Excitation, detection, and passive cooling of a micromechanical cantilever using near-field of a microwave resonator

L. Hao,^aJ. C. Gallop, and D. Cox.

National Physical Laboratory, Hampton Road, Teddington, Middlesex TW11 0LW, United Kingdom

Micro/nanomechanical resonators are important for nanoscale and quantum physics. We show that a microwave near-field coaxial resonator system allows mechanical cantilever excitation on a scale much shorter than the microwave wavelength. Thermal noise is observed in the unexcited system, enabling room temperature displacement sensitivity of ~ 70 fm/Hz^{1/2}. The measured force between near-field probe and cantilever varies with separation, in excellent agreement with theory. Uniquely, optical excitation and read-out lasers are also included. We demonstrate passive cantilever mode cooling from 300 to 100 K by frequency detuning the microwave resonator and propose pulsed cooling operation to enable several high-sensitivity applications.

Micro and nanomechanical resonators are important for nanoscale and quantum physics¹ with applications in ultralow mass,² force,³ and displacement⁴ measurement. As electromechanical resonators decrease in size the displacement oscillation amplitude scales down rapidly, thus ultrasensitive transducer techniques and low dissipation excitation schemes are required to measure their vibration at the nanoscale.⁵ Although microwave wavelengths are in the centimeter range, the evanescent microwave field in the neighborhood of a sharp conducting tip may be used to produce spatial resolution many orders of magnitude smaller than this, something not readily achievable with optical probes such as fibers, optical lever, or interferometers.

Here we report the use of a near-field microwave resonator interferometer for excitation, detection, and passive cooling of a micromechanical resonator. Our near-field system consists of a coaxial resonator with the open end of the center conductor sharpened into a protruding tip. This leads to greatly enhanced electric field in the tip region, which rapidly reduces with increasing distance from it. If the tip is brought close to a conducting or dielectric surface of a micro/nanomechanical resonator any relative movement will change their mutual capacitance C_0 . This in turn shifts the resonant frequency ω_0 and quality factor Q_0 of the microwave resonator. Small shifts in resonator frequency can be very accurately measured so this technique proves to be very sensitive and is the basis of our microwave readout method. Applying a microwave current to the resonator the resulting alternating electric field between tip and surface produces a time averaged force between them, which is proportional to rate of change of C_0 with separation. By amplitude modulating the microwave current at a frequency f_{mod} , which is comparable with mechanical resonator frequency f_m , an alternating force may be applied. Here we concentrate on readout and excitation methods based on a homodyne microwave interferometer technique – see Fig. 1.

Cantilevers were fabricated from Si or Si₃N₄ atomic force microscope tips, precoated with a Au conducting layer ~70 nm to aid optical reflection and enhance perturbation of near-field microwaves. Focused ion beam (FIB) milling was used to reduce the cantilevers to the required dimensions – a scanning electron microscope (SEM) image of one cantilever is shown in Fig. 2. The cantilever resonators described in this work have linear dimensions, which are greater than the wavelength of visible light so that it is possible to integrate a dual visible-wavelength laser system with the microwave method to interrogate or excite cantilever resonator motion. A low power cw laser is reflected from the cantilever onto a quadrant photodiode, the output voltage being proportional to the cantilever displacement. In addition a variable optical force could be applied to the cantilever using a second diode laser, normally incident on the cantilever, on which it exerts a force through a combination of photothermal and radiation pressure. Amplitude modulation of this laser applies an alternating force. The modulation frequency was swept while

a lock-in amplifier, referenced to the modulation frequency, measured the photodiode difference voltage and phase relative to the amplitude modulation signal. The eigenfrequencies of the cantilever could thus be observed as peaks in the lock-in amplifier response.

