Delay-Optimized File Retrieval under LT-Based Cloud Storage

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Abstract—Fountain-code based cloud storage system provides reliable online storage solution through placing unlabeled content blocks into multiple storage nodes. Luby Transform (LT) code is one of the popular fountain codes for storage systems due to its efficient recovery. However, to ensure high success decoding of fountain codes based storage, retrieval of additional fragments is required, and this requirement could introduce additional delay. In this paper, we show that multiple stage retrieval of fragments is effective to reduce the file-retrieval delay. We first develop a delay model for various multiple stage retrieval schemes applicable to our considered system. With the developed model, we study optimal retrieval schemes given requirements on success decodability. Our numerical results suggest a fundamental tradeoff between the file-retrieval delay and the target probability of successful file decoding, and that the file-retrieval delay can be significantly reduced by optimally scheduling packet requests in a multi-stage fashion.

Index Terms—Delay-optimal retrieval, LT codes, Cloud storage.

1 INTRODUCTION

Cloud storage systems provide a scalable online storage solution to end users who require flexible amount of storage space but do not wish to own and maintain storage infrastructure [1], [2]. Compared with traditional data storage, cloud storage has several advantages. For example, end users can access their data anywhere through Internet without bothering about carrying physical storage media. Also, different users can collaboratively contribute to the data stored in cloud storage with permission from the data owner. Due to its high popularity in industry, cloud storage has been a hot topic in cloud computing community [3].

Generally, cloud storage systems rely on thousands of storage nodes. A content is often fragmented and distributed into a set of storage nodes. To offer high reliability and availability of storage services, redundancy of contents may be employed. Fragments of

contents may be simply replicated and stored in different storage nodes to achieve redundancy. Other than naive replication, Lin *et al.* [4] proposed two QoS-aware data replication algorithms to reduce storage cost while maintaining QoS for the applications. In [5], Weatherspoon *et al.* analyzed two schemes for redundancy: replication and erasure coding, concluding that erasure-coded systems can provide higher availability with lower bandwidth and less storage space. Since then, there are plenty of works on designing erasure codes for storage systems, and they mainly focus on the reliability and availability of storage systems [6], [7], [8], [9].

Recently, motivated by the great success of rateless codes or fountain codes, which have very low decoding complexity and can generate infinite number of encoded packets, some works [10], [11] have applied the popular rateless code or LT code, into cloud storage systems and have achieved promising performance. The main advantage for a rateless code based cloud storage system is that it significantly simplifies the challenging content placement and content recovery problems that need to be addressed in erasure code based systems. This is because a rateless code based system can potentially generate infinite number of encoded packets to be placed across the storage system and to replace those unavailable packets due to node failure. However, the disadvantage of a rateless code based cloud storage system is that it incurs longer data retrieval delay, since it requires more coded packets due to its uncertainty in decoding different from its erasure code counterpart which is deterministic in decoding. Therefore, in this paper we focus on designing an optimized retrieval method for an LT code based cloud system. As stated in [12], traffic hot spots emerge at the core of a cloud storage system which is responsible for communications between the system and end users. We call this core as a portal. In this research, we focus on addressing the problem how to minimize the retrieval delay generated by the portal in an LT-based cloud storage system.

In particular, unlike the Maximum Distance Sepa-

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erable (MDS) codes such as Reed-Soloman code [13], which need deterministic number of redundant encoded packets for decoding, a storage collector operating LT decoder requires an uncertain number of encoded fragments from the storage system for successful decoding. These uncertain number of encoded packets add delay in storage retrieval. Thus, a tradeoff between the successful decoding probability and storage retrieval delay exists. In this paper, we study such a tradeoff and propose a multiple-stage storage retrieval scheme where we show that storage retrieval delay can be reduced without sacrificing the performance on decoding probability. We further demonstrate the optimal retrieval setup for the case of a two-stage retrieval.

The contributions of this paper are twofold. Firstly, we develop a model describing the multi-stage request scheme for an LT code based storage system. Secondly, we solve the optimization problem for the optimal two-stage request scheme and provide a closed-form expression for analysis.

The remainder of this paper is organized as follows. In Section 2, we briefly review the related work. In Section 3, we introduce the system architecture and present the formal definition of the optimal retrieval delay problem. A detailed description of LT codes and its decodability is presented in Section 4. Section 5 presents a rigorous analysis on the delay for different retrieval schemes together with simulation validation. Section 6 provides analysis on the optimal two-stage request scheme. We then demonstrate the tradeoff between the delay and the probability of successful decoding in Section 7. We also show some simulation results using different traffic models in Section 8. Finally, some important conclusions are drawn in Section 9.

