All superalgebras for warped AdS_2 and black hole near horizon geometries

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Abstract

We identify all symmetry superalgebras $\mathfrak{g}$ of near horizon geometries of black holes with a Killing horizon, assuming the solution is smooth and that the spatial cross section of the event horizon is compact without boundary. This includes all warped AdS_2 backgrounds with the most general allowed fluxes in 10- and 11-dimensional supergravities. If the index of a particular Dirac operator vanishes, we find that the even symmetry subalgebra decomposes as $\mathfrak{g}_0 = \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{t}_0$, where $\mathfrak{t}_0/\mathfrak{c}$ is the Lie algebra of a group that acts transitively and effectively on spheres, and $\mathfrak{c}$ is the center of $\mathfrak{g}$. If the Dirac operator index does not vanish, then the symmetry superalgebra is nilpotent with one even generator. We also demonstrate that there are no near horizon geometries, and also therefore no warped AdS_2 backgrounds, in 10- and 11-dimensions that preserve more than 16 supersymmetries.

Keywords: black holes, supergravity, superalgebras

1. Introduction

A consequence of the horizon conjecture [1] is that under certain conditions the near horizon geometries of black holes with a Killing horizon preserve

$$N = 2N_- + \text{Index} \not{D}, \quad (1)$$

supersymmetries, where $\not{D}$ is a (twisted) Dirac operator and $N_- \in \mathbb{Z}_{\geq 0}$. In addition if $N_- \neq 0$ and the near horizon geometries exhibit non-trivial fluxes, their symmetry superalgebra contains an $\mathfrak{sl}(2, \mathbb{R})$ subalgebra. This conjecture has been demonstrated for several theories in [2–6]. These include the 10- and 11-dimensional supergravities as well as the $\mathcal{N} = 2$ four- and

$\mathcal{N} = 1$ five-dimensional (gauged) supergravities⁴. The assumptions needed to prove these results will be explained below. The purpose of this paper is to identify all symmetry superalgebras of near horizon geometries that satisfy the horizon conjecture. Warped AdS_2 backgrounds with the most general allowed fluxes are special cases of near horizon geometries. As a consequence, all symmetry superalgebras of warped AdS_2 backgrounds in 10- and 11- as well as those in 4- and 5-dimensional theories will also be determined. These results together with those presented in [7] complete the list of all symmetry superalgebras of warped AdS_n backgrounds for $n \geq 2$ with the most general allowed fluxes in 10- and 11-dimensional supergravity theories⁵.

One of our results is that if $\text{Index } \mathcal{D} = 0$, then the even subalgebra $\mathfrak{g}_0$ of the symmetry superalgebra $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ of near horizon geometries decomposes as $\mathfrak{g}_0 = \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{t}_0$. Moreover, $\mathfrak{t}_0/\mathfrak{c}$ is the Lie algebra of a group which acts transitively and effectively on a sphere, where $\mathfrak{c}$ is the centre of $\mathfrak{g}$. These results allow us to identify all possible such superalgebras which have been tabulated in table 1. These superalgebras are the same as either the left or the right copy of the symmetry superalgebra of warped AdS_3 backgrounds found in [7]. Warped AdS_2 backgrounds exhibit the same symmetry superalgebras as near horizon geometries. However, the geometry of the spatial horizon sections of generic near horizon geometries is different from that of the internal spaces of warped AdS_2 backgrounds. In particular, the spatial horizon sections of generic near horizon geometries admit a $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$ Lie algebra of isometries while the internal spaces of AdS_2 backgrounds admit a $\mathfrak{t}_0$ Lie algebra of isometries.

The index of $\mathcal{D}$ vanishes for all non-chiral theories. For example it vanishes in 11-dimensional, (massive) IIA and 5D supergravities. However it may not vanish in chiral theories like the IIB supergravity. If $\text{Index } \mathcal{D} \neq 0$ and $N_- = 0$, we demonstrate that the symmetry superalgebra of the backgrounds is nilpotent, $\mathfrak{g}_0$ is 1-dimensional and the non-vanishing anti-commutators are given in (65). We also demonstrate that if $N_- \neq 0$ and the theory has non-trivial fluxes, then $\text{Index } \mathcal{D} = 0$ and thus the symmetry superalgebras are given as in table 1.

Moreover, we show that for near horizon geometries that preserve more than half of supersymmetry, the spatial horizon sections admit a transitive group action of a group G with Lie algebra $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$. This follows from an adaptation of the homogeneity theorem of [10]. Moreover the action of the subgroup with Lie algebra $\mathfrak{t}_0/\mathfrak{c}$ is effective. Similarly the internal spaces of warped AdS_2 backgrounds admit a transitive action of a group G with Lie algebra $\mathfrak{t}_0$ such that $\mathfrak{t}_0/\mathfrak{c}$ acts effectively. We use these results together with the classification of homogeneous spaces in 8 and 9 dimensions [11] to demonstrate that there are no near horizon geometries and warped AdS_2 backgrounds in 10- and 11-dimensional supergravity theories that preserve more than 16 supersymmetries.

As has already been mentioned, the proof of the above results relies on the horizon conjecture which allows us to establish the results in a theory independent way. In turn, the assumptions required to establish the horizon conjecture are that the fields are smooth and the black hole spatial horizon section is compact without boundary. The assumptions of the horizon conjecture are also needed to exclude the possibility that warped AdS_2 backgrounds can be rewritten as warped AdS_n , $n > 2$, backgrounds [12, 13]. If this possibility is

⁴For a proof of the emergence of $\mathfrak{sl}(2, \mathbb{R})$ symmetry near extreme 4- and 5-dimensional horizons under the assumption of axial $U(1)$ symmetries see [8]. However, note that *a priori* near horizon geometries may not have axial symmetries, see e.g. those with $N = 2$ supersymmetry in table 1.

⁵See [9] for the classification of symmetry superalgebras of warped AdS_4 backgrounds under different assumptions. In [7] the symmetry superalgebras were referred to as Killing superalgebras. We have changed the terminology as we use here the term Killing for the horizons.

Table 1. Symmetry superalgebras of near horizon geometries and warped AdS_2 backgrounds.

N	$\mathfrak{g}/\mathfrak{c}$	$\mathfrak{t}_0/\mathfrak{c}$
$2n$	$\mathfrak{osp}(n 2)$	$\mathfrak{so}(n)$
$4n, n > 1$	$\mathfrak{sl}(n 2)$	$\mathfrak{u}(n)$
$8n, n > 1$	$\mathfrak{osp}^*(4 2n)$	$\mathfrak{sp}^*(n) \oplus \mathfrak{sp}^*(1)$
16	$\mathfrak{f}(4)$	$\mathfrak{spin}(7)$
14	$\mathfrak{g}(3)$	$\mathfrak{g}_2$
8	$\mathfrak{D}(2, 1, \alpha)$	$\mathfrak{so}(3) \oplus \mathfrak{so}(3)$
8	$\mathfrak{sl}(2 2)/1_{4 \times 4}$	$\mathfrak{so}(3)$

not excluded, symmetry superalgebras are different and in particular they do not decompose as $\mathfrak{g}_0 = \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{t}_0$.

To identify the symmetry superalgebra of near horizon geometries, we use the methodology developed in [7] to find the symmetry superalgebras of warped AdS_n , $n > 2$, backgrounds. First, one can compute part of the symmetry superalgebra using the explicit expression of the Killing spinors in terms of the light-cone coordinates that arise in the description of near horizon geometries. For this one utilizes the geometric approach of defining the brackets proposed in [14, 15] via the use of Killing spinor bilinears and spinorial Lie derivatives which is summarized in section 2. To identify the remaining brackets, the closure of the symmetry superalgebra is imposed via the application of super-Jacobi identities. Closure of the symmetry superalgebras of supersymmetric backgrounds has been demonstrated in [16, 17] for some theories and it is thought that it is valid in general. The advantage of imposing the super-Jacobi identities is that the computation of the remaining (anti-) commutators of the symmetry superalgebra can be done generically and it does not use details of the geometry of the underlying backgrounds which may not be available.

This paper is organized as follows. In section 2, we summarize the geometry of supersymmetric near horizon geometries and review the construction of symmetry superalgebras. In section 3, we construct the symmetry superalgebras of near horizon geometries and AdS_2 backgrounds preserving any number of supersymmetries. In section 4, we prove a no-go theorem for near horizon geometries and warped AdS_2 backgrounds that preserve more than half of supersymmetry, and in section 5 we give our conclusions.

