Comparison of Different Soft Demodulation and Decoding Algorithms in Coded M-ary Orthogonal DS-CDMA Systems

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Abstract— The system under study is a convolutionally coded and orthogonally modulated DS-CDMA system in time-varying frequency selective Rayleigh fading channels. In this paper, we investigate several iterative schemes based on soft demodulation and decoding algorithms. The performance of different strategies are evaluated numerically and proved to achieve substantial performance gain compared to the conventional hard decision based scheme, especially when the soft demodulator is assisted by decision directed channel estimation and interference cancellation techniques, and also when demodulation and decoding are performed jointly in an iterative manner.

I. INTRODUCTION

In this paper, we study a coded CDMA system with orthogonal signalling formats. The signal modulation is accomplished with Walsh (Hadamard) code, which is used by CDMA users as orthogonal spreading sequences and by the coding community as an error correcting code. The use of Walsh code is widespread in practical CDMA systems. For example, it is used in IS-95 system for orthogonal modulation in the uplink and user separation in the downlink; in 3G systems, it is used for spreading and channelization.

It is well-known that multiuser detection (MUD) is an effective tool to increase the capacity of interference limited CDMA systems and alleviate some technical requirements, such as power control. Several iterative MUD schemes were proposed e.g., in [1, 2] for uncoded M-ary orthogonal systems with affordable complexity (much less than that of an optimum receiver) and performance much better than the standard receiver, especially in high-capacity networks in which the interference from other users is large.

Convolution coding is employed in this system to improve the error correcting capability and power efficiency of the system. It is believed that CDMA systems exhibit their full potential when combined with forward error correction (FEC) coding [3]. Combined with FEC coding, MUD can overcome its limitations in highly correlated multiuser systems [4]. Therefore, in some proposed systems, MUD is employed in conjunction with FEC coding to obtain greater capacity and throughput.

In [5, 6], MAP demodulator and SOVA (soft-output VA) decoder were applied to a similar system using M-ary modulation and FEC. A performance gain of about 0.6dB at a bit error rate (BER) of 10^{-3} was noticed for single user system in AWGN channel when compared against the conventional

non soft-input-soft-output demodulator and decoder. However, some important issues, e.g., channel estimation and interference mitigation were not addressed in the above references.

In this paper, we investigate different approaches to iterative soft demodulation and decoding for this serially concatenated CDMA system with orthogonal modulation and convolutional coding in a multi-user environment over multipath fading channels. Decision-directed interference cancellation and channel estimation are proposed to improve the reliability of the demodulation process. Different channel decoding algorithms, e.g., Viterbi decoding and Log-MAP decoding algorithms are considered and their performance compared. Two alternatives of interfacing inner demodulator and outer decoder, namely partitioned approach and integrated approach are studied. The former one treats demodulation and decoding separately; while the latter one performs them together in the iteration process.

II. SYSTEM MODEL

The system model is only briefly described in this section. For a more detailed description, readers are referred to [1]. The block diagram of the transmitter is shown in the upper part of Fig. 1. The k^{th} user's l^{th} information bit is denoted as $b_k[l] \in \{+1, -1\}$ ($k = 1, \dots, K$, $l = 1, \dots, L_b$, and L_b is the block length). The information bits are convolutionally encoded into code bits $\{u_k[l^n]\} \in \{+1, -1\}$, where $u_k[l^n]$ denotes the n^{th} code bit due to $b_k[l]$. For example, in case of a rate $1/3$ code, $b_k[l]$ is encoded into $u_k[l^0], u_k[l^1], u_k[l^2]$.

