THE IMPINGEMENT OF A
TWO-DIMENSIONAL SUPERSONIC JET UPON
A NORMAL GROUND SURFACE

BY

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SUMMARY

The impingement of a quasi-two dimensional supersonic jet upon a flat plate placed normal to the axis of the jet has been studied.

In order to perform the experimental investigation, an apparatus which operates in the manner of an intermittent wind tunnel was designed and built. During the testing of the characteristics of the free jets at Mach Number 1.4 the results of previous investigations in the plane jet mixing region were confirmed and some results obtained for the fully turbulent region that had not been available previously for plane supersonic jets. All of these results were shown to agree with accepted theoretical models.

The flow field of the impinging jet was studied by flow visualisation and pressure plotting techniques over an extensive range of separation distances between the nozzle exit and the flat plate. At small separation distances the stand-off shock wave in the jet is of some note and its shape is shown to be modified by disturbances within the jet flow. A study of the oblique shock wave equation is made which yields an explanation for some of the changes of shape of the wave. Also a 'blunt body' numerical procedure is used to predict the shape of the shock wave and the pressure distribution on the impingement plate at several jet Mach Numbers. Good agreement is demonstrated at higher Mach Numbers.

The experimental results for the pressure distribution on the flow boundaries are presented and discussed in some detail. The impingement plate pressure distributions at various separation distances were correlated on the basis of the characteristics of the free jets.

Some variations in the value of the peak impingement pressures were noted and attributed to disturbances within the jet structure. A special series of tests showed very good correlation on the basis of a movement of the effective origin of the jet. These latter results were shown to be in good agreement with the prediction of a well known potential flow solution to the flow field.

Finally the total loads on the flow boundaries are computed from the pressure distributions. The results show the interesting reversal of thrust upon the impingement plate, at small separation distances, similar to that which occurs in low speed jets.

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LIST OF SYMBOLS

B	Atmospheric Pressure
b	Initial Jet Width (Nozzle exit height)
d_N	Nozzle Diameter (Ref 5)
h	Separation Distance (See Fig 1)
h_o	Stagnation Enthalpy
k	Jet Width at any position
J	Total Force
J'	Isentropic Jet Thrust
M	Mach Numbers
M_j	Jet Mach Number
M_m	Maximum Mach Number at any Cross Section of the Jet
M_∞	Free stream Mach Number
P	Pressure (p Ambient Pressure)
P_1, P_2 Q_1, Q_2 }	Functions of Integral Relations Equation 6(i)5
q	Total Velocity
r_N	Nozzle Radius (Ref 27)
R	Gauge Pressure Reading ($P - p_\infty$)
R_j	_____ " _____ Pitot Reading in Free Jet
R_m	Maximum Value of R
S	Entropy
t	Time
T	Temperature
$u, \bar{u}$	Velocity and non-dimensional velocity in the x direction
U	Free Jet Velocity along the axis
U_j	Free Jet Exit Velocity

U_m	Maximum Value of U at any cross-section
$v, \bar{v}$	Velocity and non-dimensional velocity in the y direction
x, y	Cartesian: coordinates Impinging Jet see Figure 1
$\bar{x}, \bar{y}$	$\frac{x}{b}, \frac{y}{b}$
x_j, y_j	Cartesian coordinates Free Jet see Figure 2
x', y'	Cartesian coordinates Jet Mixing Region
$(y_s)_0$	Centre Line shock height
Δy	Width of Jet Mixing Region
γ	Isentropic Index
δ	Jet Half Width or shock wave turning angle (Appendix I & Fig 30 only)
ϵ	Eddy Viscosity
ζ_1	Shock Angle, (Relative to Upstream Flow)
ζ	Vorticity Appendix I
η	Non dimensional distance in free Jet ($\eta = \frac{y}{\delta}$)
θ	Stream line inclination
μ	Mach Angle
ξ	Non dimensional distance in jet mixing region ($\xi = \sigma \frac{y'}{x'}$)
ρ	Density
σ	Shock Angle (Relative to Cartesian coordinates) or Free jet Constant
τ	Shear stress
Φ	Pseudo viscosity or Damping Factor
ω_s	Inclination of sonic line

PART I

INTRODUCTION

Chapter 1

INTRODUCTION

The fluid dynamics of the flow in and around gas jets have interested investigators for many years. Recent technological advances have sustained and widened this interest to include high jet velocities. A particular problem that has received some attention but still retains considerable interest is that of the impingement of a high speed jet upon a surface. The engineering situations in which this flow phenomenon is present are many and varied. The general field of VTOL aircraft has many examples of high speed jets impinging upon ground surfaces with the consequent problems of lift loss. On a much smaller scale pressurised gas bearings often operate with their gas supply through choked orifices, especially during the stationary 'jacking' process. At these times any instability could lead to extremely dangerous bearing failures. A full knowledge of the flow field of a high speed impinging jet would therefore be useful in many engineering applications. This thesis presents some information and ideas to add to the existing knowledge of the phenomenon. The present work was not directed towards any one particular engineering situation but is an investigation into the basic impingement phenomenon.

A survey of a previous work established that there was no report of studies of jet impingement spanning the entire range from fully turbulent jet impingement to closely confined impingement in correctly expanded high speed jets. The current investigation covers the entire range and thus brings together inviscid considerations at small separation distances and the fully developed jet flow description. The

flow field of a jet of air at a Mach Number of 2.4 impinging upon a plane surface normal to the axis of the jet has been studied. Figure 1 shows the geometry and the main features of the area under consideration.

