \cdot 2 \cdot P_2] \\
&= \frac{1}{4}[P_{1I} + P_{1Q} + 2P_2]
\end{aligned}$$

where n_{a1I}, n_{a1Q} denotes the number of non-diagonal neighbouring regions that differ in 1 bit along the I/Q axis from the transmitted type a symbol, and n_{a2} is the number of diagonal neighbouring regions that differ in 2 bits from the transmitted type a symbol. $n_{b1I}, n_{b1Q}, n_{b2}, n_{c1I}, n_{c1Q}, n_{c2}$ are defined similarly for type b and type c symbols. The bit error probability is thus

$$\begin{aligned}
P_b &= \frac{1}{16}(n_a P_a + n_b P_b + n_c P_c) = \frac{1}{16}(4P_a + 8P_b + 4P_c) \\
&= \frac{1}{16}(5P_{1I} + 7P_{1Q} + 18P_2) \\
&= \frac{5}{16}Q\left[\frac{\sqrt{2}A(h_I - h_Q)}{\sqrt{N_w}}\right] + \frac{7}{16}Q\left[\frac{\sqrt{2}A(h_I + h_Q)}{\sqrt{N_w}}\right] \\
&\quad + \frac{18}{16}Q\left[\frac{\sqrt{2}A(h_I - h_Q)}{\sqrt{N_w}}\right]Q\left[\frac{\sqrt{2}A(h_I + h_Q)}{\sqrt{N_w}}\right]
\end{aligned}$$

Denote $x = h_I - h_Q, y = h_I + h_Q$. The PDFs of x, y are

given in (11). The bit error probability can be expressed as

$$\begin{aligned}
P_b(x, y) &= \frac{5}{16}Q\left[\frac{\sqrt{2}Ax}{\sqrt{N_w}}\right] + \frac{7}{16}Q\left[\frac{\sqrt{2}Ay}{\sqrt{N_w}}\right] \\
&\quad + \frac{18}{16}Q\left[\frac{\sqrt{2}Ax}{\sqrt{N_w}}\right]Q\left[\frac{\sqrt{2}Ay}{\sqrt{N_w}}\right]
\end{aligned} \tag{16}$$

where $N_w = P_1 + P_2 + N_0$. The average bit error probability $\bar{P}_b$ is derived by taking expectation of $P_b(x, y)$ expressed in (16) with respect to x and y . Going through the same procedure as we did earlier for QPSK modulation, we have

$$\begin{aligned}
P_b(y) &= \frac{5}{32} + Q\left[\frac{\sqrt{2}Ay}{\sqrt{N_w}}\right] \\
\bar{P}_b &= E[P_b(y)] = \frac{5}{32} + Q\left(\frac{\mu}{\sqrt{1 + \lambda^2}}\right)
\end{aligned} \tag{17}$$

where $\lambda = \frac{\sqrt{2}A\sigma}{\sqrt{N_w}}, \mu = \frac{\sqrt{2}Am_y}{\sqrt{N_w}}$.

IV. ANALYTICAL RESULTS AND PERFORMANCE COMPARISON

Comparison between analytical and simulated results is presented in this section to verify theoretical analysis conducted in the previous sections. The simulated curves are obtained numerically by averaging the results over 1000 channel realizations. Some statistics of the SUI-3 channel coefficients are summarized below:

$$\begin{aligned}
E[h_0] &= \sqrt{0.5} + j\sqrt{0.5} \\
P_0 &= E[|h_0|^2] = 1.5 \\
\text{var}[h_0] &= E[|h_0|^2] - |E[h_0]|^2 = 0.5 \\
E[h_1] &= E[h_2] = 0 \\
P_1 &= \text{var}[h_1] = E[|h_1|^2] = 10^{-0.5} = 0.3162 \\
P_2 &= \text{var}[h_2] = E[|h_2|^2] = 0.1
\end{aligned}$$

Based on the above information, $\sigma^2 = 0.25, s^2 = m_1^2 + m_2^2 = 0.5 + 0.5 = 1$ in equation (2), (4), (7), (8) and $\sigma^2 = 0.25 * 2 = 0.5, m_y = \sqrt{0.5} + \sqrt{0.5} = 2\sqrt{0.5}$ in equation (11), (15), (17).

Fig. 3 shows that the theoretical analysis for coherent QPSK modulation expressed by equation (4) is in close agreement with the simulated results, even with imperfect phase estimate which is produced by maximum likelihood algorithm presented in [9]. The figure also shows the close agreement between theoretical analysis for non-coherent QPSK expressed by (15) and the simulated results. It is evident that the coherent detection greatly improves the system performance compared to the non-coherent scheme. However, coherent algorithm alone can not remove the effect the intersymbol interference (ISI) which is inherent in multipath BFWA channel.

For 16-QAM illustrated in Fig. 2, the average symbol energy is

$$E_s = \frac{1}{4}[(A^2 + A^2) + 2(A^2 + 9A^2) + (9A^2 + 9A^2)] = 10A^2$$

Fig. 3. Coherent vs. non-coherent detection for QPSK modulated BFWA system.

Fig. 4. Coherent vs. non-coherent detection for 16-QAM modulated BFWA system.

In our simulations, the transmitted symbols are normalized to unit average energy, therefore, $E_s = 10A^2 = 1$, $A = \sqrt{0.1}$ in equation (7) and in (17). Fig. 4 shows slight discrepancy between the theoretical analysis for coherent 16-QAM modulation expressed by (7), (8) and the simulated results. The gap is quite small though. Coherent detection with imperfect phase estimate yields very close performance compared to the genie-aided case assuming perfect knowledge of the channel. Fig. 4 also shows the close agreement between theoretical analysis for non-coherent 16-QAM expressed by (17) and the simulated results.

Comparing Fig. 3 with Fig. 4, it is evident that QPSK modulation is more robust but supports lower data rate, while 16-QAM supports higher data rate but has worse performance. Their performance gap is much more obvious

with coherent detection than with non-coherent detection.

V. CONCLUSIONS

The QPSK and 16-QAM modulated BFWA systems with conventional detection are theoretically analyzed in this paper. Comparison with the simulated results show that the analysis is fairly accurate. Both simulation and analysis indicate that the BFWA channels are very hostile. Conventional non-coherent and coherent detection schemes will therefore not suffice. Channel coding and equalization algorithms are needed to remove the detrimental effect of ISI and improve the system performance. They are the future research topics for the authors.

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