Code bits are subsequently interleaved and each block of $\log_2 M$ coded and interleaved bits $\{u_k'[l^n]\} \in \{+1, -1\}$ is mapped into $\mathbf{w}_{i_k(j)} \in \{\mathbf{w}_0, \dots, \mathbf{w}_m, \dots, \mathbf{w}_{M-1}\}$, which is one of the M Walsh codewords. The subscript $i_k(j) \in \{0, 1, \dots, M-1\}$ denotes the k^{th} user's j^{th} Walsh symbol index. The interleaver and deinterleaver are denoted as Π and Π^{-1} , respectively, in Fig. 1 and the following figures. The purpose of interleaving is to separate adjacent code bits in time so that, ideally, each code bit will experience independent fading.

The Walsh codeword $\mathbf{w}_{i_k(j)} \in \{+1, -1\}^M$, is then repetition encoded into

$$\mathbf{s}_k(j) = \text{rep}\{\mathbf{w}_{i_k(j)}, N/\log_2(M)\} \in \{+1, -1\}^N \quad (1)$$

where $\text{rep}\{\cdot, \cdot\}$ denotes the repetition encoding operation, where its first argument is the input bits and the second one is the repetition factor. Therefore, each bit of the Walsh codeword is spread (repetition coded) into $N_c = N/M$ chips, and each Walsh symbol is represented by N chips and denoted as $\mathbf{s}_k(j)$.

Fig. 1. Transmitter, channel and receiver front end.

The Walsh sequence $\mathbf{s}_k(j)$ is then scrambled (randomized) by a scrambling code unique to each user to form the transmitted chip sequence $\mathbf{a}_k(j) = \mathbf{C}_k(j)\mathbf{s}_k(j) \in \{+1, -1\}^N$ where $\mathbf{C}_k(j) \in \{-1, 0, +1\}^{N \times N}$ is a diagonal matrix whose diagonal elements correspond to the scrambling code for the k^{th} user's j^{th} symbol. The purpose of scrambling is to separate users. In this paper, we focus on the use of long codes, e.g., the scrambling code differs from symbol to symbol. The scrambled sequence $\mathbf{a}_k(j)$ is pulse amplitude modulated using a unit-energy rectangular chip waveform $\psi(t)$ to form the baseband signal.

The baseband signal is multiplied with a carrier and transmitted over a Rayleigh fading channel with noise power spectral density $N_0/2$ and with L_k resolvable paths, having time-varying complex channel gains $h_{k,1}(t), h_{k,2}(t), \dots, h_{k,L_k}(t)$ and delays $\tau_{k,1}, \tau_{k,2}, \dots, \tau_{k,L_k}$ (see the lower part of Fig. 1). The received signal is the sum of K users' signals plus additive white complex Gaussian noise $n(t)$. After frequency down-conversion and chip matched filtering (CMF), the received signal corresponding to the k^{th} user's j^{th} transmitted Walsh sequence $\mathbf{s}_k(j)$ can be written in vector form as

$$\begin{aligned} \mathbf{r}(k, j) &= \mathbf{A}(k, j)\mathbf{h}(j) + \mathbf{n}(k, j) \\ &= \mathbf{X}_k(j)\mathbf{h}_k(j) + \text{ISI}(k, j) + \text{MAI}(k, j) + \mathbf{n}(k, j) \in \mathbb{C}^{N_k} \end{aligned} \quad (2)$$

where the columns of the matrix $\mathbf{A}(k, j)$ are the delayed version of transmitted chip sequences $\mathbf{a}_k(j)$ for $k = 1, 2, \dots, K$, one column per path. The length of the processing window N_k , is larger than the symbol interval N to account for the asynchronous and multipath nature of the channel. The columns are weighted together by $\mathbf{h}(j)$, whose elements are the path gains of all users' paths. The received vector $\mathbf{r}(k, j)$ can be written as the sum of four terms: the signal of interest $\mathbf{X}_k(j)\mathbf{h}_k(j)$, the intersymbol interference (ISI), the multiple access interference (MAI), and the noise represented by $\mathbf{n}(k, j)$ which is a vector of complex noise samples with zero mean and variance N_0 . The columns

of the matrix $\mathbf{X}_k(j)$ are essentially the shifted versions of the chips due to the k^{th} user's j^{th} symbol, one column per path (the shift is determined by the path delay). The vector $\mathbf{h}_k(j) = [h_{k,1}(jT) \ h_{k,2}(jT) \ \dots \ h_{k,L_k}(jT)]^T$ corresponds to the channel gains of the k^{th} user's paths, it is a part of $\mathbf{h}(j)$.