The flow of an impinging jet shows the jet issuing from a nozzle and widening along the jet axis. This widening may be due to simple mixing but may also be aided by effects at the nozzle lip, particularly in supersonic flow. The subsonic impingement of the jet upon the surface results in the deflection of the streamlines upstream of the surface towards a direction parallel to the surface. In supersonic flow a shock wave is formed within the jet flow with a subsonic deflecting flow taking place downstream of it. Surrounding this main flow a secondary flow is induced by the mixing at its edges. Along the impingement plate this secondary flow joins with the main mass of fluid and a wall jet type flow occurs. The general requirements of the mixing flow induce an inward flow along the nozzle plates, except at very small separation distances.

In the experimental work a rectangular jet was used in order to produce a quasi-two dimensional flow field. Two jets were used with aspect ratios of 4 and 8. This approach was adopted for various reasons. There was a lack of information on the fully turbulent characteristics of supersonic plane jets and the impingement phenomenon had not been studied previously under two dimensional conditions. The investigation was a basic flow investigation and the plane flow impingement situation is sufficiently interesting to warrant investigation in its own right.

The results of the investigation are presented in Parts III and

IV of the thesis. The experimental investigation into the properties of the free jets is reported in Part III. The characteristics of the free jets were established as a preliminary step to the investigation of the impinging jet field. In the jet mixing region the velocity distribution and the rate of spread of the region were shown to confirm the predictions of other authors and results obtained by interferometric methods. There were no comparable results for the mixed subsonic/supersonic flow in the fully turbulent region of an initially supersonic plane jet. The results obtained were in agreement with the form predicted for low speed plane jets. The velocity distribution showed close similarity with previous results for low speed jets. The rates of spread and decay were both slightly smaller than published results at lower jet velocities.

The impinging jet flow studies are reported in Part IV. The main variable in the investigation was the separation distance ($\frac{h}{b}$) between the nozzle exit and the impingement plane. A large range was covered from $\frac{h}{b} = 18$ at which the flow was fully turbulent before impingement to $\frac{h}{b} = 0.45$ at which distance the nozzle flow was no longer supersonic. In addition, several plate configurations were used in the experiments to illustrate variations in the induced flow and also aspect ratio effects.

The work is presented in four Chapters. Chapter 5 gives descriptions of the flowfield, Chapter 6 reports the results of a blunt body numerical procedure. Chapter 7 presents the results of the pressure measurements, and finally Chapter 8 gives a brief outline of a time dependant numerical solution obtained by another author.

The flow field descriptions given in Chapter 5 commence with a separation of the several modes of flow present at the various separation distances tested. This separation is essential to a physical understanding

of the flow phenomena. The structure of the immediate impingement region has been given attention. In particular the shape of the shock wave has been studied. It was found that at some separation distances, the shape varied from the expected monotonic curved shape due to disturbances in the jet. An examination of the oblique shock wave equations yielded a partial explanation for this behaviour which was due to the delicate balance between shock wave angle, upstream flow direction and downstream pressure condition at Mach Numbers around 1.4.

The shock layer numerical procedure reported in Chapter 6 has not been applied previously to the two-dimensional impingement process. Predictions of the shape of the shock wave and the pressure distributions upon the impingement plate were obtained. At the nominal test Mach Number of 1.4 the predictions were rather inaccurate but some additional experimental tests at higher Mach Numbers showed that the predictions were quite good at values of 2 and above.

The main experimental measurements were pressure measurements and the results of this work are reported in Chapter 7. The earlier parts of the chapter deal with the pressure distributions in the impingement region. The results at various separation distances were correlated successfully on the basis of free jet characteristics. The peak impingement pressure however seemed rather sensitive to the jet exit conditions. The later parts of the chapter present the results for the total loads upon the plates. At small separation distances the force on the impingement plate was shown to be directed towards the nozzle plate, a load reversal similar to that experienced with low speed jets. The separation distance at which this load reversal occurred was shown to be dependant upon the geometry of the plates.

Chapter 8 is included to show a brief comparison between the present experimental results and a time-dependant numerical technique for predicting the flow field. This technique has been developed recently by another author. The agreement between the results is good.

The thesis is concluded in Part V with some remarks upon the results of the investigation and some suggestions for future work.

Chapter 2

JETS AND JET IMPINGEMENT

The fluid dynamics of jets have received a great deal of attention from analytical and experimental investigators. Some of this work is applicable to the phenomenon studied here. This survey of the pertinent work is presented under four headings and first the applicability of these headings will be demonstrated.

An understanding of free jet behaviour is an essential prerequisite to a study of impinging jets. The development of a jet before it impinges upon a surface must follow trends similar to those in free jets. Therefore the first two sections are concerned with the general development of both subsonic and supersonic jets. The impingement studies discussed in sections 2 (iii) and 2 (iv) have direct parallels with the present work and these have fallen generally into the two categories chosen. These two categories arise either from the jet velocity or from the distance of the impingement plane from the nozzle.

Although of course any separation into categories can be questioned and alternative groupings suggested it is felt that the headings adopted here are in accord with the work presented later.

