Predicting fatigue damage in adhesively bonded joints using a cohesive zone model

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Abstract

A reliable numerical damage model has been developed for adhesively bonded joints under fatigue loading that is only dependant on adhesive system and not on joint configuration. A bi-linear traction-separation description of a cohesive zone model was employed to simulate progressive damage in the adhesively bonded joints. Furthermore, a strain-based fatigue damage model was integrated with the cohesive zone model to simulate the deleterious influence of the fatigue loading on the bonded joints. To obtain damage model parameters and validate the methodology, carefully planned experimental tests on test coupons cut from a bonded panel and separately manufactured single lap joints were undertaken. Various experimental techniques have been used to assess joint damage including the backface strain technique and in-situ video microscopy. It was found that the fatigue damage model was able to successfully predict the fatigue life and the evolving backface strain and hence the evolving damage.

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1. Introduction

Fatigue is one of the most common yet complicated failures that can cause damage to mechanical structures. Structural adhesively bonded joints are not exempt from this deleterious phenomenon and have to be assessed under fatigue loading. Mechanical behaviour of structures under fatigue loading can be studied experimentally and numerically. However, the experimental testing is often expensive and time consuming and sometimes impossible in the case of huge structures, whilst implementation of numerical models is time and cost efficient and can effectively enable engineers to optimise the experimental effort required. Nonetheless, much work has been undertaken in characterising experimentally the response of bonded joints to fatigue loads, whereas less work has been directed towards modelling fatigue failure. Moreover, numerical fatigue models found in literature are often joint geometry dependent and may not be applicable to different joint configurations.

Besides being joint geometry independent, some other key points need to be considered in a reliable and effective numerical fatigue failure model. Firstly, in order to predict residual strength, damage and the evolution of predicted damage need to be consistent with the experimentally measured damage during the fatigue loading. Secondly, whole fatigue lifetime including the initiation and propagation phases should be taken into account. This is because, in fatigue loading, either of these phases can be dominant depending on the load range and other factors such as materials, joint geometry and test environmental conditions. Thus both phases need to be considered. Hence, physically clear definitions or criteria are required to

differentiate between the initiation and propagation phases. Lastly, since fatigue is a complicated phenomenon, different fatigue aspects like the fatigue endurance limit should be incorporated into the predictive model.

In this current work, a bi-linear traction-separation description of the cohesive zone model integrated with a strain-based fatigue damage model was utilised for simulating the progressive fatigue damage in adhesively bonded joints. The approach outlined here incorporates a fatigue damage model based on maximum fatigue load conditions and hence requires a significantly less computational effort in comparison with the models based on the cycle-by-cycle analysis. Furthermore, the backface strain technique and in-situ video microscopy were employed for assessing the damage and the damage evolution in adhesive bond line of the adhesively bonded joints. The aim of this research was to develop a numerical fatigue damage model which was only dependent on the adhesive system. Thus, two joints (single lap joint and laminated doublers in bending) using the same adhesive system (i.e. identical adhesive material, surface pre-treatment and priming) but different geometries and hence different stress states and mode mixities were considered and tested under static and cyclic loading. Then, a numerical fatigue model was developed and calibrated against the experimental results obtained for the single lap joint and the same fatigue damage model with the same parameters were employed for predicting the fatigue response of the other joint (doubler in bending). The calibration process consisted of a systematic assessment of the affect of the fatigue damage parameters on the load-life and back-face strain responses of the joint and then an informed fitting of predicted and measured load-life curves

2. Background

Up until now, various methods have been employed to model the fatigue damage in adhesively bonded joints. Some methods [1] consider only total fatigue lifetime. Although the total-life approach can be useful to predict the fatigue lifetime, this method is not able to indicate the damage or the evolution of the damage during the fatigue loading. Therefore, the residual strength cannot be determined using this method. Another deficiency of the total-life approach is that the damage initiation and propagation phases of fatigue lifetime are not differentiated.

Other methods, like those based on the stress singularity, only take into account the damage initiation phase and ignore the damage propagation phase. Using such methods, the presence of the stress singularity at the damage initiation point is utilised to predict the fatigue initiation lifetime [2-5]. However, calculating the singularity parameters is not straightforward and may require cumbersome and rigorous analytical and/or numerical efforts. Moreover, this approach cannot study progressive damage during the initiation phase and since it is based on an elastic stress field, it may not be appropriate for problems with extensive plastic deformation.

In other approaches, like fracture mechanics based methods, only the damage propagation phase is considered, whilst damage initiation is disregarded. Using the fracture mechanics approach the number of cycles to failure can be obtained by integrating a fatigue crack growth law, like the Paris law, from initial to final crack length. This method predicts the fatigue propagation lifetime in three steps. First, the crack growth rate (da/dN) should be determined

as a function of applied maximum strain energy release rate (SERR). This can be achieved by conducting short-term fracture mechanics tests under cyclic loading. In the second step, the variation of applied maximum SERR as a function of crack length is determined analytically or computationally. Finally, these data are combined and the resulting fatigue crack growth equation is integrated from initial to final crack length. This approach is based on linear elastic fracture mechanics, small-scale yielding, constant amplitude loading and long cracks. Numerous modifications have been proposed to adapt Paris law-based models to a wider range of problems (e.g. [6-10]).