2. Horizons and symmetry

2.1. Near horizon geometries and warped AdS_2 backgrounds

2.1.1. Fields. Consider a supergravity theory with bosonic fields given by the metric ds^2 and some form field strengths which we shall denote collectively by F . Adapting Gaussian null coordinates [18, 19] near the Killing horizon of an extreme black hole, and after taking the near horizon limit, the near horizon metric and form field strengths F can be written [20, 21] as

$$ds^2 = 2\mathbf{e}^+ \mathbf{e}^- + \delta_{ij} \mathbf{e}^i \mathbf{e}^j = 2du(dr + rh - \frac{1}{2}r^2 \Delta du) + ds^2(\mathcal{S}), \quad (2)$$

$$F = \mathbf{e}^+ \wedge \mathbf{e}^- \wedge Y + r\mathbf{e}^+ \wedge M + X,$$

where we have introduced the frame

$$\mathbf{e}^+ = du, \quad \mathbf{e}^- = dr + rh_i \mathbf{e}^i - \frac{1}{2} r^2 \Delta du, \quad \mathbf{e}^i = e^i_J dy^J, \quad (3)$$

and

$$ds^2(\mathcal{S}) = \delta_{ij} \mathbf{e}^i \mathbf{e}^j, \quad (4)$$

is the metric of the spatial horizon section $\mathcal{S}$ given by $r = u = 0$. The vector field ∂_u is taken to be time-like and becomes null at the horizon hyper-surface $r = 0$. The dependence on the (light-cone) coordinates r, u is given explicitly. The fields $h = h_i \mathbf{e}^i$, Δ , $ds^2(\mathcal{S})$, Y , M and X depend only on the coordinates y of $\mathcal{S}$. If F is a k -form field strength, then Y , M and X are $(k-2)$ -, $(k-1)$ - and k -forms on $\mathcal{S}$, respectively. Furthermore, h and Δ are a one-form and a zero-form on $\mathcal{S}$, respectively.

The assumptions required for the validity of the horizon conjecture are that

- $\mathcal{S}$ is compact without boundary and
- the fields Δ , h , Y , M and X are globally defined and smooth on $\mathcal{S}$.

These are used in the proof of Lichnerowicz type theorems which are necessary to establish the horizon conjecture, see [1–6].

The warped AdS_2 backgrounds with the most general allowed fluxes arise as a special case of near horizon geometries. For this one sets

$$\Delta = \ell^{-2} A^{-2}, \quad h = -d \log A^2, \quad M = 0, \quad (5)$$

where A^2 is a function of $\mathcal{S}$ and is identified with the warp factor, and ℓ is the radius of AdS_2 . M is set to zero so that the fluxes F are invariant under the $\mathfrak{sl}(2, \mathbb{R})$ isometries of AdS_2 . For more details, see [27].

2.1.2. Horizon conjecture and Killing spinors. The solutions of the Killing spinor equations (KSEs) of supergravity theories along the light-cone directions r, u of a near horizon geometry given in (2) can be expressed as

$$\epsilon_1 = \phi_- + u \Gamma_+ \Theta_- \phi_- + ru \Gamma_- \Theta_+ \Gamma_+ \Theta_- \phi_-, \quad \epsilon_2 = \phi_+ + r \Gamma_- \Theta_+ \phi_+, \quad (6)$$

where $\phi_{\pm}$ are (commuting) spinors that depend only on the coordinates of $\mathcal{S}$, satisfy the light-cone projections $\Gamma_{\pm} \phi_{\pm} = 0$ and are in the same representation of the spin group as that of the supersymmetry parameters of the underlying theory. The spinors $\phi_{\pm}$ obey additional KSEs which can be thought of as the restriction of the KSEs of the theory to the spatial horizon section $\mathcal{S}$. These are theory dependent but their explicit form is not essential for the derivation of the symmetry superalgebras of the near horizon geometries. Moreover $\Theta_{\pm}$ are algebraic Clifford algebra operators along $\mathcal{S}$ which depend on the fields Δ, h, Y, M and X of the theory considered. The details of how $\Theta_{\pm}$ depend on the fields is not again essential for the arguments that follow. $\Theta_{\pm}$ are independent of $\Gamma_{\pm}$ and so preserve the light-cone chirality of the spinors. The dependence of the Killing spinors in (11) on the coordinates u, r is explicit.

It can be shown using Hopf maximum principle and the KSEs on $\mathcal{S}$ that

$$\|\phi_+\| = \text{const.} \quad (7)$$

Therefore the length⁶ of ϕ_+ Killing spinors is constant. In addition, one can show that if $\phi_- \neq 0$ solves the KSE on $\mathcal{S}$, i.e. $N_- \neq 0$, then

$$\phi_+ = \Gamma_+ \Theta_- \phi_-, \quad (8)$$

also solves the KSEs on $\mathcal{S}$. Furthermore, one can demonstrate using the Hopf maximum principle that for horizons with *non-trivial fluxes* the kernel of Θ_- vanishes

$$\text{Ker } \Theta_- = \{0\}. \quad (9)$$

So, if ϕ_- is a Killing spinor then so is $\phi_+ = \Gamma_+ \Theta_- \phi_- \neq 0$. This is the origin of the doubling of supersymmetries for near horizon geometries with $N_- \neq 0$. For the derivation of (6) as well as all the remaining statements above see [1–6].

2.1.3. $\mathfrak{sl}(2, \mathbb{R})$. For any pair of Killing spinors ϵ_1 and ϵ_2 , one can define the spacetime one-form bilinear as

$$K(\epsilon_1, \epsilon_2) = \langle (\Gamma_+ - \Gamma_-)\epsilon_1, \Gamma_M \epsilon_2 \rangle dx^M. \quad (10)$$

It is expected that all these one-forms give rise to spacetime Killing vectors that leave all the remaining fields of the theory invariant.

Next consider the theories with $N_- \neq 0$ and with non-trivial fluxes. In this case there is a ϕ_- Killing spinor and which gives rise to another Killing spinor given by $\phi_+ = \Gamma_+ \Theta_- \phi_-$. Using these to construct the spacetime Killing spinors, one finds

$$\epsilon_1 = \phi_- + u\phi_+ + ru\Gamma_- \Theta_+ \phi_+, \quad \epsilon_2 = \phi_+ + r\Gamma_- \Theta_+ \phi_+. \quad (11)$$

In particular set $K_{AB} = K(\epsilon_A, \epsilon_B)$, $A, B = 1, 2$, for the Killing spinors ϵ_A in (11). The requirement that K_{AB} are Killing leads to the conditions

$$-\Delta \|\phi_+\|^2 + 4 \|\Theta_+ \phi_+\|^2 = 0, \quad \langle \phi_+, \Gamma_i \Theta_+ \phi_+ \rangle = 0, \quad (12)$$

which in turn can be used to simplify the expression for K_{AB} as

$$\begin{aligned} K_{12} &= (2r\langle \Gamma_+ \phi_-, \Theta_+ \phi_+ \rangle + r^2 u \Delta \|\phi_+\|^2) \mathbf{e}^+ - 2u \|\phi_+\|^2 \mathbf{e}^- + W_i \mathbf{e}^i, \\ K_{22} &= r^2 \Delta \|\phi_+\|^2 \mathbf{e}^+ - 2 \|\phi_+\|^2 \mathbf{e}^-, \\ K_{11} &= (2 \|\phi_-\|^2 + 4ru\langle \Gamma_+ \phi_-, \Theta_+ \phi_+ \rangle + r^2 u^2 \Delta \|\phi_+\|^2) \mathbf{e}^+ \\ &\quad - 2u^2 \|\phi_+\|^2 \mathbf{e}^- + 2u W_i \mathbf{e}^i, \end{aligned} \quad (13)$$

where we have set

$$W_i = \langle \Gamma_+ \phi_-, \Gamma_i \Theta_+ \phi_+ \rangle. \quad (14)$$

The relations (12) also follow from the KSEs on $\mathcal{S}$ and the constancy of $\|\phi_+\|^2$. Furthermore, the Killing condition on K_{AB} implies the relations

$$i_W d\Delta = 0, \quad \tilde{\nabla}_{(i} W_{j)} = 0, \quad (15)$$

and

⁶The norm is taken with respect to an inner product $\langle \cdot, \cdot \rangle$ which is the real part of the Hermitian inner product for which gamma matrices along spatial directions are Hermitian while those along time-like directions are anti-hermitian.

$$\begin{aligned} -2 \|\phi_+\|^2 - h_i W^i + 2\langle \Gamma_+ \phi_-, \Theta_+ \phi_+ \rangle &= 0, \quad i_W(dh) + 2d\langle \Gamma_+ \phi_-, \Theta_+ \phi_+ \rangle = 0, \\ 2\langle \Gamma_+ \phi_-, \Theta_+ \phi_+ \rangle - \Delta \|\phi_-\|^2 &= 0, \quad W + \|\phi_-\|^2 h + d\|\phi_-\|^2 = 0, \end{aligned} \quad (16)$$

where $\tilde{\nabla}$ is the Levi-Civita connection on $\mathcal{S}$. Using the above conditions, one can also show that

$$\tilde{\mathcal{L}}_W h = 0, \quad \tilde{\mathcal{L}}_W \|\phi_-\|^2 = 0, \quad (17)$$

where $\tilde{\mathcal{L}}$ is the Lie derivative⁷ in $\mathcal{S}$. The invariance of all of the fields of the theory under K_{AB} also implies that W leaves invariant all the components of these fields that have support on the spatial horizon section. For example, the invariance of the form flux F under K_{AB} implies that the components Y , M and X of F are invariant⁸ under W , $\tilde{\mathcal{L}}_W Y = \tilde{\mathcal{L}}_W M = \tilde{\mathcal{L}}_W X = 0$.