2 (i) Jet Mixing

The phenomena of jet exhausting into still or moving air fall generally into the field of free turbulent flows. Many investigators have worked in this field and as a result there is a large fund of literature on the subject. Most advanced text books on viscous flow devote at least one chapter to the topic and reference to any one of these (eg. Schlichting 56 or Hinze 32) reveals the scope of the field and most contain a large number of additional references. In this

survey, only work of particular application to jet mixing will be reported.

The flow of a jet can be described by reference to the pattern shown in Figure 2. A potential core commencing at the jet exit extends downstream bounded by a mixing region between the jet and the surrounding fluid. At some distance downstream the mixing region spreads across the jet, the potential core disappears, and a fully turbulent jet structure is formed. At this stage the time mean velocity profiles within the jet appear similar. Further downstream the jet structure develops a 'self-preserving' structure, that is, the flow in the jet is similar in all respects at all streamwise locations, the similarity is not restricted to mean velocity profiles but includes all aspects of the turbulence structure. This condition of 'self preservation' enabled early investigators to show that for a plane jet the width of the jet (δ) was proportional to the distance (x_j), from the origin of the jet. It was also shown that the velocity on the centre-line of the jet (U_m) was proportional to the reciprocal of the square root of the distance

$$\delta \propto x_j \quad U_m \propto x_j^{-\frac{1}{2}} \quad \dots \quad 2(i)1$$

The classical work considers two sections of the jet, (a) the mixing region ((a) in Figure 2) and (b) the fully developed region ((b) in Figure 2).

One of the earliest approaches to the problem was by Tollmien (65) using Prandtl's Mixing Length hypothesis. He studied both of the above regions for a two-dimensional jet and the also fully turbulent region of an axially symmetric jet. A later analysis by Goerther (24) used an eddy viscosity approach in which

$$\gamma = \epsilon \frac{\partial u}{\partial y}$$

where

$$\epsilon = \alpha_1 k (u_{\max} - u_{\min})$$

Considering the mixing region Goertler obtained a velocity distribution of the form:

$$\frac{u}{U_m} = \frac{1}{2} [1 + \operatorname{erf} \xi] \quad \dots 2(i)2$$

where $\xi = \sigma \frac{y'}{\alpha}$ and

$$\operatorname{erf} t = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-t^2} dt$$

In the fully turbulent region of the plane jet the velocity distribution obtained was

$$\frac{u}{U_m} = \operatorname{sech}^2 \gamma \quad \gamma = \frac{y}{\delta} \quad \dots 2(i)3$$

Abramovich (2) using a numerical approach to the boundary

layer equations, studied the effects of compressibility upon the mixing region of a plane free jet at initial Mach Numbers of 0.5, 0.9 and 1.0. The general conclusions from the work were that for the range of Mach Numbers investigated the effects of compressibility upon the non-dimensional time mean velocity profiles were very small. It was claimed that the only noticeable effect was a small decrease in the rate of spread of the jet. Kovasznay (36) and Morkovin (42 & 43) in their pioneering work on the use of hot-wire anemometers for the analysis of turbulence in supersonic flow uphold the first conclusions. This work showed that the general behaviour of the turbulence is unaffected

by compressibility if the pressure fluctuations are small in comparison with the total pressure of the flow. This infers that the fluctuating Mach Number is small. It was reported that in boundary layers and mixing regions where the free stream Mach Number was less than 5, the fluctuating value was usually less than 0.2.

Whilst several authors have investigated the turbulence structure of jets, the present work concerned the rate of spread of the jet and the time mean velocity profiles within it. Bradbury (10 & 11) presented some simple expressions for the latter properties. The expressions were of the general form as predicted by theoretical approaches but with the various constants adjusted to give good agreement with several sets of experimental results. The results quoted for the fully turbulent plane incompressible jet issuing into still air were as follows:

$$\frac{U}{U_m} = f(\eta) = \exp[-0.6749 \eta^2 (1 + 0.0269 \eta^4)] \quad 2(1)4$$

$$\frac{U}{U_j} = 2.5 \left[\frac{x_j - x_0}{b} \right]^{-\frac{1}{2}} \quad 2(1)5$$

$$\frac{\delta}{b} = 0.109 \left[\frac{x_j - x_0}{b} \right] \quad 2(1)6$$

(Where x_j is distance along the jet axis from the origin x_0 and b is the initial width of the jet.)

The turbulent mixing of a correctly expanded supersonic jet follows the general pattern of the mixing of low speed jets. The earliest investigations into supersonic jet mixing appears to be by Gooderum, Wood and Brevoort (26) who studied a plane jet of air at Mach Number 1.6 with a Mach Zehnder Interferometer. The nozzle was designed to produce a parallel jet and throughout the tests it was operated at its design pressure ratio. The jet had an aspect ratio of 1 and the investigation covered the region from the nozzle exit to a position of $2\frac{1}{2}$

exit heights downstream (the initial mixing region only). The experimental results for the velocity distribution in the mixing region were compared with the theoretical profile due to Tollmein. The numerical solution of Tollmein was of the form:

$$\frac{u}{U_m} = f \left(\frac{y}{x} \frac{1}{\sqrt[3]{2c^2}} \right)$$

where 'c' was a constant of proportionality between the mixing length 'l' and the distance x along the jet.