2 RELATED WORK

Erasure code based cloud storage systems: An example of a successful deployment of commercial cloud storage system using erasure coding is Symform [14], [15]. In Symform, Reed-Soloman code [13] is used for redundancy generation. Its success shows erasure-code-based cloud storage systems are practical and comparable with traditional replication-based cloud storage systems. Another research effort focuses on the design of storage system operational optimization based on existing erasure codes to achieve some specific optimal performance matrices. These operational designs may include storage redundancy overhead optimization, storage allocation and repair, and others [16], [17], [18], [19]. As aforementioned, compared with erasure code based cloud storage systems, LT code based systems simplify content placement and recovery problems at the cost of longer retrieval delay.

LT code: Luby Transform (LT) code [20] enjoys relatively low computational complexity of $O(k \ln(k/\delta))$

for recovering k symbols with $O(\sqrt{k \ln^2(k/\delta)})$ additional packets. Here, $1 - \delta$ is the probability of successful recovery of all k symbols. The property of low decoding complexity makes LT code suitable for end users equipped with moderate CPU. Another property of LT code is rateless which means the redundancy ratio of data can be arbitrarily changed without additional design. This property is naturally fit with cloud storage systems with thousands of storage nodes storing various type of data. The promising performance of LT code based storage systems has been shown in [10], [11].

Data retrieval performance in storage systems: In addition to the reliability and availability issues of a storage system, another important problem raised in a storage system is data retrieval performance. Traditional distributed file systems like NFS [21] and Sprite [22] are single server based and achieve acceptable retrieval performance through file caching. However, the retrieval performance is bounded by the speed of the single storage server. Conversely, parallel file systems, such as Vesta [23], Galley [24], PVFS [25] and GPFS [26], allow data spreading across multiple storage nodes and provide parallel access to it. The high transfer rate is reached by various techniques such as I/O scheduling, content prefetching, *etc.* More recently, high retrieval performance is achieved by introducing fast storage devices like solid-state drivers (SSD) [27] and DRAM [28]. Unlike these existing work dealing with the retrieval performance problem from the application level, we consider this problem from the network level since in an LT code based storage system the probabilistic LT decoding could result in large retrieval delay due to decoding failure. In [29], [30], [31], [32], the authors discussed some mechanisms for reducing data fetching latency in distributed networks. Those mechanisms are designed for general content without considering any coding metrics, and they could be applied together with our method.

3 SYSTEM MODEL AND PROBLEM STATEMENT

In this section, we first present a schematic description of a cloud storage architecture, in which LT-encoded data packets are spread out across a pool of storage nodes for high availability. Following that, we formulate a delay-optimal file-retrieval problem, which aims to minimize the retrieval delay by strategically scheduling packet retrieval requests. The notations used in this section are summarized in Table 1.

3.1 System Architecture

In this paper, we focus on a distributed cloud storage system (e.g., a community-based cloud storage system as in Tahoe-LAFS [33]). As illustrated in Figure 1, the cloud storage system manages a set of storage

TABLE 1
Notations

Notation	Description
k	No. of original symbols.
n	No. of LT encoded symbols for certain target decoding probability.
λ_a	Arrival rate for ambient traffic.
l, σ^2	Mean and variance of length of ambient traffic packets.
θ	Arrival rate for LT traffic.
l_{LT}	Length of LT encoded symbols.
t_i	Arrival time for i th LT encoded symbol.
s	No. of stages.
n_i	No. of LT encoded symbols requested in stage i .

Fig. 1. System architecture for a cloud storage system.

nodes through a communication network. The users can access to the storage service through a dedicated portal. One particular storage application of interest in this research is the file storage.

We assume that the storage service provides both premium LT-based file storage and regular file storage. For the LT-based file storage, the portal, upon receiving a file from a user request, encodes the file with a chosen LT code into a set of LT fragments or packets¹ and spreads them across different nodes for reliability and high availability. Upon receiving a file-retrieval request, the portal sends a few packet retrieval requests to different storage nodes, and gathers sufficient encoded packets to be forwarded to the user. For the regular file storage, the portal simply stores the received user file into a chosen storage node, with some potential replication in another set of nodes.

