Angular momentum orientation at bandcrossings in rotating nuclei

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Abstract

Relative rotational $E2$ transition strengths are examined at bandcrossings, throughout the $A \approx 170$ region of well deformed nuclei. It is inferred that for the crossing band (s band) the angular momentum projection on the nuclear symmetry axis follows closely the maximum value for the two singly occupied $i_{13/2}$ neutron orbitals adjacent to the Fermi surface. The method provides direct information from the experimental data about the orientation of the angular momentum vector relative to the intrinsic nuclear symmetry axis.

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Angular momentum is a basic conserved quantity in isolated physical systems, such as atomic nuclei. Furthermore, for objects with an intrinsic axis of symmetry, the angular-momentum projection on the symmetry axis is also conserved, at least in the absence of collective rotation. However, under rotation, the intrinsic frame of reference is not an inertial frame, and Coriolis and centrifugal forces perturb the internal motion. Such is the case for deformed atomic nuclei, where the angular-momentum projection on the symmetry axis is designated K (in units of $\hbar$). Collective rotation induces K mixing and rotation alignment [1]. Nevertheless, for well deformed nuclei the K quantum number remains approximately valid, and continues to influence nuclear structure in a fundamental way [2].

Despite its basic role, the ability to determine the K quantum number is severely constrained by the limited range of observables available in experiments. For example, the K projection is commonly assumed to be equal to the total angular momentum, I , of the state of lowest spin in a rotational band, but this is of little value when the bandhead is itself not well defined. Indeed, high-spin rotational bands are typically observed as sequences of fast (collective) electric-quadrupole ($E2$) $I \rightarrow I - 2$ transitions which decay from one band to another, e.g. at bandcrossings, without reaching their respective bandheads. Although the Bohr-Mottelson collective model [3] provides a simple expression for the reduced $E2$ transition probability in a rotational band, $B(E2, I \rightarrow I') = \frac{5}{16\pi} e^2 Q_0^2 < IK20 | I'K >^2$, which clearly involves the K quantum number through the Clebsch-Gordan angular-momentum coupling coefficient, it is often assumed that $K = 0$ [4, 5].

The present work examines the relative $B(E2)$ values at bandcrossings observed in even-even nuclides in the well deformed rare-earth region. To be consistent with stable deformation, it is required that the energy ratio of the first two rotational states, $E_{4_1^+}/E_{2_1^+} > 3$ (compared to the rigid-rotor limit of $3\frac{1}{3}$). A search for appropriate γ -ray information has been made for $90 \leq N \leq 110$ and $60 \leq Z \leq 76$. Despite limited data (see later) significant sensitivity is found to K values, reflecting the role of $i_{13/2}$ neutrons in the structure of high-spin states in this region. The work builds on observations first made for the odd- N nuclide ^{179}W [6], applied now to a range of even-even nuclides with a methodology that has not previously been exploited.

A two-band-mixing analysis is employed, as outlined by Casten [7], but now including explicitly the K dependence, as given by Walker *et al.* [8]. Figure 1 illustrates the wavefunction convention, where α, β are the amplitudes at spin I (with $\beta > \alpha$) and γ, δ are

the amplitudes at spin $I - 2$ (with $\gamma > \delta$). Normalisation requires that $\alpha^2 + \beta^2 = 1$, and $\gamma^2 + \delta^2 = 1$:

$$\Psi_{+,I} = -\beta\Phi_{g,I} + \alpha\Phi_{s,I}; \quad \Psi_{-,I} = \alpha\Phi_{g,I} + \beta\Phi_{s,I} \quad (1)$$

$$\Psi_{-,I-2} = \gamma\Phi_{g,I-2} + \delta\Phi_{s,I-2}; \quad \Psi_{+,I-2} = -\delta\Phi_{g,I-2} + \gamma\Phi_{s,I-2} \quad (2)$$

In the two-component wavefunctions, the first component refers to the g band (ground-state band) and the second component to the s band (crossing band). Thus, at a bandcrossing, the predominantly s -band state of spin $I - 2$, represented by $\Psi_{+,I-2}$, is at higher energy than the predominantly g -band state, $\Psi_{-,I-2}$, while their relative energies are reversed at spin I . The mixing strength, V , which determines the amplitudes $\alpha, \beta, \gamma, \delta$, is assumed to be the same at spins I and $I - 2$.

It is usual to determine V experimentally (the magnitude, not the sign) either from the energy-level perturbations, or from the γ -ray branching ratios (or, equivalently, from the ratio of $B(E2)$ values, which include an E_γ^5 factor, where E_γ is the γ -ray transition energy). In the present context, it is notable that the extraction of V from the energy-level perturbations is independent of any K -value assumptions. A value of V so derived can then be compared with that obtained from transition branching ratios, under different K -value assumptions. This is essentially the methodology used to determine that several s bands, in nuclides with $N = 105 - 110$, have high K values, see e.g. refs [2, 9], and these bands have been called t bands to acknowledge their high- K character. However, there is no sharp division between the s and t designations, and, in the present work, the label “ s band” is used throughout for the band that crosses the g band. The quantitative K -value deduction for the s band first requires the specification of the K value of the g band, e.g. $K_g = 0$ for even-even nuclides.