Gooderum et al modified the resulting profile and wrote their velocity distribution in the form:

$$\frac{u}{U_m} = f \left(\sigma \frac{y}{x} \right)$$

With these modifications good agreement was demonstrated between the experimental results and the chosen profile when a value of $\sigma = 15$ was used for the scale factor. For plane low speed jets Schlichting (56) and Hinze (32) both reported close similarity between the various theoretical profiles and a consensus of $11 \leq \sigma \leq 12$ for good agreement with experimental data. This result was in good agreement with the conclusions of Abramovich. Bershader and Pai (7) also using an inter-ferometer obtained similar results for a correctly expanded plane jet, of aspect ratio 2 at a Mach Number of 1.7. The latter workers found that a value of $\sigma = 17$ gave a suitable scale to the results.

Subsequent investigators have concentrated on axially symmetric jets. Warren (67) and Pitkin & Glassman (48) conducted experiments into the performance of two axially symmetric nozzles at Princeton University.

Warren presented measurements of static pressure and total pressure in a jet of Mach Number 0.97 within the first 30 diameters downstream from the nozzle exit. The static pressures as measured indicated peak values approximately 4% below the ambient pressure, however, this was only 2% of the local total pressure. Although the distributions of velocity were presented no comparison was made with any theoretical work and there was insufficient detail to allow other investigators to do so. A single series of measurements in a Mach Number 2.6 jet 2.55 in. exit diameter was presented, the position chosen was 15 diameters downstream of the exit. In this latter jet the measured static pressure had a peak value 10% below the ambient, but only 1% of the local total pressure. Pitkin and Glassman presented further results for this latter jet. Pitot pressure and static pressure profiles were measured and showed good agreement with the preliminary results of Warren. The rate of spread of the jet followed the expected pattern for an axially symmetric jet $\delta \propto x_j$, but the measurements indicated a constant velocity core existing for approximately 12 diameters downstream.

A most comprehensive investigation of a correctly expanded supersonic jet was reported by Johannesen (34 and 35). A study was made of two jets of Mach Number 1.4 originating from nozzles of 0.765 ins. exit diameter and 0.739 ins. exit diameter respectively. Shadowgraph and schlieren photographs were presented showing the nozzle exit regions for both jets. Jet 1 (0.765") was seen to have a severe shock pattern but Jet 6 (0.739") produced an extremely uniform and shock-free jet. Pitot and static pressure traverses were performed within both jets. The traverses were extended further downstream in Jet 6 (to $\frac{x_j}{r_n} = 189$) than in Jet 1 (to $\frac{x_j}{r_n} = 27$). Results were presented for both jets and for

both the mixing region and the fully developed region. Within the mixing region the rate of spread of the jets was studied, several definitions of the width of the jet being used. In each jet the rate of spread was found to be directly proportional to ' x_j ' for all definitions of jet width but for Jet 1 there were two sections each with a different coefficient. The velocity profiles were compared with the error function as suggested by Goertler. Very good agreement was demonstrated in each case when an appropriate value for the scale factor (σ) was chosen, Jet 1 gave very good agreement with $\sigma = 21.9$ for the region close to the exit and $\sigma = 11.9$ further away, for Jet 6 a single average value of $\sigma = 13.7$ seemed to be appropriate. The mixing region in both jets extended downstream to the position $\frac{x_j}{T_N} = 8.1$ at which location the fully developed region was deemed to commence.

The results for the fully developed region of Jet 6 were found to exhibit similarity and showed good agreement with the expression:

$$\frac{u}{U_m} = \exp \left[-0.6932 \left(\frac{r}{r_{.5}} \right)^2 \right]$$

in which the numerical constant was chosen to make the curve pass through the point $\frac{r}{r_{.5}} = 1$, $\frac{u}{U_m} = 0.5$. This jet also demonstrated the rate of spread proportional to ' x_j ' in the fully developed flow region, and the centre-line velocity decaying as $\frac{1}{x_j}$.

In these two reports a great deal of emphasis was placed upon the measurement of static pressures within the jets. The work of Goldstein (25) and of Fage (21) was quoted. In those two articles estimations were made for the pressure differences due to the fluctuating components of velocity in low speed turbulent flows. Using the expressions suggested by the above authors, Johannesen estimated that errors of the

order of 10% were likely in measurements of static pressure by conventional means.

2(ii) Structure of Supersonic Jets

A correctly expanded supersonic parallel jet exhibits a generally similar structure to that of the low speed jet shown in figure 2, with the 'potential core' becoming a region of mixed supersonic and subsonic flow. However, small differences of pressure between the emergent jet and its surroundings lead to significant changes in the flow structures. At this stage the mixing region becomes of less concern and investigators concentrate upon the transonic sections of the jet. With large pressure ratios little similarity with the low speed jet structure remains.

At pressure ratios only slightly above that for correct expansion a repetitive pattern of disturbances appears. Figure 27 shows the general form of the likely jet structure. Early investigators were interested in the wavelength of this pattern. Emden (19) was one of the earliest investigators followed by Prandtl (49). These two authors both noted the periodic structure from experiments and Prandtl attempted a theoretical analysis which yielded an expression for the wavelength. Hartmann and Lazarus (29) also performed experiments with jets issuing from circular orifices under various conditions. Their paper contains an informative plate showing the small disturbances and then shock waves occurring as the pressure ratio was increased.