Some researchers (e.g. Wahab et al. [11], Imanaka et al. [12], Hilmy et al. [13,14]) employed continuum damage mechanics to predict the fatigue in adhesively bonded joints. In this method, a damage parameter (D) is defined which modifies the constitutive response of the adhesive. According to this theory, the damage accumulation can be expressed in terms of number of cycles to failure. Although the continuum damage mechanics based method provides a valuable engineering predictive framework, it does not give a clear definition of the fatigue initiation and propagation phases.

The cohesive zone model (CZM) has recently received considerable attention and has been employed for a wide variety of problems and materials including metals, ceramics, polymers and composites. This model was developed in a continuum damage mechanics framework and made use of fracture mechanics concepts to improve its applicability. The CZM was originally introduced by Barenblatt [15,16], based on the Griffith's theory of fracture. He assumed that finite molecular cohesion forces exist near the crack faces and described the crack propagation in perfectly brittle materials using his model. Then, Dugdale [17]

considered the existence of a process zone at the crack tip and extended the approach to perfectly plastic materials. He postulated the cohesive stresses in the CZM as constant and equal to the yield stress of material.

For the first time, Hillerborg et al. [18] implemented CZM in the computational framework of FEM. They proposed a fictitious crack model for examining crack growth in cementitious composites. Contrary to previous works, where the cohesive zone tractions had been defined as a function of the crack tip distance, they defined tractions versus the crack opening displacement and consequently, the prevailing description of the CZM in the form of a traction-separation law was formed. Other researchers then extended the model by proposing various traction-separation functions and applying it to different problems. However the fundamental concept remained essentially unchanged. For example, Needleman suggested a number of different functions such as polynomial [19] and exponential [20] for traction-separation relationship. More details about the different functions can be found in Ref. [21].

The cohesive zone model has been employed for predicting fatigue response of structures by several authors [22-30]. They coupled CZM with a fatigue damage evolution law to simulate fatigue degradation. Some authors [25-29] modelled fatigue loading cycle by cycle which was computationally expensive and practically impossible in case of high cycle fatigue. Therefore, others [22,30] tried to reduce the computational effort by employment of cyclic extrapolation techniques. Alternatively, some other researchers [23,24] developed fatigue damage models based on maximum fatigue load conditions. Robinson et al. [23] incorporated a cumulative damage model based on the maximum fatigue load into the static CZM. In this model, the fatigue model parameters need to be determined for every mode ratio. Later, Tumino et al.

[24] resolved this deficiency by considering the fatigue damage parameters as functions of the mode ratio. However, this resulted in introducing quite a few new parameters in the fatigue model and required an involved calibration process.

To experimentally determine residual strength of a joint under fatigue loading, the damage and the evolution of damage need to be evaluated. This can be done only when the complicated process of damage during the cyclic loading is clarified and this requires a localised damage assessment. This is because the localised damage such as damage initiation may not affect the overall behaviour of the joint, consequently, methods that rely on global behaviour such as the overall stiffness loss detection, may not be able to monitor fatigue damage in detail. A reliable localised damage assessment technique which can be utilised for adhesively bonded joints is the backface strain technique. In this method, strain gauges are bonded on the backface (exposed surface) of the substrate, near a site of anticipated damage. The measured strain during the onset and growth of the damage changes and this change is utilised to indicate the damage. The backface strain technique was initially employed by Abe and Satoh [31] to study crack initiation and propagation in welded structures. Later, other authors [32-38] applied this technique to adhesively bonded joints.

3. Cohesive zone model

The cohesive zone model, shown in Fig. 1, combines a strength-based failure criterion to predict the damage initiation and a fracture mechanics-based criterion to determine the damage propagation. This section outlines the key parameters of such a model (E_0 , T and G_C) as defined in Fig. 1.

The initial stiffness of cohesive zone model (E_0 , defined as traction divided by separation, having units N/m^3) should be chosen as high as possible so that the CZM does not influence the overall compliance before damage initiation, but from a numerical perspective it cannot be infinitely large otherwise it leads to numerical ill-conditioning.

Fig. 1. Schematic damage process zone and corresponding bi-linear traction-separation law in an adhesively bonded joint.

The tripping traction (T) is related to the length of the process zone and to the tensile strength of the material and is difficult to measure experimentally [39]. Therefore some researchers [40,41] treated even the tripping traction as a penalty parameter. Liljedahl et al. [42] studied the interaction of the tripping traction value and the FE mesh on the failure load and divided the tripping traction range into three regions. In the lower and higher tripping traction regions, the failure load was highly dependent on the tripping traction but in the intermediate region,

the failure load was essentially constant. Thereby, they suggested using a tripping traction from intermediate region for the traction separation law.

The fracture energy (G_C), the area beneath the traction-separation curve, is the most important parameter which is often available in literature or can be determined by means of some standard experimental tests. Some researchers (e.g. [43-45]) assumed that the influence of the shape of strain-softening branch on results can be disregarded. Other researchers [21,46] emphasised that shape of the strain-softening branch can significantly influence the response. Chandra et al. [21] investigated two softening branch shapes (bi-linear and exponential) and compared their influences on the mechanical behaviour of a push-out test. They found that the bi-linear CZM reproduced the macroscopic mechanical response and failure process in their problem whilst the exponential form did not.