We assume that the number of subscribers for the premium LT-based storage service is much smaller than that for the regular storage service. As such, it is with high probability that, at one instant, only one user will retrieve his/her LT-coded files and the majority of the file-retrieval requests are for the regular storage service. In this paper, we will analyze such a

1. Here we use the term *packet* interchangeably with *fragment* as we assume that an IP packet carries an LT encoded fragment in the system.

Fig. 2. Arrival process of LT encoded packets seen at the portal for a file-retrieval request.

simplified use case. The insights obtained through the mathematical framework and numerical analysis will shed lights on more practical application scenarios.

3.2 Traffic Model

In this architecture, the portal node observes two types of packets. The first one is packets for the regular storage service, considered as a process of ambient traffic. The ambient traffic follows a particular arrival process with an arrival rate of λ_a . Moreover, the length of ambient packets is modeled as a random variable with mean l and variance σ^2 . The second one is the LT-coded packets for the premium storage service. We assume the arrival delay of each LT-coded packet is identically distributed, each of which is a certain random variable with a mean value of θ^{-1} . The length of LT-coded packets is assumed to be a constant, denoted by l_{LT} .

When the portal requests n LT-coded packets from the storage grid, the packet arrival process is illustrated in Figure 2. We assume that the packet request is sent out at time 0 (i.e., $t_0 = 0$), and denote the arrival time of the i -th LT-coded packet as t_i . We also denote the inter-arrival time between the $i - 1$ -th packet and the i -th packet as $\tau_i = t_i - t_{i-1}$.

3.3 Problem Statement

In the considered distributed storage system, the bottleneck lies on the portal, because a large number of users can issue their file requests to the portal. In this paper, we focus on the file-retrieval delay, defined as the duration between the time for the portal receiving an LT-coded file request and the time when the last LT-coded packet is sent out by the portal. The file-retrieval delay is a good indicator of user experience. Therefore, we aim to reduce the file-retrieval delay by strategically scheduling the LT-coded packet requests.

The delay-optimal file-retrieval problem is stated as follows. We assume a probabilistic file-retrieval model. In order to achieve the probability of successful decoding of p , the portal needs to request n LT encoded packets from the storage grid. Traditionally, the n packets are requested in one shot. This scheme is referred to as a *one-stage request scheme*. In this paper, we propose to use a *multiple-stage request scheme* to improve the file-retrieval performance. Specifically, the user can divide the request into s stages, each consisting of requests for $n_1, n_2, \dots, n_s$ packets ($\sum_i n_i = n$).

If the user successfully decodes the file after stage m , it will stop requesting the rest $\sum_{i=m+1}^s n_i$ packets.

In our proposed multi-stage request scheme, the designing objective is to minimize the average file-retrieval delay, for a given number of stages. Mathematically, it can be formulated as the following optimization problem,

$$\min \quad \overline{D}_s(n_1, n_2, \dots, n_s) \quad (1)$$

$$\text{s.t.} \quad \sum_{i=1}^s n_i = n, \quad (2)$$

$$n_i \geq 0, 1 \leq i \leq s. \quad (3)$$

Here $\overline{D}_s(\cdot)$ denotes the average retrieval delay under a given request scheme.

Intuitively, multiple stages of packet retrieval incurs additional delay in storage retrieval. However, we will show that the multi-stage retrieval scheme can outperform the single-stage retrieval scheme in terms of the average file-retrieval delay. This delay reduction originates from the following observation. In the multi-stage retrieval scheme, just retrieving an adequate number of packets may already give sufficiently high success decodability, eliminating the need for further rounds of packet retrieval. Only when the file cannot be decoded with the set of received packets, the additional packet requests are sent in subsequent stages. In such a case, the single-stage retrieval scheme is a special case of our proposed multi-stage retrieval scheme, when $s = 1$. From an optimization perspective, the performance of the multi-stage retrieval scheme cannot be worse than that of the single-stage scheme.

4 LT CODES AND ITS DECODABILITY

In this section, we present a detailed description about LT-codes, including its encoding and decoding processes, and the probability of decoding files for a given number of retrieved packets.

4.1 LT Encoder/Decoder

LT codes are the first class of digital fountain codes, with nearly optimal erasure correcting capability. Its main characteristic lies in employing a simple algorithm based on the exclusive-or operation to encode and decode the message. As such, it is particularly appealing to cloud-based file storage. In the LT-based cloud storage system, a file is first broken into k source symbols, and then these k source symbols are encoded into n packets and spread across a pool of storage nodes. The number of encoded packets n can be arbitrary large depending on the duplication ratio of the system. In this subsection, we briefly explain its encoding and decoding processes.