Using the relations (16), the associated vector fields to K_{AB} are

$$\begin{aligned} K_{12} &= -2u \|\phi_+\|^2 \partial_u + 2r \|\phi_+\|^2 \partial_r + W^i \tilde{\partial}_i, \quad K_{22} = -2 \|\phi_+\|^2 \partial_u, \\ K_{11} &= -2u^2 \|\phi_+\|^2 \partial_u + (2 \|\phi_-\|^2 + 4ru \|\phi_+\|^2) \partial_r + 2u W^i \tilde{\partial}_i. \end{aligned} \quad (18)$$

One can easily compute the Lie algebra of these vector fields to find

$$\begin{aligned} [K_{AB}, K_{A'B'}] &= \|\phi_+\|^2 (\epsilon_{AA'} K_{BB'} + \epsilon_{BB'} K_{AA'} \\ &\quad + \epsilon_{BA'} K_{AB'} + \epsilon_{AB'} K_{BA'}), \end{aligned} \quad (19)$$

where $\epsilon_{AB} = -\epsilon_{BA}$ and $\epsilon_{12} = 1$. This is isomorphic to $\mathfrak{sl}(2, \mathbb{R})$. For the proof on how the $\mathfrak{sl}(2, \mathbb{R})$ symmetry arises in the context of horizons as a consequence of the horizon conjecture see [1–6].

2.2. Symmetry superalgebras

The symmetry superalgebras of supersymmetric backgrounds are constructed as follows. First one introduces for each Killing spinor ϵ_m of the background an odd generator $Q_m = Q_{\epsilon_m}$ and for each one-form bilinear $K_{mn} = K(\epsilon_m, \epsilon_n)$ an even generator $V_{mn} = V_{K_{mn}}$ for the superalgebra, respectively. The symmetry superalgebra of the background is spanned as $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 = \mathbb{R}\langle V_{mn}, Q_p \rangle$, where $\mathfrak{g}_0 = \mathbb{R}\langle V_{mn} \rangle$ is the even subalgebra and $\mathfrak{g}_1 = \mathbb{R}\langle Q_p \rangle$ is the odd subspace. The (anti-) commutators of $\mathfrak{g}$ are defined [14, 15] as

$$\{Q_m, Q_n\} = V_{mn}, \quad [V_{mn}, Q_p] = Q_{\mathcal{L}_{K_{mn}} \epsilon_p}, \quad [V_{mn}, V_{m'n'}] = V_{[K_{mn}, K_{m'n'}]}, \quad (20)$$

where $\mathcal{L}_{K_{mn}}$ is the spinorial Lie derivative⁹ along the vector field K_{mn} and the commutator in the right-hand-side in the last equation is that of two vector fields. The closure of this superalgebra for supersymmetric backgrounds of some supergravity theories which include 10- and 11-dimensional supergravities has been demonstrated in [16, 17].

3. Superalgebras of horizons and AdS_2 backgrounds

3.1. The superalgebra of $N = 2$ horizons with $\text{Index } \not{D} = 0$

As $\text{Index } \not{D} = 0$, the Killing spinors of near horizon geometries preserving two supersymmetries, $N = 2$, are given as in (11) and the one-form bilinears as in (13). The symmetry

⁷ To distinguish operations along $\mathcal{S}$ from those on the spacetime, we denote the former using a tilde.

⁸ If the fluxes are twisted like in IIB, then the invariance is up to gauge transformations.

⁹ The spinorial Lie derivative of the spinor ϵ along the Killing vector field K is $\mathcal{L}_K \epsilon = \nabla_K \epsilon + \frac{1}{8} dK \epsilon$.

superalgebra is spanned as $\mathfrak{g} = \mathbb{R}\langle V_{AB}, Q_C \rangle$. The anti-commutators $\{Q_A, Q_B\}$ can be extracted from the one-form bilinears (13). To find the commutators of even with odd generators, we have to compute the spinorial derivative of the Killing spinors (11) with respect to the Killing vectors (18). After some computation using the relations (16), one finds that the *a priori* non-vanishing spinorial Lie derivatives are

$$\begin{aligned}\mathcal{L}_{K_{12}}\epsilon_1 &= -\|\phi_+\|^2\epsilon_1 + \tilde{\mathcal{L}}_W\epsilon_1, \quad \mathcal{L}_{K_{12}}\epsilon_2 = \|\phi_+\|^2\epsilon_2 + \tilde{\mathcal{L}}_W\epsilon_2, \\ \mathcal{L}_{K_{22}}\epsilon_1 &= -2\|\phi_+\|^2\epsilon_2, \\ \mathcal{L}_{K_{11}}\epsilon_1 &= u(-2\|\phi_+\|^2\phi_- + 2\|\phi_-\|^2\Gamma_-\Theta_+\phi_+ - \mathcal{W}\Gamma_-\phi_+) + 2u\tilde{\mathcal{L}}_W\epsilon_1, \\ \mathcal{L}_{K_{11}}\epsilon_2 &= 2\|\phi_+\|^2u\epsilon_2 + 2\|\phi_-\|^2\Gamma_-\Theta_+\phi_+ - \mathcal{W}\Gamma_-\phi_+ + 2u\tilde{\mathcal{L}}_W\epsilon_2.\end{aligned}\quad (21)$$

Closure of the Killing superalgebra requires first that these spinorial Lie derivatives must be expressed in terms of the original Killing spinors ϵ_A and second that the super-Jacobi identities of the symmetry superalgebra must be satisfied. For the spinorial Lie derivatives above to satisfy these criteria additional conditions must be imposed.

The super-Jacobi identity of the symmetry superalgebra for three Q_1 generators is satisfied provided that $\mathcal{L}_{K_{11}}\epsilon_1 = 0$. This in particular gives the conditions

$$\tilde{\mathcal{L}}_W\phi_+ = 0, \quad \tilde{\mathcal{L}}_W\Theta_+\phi_+ = 0, \quad (22)$$

and

$$2\tilde{\mathcal{L}}_W\phi_- - 2\|\phi_+\|^2\phi_- + 2\|\phi_-\|^2\Gamma_-\Theta_+\phi_+ - \mathcal{W}\Gamma_-\phi_+ = 0. \quad (23)$$

Typically, the second condition in (22) follows from the first and the invariance of the fields on $\mathcal{S}$ under the action of W .

Using (23) as well as (12) and (16), one can show that $\|\tilde{\mathcal{L}}_W\phi_-\|^2 = 0$ and so

$$\tilde{\mathcal{L}}_W\phi_- = 0. \quad (24)$$

Because of this, (23) gives

$$-2\|\phi_+\|^2\phi_- + 2\|\phi_-\|^2\Gamma_-\Theta_+\phi_+ - \mathcal{W}\Gamma_-\phi_+ = 0, \quad (25)$$

and (22) together with (24) lead to

$$\tilde{\mathcal{L}}_W\epsilon_1 = \tilde{\mathcal{L}}_W\epsilon_2 = 0. \quad (26)$$

The non-vanishing spinorial Lie derivatives can now be re-arranged as

$$\begin{aligned}\mathcal{L}_{K_{12}}\epsilon_1 &= -\|\phi_+\|^2\epsilon_1, \quad \mathcal{L}_{K_{12}}\epsilon_2 = \|\phi_+\|^2\epsilon_2, \\ \mathcal{L}_{K_{22}}\epsilon_1 &= -2\|\phi_+\|^2\epsilon_2, \quad \mathcal{L}_{K_{11}}\epsilon_2 = 2\|\phi_+\|^2\epsilon_1.\end{aligned}\quad (27)$$

Note that to establish the above spinorial Lie derivatives, we did not make use of the details of the geometry of the underlying spacetime. Furthermore notice that (25) can be seen as the inverse transformation to $\phi_+ = \Gamma_+\Theta_-\phi_-$ as it expresses implicitly ϕ_- in terms of ϕ_+ .