We have shown that the near-field microwave method provides a very sensitive technique both for excitation and readout of the cantilever motion, complementary to the better known optical techniques. All combinations of optical and microwave excitation and readout have been demonstrated. The signal to noise achieved is exceptionally good – see Fig. 2. This means that for a resonator with a Q_m value of around 3000 the reproducibility of the center frequency of the resonance is better than 1 ppm or ~ 0.25 Hz for the Si resonator which has a lowest resonant frequency at around 0.25 MHz. The system is also capable of observing the thermal fluctuation displacement noise of the cantilever at room temperature. The thermally generated displacement noise is amplified in the vicinity of the resonant frequency so that the noise amplitude versus frequency can also be fitted by a Lorentzian line shape. The signal to noise obtained with our system allows displacement sensitivity as small as $70 \text{ fm}/\sqrt{\text{Hz}}$ to be achieved. The force between the open-ended microwave resonator and cantilever varies with separation, being attractive at large separation. In addition to a deflection this force also shifts the cantilever resonant frequency by an amount dependent on the electric field strength and therefore on the microwave power fed to the resonator. We have attempted to model the dependence of this force on the separation x between microwave tip and cantilever. The simplest assumption is that the near-field interaction between resonator and cantilever can be treated as an additional parallel capacitance C_x , which is added to the microwave resonator's lumped circuit representation and which varies with x . The variation of C with x should directly predict the functional form of the microwave resonant frequency shift with x . To predict the force exerted by the microwave field note that the amplitude of the cantilever vibration – as measured by the quadrant photodiode output – is proportional to deflection, which is in turn proportional to force applied, rather than to capacitance. The stored energy in the near-field capacitor is $1/2 C V_{\text{rf}}^2$, where V_{rf} is the mean rms microwave voltage on the capacitor. We can also estimate the total stored energy in the whole coaxial resonator as $1/2 C_0 V^2 L$, where C_0 is the capacitance per unit length of the coaxial line and L is the resonator wavelength.

$$F = -\frac{d}{dx}U = -\frac{V^2}{2} \frac{dC}{dx} = \frac{U}{C_0 L} \frac{dC}{dx} = \frac{\beta Q P}{\omega_0 C_0 L} \frac{dC}{dx}. \quad (1)$$

The tip of the sharpened coaxial center conductor is not perfectly sharp but approximates to a hemispherical end with radius R and this tip region dominates the near-field capacitance. The capacitance between a hemisphere and an infinite conducting plane may be calculated by the iterative image charge method.⁶ Thus the net force can be approximated by the electrostatic force F ,

$$F \propto 2\pi\epsilon_0 V_{\text{rf}}^2 \sum_{n=1}^{\infty} \frac{[\coth(u) - n \coth(nu)]}{\sinh(nu)}, \quad (2)$$

where $\cosh u = 1 + x/R$. Here ϵ_0 is the permittivity of free space, V_{rf} is the voltage applied to the tip, and x is the separation between plane and sphere. We have fitted our experimental results using Eq. [2](#) where the first six terms in the summation are used [see Fig. 3](#). It is clear that the expression from Smythe⁶ gives agreement with experiment to within a few percent, the only free parameter being R which is 40 nm , in reasonable agreement with estimates made from optical microscope images.

There is great interest at present in cooling a mechanical resonator into the ground state of one of its vibrational eigenmodes.⁷⁻¹¹ Steady state cryogenic cooling requires a temperature T such that $\hbar f / k_B$, where h is Planck's constant, k_B is Boltzmann's constant, and frequency f is the eigenfrequency. Although submillikelvin temperatures have been achieved in a variety of steady-state systems there are extreme requirements to attain and maintain these ultralow temperatures. Using the excitation and readout systems described here we have demonstrated out-of-equilibrium cooling [generally referred to as passive cooling](#) of the lowest mode of a cantilever. A quasistatic electromagnetic force applied to the mechanical resonator will shift its resonant frequency by altering its effective spring constant and also change the cantilever damping. [Figure 4](#) shows cantilever resonance center frequency and Q_m value [inverse linewidth](#) changes brought about by variations in microwave resonator frequency and power. The unperturbed microwave resonator frequency ω_0 is 11.020 GHz . There is a linear dependence of cantilever frequency on microwave power level at fixed microwave frequency of 11.01 GHz . The results are in good qualitative agreement with theory for the variation of cantilever resonant frequency with both microwave drive frequency and microwave power.^{8,10}