III. DEMODULATION AND DECODING SCHEMES

The task of the receiver is to detect the information bits transmitted from all users, i.e., detect $b_k[l]$ ($l = 1, 2, \dots, L_b$ and $k = 1, 2, \dots, K$) given the observation $\mathbf{r}(k, j)$, for $k = 1, 2, \dots, K$ and $j = 1, 2, \dots, \frac{L_b}{\log_2 M}$. To this end, we first need to demodulate the received signal to transmitted Walsh sequence $\mathbf{s}_k(j)$ or directly to the code bits $\{u'_k[l]\}$ which are subsequently convolutionally decoded to obtain an estimate of $\{b_k[l]\}$. Four different demodulation and decoding strategies are studied in this paper and outlined in Fig. 2. The first one is the conventional scheme with partitioned hard demodulation and decoding with hard input Viterbi algorithm (HIVA); the second one is still partitioned approach, but with soft demodulation and decoding with soft input Viterbi algorithm (SIVA); the third scheme is an integrated approach with soft demodulation and SIVA decoding; the last one is an integrated approach with soft demodulation and Log-MAP decoding. The design and implementation of different algorithms are discussed next.

A. Conventional partitioned hard demodulation, HIVA decoding scheme

The conventional scheme is delineated in Fig. 3. Based on the received observation $\mathbf{r}(k, j)$, the receiver makes hard decisions on the transmitted sequences $\hat{\mathbf{s}}_k(j) \in \{\mathbf{s}_0, \dots, \mathbf{s}_{M-1}\}$ using symbol matched filter or more advanced multiuser detectors. The detected symbols are converted to bits $\{\hat{u}'_k[l]\}$, which are then deinterleaved and decoded with the HIVA decoder.

In case a multistage multiuser detector is used, the iteration is only inside the demodulation block. The per-

Fig. 2. Different demodulation and decoding schemes.

Fig. 3. Conventional demodulation and decoding (scheme 1).

formance is improved in an iterative manner due to the fact that the channel is more accurately measured and interference is better detected (meaning better interference mitigation), as the iteration goes on.

Since only hard decisions are passed from demodulator to decoder, performance loss is inevitable. It is a well known fact that for VA, hard-decision decoding is 2 ~ 3dB inferior to soft-decision decoding in an additive white Gaussian noise channel and the gap is even greater in presence of fading. In order to enable soft decoding, three soft demodulation algorithms were proposed in [2]. In this paper, we focus on different ways of interfacing the soft demodulator and channel decoder. They are presented in order of increased complexity and improved performance.

B. Partitioned soft demodulation, SIVA decoding scheme

Instead of estimating the transmitted M -ary sequence $\mathbf{s}_k(j)$ and then converting them into bits $\{u'_k[l]^n\}$, we can directly derive soft reliability value for each bit $u'_k[l]^n$ from the received vector $\mathbf{r}(k, j)$. Assuming bits $+1$ and -1 , are equally probable, i.e., $P(u'_k[l]^n = +1) = P(u'_k[l]^n = -1)$, a posteriori log-likelihood ratio (LLR) for a transmitted $+1$ and a transmitted -1 in the bit sequence $\{u'_k[l]^n\}$ is defined as [7]

$$\begin{aligned} \lambda(u'_k[l]^n; O) &= \ln \frac{f(u'_k[l]^n = +1 | \mathbf{r})}{f(u'_k[l]^n = -1 | \mathbf{r})} = \ln \frac{f(\mathbf{r} | u'_k[l]^n = +1)}{f(\mathbf{r} | u'_k[l]^n = -1)} \\ &= \ln \frac{\sum_{m: u'_k[l]^n = +1} f(\mathbf{r} | \mathbf{s}_m)}{\sum_{m: u'_k[l]^n = -1} f(\mathbf{r} | \mathbf{s}_m)} \end{aligned} \quad (3)$$