The encoding and decoding processes of LT codes can be illustrated via an example, as shown in Figure 3.

Fig. 3. An example of LT encoding and decoding procedures for $k = 4$ and $n = 5$. (a) LT encoding: encode 4 source symbols, S_1-S_4 , into 5 packets, P_1-P_5 . (b)-(h) LT decoding to recover the 4 source symbols.

An LT encoder follows a three-step process to generate one encoded packet. First, a degree $d \in [1, k]$ is randomly chosen from a degree distribution of Ω_d . Second, d distinct source symbols are chosen, uniformly at random, from the set of source symbols. Finally, the encoded packet is equal to the bitwise sum, modulo 2, of those d chosen source symbols. The information about the set of chosen source symbols is called coding vector and it is recorded in the head of the encoded packet. The same process repeats until n encoded packets are generated. In Figure 3, Panel (a) demonstrates an example of LT encoding procedure with 4 binary source symbols (*i.e.* $k = 4$), denoted by $S_1, S_2, \dots, S_4$, and 5 LT encoded packets (*i.e.* $n = 5$), denoted by $P_1, P_2, \dots, P_5$ with value 10110.

The LT decoding process employs a simple message passing algorithm [34], which iteratively decodes source symbols from degree-1 encoded packets. Consider the same example illustrated in Figure 3. At the first iteration, the only encoded packets with degree equal to 1 is P_3 (refer to Panel (a)). Thus, the source symbol S_2 which is connected to P_3 is set to 1 (refer to Panel (b)). After S_2 is recovered, the encoded packets which have connections with S_2 are extracted by the value of S_2 and the connections are removed (refer to Panel (c)). After this step, a new symbol S_4 can be recovered and the same decoding procedure is executed iteratively until no more degree-1 encoded packets can be found. If there are still some encoded packets which are not recovered, a decoding failure is reported. In order to recover the rest of source symbols, one can request more encoded packets. Clearly, the LT decoding process cannot guarantee to recover

Fig. 4. Fitting results versus simulation results.

all the source symbols. In this example, if P_5 is missing, the decoding process will be terminated at Panel (b) since no more degree-1 encoded packets can be found after recovering S_2 .

4.2 LT Decodability

As shown in the example in Section 4.1, the LT decoding process is probabilistic. Specifically, given a set of packets with a full-rank coding matrix, the decoding process could still fail with a certain probability. Moreover, the successful probability of decoding increases as the number of received packets n increases. In this case, LT decodability, denoted by $f_k(n)$, is defined as the probability of successful decoding k source symbols from a set of n encoded packets ($n \geq k$).

Existing research on characterizing the LT decodability has not resulted in a closed-form solution yet. In [35], the authors proposed a dynamic programming algorithm to calculate the LT decodability, with a computing complexity of $O(n^3 \log^2 n)$. Another algorithm in [36] reduces the computing complexity to $O(k^2 \log k)$. However, none of these previous research efforts resulted in a closed-form solution to the LT decodability, while relying on numerical evaluation using dynamic programming.

TABLE 2
Fitting results for α and β

k	100	200	300	400	1000
α	6.108	10.268	14.907	17.912	35.65
β	1.263	1.3315	1.3373	1.3235	1.286
SSE	0.003096	0.005643	0.01805	0.01113	0.02308

In this paper, we derive an empirical model for the LT decodability, by leveraging the numerical evaluation techniques from [36]. Note that, in an LT-based cloud storage system, the decodability under 50% is of less engineering interest. As a result, we focus on the decodability over 50% in the rest of this paper. We use a two-step procedure to derive the empirical model for the LT decodability, as follows:

- Firstly, we plot the LT decodability as a function of the number of encoded packet, by using the numerical technique suggested in [36].
- Secondly, we use a generic function of $f_k(n) = 1 - e^{-\alpha(\frac{n}{k}-\beta)}$ to curve fit the numerical results, where α and β are two fitting parameters related to k .

We illustrate this approach in Figure 4. First, we obtain the simulated LT decodability for the cases of $k = 100$, $k = 200$, $k = 300$, $k = 400$ and $k = 1000$ as illustrated with the solid curves. Second, using MATLAB's "Curve Fitting Tool" with a specific fitting function of $f_k(n) = 1 - e^{-\alpha(\frac{n}{k}-\beta)}$, we obtain the fitting results of α and β together with the sum of square errors (SSE) in Table 2. Finally, we compare the LT decodability between the simulation results and the fitting results (the dotted curves) for $k = 100, 200, 300, 400$, and 1000 , in Figure 4. It can be observed that the empirical model captures the simulation results with a decent accuracy.