Pack in two later papers, (46 & 47) used two analytical approaches to the problem. In reference 47 the linearised equations of motion were used to predict the wave-length of the oscillations for both

plane and axisymmetric jets. Packs approach whilst similar to that of Prandtl gave much better agreement with experimental results for sonic jets. Reference 46, the earlier of these two papers by Pack, together with a report by Owen and Thornhill (45) marked a significant change in approach to the problem. These two articles were the first of many to use the 'Method of Characteristics' to predict the flowfield of a supersonic jet. The Method of Characteristics has now become well established and standard text books give an ample exposition of the theory (see Sear 57) or Shapiro 58). Pack considered a plane jet of Mach Number 1.5 at two values of pressure ratio. The jet exit pressure in both cases was chosen to be higher than the local ambient pressure thus an 'under expanded' pattern was produced. A numerical method was used for the solution. The results showed how the coalescence of characteristics of the same family created shock waves within the jet. At the increased pressure ratio the shocks were shown to change from weak compression waves to intersecting shocks. Owen and Thornhill used the numerical method of characteristics to calculate the flowfield of a steady axisymmetric jet of air issuing into a vacuum. The Mach Angle of the flow at the orifice was chosen as 85° ($M=1.004$) the largest value consistent with avoiding the problems associated with exactly sonic conditions. The results were presented in a readily usable manner. From about 1950 onwards Love et al (40) were conducting a comprehensive theoretical and experimental study of the flow in axisymmetric free jets ($M>1$). A large number of nozzles were constructed and schlieren photographs were taken of the flows produced under various pressure conditions. The controlling parameters for the investigations were Jet Exit Mach Number and Jet Exit Flow Angle for an extensive range of pressure ratios.

The analytical work comprised calculations by the method of characteristics for each test condition. Good agreement was obtained between theory and experimental results for the plume shape of the jets and the position and shape of the intersecting shock. The results of this work have been used as basic data by many investigators developing approximate methods for predicting jet shapes and structures.

The three references above constitute the main work using the method of characteristics and few substantial advances have been presented subsequently. However the work of Eastman and Radtke (16 & 17) on an axisymmetric jet and the work of Sheeran and Dosanjh (59 & 60) on a two-dimensional sonic jet compliment the results. In all of the above articles the plume shape and the intersecting shock shapes and positions had been consistent with the predicted conditions. However it had not been possible in any of the cases to predict the position of the Rieman wave (or Mach Disc) because this originated at a 'triple point' on the intersecting shock wave with a slip surface downstream. In turn this meant that the solutions predicted that the intercepting shock would continue to the axis of the jet at which stage the solution would be terminated or if the shock were very weak, regular reflection might take place. Abbett (1) suggested a flow model which enabled the axial position of the triple point to be estimated and then used as an initial condition for the solution downstream. A test case was calculated and the results compared favourably with data quoted by Love (40).

Other authors had attempted to estimate the position of the Mach Disc. One of the more successful attempts was reported by Adamson and Nicholls (3). Using the data provided by Owen and Thornhill they postulated an artificial nozzle with isentropic flow extending along the jet, and normal shock

recovery to ambient pressure at the exit from this nozzle. The length of the nozzle extension corresponded to the distance to the Mach Disc. This approximation was in good agreement with the experimental results of other authors.

A report of an experimental investigation of the properties of the Mach Disc was published by Crist, Sherman & Glass (14). A study was made of the flow in a highly under expanded sonic jets. Several gases were used which had different values of the specific heat ratio, and the pressure ratios of the flow took various values between 10 and 10^5 , with appropriate stagnation temperatures. It was reported that the location of the Mach Disc was primarily dependant upon the pressure ratio and was insensitive to condensation, the specific heat ratio and the absolute pressure level. The diameter of the disc was however shown to increase with the specific heat ratio and the onset of condensation within the flow.

It would appear therefore that it was possible to predict the jet structure upstream of a Mach Disc by means of the Method of Characteristics. The location of the Mach Disc and its size were not so readily predicted and alternative methods were still required.

2(iii) Low Speed Jet Impingement

As early as the beginning of the nineteenth century experiments were performed to study the phenomenon of jet impingement. Willis (63) in reporting his own work refers to earlier work in France. The careful work of Willis was directed towards determining the magnitude and direction of the force exerted upon a plate by a small jet of air issuing from an orifice in a plate. This work was an early demonstration that with the impingement

plate close to the orifice the force exerted upon the plate could change from one of repulsion to one of attraction.

In addition to experimental investigations a mathematical interest had been shown in jet impingement as an application within the field of free streamline flow in two-dimensional classical hydrodynamics. A solution to the particular problem had become quite well known by the time that Milne-Thomson included it in his book (41).

The advent of jet aircraft with vertical take-off capabilities presented a demand for greater knowledge of the flow processes. Various experimental investigations were therefore performed, two examples of which were the work of Ludwig and Brady (12) and Murdin (44). Recently Bradbury (11) presented a dimensional argument for correlating the results for different nozzle conditions and different heights above the ground. A similarity argument was used between the conditions in the free jet and impinging jet at appropriate axial locations. Experimental results were reported for various types of nozzle at velocities high enough for some compressibility effects to be apparent. The results were shown to confirm the similarity argument.