4. Experimental

Two different types of adhesively bonded joints, namely single lap joints (SLJ) and laminated doublers in bending (LDB), shown in Fig. 2, were tested under static and fatigue loading to obtain damage model parameters and validate the methodology. The SLJ was a standard single lap joint made of Aluminium 2024-T3 substrates bonded with FM® 73M OST toughened epoxy film adhesive. The LDB was made of a multi-layered laminated aluminium 2024-T3 substrate stiffened with a T shape 2024-T3 aluminium stringer. The laminated substrate consisted of six aluminium sheets with FM® 73M OST adhesive between them and was bonded to the stiffener using FM® 73M OST adhesive. In all cases the aluminium was pre-treated prior to bonding. This pre-treatment consisted of a chromic acid etch (CAE) and

phosphoric acid anodise (PAA) followed by the application of BR® 127 corrosion inhibiting primer to maximise environmental resistance and bonding durability. For both of the joints the same adhesive system (i.e. identical adhesive material, surface pre-treatment and priming) was employed so that a single fatigue damage model can be developed for them.

Before fatigue testing, static tests were conducted on the joints to study the static failure behaviour of the joints and define the cohesive zone model. The static tests were executed in displacement control and with the rate of 0.1 mm/min and the corresponding load level and backface strain data were recorded. Hence, average static strengths of 10.0 KN and 5.8 KN were obtained for the SLJ and LDB, respectively. Moreover, cohesive failure was observed for the both joints.

Fig. 2. Test coupons, a) SLJ, b) LDB.

Fatigue testing was carried out at 5Hz with a load ratio of 0.1. To assess the damage evolution in the adhesive bond line during the fatigue loading, the backface strain technique and in-situ video microscopy were utilised. Moreover, to maximise the backface strain technique sensitivity, numerical analyses were performed to determine the optimum positions of the strain gauges, where the maximum change in strain can be recorded during damage growth. Then the backface strain data were used to assess the validity of the damage model by comparing the predicted and measured backface strain changes. The strain gauges were placed at 1 and 3 mm inside the overlap for the SLJ and 2 and 4 mm inside the overlap for the LDB. The strain gauges were connected to a Wheatstone bridge unit with a maximum capacity of six strain gauges and the change in output voltage were amplified and then recorded using a bespoke software package developed on a Labview platform. This software recorded maximum and minimum voltage values of the strain gauges and preset sequences of complete cycles. Moreover, video microscopy images were used as supporting evidence of damage evolution and the backface strain technique.

Fatigue tests were conducted in load control at various maximum fatigue load levels. The maximum fatigue load levels were 40% and 50% of the average static strength for the SLJ and 40%, 50% and 60% of the average static strength for the LDB. These load levels were selected to give a representative range of fatigue lives. The single lap joints averagely sustained 132,400 and 26,600 loading cycles at 40% and 50% maximum fatigue load levels, respectively. Whereas, the laminated doubler in bending joints averagely endured 45,000, 10,000 and 2,200 loading cycles at 40%, 50% and 60% maximum fatigue load levels, respectively. Detailed results are presented later. Observing the failure surfaces revealed that

the locus of failure was always cohesive within the adhesive. Moreover, at lower loads, and hence longer lives, the cohesive failure appeared to be closer to the interface.

5. Modelling

A significant programme of parametric FE modelling was undertaken. Finite element models, shown in Fig. 3, were developed in ABAQUS Standard finite element code to predict the SLJ and LDB behaviours under static and fatigue loading. For the SLJ, four noded plane stress elements were used for the substrates and to study the progressive damage in the adhesive, four noded cohesive elements with the bi-linear traction-separation description were utilised. Moreover, one end of the substrate was constrained by an encastre constraint, while the transverse displacement and rotation about the out of plane axis of the other end was constrained. A higher mesh density was used near the cohesive elements to obtain the more accurate results. The size of the cohesive elements was 0.2×0.2 mm. For the LDB, it was determined from the experimental observations that the failure always occurred in the adhesive between the stringer and the laminate. Therefore, four-node cohesive elements with a bi-linear traction-separation description were employed for the adhesive bond line between the stringer and the laminate and damage free four-node plane strain elements were used for aluminium and other adhesive layers. Moreover, to minimise the computational effort, the symmetry of the joint was exploited and only half of the joint was modelled. It is noteworthy that by comparing the three-dimensional and two-dimensional analyses results, it was determined that the plane stress state for the SLJ and plane strain state for the LDB provided

the most accurate 2D representations. The reason for this is not fully understood but may be to do with the difference in transverse deformation between a solid and a laminated substrate.

Fig. 3. Finite element mesh and boundary conditions of a) SLJ, b) LDB.

Initially, a non-fatigue damaged bi-linear traction-separation response was determined by simulating the static strength of the two joint configurations. Thus, a study was made investigating the effect of different damage initiation and growth criteria and the interaction of mesh size, tripping tractions and fracture energies. It has been shown in previous work [42] that, for a given tripping traction, a minimum element size is required to operate with a continuous process zone. Such assessment studies with varying size elements were undertaken to assess then minimum element size and ensure that the elements used were smaller than this critical value. The fine mesh that is used can be seen in Fig 3.