The spinorial Lie derivatives (27) specify all the commutators between the even and odd generators of the symmetry superalgebra. Collecting all non-vanishing (anti-) commutators together we have

$$\begin{aligned}\{Q_A, Q_B\} &= V_{AB}, \quad [V_{AB}, Q_C] = -\epsilon_{CA}Q_B - \epsilon_{CB}Q_A, \\ [V_{AB}, V_{A'B'}] &= \epsilon_{AA'}V_{BB'} + \epsilon_{BB'}V_{AA'} + \epsilon_{BA'}V_{AB'} + \epsilon_{AB'}V_{BA'},\end{aligned}\quad (28)$$

where we have set $\|\phi_+\|^2 = 1$. Therefore the symmetry superalgebra $\mathfrak{g} = \mathbb{R}\langle V_{AB}, Q_C \rangle$ of near horizon geometries preserving strictly two supersymmetries is isomorphic to $\mathfrak{osp}(1|2)$. Note that there is no superalgebra generator corresponding to the Killing vector field W . Instead the generators V_{AB} are associated with vector fields which have components both along the lightcone coordinates and along directions tangent to $\mathcal{S}$. This is because W is not associated with a spacetime one-form spinor bilinear.

3.2. Superalgebra of $N = 2$ AdS_2 backgrounds with Index $\mathcal{D} = 0$

Warped AdS_2 backgrounds with the most general allowed fluxes are special cases of near horizon geometries. These arise whenever $W = 0$. Substituting this into the last equation in (16) gives

$$\Delta = \ell^{-2} \|\phi_-\|^{-2}, \quad h = -d \log \|\phi_-\|^2, \quad \|\phi_+\|^2 = \frac{1}{2\ell^2}. \quad (29)$$

Therefore spacetime is a warped product AdS_2 space, $AdS_2 \times_w \mathcal{S}$, with warp factor $A^2 = \|\phi_-\|^2$ [13]. The symmetry superalgebra of warped AdS_2 backgrounds preserving two supersymmetries is again $\mathfrak{osp}(1|2)$. The only difference between AdS_2 backgrounds and generic near horizon geometries is that in the former case the orbits of $\mathfrak{sl}(2, \mathbb{R})$ subalgebra are 2-dimensional and so all three associated vector fields (18) are tangent to the AdS_2 subspace, while in the latter case the orbits of $\mathfrak{sl}(2, \mathbb{R})$ can be 3-dimensional.

3.3. Horizons and AdS_2 backgrounds with $N > 2$ and Index $\mathcal{D} = 0$

3.3.1. Killing spinors and spinorial Lie derivatives. Suppose now that a near horizon geometry of an extreme black hole with a Killing horizon admits $N = 2k$ supersymmetries and exhibits non-trivial fluxes. The horizon conjecture implies that the Killing spinors can be chosen as

$$\begin{aligned} \epsilon_1^s &= \phi_-^s + u\phi_+^s + r\Gamma_- \Theta_+ \phi_+^s, \\ \epsilon_2^s &= \phi_+^s + r\Gamma_- \Theta_+ \phi_+^s, \end{aligned} \quad (30)$$

where $\phi_+^s = \Gamma_+ \Theta_- \phi_-^s$ and ϕ_-^s , $s = 1, \dots, k$ are linearly independent Killing spinors on the spatial horizon section $\mathcal{S}$. In addition the horizon conjecture implies that

$$\langle \phi_+^r, \phi_+^s \rangle = \text{const}, \quad (31)$$

and that the matrix $(\langle \phi_+^r, \phi_+^s \rangle)$ is non-degenerate because of (9). In what follows without loss of generality we shall take $(\langle \phi_+^r, \phi_+^s \rangle)$ to be proportional to the identity matrix though we shall leave $\langle \phi_+^r, \phi_+^s \rangle$ in the formulae below as this illustrates better the origin and the meaning of some of the terms.

To proceed, one uses (10) to define the one-form Killing spinor bi-linears $K_{AB}^{rs} = K(\epsilon_A^r, \epsilon_B^s)$. The Killing condition on K_{AB}^{rs} gives

$$4\langle \Theta_+ \phi_+^r, \Theta_+ \phi_+^s \rangle = \Delta \langle \phi_+^r, \phi_+^s \rangle, \quad \langle \phi_+^r, \Gamma_i \Theta_+ \phi_+^s \rangle = 0, \quad (32)$$

which can be used to simplify the expression for the bilinears as

$$\begin{aligned}
K_{12}^{rs} &= (2r\langle\Gamma_+\phi_-^r, \Theta_+\phi_+^s\rangle + r^2u\Delta\langle\phi_+^r, \phi_+^s\rangle)\mathbf{e}^+ - 2u\langle\phi_+^r, \phi_+^s\rangle\mathbf{e}^- + W^{rs}, \\
K_{22}^{rs} &= r^2\Delta\langle\phi_+^r, \phi_+^s\rangle\mathbf{e}^+ - 2\langle\phi_+^r, \phi_+^s\rangle\mathbf{e}^-, \\
K_{11}^{rs} &= (2\langle\phi_-^r, \phi_-^s\rangle + 4ru\langle\Gamma_+\phi_-^r, \Theta_+\phi_+^s\rangle + r^2u^2\Delta\langle\phi_+^r, \phi_+^s\rangle)\mathbf{e}^+ \\
&\quad - 2u^2\langle\phi_+^r, \phi_+^s\rangle\mathbf{e}^- + 2uW^{(rs)},
\end{aligned} \tag{33}$$

where

$$W^{rs} = \langle\Gamma_+\phi_-^r, \Gamma_i\phi_+^s\rangle\mathbf{e}^i. \tag{34}$$

The Killing condition in addition requires

$$i_{W^{rs}}d\Delta = 0, \quad \tilde{\nabla}_{(i}W_{j)}^{rs} = 0, \tag{35}$$

and

$$\begin{aligned}
-2\langle\phi_+^r, \phi_+^s\rangle - i_h W^{rs} + 2\langle\Gamma_+\phi_-^r, \Theta_+\phi_+^s\rangle &= 0, \quad i_{W^{rs}}(dh) + 2d\langle\Gamma_+\phi_-^r, \Theta_+\phi_+^s\rangle = 0, \\
2\langle\Gamma_+\phi_-^r, \Theta_+\phi_+^s\rangle - \Delta\langle\phi_-^r, \phi_-^s\rangle &= 0, \quad W^{(rs)} + \langle\phi_-^r, \phi_-^s\rangle h + d\langle\phi_-^r, \phi_-^s\rangle = 0.
\end{aligned} \tag{36}$$

Therefore all vector fields W^{rs} are Killing on $\mathcal{S}$. It follows from (36) that

$$\tilde{\mathcal{L}}_{W^{rs}}h = 0. \tag{37}$$

Furthermore in all theories for which the invariance of the remaining fields F of the theory under K_{AB}^{rs} has been investigated, it has been found to imply that Y, M and X are invariant under W^{rs} on $\mathcal{S}$.

To find the commutators of even and odd generators, we must compute the spinorial Lie derivatives of the Killing spinors (30) with respect to the one-form bilinears (33). After some computation, one finds that the *a priori* non-vanishing spinorial Lie derivatives are

$$\begin{aligned}
\mathcal{L}_{K_{12}^{rs}}\epsilon_1^t &= -\langle\phi_+^r, \phi_+^s\rangle\epsilon_1^t + \tilde{\mathcal{L}}_{W^{rs}}\epsilon_1^t, \quad \mathcal{L}_{K_{12}^{rs}}\epsilon_2^t = \langle\phi_+^r, \phi_+^s\rangle\epsilon_2^t + \tilde{\mathcal{L}}_{W^{rs}}\epsilon_2^t, \\
\mathcal{L}_{K_{22}^{rs}}\epsilon_1^t &= -2\langle\phi_+^r, \phi_+^s\rangle\epsilon_2^t, \\
\mathcal{L}_{K_{11}^{rs}}\epsilon_1^t &= u(-2\langle\phi_+^r, \phi_+^s\rangle\phi_-^t + 2\langle\phi_-^r, \phi_-^s\rangle\Gamma_-\Theta_+\phi_+^t - W^{(rs)}\Gamma_-\phi_+^t) + 2u\tilde{\mathcal{L}}_{W^{(rs)}}\epsilon_1^t, \\
\mathcal{L}_{K_{11}^{rs}}\epsilon_2^t &= 2u\langle\phi_+^r, \phi_+^s\rangle\epsilon_2^t + 2\langle\phi_-^r, \phi_-^s\rangle\Gamma_-\Theta_+\phi_+^t - W^{(rs)}\Gamma_-\phi_+^t + 2u\tilde{\mathcal{L}}_{W^{(rs)}}\epsilon_2^t.
\end{aligned} \tag{38}$$