Cartwright and Russell (13) investigated the impingement of a slot jet (aspect ratio 24) and presented experimental results. Measurements of static pressure upon the impingement plate were recorded and presented non-dimensionally. The results were collapsed onto a single graph in a manner which was effectively the argument as suggested by Bradbury.

Whilst no complete theory has been found for the impingement of a subsonic jet upon a flat plate, ample argument and experimental confirmation has been published for the comparison of various configurations.

2(iv) High Speed Jet Impingement

High speed jet impingement has been studied as an extension to the search for knowledge in VTOL conditions and in its own right for rocket launching and lunar landing conditions.

The lunar landing programme prompted a great deal of work with sonic and supersonic jets exhausting under simulated lunar conditions, then impinging upon a surface. An extensive survey of the NASA sponsored work was given by Vick Cabbage and Andrews (66). Although in essence this work was similar to the current work the lunar simulation led to the use of extremely high pressure ratios and therefore very highly under-expanded jet planes. The articles of Roberts (52) and Stitt (64) indicated that the flow-field under these conditions was vastly different from that at pressure ratios approaching the correct expansion ratio. Therefore most of the approximations used within their approaches were not appropriate to low pressure ratios. The interesting suggestions by Eastman and Radtke (18) and Yoshihara (69) suffered from the same limitations. The majority of the work derived from the lunar landing programme was therefore not of direct pertinence to the current investigations.

Other approaches to the problem have not been in such concerted manner as the NASA work.

Roberts (51) attempted an analytical solution to the problem as a limiting case of a wedge in a supersonic jet with a 'stand-off' wave. A hodograph transformation was used and a solution for the stream-function in the form of a hypergeometric series was sought within the transonic region. Unfortunately the boundary conditions on the shock-polar were such that it was not possible to obtain stable results in the physical plane.

Henderson (31) also investigated the hodograph plane representation of the flow, in conjunction with an experimental programme. The work was concerned with a supersonic axisymmetric jet impinging upon a circular flat plate which was positioned at various angles to the jet axis. A hodograph description was presented for the development of the flow pattern up to correct expansion conditions and impingement upon a plate normal to the axis of the jet. A further description was included for the pattern as the plate angle was inclined from the normal to the axis towards the tangential condition. The experimental programme involved the investigation of the flow from three nozzles ($M = 1.8, 2.01, \text{ \& } 2.14$) impinging upon a plate at several inclination angles between 90° and 20° . Some shadow-graph photographs were shown of the conditions with plate inclinations of 20° to 30° at a position two diameters downstream from the jet exit. Also some pressure distribution results were included but the relatively small number of pressure tappings used meant that they were an indication only of the likely distribution. The main interest would seem to have been in the movement of the stagnation point with the inclination of the plate.

Donaldson and Snedeker (15) also performed an experimental investigation with a free axisymmetric jet impinging upon a flat plate at various inclinations. These results were given in rather more detail than those of Henderson but confirmed the trends shown. The nozzle used in this case was convergent only and was operated at three different pressure ratios corresponding to a subsonic jet and a sonic jet moderately under-expanded and highly under-expanded. An exploration of the conditions within the free jet had been undertaken prior to the impingement studies. In the cases of the sonic jet a severe shock structure

existed due to the under-expansion even in the free jet.

Gummer and Hunt (27) presented a very interesting approach to the problem following an experimental and theoretical investigation. The theoretical approach was based upon utilising the method of integral relations and polynomial approximation: developed in the work upon supersonic blunt body calculations. The origins of these methods appeared in the work of Belotserkovskiy (6) and were surveyed by Hayes and Probst (30). A similarity was suggested between the impingement of a jet upon a plate and the flow around a flat-faced disc. In the case of the disc the controlling scale was given to the problem by the diameter of the disc, in the jet case the jet diameter became the controlling parameter. A single strip version of the polynomial approximation was used. The results of calculations for the shock shapes and the centre-line shock heights were presented for several jet Mach Numbers. At higher jet Mach Numbers the results compared very favourably with experimental results taken from shadowgraph photographs. At lower Mach Numbers the agreement was not so good, an error exceeding 50% in the centre-line shock height was reported for a jet Mach Number of 1.64.

Sinha, Zakkay and Erdos (62) presented an alternative approach to the calculation of the flowfield of an impinging supersonic jet. In their solution they proposed that the steady-state solution of a mixed subsonic/supersonic flow could be considered as the asymptotic limit of a transient flow. Using a finite difference technique of the Lax-Wendroff type a lengthy calculation for an under-expanded sonic impinging jet was performed. A very detailed picture of the flow-field was obtained and it was compared with an interferogram obtained for the same initial conditions, by Sterrett and Barber (63).. The comparison was most success-

ful. The nature of the solution marked a notable step forward in both the impinging jet and the free jet which was also calculated, since the location and the shape of the Mach disc was obtained within the solution without recourse to semi-empirical criteria.

A work of direct comparison with the current work was performed by Artingstall (5). Using a nozzle designed and used by Johannesen (34 & 35) a study was made of this jet issuing from a circular plate and impinging upon a second one. The nominal jet Mach Number was 1.4. The separation distance between the nozzle exit and the impingement plate was varied between $\frac{h}{d_N} = 0.14$ and $\frac{h}{d_N} = 3.38$. The pressure distribution upon the impingement plate was presented for each separation distance quoted. The resultant force upon the impingement plate was shown to change direction as in the case of the low speed jet. Schlieren photographs were also included for the smaller separation distance. Some experiments at a separation distance $\frac{h}{d_N} = 3.38$ investigating the flow along the plate, showed agreement with the wall-jet phenomena suggested by Glauert (23).