In the rest of this paper, we will use this empirical model of the LT decodability as a basis for our optimization framework.

5 FILE-RETRIEVAL DELAY ANALYSIS

In this section, we characterize the average file-retrieval delay for different request schemes in the presence of ambient traffic. To validate our analytical results, we run computer simulation written in Java using NetBeans 7.1. All the simulation results are obtained by averaging 1000 simulation instances. In simulations, we set that the length of packets in the ambient traffic follows an exponential distribution. Although it is a simplified model, the insights obtained with this simple model can be applied to guide practical system design. Table 3 lists the default values of some parameters in the simulator.

TABLE 3
Default values in Simulation

Notation	Physical Meaning	Value
l	Length of packet for ambient traffic	1024 Bytes
l_{LT}	Length of LT-coded packet	1024 Bytes
r	Transmission rate	800 kbps

5.1 Delay Analysis for One Stage Request Scheme

We first investigate the file-retrieval delay $D_1(n)$ for n LT packets in the one-stage request case. This delay consists of two parts as shown in Figure 5(a). The first part arises from the traffic arriving before the LT request and the second part arises from the traffic arriving after the LT request. In this analysis, we assume Poisson arrival process for ambient traffic and LT-coded packets. The first part only contains ambient traffic and it is treated as a classical M/G/1 queue²

2. More information on M/G/1 queue can be found in [37].

Fig. 5. Components of delay.

with delay denoted by W . The second part can be further divided into two sub-parts. Firstly, it contains a constant which is the transmission time of n LT packets, which can be derived as $\frac{nl_{LT}}{r}$. The rest is the time to process the ambient traffics which arrive at the portal during the inter-arrival time of each LT packet. This part is denoted by T . Thus, the delay $D_1(n)$ can be expressed as

$$D_1(n) = W + \frac{nl_{LT}}{r} + T. \quad (4)$$

Taking an expectation on both sides, we obtain

$$\begin{aligned} \bar{D}_1(n) &= \bar{W} + \bar{T} + \frac{nl_{LT}}{r} \\ &= \frac{\lambda_a(\sigma^2 + l^2)}{2r^2(1 - \frac{\lambda_a l}{r})} + \frac{\lambda_a l}{r} \sum_{i=1}^n \tau_i + \frac{nl_{LT}}{r} \\ &= \Gamma + \frac{\lambda_a l}{r} \sum_{i=1}^n \frac{1}{i\theta} + \frac{nl_{LT}}{r} \\ &\simeq \Gamma + \frac{\lambda_a l}{r\theta} (\ln(n) + 1) + \frac{nl_{LT}}{r}, \end{aligned} \quad (5)$$

where Γ is a constant when the parameters for the traffic are determined.

There are three terms in Eq. (5), Γ , the logarithm term and the linear term. In order to ensure the portal has a stable state, λ_a should be less than 100 packets per second under our setting stated in Table 3. Under this condition, the order of magnitude of Γ is 10^{-3} . The number of source symbols k usually exceeds 100 which implies the number of requested packets n is also larger than 100. Thus, compared to the linear term which has the order of magnitude equal to 10 or 10^2 , Γ is negligible. If we assume that λ_a and θ are comparable (e.g. $\frac{\lambda_a}{\theta} \approx 1$), the logarithm term has the order of magnitude equal to 10^{-2} which is also negligible. As a result, the linear term $\tilde{D}_1(n) = \frac{nl_{LT}}{r}$ can be used to approximate $\bar{D}_1(n)$. In Figure 6, we plot the values of $\bar{D}_1(n)$ and $\tilde{D}_1(n)$ for n varying from 100 to 800 when $\lambda_a = 30$ and $\theta = 50$. We can observe that Γ and the logarithm term contribute little to the retrieval delay. So it is reasonable to use $\tilde{D}_1(n)$

Fig. 6. Comparison between $\bar{D}_1(n)$ and $\tilde{D}_1(n)$.

to approximate $\bar{D}_1(n)$. Similar results are found with different settings of λ_a and θ . For brevity, those results are not reported.

Fig. 7. Delay in one-stage request. 1: $\lambda_a = 10$, $\theta = 25$; 2: $\lambda_a = 30$, $\theta = 50$; 3: $\lambda_a = 60$, $\theta = 50$.