Closure of the Killing superalgebra requires that $\mathcal{L}_{K_{11}^{rs}}\epsilon_1^t$ should be expressed in terms of the original Killing spinors ϵ_A^t . However, this is not possible as its dependence on the lightcone coordinates u, r is in conflict with that of the Killing spinors. This is unless all the coefficients vanish. Thus we have that

$$\begin{aligned}
\tilde{\mathcal{L}}_{W^{(rs)}}\phi_+^t &= \tilde{\mathcal{L}}_{W^{(rs)}}\Theta_+\phi_+^t = 0, \\
2\tilde{\mathcal{L}}_{W^{(rs)}}\phi_-^t &= 2\langle\phi_+^r, \phi_+^s\rangle\phi_-^t - 2\langle\phi_-^r, \phi_-^s\rangle\Gamma_-\Theta_+\phi_+^t + W^{(rs)}\Gamma_-\phi_+^t.
\end{aligned} \tag{39}$$

Moreover as $\phi_+^r = \Gamma_+\Theta_-\phi_-^r$, we have

$$\tilde{\mathcal{L}}_{W^{(rs)}}\phi_+^t = 0 \implies \Theta_-\tilde{\mathcal{L}}_{W^{(rs)}}\phi_-^t = 0 \implies \tilde{\mathcal{L}}_{W^{(rs)}}\phi_-^t = 0, \tag{40}$$

where we have used that h, Δ and the fluxes Y, M, X are invariant under $W^{(rs)}$, and that for near horizon geometries with non-trivial fluxes $\text{Ker } \Theta_- = \{0\}$. Hence (30) and (39) imply that

$$\tilde{\mathcal{L}}_{W^{(rs)}}\epsilon_A^t = 0, \quad 2\langle\phi_+^r, \phi_+^s\rangle\phi_-^t - 2\langle\phi_-^r, \phi_-^s\rangle\Gamma_-\Theta_+\phi_+^t + W^{(rs)}\Gamma_-\phi_+^t = 0. \tag{41}$$

Then the non-vanishing spinorial Lie derivatives can be re-arranged as

$$\begin{aligned}\mathcal{L}_{K_{12}^{rs}}\epsilon_1^t &= -\langle\phi_+^r, \phi_+^s\rangle\epsilon_1^t + \tilde{\mathcal{L}}_{Z^{rs}}\epsilon_1^t, \\ \mathcal{L}_{K_{12}^{rs}}\epsilon_2^t &= \langle\phi_+^r, \phi_+^s\rangle\epsilon_2^t + \tilde{\mathcal{L}}_{Z^{rs}}\epsilon_2^t, \\ \mathcal{L}_{K_{22}^{rs}}\epsilon_1^t &= -2\langle\phi_+^r, \phi_+^s\rangle\epsilon_2^t, \\ \mathcal{L}_{K_{11}^{rs}}\epsilon_2^t &= 2\langle\phi_+^r, \phi_+^s\rangle\epsilon_1^t,\end{aligned}\tag{42}$$

where we have set $Z^{rs} = W^{[rs]}$.

Before we proceed to describe the superalgebra observe that the vector fields $W^{rs} = (W^{rs})^i \tilde{\partial}_i$ are both isometries of the spatial horizon section and also of the spacetime.

3.3.2. $\mathfrak{g}_0 = \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{t}_0$. To demonstrate that the even subalgebra decomposes as indicated above, we write

$$K_{AB}^{(rs)} = \langle\phi_+^r, \phi_+^s\rangle K_{AB} + \tilde{K}_{AB}^{(rs)},\tag{43}$$

where $\tilde{K}^{(rs)}$ is traceless with respect to the non-degenerate positive definite matrix $(\langle\phi_+^r, \phi_+^s\rangle)$. Observe that the Killing vectors K_{AB} have components both along the light-cone and spatial horizon section directions which generically cannot be separated.

We shall now demonstrate that

$$\tilde{K}_{AB}^{(rs)} = 0.\tag{44}$$

This is best done in the language of symmetry superalgebras. For this introduce odd generators Q_A^r for each spacetime Killing spinor ϵ_A^r and even generators V_{AB}^{rs} for each one-form bilinear K_{AB}^{rs} . Writing

$$V_{AB}^{rs} = V_{AB}^{(rs)} + \epsilon_{AB} \tilde{V}^{rs} = \langle\phi_+^r, \phi_+^s\rangle V_{AB} + \tilde{V}_{AB}^{(rs)} + \epsilon_{AB} \tilde{V}^{rs},\tag{45}$$

where $V_{AB}^{rs} = \epsilon_{AB} \tilde{V}^{rs}$, we have the (anti)-commutators

$$\{Q_A^r, Q_B^s\} = \langle\phi_+^r, \phi_+^s\rangle V_{AB} + \tilde{V}_{AB}^{(rs)} + \epsilon_{AB} \tilde{V}^{rs},\tag{46}$$

and

$$\begin{aligned}[V_{AB}, Q_C^r] &= -\epsilon_{CA} Q_B^r - \epsilon_{CB} Q_A^r, \\ [\tilde{V}_{AB}^{(rs)}, Q_C^r] &= 0.\end{aligned}\tag{47}$$

The last commutator above follows from the spinorial Lie derivatives in (42). As the $\tilde{V}_{AB}^{(rs)}$ commute with the Q 's, they are all central in the symmetry superalgebra $\mathfrak{g}$.

To continue observe that a super-Jacobi identity implies that

$$[V_{AB}^{rs}, V_{A'B'}^{r's'}] = \{Q_{A'}^{r'}, [V_{AB}^{rs}, Q_B^{s'}]\} + \{Q_B^{s'}, [V_{AB}^{rs}, Q_{A'}^{r'}]\}.\tag{48}$$

Next set $V_{AB}^{rs} = \langle\phi_+^r, \phi_+^s\rangle V_{AB}$ and $V_{A'B'}^{r's'} = \tilde{V}_{A'B'}^{(r's')}$ in (48). This gives

$$[V_{AB}, \tilde{V}_{A'B'}^{(r's')}] = \epsilon_{AB'} \tilde{V}_{BA'}^{(r's')} + \epsilon_{BB'} \tilde{V}_{AA'}^{(r's')} + \epsilon_{AA'} \tilde{V}_{BB'}^{(r's')} + \epsilon_{BA'} \tilde{V}_{AB'}^{(r's')}.\tag{49}$$

As $\tilde{V}_{A'B'}^{(r's')}$ are central, they must commute with V_{AB} and so this commutator must vanish. This is the case provided that $\tilde{V}_{A'B'}^{(r's')} = 0$, which in turn implies that $\tilde{K}_{A'B'}^{(r's')} = 0$. This establishes (44) and so

$$K_{AB}^{rs} = \langle\phi_+^r, \phi_+^s\rangle K_{AB} + \epsilon_{AB} Z^{rs}, \quad W^{rs} = \langle\phi_+^r, \phi_+^s\rangle \tilde{W} + Z^{rs}.\tag{50}$$

It remains to show that $[K_{AB}, Z^{rs}] = 0$. To demonstrate this, we use again (48) with $V_{AB}^{rs} = \langle \phi_{+}^r, \phi_{+}^s \rangle V_{AB}$ and $V_{A'B'}^{r's'} = \epsilon_{A'B'} \tilde{V}^{r's'}$. After applying (47), a brief computation reveals that $[V_{AB}, \tilde{V}^{r's'}] = 0$ which establishes the result. In particular, this implies that

$$[Z^{rs}, \tilde{W}] = 0. \quad (51)$$

This will be used later in the identification of the Lie algebras of groups that act transitively on the spatial horizon section.