PART II

EXPERIMENTAL APPARATUS

Chapter 3

THE DESIGN AND OPERATION OF THE EXPERIMENTAL APPARATUS

The general arrangement of the experimental apparatus is shown in Figures 3 & 4. In outline it was similar in structure and operation to an intermittent blow-down wind tunnel. Air from the compressor plant was accumulated in the storage vessel over an extended period and then discharged rapidly through the settling chamber and thence the nozzle, the rate of flow being modulated by the control valve. The duration of a test run was determined by the initial storage state and the nozzle flow conditions. In all cases the useful portion of a test run exceeded 35 seconds.

During a test run the experimental observations were recorded for subsequent analysis.

The entire apparatus was installed in a large laboratory (32,000 ft³ volume) which could be made light-proof and was freely vented to the outside of the building. Since the air was exhausted from the apparatus directly into the room it was important that any rise of pressure within the room should be negligible. This condition was found to be satisfied.

3(i) Compressed Air Plant

The compressed air was supplied by a two stage reciprocating compressor which delivered 200 ft³ free air/minute at a pressure of 450 lbf/in². On leaving the compressor the air passed through an after-cooler-separator and then an adsorber. The adsorber was an automatic 'two tower' type operating at 450 lbf/in². The manufacturer's data for the adsorber indicated that saturated air entering at 450 lbf/in² and

and 20 C would be dried to an outlet moisture content of 5 grains/lb at 25C (giving a dew point of approximately - 55C)) The maximum working pressure of the storage vessel was 450 lbf/in² and its volume was 500 ft. The vessel was fitted with a vertical stand-pipe of 4 in. nominal bore terminated in a conventional gate-valve to which the test equipment was connected.

The entire plant was located in a basement immediately below the laboratory thus keeping the pipe length between the storage vessel and the apparatus to a minimum.

3 (ii) Control Valve

The control valve was mounted directly upon the gate-valve on the vessel standpipe. A Hammel-Dahl 'Venturiflo' Control Valve of 4 in nominal bore was used. In this type of valve the inlet and exhaust axes were at right angles so that air entered vertically upwards and was discharged in a horizontal direction. The internal passages of the valve were smooth and flow control was achieved by movement of a profiled plug away from a fixed seating. The actuation for the valve was a manual/pneumatic system. A stop was positioned along the plug shaft by operation of a handwheel and air pressure on a diaphragm maintained the plug in position against the stop. Movement of the plug stop during a run enabled steady flow under predetermined conditions to be maintained in the settling chamber and nozzle.

Fast opening of the valve was achieved by setting the plug stop at a partially open position and suddenly admitting air to the diaphragm chamber by the operation of a solenoid valve. The pressure on the diaphragm rapidly moved the plug shaft in the opening direction to the stop, thus opening the valve. In an emergency the air behind the diaphragm

could be vented rapidly by opening a second solenoid valve, the plug would then be returned to its seat by spring pressure. The air supply to the valve was completely independent of the air storage.

This method of flow control enabled the pressure in the settling chamber to be controlled within 0.1 lbf/in^2 of the nominal value throughout a test run (typical values 32.4 to 32.2 lbf/in^2 gauge).

3(iii) Settling Chamber and Initial Contraction

The settling chamber for the nozzle was constructed from a circular tube 10 inches internal diameter and 54 inches long. The supply from the control valve to the settling chamber was 4 inches diameter and had a simple conical transition section.

The pressure of the air inside the settling chamber was displayed on a large dial Standard Test Gauge for the operator of the control valve and was continuously monitored with a pressure transducer for recording purposes. A thermocouple, connected to an Electronic Thermometer, was also inserted in the settling chamber. This thermocouple given an indication of the total temperature of the flow. The indicated temperature showed a gradual drop of about 25 C. during a test run of 40-50 seconds. No other temperatures were measured.

Since it was advisable that one should be able to change the nozzle configuration without major alterations to the structure it was decided to form an initial contraction within the settling chamber, upstream and independent of the nozzle. This initial contraction was 10 inches diameter at its inlet and 4 inches wide by $1\frac{1}{2}$ inches high at its exit (an area ratio of 13:1). The overall length of this contraction was

12 inches. The exit led to a parallel duct formed between the side walls (or windows) of the rig and two fixed walls.

Various design methods for the profile of the contraction were contemplated but an original design was not pursued because centre-line coordinates for the appropriate dimensions were available in reference 28. The final shape was achieved using longitudinal stringers and sheet aluminium, covered and filled with quick setting plaster. The plaster was then rubbed down and covered with several coats of hard gloss paint which was similarly burnished.

3 (iv) Nozzle

A general arrangement of the apparatus downstream of the initial contraction is shown in Figure 5. The nozzle was formed within the parallel duct from the exit of the contraction. The side walls were parallel and the nozzle profile was formed on the upper and lower walls only.

The convergent-divergent nozzle was constructed from a solid throat of circular arc shape attached to a flexible plate and rigidly mounted at the exit. The height of the throat blocks was adjustable by means of screw jacks and the exit heights could be set with shims. The deflected shape of the flexible plate formed the 'cancellation' portion of the nozzle.