In Figure 7, we plot the numerical retrieval delay $\tilde{D}_1(n)$, compared with the simulation results, as a function of the number of request packets under different λ_a and θ settings. Notice that the numerical results match the simulation results well, confirming that using $\tilde{D}_1(n)$ to approximate Eq. (5) is applicable.

5.2 Delay Analysis for Multiple Stage Request

In this subsection, we first investigate the file-retrieval delay for the two-stage request scheme. The packet arrival process is illustrated in Figure 5(b).

Suppose in the first stage, the user requests n_1 LT encoded packets. The delay for the first stage is $D_1(n_1)$. After decoding these n_1 packets, if the user fails to decode the original file, it continues to request the n_2 packets. Here we assume that during the decoding process of n_1 packets, the transmission queue in the portal has already returned to the steady state. This assumption is reasonable since it does take some time for the user to determine if the n_1 LT-encoded packets are decodable. Thus, the delay for

the second stage is identical to the first stage except for the number of encoded packets the user requests. As a result, the overall file-retrieval delay for the two stage request case is given by

$$\begin{aligned} D_2(n_1, n_2) &= f_k(n_1)D_1(n_1) \\ &+ (1 - f_k(n_1))(D_1(n_1) + D_1(n_2)) \\ &= D_1(n_1) + (1 - f_k(n_1))D_1(n_2), \end{aligned} \quad (6)$$

where $f_k(n)$ is the probability of successful decoding with n LT encoded packets. Using the fitting results presented in Section 4.2, we can calculate $f_k(n)$ approximately. Taking expectation on both sides, we have

$$\bar{D}_2(n_1, n_2) = \bar{D}_1(n_1) + (1 - f_k(n_1))\bar{D}_1(n_2). \quad (7)$$

We plot in Figs. 8-9 the average file-retrieval delay for the two-stage request scheme, as a function of the number of packets requested in the first stage, under different settings of k , n and traffic loads. Notice that the numerical results match the simulation results well, regardless of the traffic loads. Moreover, in all results, the file-retrieval delay first decreases and then increases as the number of packets requested in the first stage increases. This consistent trend indicates a potential for file-retrieval delay minimization by choosing an appropriate number of packets requested in the first stage. We shall address this optimization problem in Section 6.

Fig. 8. Delay in two-stage request when $k = 100$ and $n = 200$ under different traffic loads. Both numerical and simulation results confirm that the minimal delay exists.

The result for the two-stage request scheme can be generalized into arbitrary t -stage request. Specifically, the delay expression for t -stage request can be defined

Fig. 9. Delay in two-stage request when $k = 400$ and $n = 700$ under different traffic loads. Both numerical and simulation results confirm that the minimal delay exists.

recursively,

$$\begin{aligned} D_t(n_1, n_2, \dots, n_t) &= (1 - f_k(n_1))(D_1(n_1) \\ &+ D_{t-1}(n_2, \dots, n_t)) \\ &+ f_k(n_1)D_1(n_1), \end{aligned}$$

where the initial values for D_1 , D_2 are defined in Eq. (4) and Eq. (6).

In Figure 10 and 11, we plot the optimal delay comparison obtained by different request schemes for different number of original symbols under different traffic loads. Both figures show that the multiple stage request schemes outperform traditional one-stage request. More specifically, the two-stage request scheme can reduce retrieval delay by 15% compared with the one-stage request. Whilst, the three-stage request scheme further reduces the delay 4% only. It is obvious that to find an optimal request in three-stage request is much more difficult than that in two-stage request and the improvement of three-stage request is not so significant. Consequently, we will only focus on finding an optimal two-stage request scheme in the rest of this paper.

6 OPTIMAL SCHEDULING FOR PACKET RETRIEVAL

In this section, we will investigate the optimal scheduling of packet requests to minimize the file-retrieval delay for two-stage request scheme.

The results in Figure 8 and Figure 9 suggest an optimal two-stage request scheme from Eq. (6). Another observation is that for a rational target successful probability (larger than 90%), the optimal number of LT-coded packets requested in the first stage for a two-stage request scheme yields $f_k(n_1) > 50\%$. Thus,

Fig. 10. Minimal delay comparison among one-stage request, two-stage request and three-stage request when $k = 100$.