3.3.3. The commutator $[\tilde{V}^{rs}, Q_A^t]$. To determine the symmetry superalgebra of near horizon geometries, it remains to find the commutator $[\tilde{V}^{rs}, Q_A^t]$. This can be done via an analysis similar to that presented in [7] for warped AdS₃ backgrounds. Here we summarize some of the key points as there are some differences with the analysis presented for AdS₃ backgrounds. The computation for the $N = 4$ case is identical to that presented for AdS₃ backgrounds. To identify the superalgebra for the $N > 4$ cases, one has to determine $\tilde{\mathcal{L}}_{Z^{rs}} \epsilon_A^p$ for $p \neq r, s$. First observe that $\tilde{\mathcal{L}}_{Z^{rs}}$ commutes with the lightcone coordinates and so preserves the type of Killing spinors. Thus

$$\tilde{\mathcal{L}}_{Z^{rs}} \epsilon_A^p = \alpha^{rsp}_q \epsilon_A^q, \quad p \neq r, s. \quad (52)$$

The constants α can be different for ϵ_1^p and ϵ_2^p Killing spinors. However this is not the case as a consequence of the super-Jacobi identities involving V_{AB} , $\tilde{V}^{rs}$ and Q_A^p . In addition the super-Jacobi identities for Q_A^r , Q_B^s and Q_A^p for $B \neq A$ imply that

$$\alpha^{rsp}_q = \alpha^{prs}_q, \quad (53)$$

which together with $\alpha^{rsp}_q = -\alpha^{srp}_q$ give $\alpha^{rsp}_q = \alpha^{[rsp]}_q$. Furthermore, the spinorial Lie derivative identity

$$\tilde{\mathcal{L}}_{Z^{rs}} \langle \epsilon_2^p, \epsilon_2^q \rangle = \langle \tilde{\mathcal{L}}_{Z^{rs}} \epsilon_2^p, \epsilon_2^q \rangle + \langle \epsilon_2^p, \tilde{\mathcal{L}}_{Z^{rs}} \epsilon_2^q \rangle, \quad (54)$$

together with

$$\tilde{\mathcal{L}}_{Z^{rs}} \langle \epsilon_2^p, \epsilon_2^q \rangle = \tilde{\mathcal{L}}_{Z^{rs}} (\langle \phi_+^p, \phi_+^q \rangle + \frac{1}{2} r^2 \Delta \langle \phi_+^p, \phi_+^q \rangle) = 0, \quad (55)$$

which follows from (35), the invariance of Δ and (31), give that $\alpha^{rspq} = \alpha^{rs[pq]}$, where the indices are raised with $\langle \phi_+^p, \phi_+^q \rangle$. Putting all the properties of α together we find that

$$\alpha^{rspq} = \alpha^{[rspq]}. \quad (56)$$

So α is an four-form. This conclusion is the starting point of the analysis for the identification of the symmetry superalgebras of AdS₃ backgrounds. The results are summarized in the section below.

3.3.4. The symmetry superalgebras of horizons with Index $\mathcal{D} = 0$. Collecting all the results that we have established so far, the non-vanishing (anti-) commutators of the symmetry superalgebras of near horizon geometries are

$$\begin{aligned}
\{Q_A^r, Q_B^s\} &= \delta^{rs} V_{AB} + \epsilon_{AB} \tilde{V}^{rs}, \quad [V_{AB}, Q_C^r] = -\epsilon_{CA} Q_B^r - \epsilon_{CB} Q_A^r, \\
[\tilde{V}^{rs}, Q_A^t] &= -(\delta^{tr} Q_A^s - \delta^{ts} Q_A^r - \alpha^{rst} Q_A^t), \\
[V_{AB}, V_{A'B'}] &= \epsilon_{AA'} V_{BB'} + \epsilon_{BB'} V_{AA'} + \epsilon_{BA'} V_{AB'} + \epsilon_{AB'} V_{BA'}, \\
[\tilde{V}^{rs}, \tilde{V}^{r's'}] &= \delta^{rs'} \tilde{V}^{sr'} - \delta^{ss'} \tilde{V}^{rr'} - \delta^{rr'} \tilde{V}^{ss'} + \delta^{sr'} \tilde{V}^{rs'} \\
&\quad - \alpha^{rss'} \tilde{V}^{tr'} + \alpha^{rsr'} \tilde{V}^{ts'},
\end{aligned} \tag{57}$$

where we have set without loss of generality $\langle \phi_+^r, \phi_+^s \rangle = \delta^{rs}$ to simplify the expressions.

The structure constants that remain to be determined are those that appear in the commutators $[\tilde{V}^{rs}, Q_A^t]$. It turns out that $\alpha_{rst\ell}$ is a four-form, where the indices are raised and lowered with δ_{rs} . The symmetry superalgebra $\mathfrak{g}$ defined in (57) may have central generators. Writing

$$[\tilde{V}^{rs}, Q_A^t] = D(V^{rs})^t{}_\ell Q_A^\ell, \tag{58}$$

the centre $\mathfrak{c}$ is spanned by those elements $a_{rs} \tilde{V}^{rs} \in \mathfrak{t}_0$ which commute with all the Q 's, i.e. $a_{rs} D(\tilde{V}^{rs})^t{}_\ell = 0$. It has been shown in the context of AdS_3 backgrounds [7] that $\mathfrak{t}_0/\mathfrak{c}$ can be identified with the Lie algebra of a group that acts transitively on a sphere with the D representation. Moreover α is invariant under D . This together with the (anti-) commutators collected in (57) allow to identify all possible symmetry superalgebras of near horizon geometries that exhibit non-trivial fluxes and $\text{Index} \not= 0$. The results have been tabulated in table 1. The same symmetry superalgebras occur in warped AdS_2 backgrounds with the most general allowed fluxes, i.e. the fluxes that are invariant under the $\mathfrak{sl}(2, \mathbb{R})$ isometries of AdS_2 subspace.

3.3.5. Aspects of geometry. We summarize the conditions we have obtained so far from imposing the closure of the symmetry superalgebra to simplify the relations (35) and (36). In particular, one finds that

$$i_{Z^{rs}} d\Delta = 0, \quad i_{\tilde{W}} d\Delta = 0, \quad \tilde{\nabla}_{(i} Z_{j)}^{rs} = 0, \quad \tilde{\nabla}_{(i} \tilde{W}_{j)} = 0, \tag{59}$$

and

$$\begin{aligned}
-2\langle \phi_+^r, \phi_+^s \rangle - \langle \phi_+^r, \phi_+^s \rangle i_h \tilde{W} + 2\langle \Gamma_+ \phi_-^{(r}, \Theta_+ \phi_+^{s)} \rangle &= 0, \quad -i_h Z^{rs} + 2\langle \Gamma_+ \phi_-^{[r}, \Theta_+ \phi_+^{s]} \rangle = 0 \\
\langle \phi_+^r, \phi_+^s \rangle i_{\tilde{W}} (dh) + 2d\langle \Gamma_+ \phi_-^{(r}, \Theta_+ \phi_+^{s)} \rangle &= 0, \quad i_{Z^{rs}} (dh) + 2d\langle \Gamma_+ \phi_-^{[r}, \Theta_+ \phi_+^{s]} \rangle = 0, \\
2\langle \Gamma_+ \phi_-^{(r}, \Theta_+ \phi_+^{s)} \rangle - \Delta \langle \phi_-^r, \phi_-^s \rangle &= 0, \quad \langle \phi_+^r, \phi_+^s \rangle \tilde{W} + \langle \phi_-^r, \phi_-^s \rangle h + d\langle \phi_-^r, \phi_-^s \rangle = 0.
\end{aligned} \tag{60}$$

It follows that

$$\mathcal{L}_{\tilde{W}} h = 0, \quad \mathcal{L}_{Z^{rs}} h = 0. \tag{61}$$

Moreover, the invariance conditions of the Killing spinors (39) and (41) imply that

$$\tilde{\mathcal{L}}_{\tilde{W}} \phi_\pm^t = 0, \quad 2\langle \phi_+^r, \phi_+^s \rangle \phi_-^t - 2\langle \phi_-^r, \phi_-^s \rangle \Gamma_- \Theta_+ \phi_+^t + \langle \phi_+^r, \phi_+^s \rangle \tilde{W} \Gamma_- \phi_+^t = 0. \tag{62}$$

In particular, the inner products of $\phi_\pm^r$ are invariant under $\tilde{W}$. Notice also that the inner product $\langle \phi_-^r, \phi_-^s \rangle$ is proportional to $\langle \phi_+^r, \phi_+^s \rangle$. For this set $r = s = 1$ in the above equation and take the inner product with ϕ_-^t . After using the first equation in (60), one can express the inner product of ϕ_- 's in terms of that of ϕ_+ 's.

3.3.6. Superalgebras of AdS_2 backgrounds. The AdS_2 backgrounds that preserve $N > 2$ supersymmetries are special cases of near horizon geometries for which $\tilde{W} = 0$. The conditions on the geometry of internal spaces can be found after setting $\tilde{W} = 0$ in (59) and (60).