where the first equation holds due to the assumption $P(u'_k[l]^n = +1) = P(u'_k[l]^n = -1)$. The conditional probability $f(\mathbf{r} | \mathbf{s}_m)$ or equivalently, $f(\mathbf{r} | \mathbf{w}_m)$ can be approximated as $f(\mathbf{r} | \mathbf{s}_m) = A \exp\{B z_k(m)\}$, where $z_k(m)$ is the decision

statistic from demodulator, based on the hypothesis that the m^{th} Walsh symbol is transmitted from user k , and A and B are some constants. The derivation of $z_k(m)$ for different soft demodulators is thoroughly studied in [2]. Here, we only describe the basic principles of soft demodulation. In the above equation, we denote $m : u'_k[l]^n = \pm 1$ as the set of Walsh sequences $\{\mathbf{s}_m\}$ that correspond to the code bit $u'_k[l]^n = \pm 1$. Typically, one term will dominate each sum in (3), which suggests the “dual-maxima” rule [8]

$$\begin{aligned} \lambda(u'_k[l]^n; O) &\approx \ln \frac{\max_{m: u'_k[l]^n = +1} f(\mathbf{r} | \mathbf{s}_m)}{\max_{m: u'_k[l]^n = -1} f(\mathbf{r} | \mathbf{s}_m)} \\ &= \max_{m: u'_k[l]^n = +1} \{z_k(m)\} - \max_{m: u'_k[l]^n = -1} \{z_k(m)\} \end{aligned} \quad (4)$$

We use the notations $\lambda(\cdot; I)$ and $\lambda(\cdot; O)$ at the input and output ports of a soft-input, soft-output (SISO) device. They refer to the unconstrained LLRs when the second argument is I , and modified LLRs according to the code constraints when it is O .

The scheme based on this soft decision rule is shown in Fig. 4. Note that there are two outputs from the soft demodulator: the soft decision about the transmitted Walsh symbol $z_k(m)$ and the LLRs for code bits $\lambda(u'_k[l]^n; O)$. The former one is used to make hard decision on the Walsh symbol index $i_k(j)$ or transmitted Walsh sequence $\mathbf{s}_k(j)$ which is needed for estimating the multipath complex channel gains and interference cancellation assisted soft demodulation. The deinterleaved LLR value $\lambda(u_k[l]^n; I) = \Pi^{-1}\{\lambda(u'_k[l]^n; O)\}$ is delivered as soft input to the channel decoder.

Compared to the traditional scheme illustrated in Fig. 3, the hard demodulator is replaced by a soft demodulator so that soft-input VA (SIVA) decoding can be implemented.

Fig. 4. Partitioned soft demodulation, SIVA decoding (scheme 2).

Since the soft demodulator directly outputs the soft value for the code bits $\{u'_k[l]^n\}$, the symbol-to-bit converter is not needed.

The above two schemes are not optimized in the sense that the demodulator can not benefit from information derived from the channel decoder. Demodulation and decoding are strictly partitioned into two blocks. In the following, we present the integrated approach, where the problem of joint soft demodulation and decoding is approached by expanding the iteration loop over the concatenation of demodulation and decoding blocks.

C. Integrated soft demodulation and SIVA decoding

The first integrated scheme is illustrated in Fig. 5. As we did earlier, the computed soft metrics $\{\lambda(u'_k[l]^n; O)\}$ are de-interleaved and decoded to get the estimated information bits $\{\hat{b}_k[l]\}$. Then we go through the same encoding, interleaving and modulation processes as in the transmitter to yield an estimate of the code bits $\{\hat{u}'_k[l]^n\}$ and Walsh sequences $\hat{s}_k(j)$.