Fig. 11. Minimal delay comparison among one-stage request, two-stage request and three-stage request when $k = 400$.

we can apply the fitting function for $f_k(n)$ to Eq. (6) and simplify the optimization problem presented in Section 3.3 for the two-stage request. For the convenience of discussion, we formalize the simplified optimization problem as:

$$\begin{aligned} \min \quad & \bar{D}_2(n_1) = \bar{D}_1(n_1) + e^{-\alpha(\frac{n_1}{k}-\beta)} \bar{D}_1(n-n_1) \\ \text{s.t.} \quad & k\beta \leq n_1 \leq n, n_1 \in \mathbb{N}. \end{aligned} \quad (8)$$

By solving the above optimization problem, we can determine the optimal number of packets requested in the first stage for a two-stage request scheme. Instead of finding the exact solution, we relax the integer constraint, and assume n_1 is a positive real number to solve the approximate problem without integer constraint. Note that the minimum $\bar{D}_2(n_1)$

without the integer constraint is a lower bound of the minimum $\bar{D}_2(n_1)$ with integer constraint.

We observed in Section 5.1 that $\bar{D}_1(n)$ can be approximated as $\frac{Lx}{r}n$. Thus, $\bar{D}_2(n_1)$ in Eq. (8) is convex.

Now, taking differentiation with respect to n_1 on both sides of Eq. (8) and set $\frac{\partial \bar{D}_2}{\partial n_1} = 0$, we get the equation $e^{\frac{\alpha}{k}(n_1-k\beta)} = \frac{\alpha}{k}(n-n_1) + 1$. The solution to this equation is

$$n_1^* = n + \frac{k}{\alpha} - \frac{k}{\alpha} W(e^{\alpha(\frac{n}{k}-\beta)+1}). \quad (9)$$

Here, $W(x)$ is Lambert function [38] which is defined as $x = W(x)e^{W(x)}$. Note that $W(x)$ cannot be expressed in terms of elementary functions, and thus calculating the real value of $W(x)$ is not an easy task. There exists literatures on designing algorithm to compute $W(x)$ [39], [40]. However, in this paper, we are more interested in the analytical bound of $W(x)$.

In [41], the authors proved the following bound of $W(x)$,

Lemma 1. For $x > 1$ we have

$$W(x) \geq \frac{\log x}{1 + \log x} (\log x - \log \log x + 1), \quad (10)$$

with equality only for $x = e$.

Fig. 12. Comparison between $W(x)$ and its bound.

Figure 12 shows the comparison between the true value of $W(x)$ and the lower bound in Eq. (10). It is clear that this lower bound is tight enough and we can use this bound to approximate $W(x)$.

By applying Eq. (10) on Eq. (9), we have

$$\begin{aligned} n_1^* &= n + \frac{k}{\alpha} - \frac{k}{\alpha} W(e^{\alpha(\frac{n}{k}-\beta)+1}) \\ &\approx k\beta + \frac{k}{\alpha} \frac{(\alpha(\frac{n}{k}-\beta)+1) \cdot \log(\alpha(\frac{n}{k}-\beta)+1)}{\alpha(\frac{n}{k}-\beta)+2}. \end{aligned} \quad (11)$$

It is trivial that $n_1^* > k\beta$, and

$$\begin{aligned} n_1^* &\leq k\beta + \frac{\alpha}{k} \log(\alpha(\frac{n}{k}-\beta)+1) \\ &\leq k\beta + \frac{k\alpha(\frac{n}{k}-\beta)}{\alpha} \\ &= n. \end{aligned} \quad (12)$$

Eq. (12) shows $n_1^* \leq n$. Thus, we confirm that n_1^* is the optimal request in the first stage which minimizes the two-stage file-retrieval delay. In Figure 13, we show

Fig. 13. Comparison of the optimal first stage request between simulation and approximation under different k .

the comparison of the optimal first stage request between simulation and approximation under different k . In the simulation, the traffic parameters λ_a and θ are set to 30 packets per second and 50 packets per second respectively. The results confirm that the approximation of the optimal first stage request obtained from Eq. (11) is practical. In Eq. (11), the optimal first-stage request is unrelated to traffic load (*i.e.* λ_a, θ). When the number of original symbols k is fixed (*i.e.* the parameters a and c are known), the optimal first-stage request is determined. This is consistent with the results shown in Figure 8 and 9. This phenomena can be explained by the following reason. Compared with the LT encoded packets requested in each stage, the packets belonging to ambient traffic are negligible. The file-retrieval delay is mainly caused by the processing the requested LT encoded packets.