The symmetry superalgebras are again given in table 1. One difference is that the vector fields generated by the $\mathfrak{sl}(2, \mathbb{R})$ subalgebra are all tangent to the AdS_2 subspaces.

Furthermore, setting $\tilde{W} = 0$ in (60), one finds that $\langle \Gamma_+ \phi_-^{(r)}, \Theta_+ \phi_+^{(s)} \rangle = \langle \phi_+^r, \phi_+^s \rangle$ and so

$$\langle \phi_-^r, \phi_-^s \rangle = 2\Delta^{-1} \langle \phi_+^r, \phi_+^s \rangle, \quad h = d \log \Delta^2, \quad (63)$$

where Δ is nowhere vanishing. The remaining independent conditions on the geometry are that $i_{Z^s} d\Delta = 0$ and Z^s are Killing vectors of the internal space. Also for $\tilde{W} = 0$, the relation of ϕ_- to ϕ_+ spinors in (62) considerably simplifies.

3.4. Superalgebras of horizons with Index $\mathcal{D} \neq 0$

3.4.1. $N_- = 0$. So far we have investigated the superalgebras of near horizon geometries and warped AdS_2 backgrounds provided that $\text{Index} \mathcal{D} = 0$. Now instead let us take $N_- = 0$ and $\text{Index} \mathcal{D} \neq 0$ in (1). In this case, all the Killing spinors are of the form

$$\epsilon^s = \phi_+^s + r\Gamma_- \Theta_+ \phi_+^s, \quad s = 1, \dots, N. \quad (64)$$

The only non-vanishing bilinear is K_{22}^{rs} given in (33). The spinorial derivatives can be computed in a straightforward way to reveal that the symmetry superalgebra that arises is of Poincaré type, i.e. the only non-vanishing (anti-) commutators are

$$\{Q^r, Q^s\} = \langle \phi_+^r, \phi_+^s \rangle V, \quad (65)$$

where Q^r are the odd generators associated to the Killing spinors (64) and V is an even generator associated to the vector field ∂_u . The symmetry superalgebra is nilpotent and it can be identified as a supersymmetric version of a Heisenberg type of algebra.

3.4.2. $N_- \neq 0$. It remains to investigate the cases with non-trivial fluxes for which there is at least one Killing spinor ϕ_- , i.e. $N_- \neq 0$. We shall demonstrate that this case $\text{Index} \mathcal{D} = 0$, and so the symmetry superalgebras of such backgrounds are tabulated in table 1.

To show this, let ϕ_- be a Killing spinor. Define $\phi_+ = \Gamma_+ \Theta_- \phi_-$ and consider another Killing spinor $\hat{\phi}_+$ which without loss of generality can be chosen orthogonal to ϕ_+ and cannot be written as $\hat{\phi}_+ \neq \Gamma_+ \Theta_- \hat{\phi}_-$ for some $\hat{\phi}_-$. In this case, there is an $\mathfrak{sl}(2, \mathbb{R})$ symmetry generated by the spacetime Killing spinors ϵ_A in (11) constructed from $\phi_{\pm}$ as for the $N = 2$ near horizon geometries described in section 2.1.3.

Consider the spacetime Killing spinor $\hat{\epsilon}_2 = \hat{\phi}_+ + r\Gamma_- \Theta_+ \hat{\phi}_+$; one finds that

$$\mathcal{L}_{K_{11}} \hat{\epsilon}_2 = 2u \|\phi_+\|^2 \hat{\epsilon}_2 + 2 \|\phi_-\|^2 \Gamma_- \Theta_+ \hat{\phi}_+ - W \Gamma_- \hat{\phi}_+ + 2u \tilde{\mathcal{L}}_W \hat{\epsilon}_2, \quad (66)$$

where K_{11} is given in (13). As the superalgebra must close on the Killing spinors ϵ_1, ϵ_2 and $\hat{\epsilon}_2$ we write $\mathcal{L}_{K_{11}} \hat{\epsilon}_2 = \alpha \epsilon_1 + \beta \epsilon_2 + \gamma \hat{\epsilon}_2$ for some constants α, β and γ . This in particular yields

$$2\tilde{\mathcal{L}}_W \hat{\phi}_+ + 2 \|\phi_+\|^2 \hat{\phi}_+ = \alpha \phi_+. \quad (67)$$

Taking the inner product with $\hat{\phi}_+$, one finds that $\|\phi_+\| \|\hat{\phi}_+\| = 0$ and so either ϕ_+ or $\hat{\phi}_+$ must vanish which is a contradiction. This generalizes to backgrounds with non-trivial fluxes and $N_- > 1$, and any number of $\hat{\epsilon}_2$ Killing spinors. We conclude that if there is a Killing spinor ϕ_- , then for near horizon geometries with non-trivial fluxes all spacetime spinors are generated by pairs (ϕ_+, ϕ_-) with $\phi_+ = \Gamma_+ \Theta_- \phi_-$. Hence the index of the Dirac operator vanishes

for all such backgrounds¹⁰. Perhaps this is not unexpected, as the presence of the $\mathfrak{sl}(2, \mathbb{R})$ symmetry pairs up the ϵ_1 and ϵ_2 Killing spinors.

4. Near horizon geometries preserving more than half of supersymmetry

4.1. The spatial horizon sections are homogeneous

The main task of this section is to show that the spatial horizon section $\mathcal{S}$ of near horizon geometries preserving more than half of supersymmetry of a theory is a homogeneous space admitting a transitive action of a group with Lie algebra $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$. The proof relies on an adaptation of the homogeneity theorem of [10].

First observe that as the near horizon geometries are required to preserve more than half of the supersymmetry, at least one of the spacetime Killing spinors is constructed from a ϕ_- type spinor on $\mathcal{S}$. Moreover demanding that some of the fluxes do not vanish¹¹, we conclude from the results of the previous section that all spacetime Killing spinors are constructed from pairs (ϕ_-, ϕ_+) with $\phi_+ = \Gamma_+ \Theta_- \phi_-$ and the symmetry superalgebras are given in table 1.

The Killing vectors on $\mathcal{S}$ constructed as bilinears can be rewritten as

$$W^{rs} = -\langle \phi_-^r, \Gamma^i \Gamma_- \phi_+^s \rangle \tilde{\partial}_i, \quad (68)$$

and $\phi_+^s = \Gamma_+ \Theta_- \phi_-^s$. We shall show that for near horizon geometries that preserve more than half of the supersymmetry, the vectors W^{rs} span the tangent space of $\mathcal{S}$ at every point $p \in \mathcal{S}$.

It suffices to do the calculation pointwise. For this define the subspace $\mathcal{W}_p = \mathbb{R}\langle W^{rs}|_p \rangle \subseteq T_p \mathcal{S}$ and $\mathcal{W}_p^\perp$ its orthogonal complement in $T_p \mathcal{S}$. If the W^{rs} do not span $T_p \mathcal{S}$, then there is a $v \in \mathcal{W}_p^\perp$, $v \neq 0$, such that

$$v^r W_i^{rs}|_p = -v^i \langle \phi_-^r, \Gamma_i \Gamma_- \phi_+^s \rangle|_p = -\langle \phi_-^r, \not{v} \Gamma_- \phi_+^s \rangle|_p = 0. \quad (69)$$

This in particular implies that all the spinors $\not{v} \Gamma_- \phi_+^s$ are orthogonal to the Killing spinors ϕ_-^r . Let $\mathfrak{S}$ be the spin bundle with sections ϕ_- . Denoting the span of Killing spinors with $\mathcal{K}_p = \mathbb{R}\langle \phi_-^r|_p \rangle$ in the fibre $\mathfrak{S}_p$ of $\mathfrak{S}$ over p and $\mathcal{K}_p^\perp$ its complement, as the solutions preserve more than half of the supersymmetry, $\dim \mathcal{K}_p > \dim \mathcal{K}_p^\perp$.

On the other hand $\not{v} \Gamma_- \phi_+^s = \not{v} \Gamma_- \Gamma_+ \Theta_- \phi_-^s = 2\not{v} \Theta_- \phi_-^s$. Moreover it follows from (69) that $\not{v} \Theta_-|_p : \mathcal{K}_p \rightarrow \mathcal{K}_p^\perp$. In all supergravities for which the horizon conjecture applies and for which the fluxes are non-trivial, $\text{Ker } \Theta_- = \{0\}$. Also $\text{Ker } \not{v} \Theta_- = \{0\}$ provided¹² that $v \neq 0$ as $\not{v}^2 = v^2 \mathbf{1} \neq 0$. So $\not{v} \Theta_-$ is an injection, but this is not possible as $\dim \mathcal{K}_p > \dim \mathcal{K}_p^\perp$. This leads to a contradiction unless $v = 0$ and so $\mathcal{W}_p$ spans all $T_p \mathcal{S}$. Therefore $\mathcal{S}$ must be a homogeneous space. The group G that acts transitively on $\mathcal{S}$ has Lie algebra $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$. The Killing vector fields generated by $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$ are Z^{rs} and $\tilde{W}$. In particular $\tilde{W}$ is generated by the $\mathfrak{so}(2)$ subalgebra.