Here we still use SIVA decoder to yield hard decision on $\{b_k[l]\}$. It differs from the aforementioned algorithms in that the decisions from the channel decoder are fed back to the demodulator. The estimate of $\hat{s}_k(j)$ needed for channel estimation and interference mitigation is not delivered from the demodulator itself, but from the output of the channel decoder. As will be clear later on, spanning the iteration loop over the two blocks is really crucial in improving the system performance. The price to pay is the added complexity instead of doing it once for all. Going through convolutional encoding, interleaving and modulation processes every time also slightly increase the complexity.

D. Integrated soft demodulation and Log-MAP decoding

A further enhancement idea is to replace the VA decoder with some SISO decoder which produces soft output for both information bits $\{b_k[l]\}$ and code bits $\{u_k[l]^n\}$. Based on the soft input $\lambda(u_k[l]^n; I)$ and the trellis structure of the convolutional code, the k^{th} user's SISO channel decoder computes a posteriori LLR of each information bit

$\lambda(b_k[l]; O)$ and each code bit $\lambda(u_k[l]^n; O)$ as

$$\lambda(b_k[l]; O) = \ln \frac{P[b_k[l] = +1 | \lambda(u_k[l]^n; I)]}{P[b_k[l] = -1 | \lambda(u_k[l]^n; I)]} \quad (5)$$

$$\lambda(u_k[l]^n; O) = \ln \frac{P[u_k[l]^n = +1 | \lambda(u_k[l]^n; I)]}{P[u_k[l]^n = -1 | \lambda(u_k[l]^n; I)]} \quad (6)$$

where $\lambda(b_k[l]; O)$ is used to make decision on the transmitted information bit at the final iteration, while $\lambda(u_k[l]^n; O)$ is used for channel estimation and interference cancellation in the demodulator at the next iteration.

Several SISO algorithms can be used to compute the channel decoder outputs (5) and (6). We consider the use of Log-MAP for the purpose of this study. For a complete treatment on different SISO algorithms, their similarities, differences, performance and complexity comparisons, readers are recommended to consult [9].

The improved iterative decoding schemes are shown in Fig. 6. The demodulator and decoder are each implemented with a SISO algorithm and operate in an iterative feedback mode where the information derived by the channel decoder is fed back to demodulator. The soft metrics from demodulator are then deinterleaved and decoded to carry on with the next iteration. It is similar to the previous one shown in Fig. 5, except that the SIVA decoder is replaced by a Log-MAP decoder and the code bits $\{\hat{u}'_k[l]^n\}$ are estimated by making hard decisions on the outputs of the channel decoder, i.e., $\{\hat{u}'_k[l]^n\} = \text{sgn}(\Pi\{\lambda(u_k[l]^n; O)\})$. A Log-MAP decoder is more complex than a VA decoder due to the need for forward and backward recursions.

IV. NUMERICAL RESULTS

In the simulations, we employ a rate $R_c = 1/3$ Maximum Free Distance (MFD) convolutional code [10] with constraint length 5 and generator polynomials (25, 33, 37) in octal form for all the users. Block interleaving is applied to the convolutionally encoded bits to decorrelate the fading effect. Each block of $\log_2 8 = 3$ interleaved bits from each user is then converted into one of $M = 8$ Walsh codewords spread to a total length of $N = 64$ chips. The number of chips per inner code bit is $N_c = N/M = 8$.

Fig. 5. Integrated soft demodulation and SIVA decoding (scheme 3).

Fig. 6. Integrated soft demodulation and Log-MAP decoding (scheme 4).