We have measured the thermally generated displacement noise of the cantilever at room temperature, fitting a Lorentzian lineshape to the data. When the open-ended resonator is brought close to the cantilever

with a microwave signal at frequency ω we find that the thermal noise response is modified, depending on the detuning frequency of the applied microwave signal relative to the resonant frequency. For negative detuning the noise linewidth is reduced but the noise amplitude is increased. The opposite conditions apply for $\omega > \omega_m$. An effective temperature for the cantilever mode is calculated by integrating the noise amplitude over the linewidth. This produces effective heating of the mode for negative detuning and mode cooling for positive detuning, as shown in Fig. 5. Cooling by a factor of 3, as shown in the Fig. 5_b, is already significant – the microwave resonator Q is 80 for the mode selected – but considerably greater cooling could be achieved by increasing the microwave resonator Q , for example. Our present setup is firmly in the bad cavity regime¹⁰ where the cantilever resonance frequency is much less than the linewidth of the microwave resonance. In this case the minimum temperature is given by

$$T_{\text{eff}} \approx \frac{\left(\frac{h\Gamma_m\gamma_m}{8\pi k_B} \right) + \gamma_m T}{(\Gamma_m + \gamma_m)}, \quad (3)$$

where $\gamma_m = \Gamma_m/Q_m$ is the additional cantilever damping rate due to the microwave radiation and Γ_m is the damping rate in the absence of any applied microwave power. Since for the bad cavity case the first term in the numerator is much less than the second, Eq. 3 becomes $T_{\text{eff}} \approx T / (1 + \gamma_m/\Gamma_m)$, which we use to calculate cantilever T_{eff} which is shown in Fig. 5_b.

In conclusion we have demonstrated that coupling between the near-field region of an open-ended microwave resonator and a conducting mechanical cantilever provides an effective means for both excitation and detection of the motion of the cantilever at the equilibrium thermal limit. Quantitative fitting of the force versus separation distance behavior has been achieved and passive mode cooling using detuned microwave excitation from 300 to 100 K has also been demonstrated. Greater cooling is to be expected in future, by optimizing microwave resonator Q value, microwave power level, and improved near-field electromagnetic enhancement. Higher cantilever resonant frequencies will also assist in this aim and we intend to extend to the frequency to as high as a few gigahertz, using both smaller FIB-milled nanocantilevers and carbon nanotubes. Pulsed operation of the passive cooling technique may have important applications in the near-simultaneous measurement of conjugate physical quantities such as position and momentum.

FIG. 1. _a_ Schematic circuit diagram of room temperature microwave near-field system including dual microwave and optical excitation and readout techniques. _b_ Photo of overall system inside vacuum chamber mounted on antivibration table.

FIG. 2. Amplitude vs frequency plot. Full line is a leastsquares Lorentzian fit to the amplitude vs frequency data. Inset is a Scanning Electron Microscope _SEM_ image of one Si cantilever resonator.

FIG. 3. Experimental confirmation of distance dependence of microwave near-field force on cantilever. Open circles are experimental data and solid line is the fit to Eq. 2—see Ref. 6—, 113501-2 Hao, Gallop, and Cox Appl. Phys. Lett. 95, 113501 (2009).

FIG. 4. Cantilever resonance frequency as a function of microwave frequency at a range of power levels.

FIG. 5. Color online Experimental results on cantilever passive cooling by microwaves. a Plot of the cantilever frequency shift from 10.3055 kHz filled squares and damping rate open circles for the cantilever resonance as a function of microwave drive frequency at fixed +20 dBm drive power. b Effective temperature vs microwave frequency. 113501-3 Hao, Gallop, and Cox Appl. Phys. Lett. 95, 113501 (2009)

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