The calibrated traction-separation response employed for modelling is outlined in Table 1.

Table 1. Calibrated traction-separation response

Tripping traction normal (shear) MPa	Fracture energy mode I (mode II) kJ/m²	Initiation criterion	Propagation criterion
114 (66)	1.4 (2.8)	Maximum nominal stress criterion	Benzeggagh-Kenane (BK) (with $\eta=2$)

The Maximum nominal stress criterion (Eq. 1) signifies that damage is assumed to initiate when either of the peel or shear components of traction (t_I or t_{II}) exceeds the respective critical value (T_I or T_{II}).

$$\max\left\{\frac{\langle t_I \rangle}{T_I}, \frac{t_{II}}{T_{II}}\right\} = 1 \quad \text{Eq. 1}$$

in which subscripts I and II denote normal and shear directions respectively, and $\langle \rangle$ is the Macaulay bracket meaning that the compression stress state does not lead to the damage initiation. The Benzeggagh-Kenane (BK) [47] criterion is defined in Eq. 2.

$$G_{IC} + (G_{IIC} - G_{IC}) \left(\frac{G_{II}}{G_I + G_{II}} \right)^\eta = G_I + G_{II} \quad \text{Eq. 2}$$

where G_I and G_{II} are the energies released by the traction due to the respective separation in normal and shear directions, respectively and G_{IC} and G_{IIC} are the critical fracture energies required for the failure in normal and shear directions, respectively and η is a material property.

Fig. 4 shows a schematic of a mixed-mode cohesive zone model. The bi-linear traction-separation responses under peel, shear and mixed-mode stress states are illustrated. Points A and B corresponding to the damage initiation and full failure conditions of mixed-mode response are defined based on the mixed-mode damage initiation (Eq. 1) and propagation (Eq. 2) criteria, respectively. Then the position of the mixed-mode response between the mode I and mode II responses is determined based on the mode ratio ($G_{II} / G_I + G_{II}$). This can be done by ABAQUS at each element integration point. A typical mixed-mode response is depicted in Fig. 4.

Fig. 4 Mixed-mode bi-linear traction-separation law.

The predicted and measured static strengths are summarised in Table 2. As can be seen, these parameters gave accurate results for the static strength of the SLJ and the LDB. Moreover, the calibrated traction-separation model operated in the energy controlled region which was mesh independent.

Table 2. The experimental and predicted static strength.

Joint	static strength (KN)		Error
	Experimental	Predicted	
SLJ	10.0	9.93	-0.7%
LDB	5.8	5.6	-3.4%

It was necessary to use a small level of viscous damping in the constitutive equation of the cohesive elements to avoid analysis termination at the point of failure. This was required

because the numerical simulations of interfacial degradation using cohesive elements are often accompanied by numerical instability, particularly when the simulation is close to catastrophic failure. At the point of instability, the simulation terminates and the complete failure response may remain undefined. To obtain appropriate viscosity values for the SLJ and LDB, parametric studies with decreasing levels of viscous damping were implemented. It should be noted that by incorporating a fictitious viscous damping, the overall behaviour of the structure should remain essentially unchanged, i.e. the energy dissipated due to the viscous damping should be negligible. Fig. 5 shows the effect of the viscous damping coefficient on the predicted load-displacement curve of SLJ. It can be seen that the value used (???) hardly affects the static response of the structure

Fig. 5. The effect of viscous damping coefficient (μ) on the predicted load-displacement curve of SLJ

The deleterious influence of fatigue was simulated by degrading the traction-separation response. This degradation process was implemented by incorporating a fatigue damage parameter that evolved during fatigue and was based on a fatigue damage evolution law (Eq. 3) expressed as the cyclic damage rate. A simpler form of this has been used successfully

elsewhere [48]. However, they did not employ cohesive zone model and just degraded the elastic-plastic material properties of the conventional continuum elements. The advantage of using cohesive zone approach is that it can accommodate progressive damage in static as well as fatigue situation.

$$\frac{\Delta D}{\Delta N} = \begin{cases} \alpha(\epsilon_{\max} - \epsilon_{th})^\beta, & \epsilon_{\max} > \epsilon_{th} \\ 0, & \epsilon_{\max} \leq \epsilon_{th} \end{cases} \quad \text{Eq. (3)}$$

$$\epsilon_{\max} = \frac{\epsilon_n}{2} + \sqrt{\left(\frac{\epsilon_n}{2}\right)^2 + \left(\frac{\epsilon_s}{2}\right)^2}$$

where ΔD is the increment of damage, ΔN is the cycle increment, $\epsilon_{\max}$ is the maximum principal strain in the cohesive element which is a combination of normal and shear components of strain (ϵ_n and ϵ_s), ϵ_{th} is the threshold strain (a critical value of $\epsilon_{\max}$ below which no fatigue damage occurred) and α and β are material constants. The parameters ϵ_{th} , α and β need to be calibrated against the experimental tests. It should be noted that in this paper strain refers to average strain and is defined as below:

$$\epsilon_{n,s} = \frac{\delta_{n,s}}{t_{Adh}} \quad \text{Eq. (4)}$$

in which t_{Adh} is the thickness of the adhesive bond line, δ is the separation and the subscripts n and s denote normal and shear directions, respectively.