The present approach is more selective, but considerably more transparent, especially regarding experimental uncertainties. Energy-level perturbations are not used to determine the mixing strength, V . Instead, it is required that, at a given bandcrossing, the relative intensities of all four $E2$ γ -ray transitions (as illustrated in Figure 1) have been measured, yielding two independent $B(E2)$ ratios. Consideration is restricted to cases where all relative intensities have uncertainties $< 50\%$, since otherwise little can be concluded. For a pair of crossing bands with different $B(E2)$ ratios it will be immediately evident that either the intrinsic quadrupole moments, or the K values (or both) must differ. Since the present

study is focused on well deformed nuclei, it is reasonable to assume equality of the intrinsic quadrupole moments of the two crossing bands, but this point will be returned to later.

To clarify their relationship in a two-band-mixing scenario, each out-of-band:in-band $B(E2)$ ratio can most simply be written as:

$$R[B(E2)] = [(c\beta\delta + \alpha\gamma)/(c\beta\gamma - \alpha\delta)]^2 \quad (3)$$

where c is the ratio of Clebsch-Gordan coefficients:

$$c = [(I-K_1)(I-K_1-1)(I+K_1)(I+K_1-1)/((I-K_2)(I-K_2-1)(I+K_2)(I+K_2-1))]^{1/2} \quad (4)$$

Here, K_1 corresponds to the dominant component, K_s or K_g , of the initial state. If $K_s = K_g$, then $c = 1$ and the two $B(E2)$ ratios are equal. However, if, for example, $I = 18$, $K_g = 0$ and $K_s = 10$, then $c = 0.67$ or 1.49 , depending on which $B(E2)$ ratio is calculated. Assuming $K_g = 0$, there is then a unique combination of V and K_s that reproduces the two experimental branching ratios. A summary of the actual results is presented in Table I, with data from refs [9–14]. Since K mixing is implied, we now refer to average values, $\langle K \rangle$. The $\langle K_s \rangle$ column gives the “pure” s -band K value deduced for each nuclide, while the $\langle K_{obs} \rangle$ column gives the K value of the lowest-energy observed state of the given spin, I , with $\langle K_{obs} \rangle = \beta^2 \langle K_s \rangle$.

The most obvious feature of Table I, given the many even-even nuclides in the range $90 \leq N \leq 110$ and $60 \leq Z \leq 76$, is the small number of cases. This is especially surprising in view of the high sensitivity of the γ -ray spectrometers currently in use worldwide. Nevertheless, significant deductions can be made.

The resulting crossing-band K -value dependence on neutron number is illustrated in Figure 2. Note that, for $^{154}\text{Gd}_{90}$, no intensity uncertainties are given in the literature [11]. However, uncertainties are given for angular correlation ratios, and for the four transitions under consideration all these uncertainties are $< 10\%$. Based on this, 5% uncertainties are now estimated for the $B(E2)$ ratios, leading to the $\langle K_s \rangle$ uncertainty specified in Table I and Figure 2.

Also shown in Figure 2 (full line) is the sum of the Ω values for the $i_{13/2}$ neutron orbitals on either side of the Fermi surface, as obtained from the systematics of Jain *et al.* [15]. Ω is the individual-nucleon angular-momentum projection on the symmetry axis, specified by the deformed shell model (Nilsson model) in the absence of collective rotation. In this mass

region, the $i_{13/2}$ neutrons are well known [1] to be strongly influenced by rotation, through the Coriolis force, which causes rotation alignment. Bandcrossings are interpreted as being due to the neutron $(i_{13/2})^2$ configurations of the crossing bands. Usually, it is the degree of individual-nucleon angular-momentum alignment along the collective rotation axis that is analysed, on the basis of experimental data, rather than the effective $\langle K \rangle$ projection on the symmetry axis (perpendicular to the rotation axis). The present study therefore adds a contrasting perspective regarding the orientation of the angular-momentum vector. It is evident from Figure 2 that the $\langle K_s \rangle$ values (data points with error bars) closely match the sums of the Ω values (full line) for $(i_{13/2})^2$ configurations in the absence of rotation. In the presence of rotation, these values for $(i_{13/2})^2$ configurations are, arguably, the maximum limits, as they should be reduced by rotation alignment. The implied persistence of the K quantum number, in conjunction with rotation alignment, was first demonstrated in $^{179}\text{W}_{105}$ [6] and interpreted as being due to a Fermi-aligned, high- K coupling of the two $i_{13/2}$ neutrons [6, 8, 16]. This is entirely in accord with the present work, but now a consistent analysis is applied over the whole of the rare-earth region of well deformed nuclei. According to this analysis, the effects of the common approximation $\langle K_s \rangle \approx 0$ [4, 5, 10] need to be re-evaluated.