Based on Eq. (11), if we increase the target successful probability (*i.e.* increase the number of requested packets n), the optimal number of packets requested in first stage n_1^* will be also increased. The ratio $\frac{n_1^*}{n}$ is decreased when n is increased. We plot the ratio in Figure 14. This ratio dropping phenomena can be understood as follows. From the aspect of the decoding probability, every packet contributes in the decoding process. From Figure 4, we see an unequal contribution of each packet to the decoding probability. The contribution of an additional packet to the decoding probability is high when the probability stays at a lower level. As the probability progresses higher, the contribution of each additional packet drops. In other words, to make a same gain in the decoding probability, more packets are needed when the probability is at

a higher level. Notice from Figure 6 that the delay in the one-stage request scheme increases linearly with the number of requested packets. Consequently, trade-off exists between decodability and delay. Instead of requesting all the packets required by the target decoding probability at once, requesting a small portion of the number of packets needed in the first stage will generate a decoding probability that contributes most to the final decoding probability; while the marginal contribution from the second stage is relatively small. As the decodability increases, the contribution from the second stage becomes smaller. In other words, a proper number of packets requested in the first stage will give us a relatively high decoding probability. As a result, setting a very high decoding probability is not rational.

Fig. 14. Optimal first stage request ratio when $k = 100$ and $k = 400$.

7 DELAY-DECODABILITY TRADEOFF

We now investigate the fundamental tradeoff between the targeted decoding probability and the file-retrieval delay for different request schemes. This delay-decodability tradeoff is important since it helps users to make a rational request scheme depending on the requirement of their applications.

Fig. 15. Tradeoff between decoding probability and delay when $k = 100$ and $k = 400$.

In Figure 15, we plot the delay-decodability tradeoff for $k = 100$ and $k = 400$ respectively. The numerical result is obtained by combining Eq. (6) and Eq. (11). Since traffic parameters do not have strong effect on the optimal retrieval delay (refer to the figures shown in Section. 5), we choose $\lambda_a = 30, \theta = 50$ as an example. As shown in Fig. 15, the delay for one-stage request increases sharply when the decoding

probability crosses 95%. In comparison, for the two-stage request scheme, the delay is consistently lower. This advantage reflects the gain from being able to decode in the first stage with an adequate instead of excessive number of LT-coded packets. Based on the reported delay-decodability tradeoff, users can choose an appropriate target of decoding probability, *i.e.* for some delay sensitive applications like stream video, a lower decoding probability can be adopted such that the average retrieval delay can be maintained under a certain threshold.

8 GENERAL TRAFFIC MODEL

In Sections 6 and 7, we assume that the arrival process of ambient traffic follows Poisson process. In this section, we use a different traffic model to examine how our two-stage request scheme performs.

The Markov-modulated Poisson process (MMPP) [42] has been extensively used for modeling processes whose arrival rates vary randomly over time [43]. We use MMPP to model the ambient traffic here since it is more general than simple Poisson process and can capture some features to study burstiness of ambient traffic.

In our simulations, the MMPP traffic has arrival rates of λ_1 and λ_2 for the two phases, and an exponentially distributed rate switching duration with a mean value equal to 0.1s. We fix $\theta = 50$ and vary the value of λ_1 , λ_2 and n . The simulation results are shown in Figure 16.

Fig. 16. Simulation result of MMPP model. (a) $k = 100$, $n = 200$. (b) $k = 100$, $n = 250$. (c) $k = 400$, $n = 700$. (d) $k = 400$, $n = 800$.

Comparing with the simulation results shown in Figure 8 and Figure 9, we can see that our two-stage

request scheme exhibits the same behavior even when the bursty MMPP traffic model is used. With a different decoding probability, the best first-stage request can still be found. Only the average delay is affected since more packets are needed to achieve a higher decoding probability. This observation confirms the optimal request in first stage in Eq. (11) which is independent of the property of ambient traffic. We believe that different models of ambient traffic only affect on the overall delay while using our two-stage request scheme can always give the minimal delay.

9 CONCLUSION

In this paper, we have investigated the problem of delay optimal file-retrieval under a distributed cloud storage system. The file is first LT-encoded and spread out into a list of distributed storage nodes. During retrieval, the user schedules the packet request in a multi-stage manner, with an objective to minimize the average file-retrieval delay. We developed an accurate model to characterize the average file-retrieval delay for different request strategies. Using this model, we derived an optimal two-stage request scheme for a given decoding probability. Both simulation and numerical results confirm that this optimal scheme can reduce the average delay dramatically. Our analysis offers a way for storage system operators to design an optimized storage retrieval scheme for LT-based distributed cloud storage systems.

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