In order to simplify the numerical process, the fatigue load was characterised by the maximum load and this was applied to the FE models of the joints. Then, the proposed fatigue damage was accumulated through numerical integration of the cyclic damage rate (Eq. 3),

which is dependent on the number of cycles and maximum principal strain. Moreover, damage only occurred if the maximum principal strain in the cohesive elements exceeded the threshold strain.

The fatigue damage was modelled by degrading the bi-linear traction-separation response and was implemented by coupling the ABAQUS Standard finite element code with a FORTRAN subroutine. The material degradation process is illustrated in Fig. 6. As shown, the fatigue modelling consisted of two steps. In the first step, the maximum fatigue load was applied and the intact joint was analysed with a static finite element analysis. This provided the state at the beginning of the fatigue test. Then the maximum principal strains of the cohesive elements were obtained from the finite element analysis results by using the utility subroutine *GETVRM. In the second step, a fatigue damage variable at each element integration point was introduced into the model. This variable was updated according to the strain-based fatigue damage law (Eq. 3) for each increment of cycles (ΔN). Then the tripping tractions and fracture energies in mode I and mode II for the cohesive elements were reduced linearly based on this damage variable. Following the material degradation, the maximum principal strains of the cohesive elements were again calculated by ABAQUS for the next cyclic increment (ΔN) and the fatigue damage variable was again updated. This material degradation process was repeated until the damaged joint can no longer sustain the applied maximum fatigue load. This then provides the predicted fatigue life. For the sake of simplicity, material degradation due to fatigue in different modes was assumed to be the same, i.e. cohesive properties in mode I and II were reduced with the same rate after each increment of cycles (ΔN). This necessary may not be the case, as the fatigue effect on one mode might be more damaging than on the other one.

Fig. 6. Fatigue degradation of cohesive element properties.

Each element integration point has two damage variables corresponding to static damage (D_S) and fatigue damage (D_F) or in ABAQUS terminology SDEG and SDV, respectively. As shown in Fig. 7, the fatigue damage variable was used to determine the degraded traction-separation response whereas the static damage parameter was utilised to define the material status within that traction-separation response. Furthermore, the fatigue damage variable was calculated by numerical integration of fatigue damage evolution law (Eq. 3) using the FORTRAN subroutine and the static damage variable was obtained by finite element analysis using ABAQUS. The element was removed if either of the damage variables became one. This enabled the model to account for the catastrophic static failure as well as the gradual fatigue failure. For instance, for some bonded joints (e.g. SLJ) by growing the damage, the load bearing capacity of the joint diminished. This continued until the load bearing capacity dropped below the maximum fatigue load level which gave rise to catastrophic static failure. Basically, after each cyclic increment (ΔN), having determined the degraded cohesive zone properties of the elements, ABAQUS treated the problem as a static analysis and checked whether the structure can sustain the maximum fatigue load level.

Fig. 7 Fatigue damage (D_F) and traction-separation response damage (D_S) parameters.

Obviously, in the course of fatigue material degradation process, the tripping traction of some elements might drop below the applied level of traction. This would give rise to an increase of the static damage variable for those elements. However, the fatigue material degradation continues until the element is removed (i.e. either of the static or fatigue damage variable becomes one).

As numerical integration was used to accumulate the fatigue damage, a study was undertaken to establish the maximum size of cycle increment that can be used for each configuration. The result of this study is shown in Fig. 8. This shows that the maximum cycle increment size needs to be reasonably small and the size can be obtained by undertaking a simple convergence study on the maximum cycle increment size.

Fig. 8. Maximum cycle increment size effect on the predicted fatigue lifetime.

Finally, a parametric study was undertaken to study the effect of fatigue damage model parameters on the load-life curves and appropriate values for the fatigue damage model parameters were obtained. The model parameters are summarised in Table 3.

Table 3. Damage model parameters.

α	β	ϵ_{th}
1.5	2	0.0319

It was observed that increasing the constant α accelerated the damage evolution and consequently decreased the predicted fatigue lifetime, conversely, increasing the power β and the threshold strain (ϵ_{th}) decelerated the damage evolution and increased the lifetime. Moreover, changing the constant α had a similar effect on the predicted fatigue lifetime for different load levels, leading to a shift of S-N curve in horizontal direction, but the effect of increasing β tended to decelerate the damage more at lower strain (load) levels thus decreased the slope of the S-N curve. This is because, the lower the load level, the lower the strain values and this can be intensified by increasing the power β . Table 4 outlines the sensitivity of the fatigue model to the change of fatigue model parameters for SLJ. The sensitivity parameter was obtained by changing each of the fatigue model parameter at a time by 5% of calibrated values (see table 3) and keeping the other parameters constant.