Channels are independent multipath Rayleigh fading channels. The channel gain $h_{k,l}(t)$ is a complex circular Gaussian process with autocorrelation function $E[h_{k,l}^*(t)h_{k,l}(t+\tau)] = P_{k,l}J_0(2\pi f_D\tau)$ where f_D is the maximum Doppler frequency, $J_0(x)$ is the zeroth order Bessel function of the first kind, and $P_{k,l}$ is the power of $h_{k,l}(t)$. Here, the normalized Doppler frequency is assumed to be $f_D T = 0.01$. Perfect slow power control is assumed in the sense that $P_k = \sum_{l=1}^{L_k} P_{k,l}$, the average received power, is equal for all users. The number of multipath channels L_k is set to be 3, ($L_k = L = 3$) for k . Channel estimation is conducted with the ML algorithm presented in [1].

The long scrambling codes $\mathbf{C}_k$ are randomly assigned. The noise variance N_0 , and $\mathbf{C}_k$ as well as path delays $\tau_{k,1}, \tau_{k,2}, \dots, \tau_{k,L_k}$ are assumed to be known to the receiver. For each block, 4620 code bits are passed through a block interleaver of size 66×70 . The simulation results are averaged over random distributions of fading, noise, delay, and scrambling code with minimum of 10 blocks of data transmitted and at least 100 errors generated. To study the behavior of each algorithm, the number of iterations is usually set to 6 or 7, since it is observed that almost all the algorithms would converge after 5 or 6 iterations.

In Fig. 7, we compare the performance of all the discussed algorithms with interference cancellation (IC) assisted demodulator in a 12-user system. Apparently, the most significant gain (over 2 dB at BER $< 10^{-2.5}$) is obtained by using soft demodulator instead of hard demodulator. Replacing partitioned approach with integrated approach also gives substantial improvement (around 1 dB). The improvement by replacing VA with Log-MAP decoding is relatively small, but still noticeable.

Fig. 7. Different schemes with IC assisted demodulator.

The behavior and convergence property of two integrated schemes are further investigated in Fig. 8 and Fig. 9. As indicated in Fig. 8, the performance gain by replacing SIVA decoding with Log-MAP decoding is obvious during the first 3 iterations; however, the gap tends to decrease as they reach convergence, at which point, the difference is 0.3/0.4/0.5 dB at BER of $10^{-4}/10^{-3}/10^{-2}$. For both schemes, the gain at each iteration increases with increased SNR. A reasonable level of SNR needs to be maintained in order to benefit from the iteration process. It can be seen from Fig. 9 that with $E_b/N_0 = 9$ dB and target BER of 10^{-3} , 10/6, 19/14, 23/19, 27/23, 30/27 users can be supported by 1, 2, 3, 4, 5 stages of IC soft demodulation with

Fig. 8. Comparison of two integrated approaches with SIVA/Log-MAP decoding.

Fig. 9. Test of system capacity.

Log-MAP/SIVA decoding. The difference in capacity gets smaller as two algorithms converge. Also, the gain through iteration becomes smaller as the system becomes more heavily loaded. We observe from these plots that all the algorithms converge to maximum achievable performance after 4 ~ 6 iterations, beyond which improvement through the iteration process becomes insignificant.

V. CONCLUSIONS

In this paper, we analyzed and compared the performance and complexity of several strategies for demodulating and decoding orthogonally modulated and convolutionally coded signals in frequency selective channels. We have shown that of all the presented enhancement ideas, the replacement of the hard demodulator with a soft demodulator is most crucial in the improvement of system performance.

In order to remove deteriorative effect of interference, we need to perform interference cancellation and channel estimation which can be implemented using decisions either

from the output of demodulator or channel decoder. In the former case, the demodulation and decoding blocks are partitioned as in the conventional system; in latter case, the two blocks are connected in an iterative fashion so that the output of the channel decoder can be fed back to the demodulator. The integrated approach outperforms the partitioned approach, at cost of higher complexity.

The convolutional decoder can be implemented either with the VA or the Log-Map algorithm. Noticeable gain is obtained by replacing the VA with the Log-MAP, especially at the first few iterations. It is attributed to fact that the Log-MAP is designed to minimize symbol (bit) error probability, thus yielding a better estimate of information bits as well as a better estimate of code bits for channel estimation and interference mitigation.

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