It is pertinent to comment here on the assumption that the intrinsic quadrupole moments are equal in the two crossing bands, i.e. that the β deformations are equal. While there is limited information available, potential energy surface calculations indicate stable β deformations for the ^{184}Os and ^{174}W cases [9, 10]. If the s band were to be slightly more deformed than the g band, which is perhaps indicated for ^{174}W [10], then the s -band transition rates would be correspondingly enhanced, according to the above Bohr-Mottelson formula. This in turn would require lower $\langle K_s \rangle$ values in order to reproduce the experimental branching ratios. Therefore, the deduced ^{174}W $\langle K_s \rangle$ value given in Table I and Figure 2 might be a slight underestimate. However, considering also the restriction $E_{4_1^+}/E_{2_1^+} > 3$, only small effects are expected, which would not change the general appearance of Figure 2. Nevertheless, it would be worthwhile to gain a better understanding of the degree of shape stability if the present methodology is to be more deeply investigated.

An implicit assumption in the foregoing presentation is the validity of the two-band-mixing analysis. Additional band interactions could be included, such as through the Coriolis force, with $\Delta K = 1$ matrix elements that can be large (>1 MeV at $I \approx 18$). However, the

key point in the present analysis of each nuclide is the consideration of a *crossing* between two bands, and the unique sensitivity that this gives to K -value differences. Notwithstanding this assertion, it would be worthwhile in future work to establish, through comparison with more-sophisticated treatments, e.g. ref. [17], the degree of validity of the two-band-mixing approach. An interesting case for further clarification is ^{184}Os , where a phenomenological three-band-mixing analysis has been applied with considerable success [18], albeit with additional parameters, implying $\langle K \rangle \approx 3.5$ for the lowest-energy $I = 16$ state, compared to $\langle K_{obs} \rangle \approx 8$ found here. In addition, extension of the present two-band-mixing methodology to the $A \approx 130$ deformed region, for example, might present problems due to competing proton and neutron s -band structures, and associated shape changes [19].

The effective $\langle K_s \rangle$ values illustrated in Figure 2 can have a strong influence on other nuclear properties, especially the interpretation of K -forbidden transitions, as already discussed in the ^{179}W case [6, 8], for example. Also, the apparent breakdown of the K quantum number in the g band of ^{178}Hf for $I > 12$ [20] could be caused by s -band mixing. While K -forbidden transition rates depend sensitively on the low-amplitude overlap of wavefunction extrema, the present analysis depends on the bulk components of the g -band and s -band wavefunctions (as well as the mixings between them). These are important for the interpretation of other observables, such as static electromagnetic moments, in addition to transition rates.

In summary, relative $E2$ transition rates at bandcrossings in $A \approx 170$ well deformed nuclides are shown to yield a range of K values for the crossing bands, depending on the neutron Fermi level. This leads to a new quantitative understanding of the orientation of the total angular momentum vector in rotating nuclei. For total angular momenta of $I = 16 - 18$, the symmetry-axis projection for s bands is in the range $K = 0 - 10$, consistent with neutron $(i_{13/2})^2$ structure. While the available experimental data are sparse, for the kind of B(E2)-ratio analysis presented here, it is to be hoped that additional cases will be forthcoming, perhaps even through re-analysis of existing data.

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TABLE I: Bandcrossing analysis details.

	I ($\hbar$)	$R[B(E2)]_s$	$R[B(E2)]_g$	V (keV)	$< K_s >$	$< K_{obs} >$
$^{154}\text{Gd}_{90}$	18	0.66 ± 0.03	0.63 ± 0.03	$24.3^{+0.3}_{-0.2}$	$2.2^{+1.3}_{-2.2}$	1.9
$^{164}\text{Yb}_{94}$	16	3.09 ± 0.75	3.76 ± 0.34	$93.6^{+0.1}_{-0.2}$	< 3	< 3
$^{174}\text{W}_{100}$	18	2.54 ± 0.23	1.36 ± 0.13	62.2 ± 0.6	$8.1^{+0.7}_{-0.9}$	7.2
$^{176}\text{W}_{102}$	18	1.20 ± 0.37	0.97 ± 0.16	$29.3^{+0.8}_{-1.1}$	$5.7^{+2.7}_{-5.7}$	5.3
$^{178}\text{Os}_{102}$	18	0.85 ± 0.18	0.62 ± 0.20	$29.8^{+1.5}_{-1.8}$	$6.7^{+2.9}_{-6.7}$	6.3
$^{184}\text{Os}_{108}$	16	1.12 ± 0.07	0.71 ± 0.04	44.6 ± 0.4	$8.1^{+0.7}_{-0.8}$	7.8

FIG. 1: Illustration of energy levels and $E2$ transitions at a bandcrossing, together with the wavefunction convention. The left-hand states are predominantly of g -band character, while the right-hand states are predominantly of s -band character, i.e. $\beta > \alpha$ and $\gamma > \delta$.

FIG. 2: Graph showing how the average K projection in the s band varies with neutron number. See Table 1 for more details. The full line represents the sum of the Ω values for the $i_{13/2}$ neutron orbitals on either side of the Fermi surface.