Table 4. Sensitivity of the model to the change of fatigue model parameters

P_{max}/P_S	S_α	S_β	$S_{\epsilon th}$
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40%	-0.8	10.0	10.3
50%	-0.8	8.5	4.6

where S_λ ($\lambda = \alpha, \beta$ and ε_{th}) is sensitivity parameter and is defined as below:

$$S_\lambda = \frac{d\left(\frac{N_\lambda}{N_0}\right)}{d\left(\frac{\lambda}{\lambda_0}\right)} \bigg|_{\lambda=\lambda_0}, \quad \lambda = \alpha, \beta, \varepsilon_{th} \quad \text{Eq. (5)}$$

in which α , β and ε_{th} are fatigue model parameters and N_0 and N_λ are predicted fatigue lives using baseline fatigue model parameters (λ_0) and new fatigue damage parameters (λ), respectively. The calibrated fatigue model parameters (see table 3) were considered as the baseline parameters. For instance, for the fatigue load with the maximum level of 40% of static strength, by changing each of α , β and ε_{th} at a time by 100% from the baseline values and fixing the other two, predicted fatigue life changed by factors of -0.8, 10.0 and 10.3, respectively. It is evident from table 4 that the model is more sensitive to β and ε_{th} than to α . Moreover, by increasing the maximum fatigue load level, the sensitivity of the model to β and ε_{th} decreased whilst the sensitivity of the model to α remained unchanged.

The variations of fatigue damage in terms of the length along the overlap for the joints at 0, 1/3, 2/3 and all of the total fatigue number of cycles are shown in Fig. 9. The damage values of 0 and 1 imply undamaged and fully damaged material, respectively. As can be seen, for both joints, fatigue damage was initially zero and by increasing the number of cycles, damage occurred from both bond line ends and then damage evolution accelerated towards the middle of the bond line.

Fig. 9. Damage variations vs. length along the overlap of a) SLJ, b) LDB at Max fatigue load of 50% of static strength.

The damage and corresponding von-Mises stress contour plots for the joints at the beginning ($N = 0$) and the end of the fatigue life ($N = N_f$) just before final, fast rupture are shown in Fig. 10.

Fig. 10 - Damage and von-Mises contour plots of, a) SLJ, b) LDB.

The regions with $SDV1=1$ in the damage contours indicate the fully damaged material signifying no fatigue resistance. As can be seen, the highest stress level in laminated substrate of the LDB corresponded to the adhesive layer between the stringer and the laminate.

6. Discussion

The monolithic single lap joint and laminated doubler in bending were examined numerically under fatigue loading using the fatigue damage model. The predicted load-life data correlated well against the corresponding experimental data, as shown in Fig. 11. The fatigue load has been expressed (normalised) as a fraction of the static failure load of the particular configuration.

Fig. 11. Load-life fatigue data.

Correlation between the experimental and numerical backface strain data can provide an independent validation of the damage model. As shown in Fig. 12, the predicted and measured backface strains of both joints agreed very well. This confirms that the proposed damage model successfully predicted the damage evolution consistent with the corresponding experimental damage. Several tests were conducted for backface strain correlation and typical ones are shown in Fig. 12 for the SLJ and LDB. In case of the both joints, the backface strain increased initially followed by a decrease. This backface strain reduction was due to a local deformation relaxation at the location of the strain gauge as the crack passed under the strain gauge position.

Fig. 12. Comparison of the predicted and measured backface strain variations, a) SLJ at 1 mm inside the overlap and maximum fatigue load of 50% of static strength, b) LDB at 2 mm inside the overlap and maximum fatigue load of 60% of static strength.

The tests were stopped at certain cycles and the bond line was examined using a travelling video microscope. The video microscopy images are shown in Fig. 13. It should be noted that the spew adhesive was not fully removed as this provided a convenient method of enhancing the view of the cracks.

**Fig. 13. Video-microscopy images of SLJ fatigue test
at maximum fatigue load of 40% of static strength.**

To characterise this visually observed damage, the variation of backface strain with crack length (Fig. 14 (b)) was determined numerically and the results were compared with the experimentally obtained backface strain variations in terms of the number of cycles (Fig. 14 (a)). As is evident from Fig. 14, the strain gauge recorded the value of 930 micro strain at 20,000 fatigue cycles and the numerical analysis predicted ~1.2 mm crack length for this strain value which correlated very well with the observed damage in Fig. 13 at 20,000 cycles.

Fig. 14. The backface strain (BS) variations for fatigue test of SLJ, a) Experimental BS vs. number of cycles, b) Numerical BS vs. crack length.

The bonded joints investigated in this work (SLJ and LDB) represented different mode-ratios with the average values of $t_{II}/(t_I+t_{II})=0.5$ and 0.3, respectively. Although, in both cases, the mode-ratio changed with crack growth, the variations were not considerable. In future work it is planned to apply the fatigue model, developed in this work, for other joints with more diverse mode-ratios.

To implement the proposed fatigue model, two sets of parameters including the static (CZM) parameters and the fatigue model parameters need to be obtained. There are several ways of calibrating the CZM parameters. The most common one is obtaining the fracture energy from standard fracture mechanics tests like DCB (for mode I) and ENF (for mode II) and obtaining the tripping traction by the correlation between the simulated and the experimental failure

load. The initial stiffness can be assumed so that the compliance of the joint is negligible before the damage initiation. For the fatigue model parameters, the parameters can be obtained by correlation between the simulated and the experimental load-life curves and the backface strain data. This can assure that the model can give a consistent match with the experimental fatigue response in terms of life and progressive damage growth.