We remark that for warped $\text{AdS}_2 \times_w \mathcal{S}$ backgrounds $\tilde{W} = 0$. In such a case, the internal space $\mathcal{S}$ is homogeneous and the group G that acts transitively on it has Lie algebra $\mathfrak{t}_0$.

¹⁰ The vanishing of the index of the Dirac operator as a consequence of a symmetry argument is reminiscent of the results of [25].

¹¹ If the fluxes vanish and the near horizon geometries exist, they are locally products $\mathbb{R}^2 \times \mathcal{S}$, where $\mathcal{S}$ is a product of typically special holonomy manifolds that admit parallel spinors. Apart from perhaps $\mathbb{R}^2 \times T^{D-2}$ all the rest preserve less than half of supersymmetry. Also such solutions may not be considered as near horizon geometries of black holes.

¹² The signature of the metric of $\mathcal{S}$ is Euclidean and so the only vector with zero length is the zero vector.

4.2. A no-existence theorem for $N > 16$ AdS_2 backgrounds and black hole horizons

Suppose that the AdS_2 backgrounds and the near horizon geometries preserve more than 16 supersymmetries in 10- and 11-dimensional supergravities and have non-trivial fluxes. In this case the internal space or the spatial horizon section has dimension either 8 or 9 and must be homogeneous. Furthermore, an inspection of table 1 reveals that the only possible choices of the $\mathfrak{t}_0/\mathfrak{c}$ subalgebra are

$$\begin{aligned} &\mathfrak{so}(N/2) \ (N = 18, 20, 22, 24, 26, 28, 30); \quad \mathfrak{u}(N/4) \ (N = 20, 24, 28,); \\ &\mathfrak{sp}(3) \oplus \mathfrak{sp}(1) \ (N = 24). \end{aligned} \quad (70)$$

Near horizon geometries and warped AdS_2 backgrounds that preserve 32 supersymmetries have already been excluded in [26]. Moreover there are no such backgrounds with 30 supersymmetries in 11-dimensional supergravity [22] and with ≥ 28 supersymmetries in IIB [23].

If there are any near horizon geometries that preserve more than 16 supersymmetries, the near horizon section $\mathcal{S}$ must admit a transitive group action with Lie algebra $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$, where $\mathfrak{t}_0/\mathfrak{c}$ is given in (70). Similarly, the internal space of a warped AdS_2 background with the most general allowed fluxes that preserves more than 16 supersymmetries must admit a transitive group action of a group G with Lie algebra $\mathfrak{t}_0$, where again $\mathfrak{t}_0/\mathfrak{c}$ is given in (70). In both cases $\mathfrak{t}_0/\mathfrak{c}$ must act effectively on either the spatial horizon section or the internal space. This is because even generators in the symmetry superalgebra are introduced whenever there is a non-vanishing vector spinor bilinear leading to a non-trivial action on the spacetime. Moreover, the even generators of $\mathfrak{t}_0/\mathfrak{c}$ act effectively on $\mathfrak{g}_1$ and so they are not allowed to vanish.

The relevant 8- and 9-dimensional homogeneous spaces have been classified in [11]. An inspection reveals that the only homogeneous spaces that satisfy these criteria are $S^9 = SO(10)/SO(9)$ in 9-dimensions, and $S^8 = SO(9)/SO(8)$ and $\mathbb{CP}^4$ in 8-dimensions. $\mathbb{CP}^4$ does not admit a spin structure so it should be excluded. The remaining cases can also be excluded. As S^9 is a symmetric space and does not admit invariant two-, three- and four-forms, it follows that the four-form flux of 11-dimensional supergravity vanishes. As a result, the spacetime field equations cannot be satisfied for non-zero Δ or h . Potentially, there is an additional case in 11-dimensional supergravity for $\mathcal{S} = S^8 \times S^1$, where S^1 may arise due to an element in $\mathfrak{c}$. However as the relevant superalgebra here is $\mathfrak{osp}(9|2)$, $\mathfrak{c} = \{0\}$. Alternatively, one can verify by an explicit computation that there is no solution with either Δ or h non-vanishing and $\mathcal{S} = S^8 \times S^1$.

Similarly, S^8 is symmetric, and all invariant forms are parallel. Furthermore, as a consequence of the homogeneity theorem, all scalar fields in IIB and (massive) IIA supergravity must be constant for $N > 16$ supersymmetries. An inspection of the IIB Einstein equation in [1] along the spatial horizon section $\mathcal{S}$ implies that it must be Ricci flat which is a contradiction as S^8 has a non-vanishing Ricci tensor. In (massive) IIA theory, the only non-vanishing flux is associated with the electric component of the two-form field strength. An inspection of the dilatino KSE along $\mathcal{S}$ in [5] reveals that there are no solutions that preserve $N > 16$ supersymmetries unless both this component, and the mass parameter (of the massive theory), vanish. In this case, the Ricci tensor of $\mathcal{S}$ vanishes which is a contradiction as S^8 has a non-vanishing Ricci tensor. Also there are no near horizon geometries with non-trivial fluxes and AdS_2 backgrounds in heterotic supergravity that preserve more than 8 supersymmetries [24].

Collecting the results above, we conclude that there are no smooth near horizon geometries with non-trivial fluxes and warped AdS_2 backgrounds in 10- and 11-dimensions that preserve more than half of the supersymmetry provided that either the spatial horizon section or the internal manifold, respectively, are compact without boundary. This also excludes the possibility of constructing solutions by taking discrete identifications. However if the above

assumptions are removed, there are solutions. For example the maximally supersymmetric $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ [34–36] solutions of 11-dimensional supergravity can be re-interpreted as warped AdS_2 solutions. In this case the internal space is not compact.

5. Concluding remarks

We have identified the symmetry superalgebras $\mathfrak{g}$ of near horizon geometries for all theories for which the horizon conjecture applies. These include all 10- and 11-dimensional supergravity theories as well as $\mathcal{N} = 2$ four- and $\mathcal{N} = 1$ five-dimensional (gauged) supergravities. We have found that either the even subalgebra decomposes as $\mathfrak{g}_0 = \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{t}_0$ and $\mathfrak{t}_0/\mathfrak{c}$ is the Lie algebra of a group acting transitively and effectively on spheres, where $\mathfrak{c}$ is the center, or $\mathfrak{g}$ is nilpotent. The latter case arises whenever the number of supersymmetries is equal to the index of a Dirac operator. The symmetry superalgebras can be found in table 1 and in (65), respectively.

The warped AdS_2 backgrounds with the most general allowed fluxes are special cases of near horizon geometries for which the internal space is identified with the spatial horizon section and the metric and fields are restricted appropriately. Because of this the symmetry superalgebras of warped AdS_2 backgrounds are identified in the same way as those of near horizon geometries. The only difference is that for generic near horizon geometries the spatial horizon section admits the action of a group with Lie algebra $\mathfrak{t}_0 \oplus \mathfrak{so}(2)$ while for AdS_2 backgrounds the internal space admits the action of a group with Lie algebra $\mathfrak{t}_0$. The results of this paper together with those in [7] lead to the identification of all symmetry superalgebras of AdS backgrounds in 10- and 11-dimensional supergravities, see also [9] for results in the AdS_4 case.

Furthermore we demonstrated that there are no near horizon geometries and AdS_2 backgrounds that preserve $N > 16$ supersymmetries in 10- and 11-dimensional supergravity theories under the same assumptions as those utilized to prove the horizon conjecture. This together with the results of [26, 28, 29] classify all the warped AdS_n backgrounds, for $n = 2, 4, 5$ in 10- and 11-dimensional supergravity theories that preserve $N > 16$ supersymmetries up to at most discrete identifications. Product solutions $\text{AdS}_n \times M^{D-n}$ with M^{D-n} symmetric space, $D = 10, 11$, have been classified in [30–33]. It is clear from these that there are strong constraints in the existence of AdS backgrounds, especially those that preserve more than half of supersymmetry. In the context of AdS/CFT, this implies that there are very few local geometries that can be identified as gravitational duals of superconformal theories with a large

AQ4 number of supersymmetries.

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