7. Conclusions

A numerical fatigue damage model was developed and employed for two bonded joints having the same adhesive system but different geometries. A bi-linear traction-separation description of the cohesive zone model was integrated with a strain-based fatigue damage to predict the fatigue behaviour of the adhesively bonded joints. The fatigue damage was simulated by degrading the traction-separation properties. The proposed fatigue damage model gave a consistent match with the experimental fatigue response in terms of life, backface strain and predicted damage growth. The use of progressive damage modelling based on a cohesive zone model showed considerable potential for predictive modelling of bonded structures without the limitations attributed to some of the other methods.

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References

REFERENCES

1. A.D. Crocombe, G. Richardson, Assessing stress state and mean load effects on the fatigue response of adhesively bonded joints, *International Journal of Adhesion and Adhesives* **19** (1999) 19-27.
2. D.R. Lefebvre, D.A. Dillard, A stress singularity approach for the prediction of fatigue crack initiation in adhesive bonds. PART 1: Theory, *Journal of Adhesion* **70** (1999) 119-138.
3. K. Ishii, M. Imanaka, H. Nakayama, H. Kodama, Evaluation of the fatigue strength of adhesively bonded CFRP/metal single and single-step double-lap joints, *Composites Science and Technology* **59** (1999) 1675-1683.
4. P. Lazzarin, M. Quaresimin, P. Ferro, A two-term stress function approach to evaluate stress distributions in bonded joints of different geometries, *Journal of Strain Analysis for Engineering Design* **37** (2002) 385-398.
5. M. Quaresimin, M. Ricotta, Life prediction of bonded joints in composite materials, *International Journal of Fatigue* **28** (2006) 1166-1176.
6. C.J. Gilbert, R.H. Dauskardt, R.O. Ritchie, Microstructural mechanisms of cyclic fatigue-crack propagation in grain-bridging ceramics, *Ceramics International* **23** (1997) 413-418.
7. M.H. Elhaddad, T.H. Topper, K.N. Smith, Prediction of Non Propagating Cracks, *Engineering Fracture Mechanics* **11** (1979) 573-584.

8. D.C. Drucker, L. Palgen, On Stress-Strain Relations Suitable for Cyclic and Other Loading, *Journal of Applied Mechanics-Transactions of the Asme* **48** (1981) 479-485.
9. G. Xu, A.S. Argon, M. Ortiz, Nucleation of Dislocations from Crack Tips Under Mixed-Modes of Loading - Implications for Brittle Against Ductile Behavior of Crystals, *Philosophical Magazine A-Physics of Condensed Matter Structure Defects and Mechanical Properties* **72** (1995) 415-451.
10. R.G. Forman, V.E. Kearney, R.M. Engle, Numerical Analysis of Crack Propagation in Cyclic-Loaded Structures, *Journal of Basic Engineering* **89** (1967) 459-&.
11. M.M.A. Wahab, I.A. Ashcroft, A.D. Crocombe, S.J. Shaw, Prediction of fatigue thresholds in adhesively bonded joints using damage mechanics and fracture mechanics, *Journal of Adhesion Science and Technology* **15** (2001) 763-781.
12. M. Imanaka, T. Hamano, A. Morimoto, R. Ashino, M. Kimoto, Fatigue damage evaluation of adhesively bonded butt joints with a rubber-modified epoxy adhesive, *Journal of Adhesion Science and Technology* **17** (2003) 981-994.
13. I. Hilmy, M.M.A. Wahab, I.A. Ashcroft, A.D. Crocombe, Measuring of damage parameters in adhesive bonding, *Key Engineering Materials* **324-325** (2006) 275-278.
14. I. Hilmy, M.M.A. Wahab, A.D. Crocombe, I.A. Ashcroft, A.G. Solana, Effect of triaxiality on damage parameters in adhesive, *Key Engineering Materials* **348-349** (2007) 37-40.
15. G.I. Barenblatt, Equilibrium Cracks Formed on A Brittle Fracture, *Doklady Akademii Nauk Sssr* **127** (1959) 47-50.

16. G.I. Barenblatt, The mathematical theory of equilibrium cracks in brittle fracture, *Advances in Applied Mechanics* **7** (1962) 55-129.
17. D.S. Dugdale, Yielding of Steel Sheets Containing Slits, *Journal of the Mechanics and Physics of Solids* **8** (1960) 100-104.
18. A. Hillerborg, M. Modeer, P.E. Petersson, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cement and Concrete Research* **6** (1976) 773-781.
19. A. Needleman, A Continuum Model for Void Nucleation by Inclusion Debonding, *Journal of Applied Mechanics-Transactions of the Asme* **54** (1987) 525-531.
20. A. Needleman, An Analysis of Tensile Decohesion Along An Interface, *Journal of the Mechanics and Physics of Solids* **38** (1990) 289-324.
21. N. Chandra, H. Li, C. Shet, H. Ghonem, Some issues in the application of cohesive zone models for metal-ceramic interfaces, *International Journal of Solids and Structures* **39** (2002) 2827-2855.
22. A. Turon, J. Costa, P.P. Camanho, C.G. Davila, Simulation of delamination in composites under high-cycle fatigue, *Composites Part A-Applied Science and Manufacturing* **38** (2007) 2270-2282.
23. P. Robinson, U. Galvanetto, D. Tumino, G. Bellucci, D. Violeau, Numerical simulation of fatigue-driven delamination using interface elements, *International Journal for Numerical Methods in Engineering* **63** (2005) 1824-1848.

24. D. Tumino, F. Cappello, Simulation of fatigue delamination growth in composites with different mode mixtures, *Journal of Composite Materials* **41** (2007) 2415-2441.
25. S. Maiti, P.H. Geubelle, A cohesive model for fatigue failure of polymers, *Engineering Fracture Mechanics* **72** (2005) 691-708.
26. O. Nguyen, E.A. Repetto, M. Ortiz, R.A. Radovitzky, A cohesive model of fatigue crack growth, *International Journal of Fracture* **110** (2001) 351-369.
27. K.L. Roe, T. Siegmund, An irreversible cohesive zone model for interface fatigue crack growth simulation, *Engineering Fracture Mechanics* **70** (2003) 209-232.
28. T. Siegmund, A numerical study of transient fatigue crack growth by use of an irreversible cohesive zone model, *International Journal of Fatigue* **26** (2004) 929-939.
29. Y.J. Xu, H. Yuan, Computational analysis of mixed-mode fatigue crack growth in quasi-brittle materials using extended finite element methods, *Engineering Fracture Mechanics* **76** (2009) 165-181.
30. Ural A., Papoulia K.D., 2004, Modeling of fatigue crack growth with a damage-based cohesive zone model, *European Congress on Computational Methods in Applied Sciences and Engineering*, Finland.
31. H. Abe, T. Satoh, Non-destructive detection method of fatigue crack in spot-welded joints, *Yosetsu Gakkai Ronbunshu/Quarterly Journal of the Japan Welding Society* **4** (1986) 666-673.
32. Z.H. Zhang, J.K. Shang, F.V. Lawrence, A Backface Strain Technique for Detecting Fatigue-Crack Initiation in Adhesive Joints, *Journal of Adhesion* **49** (1995) 23-36.

33. M. Imanaka, K. Haraga, T. Nishikawa, Fatigue-Strength of Adhesive Rivet Combined Lap Joints, *Journal of Adhesion* **49** (1995) 197-209.
34. A.J. Curley, H. Hadavinia, A.J. Kinloch, A.C. Taylor, Predicting the service-life of adhesively-bonded joints, *International Journal of Fracture* **103** (2000) 41-69.
35. A.D. Crocombe, C.Y. Ong, C.M. Chan, M.M.A. Wahab, I.A. Ashcroft, Investigating fatigue damage evolution in adhesively bonded structures using backface strain measurement, *Journal of Adhesion* **78** (2002) 745-776.
36. H. Hadavinia, A.J. Kinloch, M.S.G. Little, A.C. Taylor, The prediction of crack growth in bonded joints under cyclic-fatigue loading I. Experimental studies, *International Journal of Adhesion and Adhesives* **23** (2003) 449-461.
37. G. Kelly, Quasi-static strength and fatigue life of hybrid (bonded/bolted) composite single-lap joints, *Composite Structures* **72** (2006) 119-129.
38. A.G. Solana, A.D. Crocombe, M.M.A. Wahab, I.A. Ashcroft, Fatigue initiation in adhesively-bonded single-lap joints, *Journal of Adhesion Science and Technology* **21** (2007) 1343-1357.
39. G. Alfano, On the influence of the shape of the interface law on the application of cohesive-zone models, *Composites Science and Technology* **66** (2006) 723-730.
40. T. Diehl, On using a penalty-based cohesive-zone finite element approach, Part I: Elastic solution benchmarks, *International Journal of Adhesion and Adhesives* **28** (2008) 237-255.

41. T. Diehl, On using a penalty-based cohesive-zone finite element approach, Part II: Inelastic peeling of an epoxy-bonded aluminum strip, *International Journal of Adhesion and Adhesives* **28** (2008) 256-265.
42. C.D.M. Liljedahl, A.D. Crocombe, M.A. Wahab, I.A. Ashcroft, Damage modelling of adhesively bonded joints, *International Journal of Fracture* **141** (2006) 147-161.
43. V. Tvergaard, J.W. Hutchinson, The Relation Between Crack-Growth Resistance and Fracture Process Parameters in Elastic Plastic Solids, *Journal of the Mechanics and Physics of Solids* **40** (1992) 1377-1397.
44. I. Mohammed, K.M. Liechti, Cohesive zone modeling of crack nucleation at bimaterial corners, *Journal of the Mechanics and Physics of Solids* **48** (2000) 735-764.
45. P. Rahulkumar, A. Jagota, S.J. Bennison, S. Saigal, Cohesive element modeling of viscoelastic fracture: application to peel testing of polymers, *International Journal of Solids and Structures* **37** (2000) 1873-1897.
46. Rots J. G., 1986, Strain-softening analysis of concrete fracture specimens. In F. H. Wittmann (Ed.), *Fracture toughness and fracture energy of concrete*: 137-148. Amsterdam: Elsevier Science Publishers.
47. M.L. Benzeggagh, M. Kenane, Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites with mixed-mode bending apparatus, *Composites Science and Technology* **56** (1996) 439-449.

48. A.G. Solana, A.D. Crocombe, I.A. Ashcroft, Fatigue life and backface strain predictions in adhesively bonded joints, *International Journal of Adhesion and Adhesives* **30** (2010) 36-42.