The designs of the nozzle profile was of the type proposed by Foelsch (22). Foelsch's design method assumed a source flow, from a virtual origin, confined within diverging walls, expanding to a supersonic condition. The diverging portion, tangentially joining the arc of the

throat block at one end, was joined by a cancellation section at the other to finally produce a parallel flow along the central axis. The shape of the cancellation portion was determined using the Method of Characteristics commencing from the radial flow and arc of constant Mach Number at the end of the diverging section. Using this approach Rosen (53 & 54) showed that by a judicious choice of the ratio of divergence length to cancellation length, satisfactory wind tunnel nozzles may be designed from flexible plate contours. The geometry of the nozzle used is shown in Figure 6. Rosen suggested using tapered plates but calculations for this case showed that the differences between the deflected shapes of a parallel plate and a tapered plate were minimal. Parallel plates 1/16 inch thick were therefore used for this section of the nozzle, the material chosen was Wardson Precision Ground Flat Stock which had properties enabling the plates to be deflected to the required shape without permanent distortion.

It was necessary in this application to maintain the nozzle exit in a fixed position, therefore the supports for the throat blocks were allowed axial movement during adjustment of the profile.

The design Mach Number for the nozzle was chosen to be 1.4 at this value the isentropic flow curve of ρq^2 versus Mach Number shows a portion with the thrust varying only slowly with Mach Number.

The procedure for fixing the nozzle profile was to set the exit height at the appropriate magnitude and centrally in the duct by means of a long engineers parallel bar, slip gauges and a clinometer. The throat blocks were then symmetrically adjusted to give the required nominal area ratio. Test runs were then performed, initially with the settling chamber pressure and the pressure registered by the central exit pressure tapping as the operation criteria. When the settings were such that the correct

ratio of these pressures and the ambient pressures had been obtained, total pressure traverses were made in the exit plane of the nozzle.

Two nozzle exit heights were used during the tests, 1 inch and $\frac{1}{2}$ inch giving aspect ratios of 4 and 8 respectively.

Comprehensive traverses in the vertical and horizontal directions in the exit plane of the 1 inch jet exhibited only very small variations in total pressure except at the very edges. For the $\frac{1}{2}$ inch jet fewer traverses were performed but these too indicated very small variations only. Schlieren photographs of the jet show weak disturbance at the exit arising from the filled heads of the attachment screws for the nozzle. Generally the jets produced by both nozzles appeared satisfactory.

All tests were conducted at settling chamber pressures that would give correct expansion of the nozzle and jet to ambient pressure, i.e.

$$\frac{P_o}{P_\infty} = 3.182 \text{ for the design Mach Number of 1.4.}$$

3 (v) Impingement Plates and Nozzle Plates

The Impingement and Nozzle Plates were mounted as shown in Figure v5. All of the plates were manufactured from $\frac{1}{4}$ inch thick duralumin.

Two impingement plates were used, one equipped with pressure tappings in the positions as shown in Figure 7b and one plain for use during photography. The nozzle plates were rigidly mounted at the nozzle exit and fitted with pressure tappings as shown in Figure 7b. All of the pressure points on the impingement plate and most of those on the nozzle were formed by drilling holes through the plate such that 1.5 mm hyperdermic tubing was an interference fit in the hole. The tube was then sealed in position with Araldite on the back of the plate and the slightly protruding hyperdermic tubing was carefully 'stoned' off on the face to

give a flush square-edged hole without burrs. The remaining holes on the nozzle plate were formed by burying plastic tubing in slots in the plate and covering the tube entirely in Araldite. The surplus filler was then removed and the surface dressed flat. A single hole was then drilled perpendicular to the surface through the Araldite into the tube. No problems were experienced with leaks whilst using these arrangements.

Two plate heights were used, 24 inches and 12.25 inches. The separation distance between the nozzle exit and the impingement plate was variable to about 11 inches. The setting of the separation distance was measured with slip gauges for distances less than 1.5 inches and with an internal micrometer for greater ones. The duralumin plates were flat to ordinary engineering tolerances, no additional measures were taken to flatten them. The separation distances were therefore set to an accuracy of .002 inches, since no discernible deflections took place during a test run it was felt that this accuracy was generally sustained.

3(vi) Pressure Measurements

The pressure measuring system was a combination of electronic transducers, standard test gauges, and a multi-tube manometer.

The extreme pressures experienced were monitored with encapsulated capacitance type pressures transducers operating from a 10 volt d.c. supply and producing a d.c. output linearly proportional to the applied pressure. Within the unit there was an oscillator-demodulator arrangement monitoring capacitance changes. The output signal from the transducer was recorded in an Ultra-Violet Galvanometer Recorder. Six channels were available on the recorder and the galvanometers used had natural frequencies of either 100 Hz (B100) or 150 Hz (B150).

Throughout a test run a continuous record was made of the settling chamber pressure with one transducer whilst two others were used to record pressure obtained by scanning a series of the static pressure holes. Two means of scanning were used, for the centre eleven holes on the impingement plate an automatic scanning switch was designed and built, a further ten holes could be scanned by manually switching a pair of Drallim valves. The automatic switch, shown in Figure 8, was driven by a 65 watt electric motor rotating at 46 rev/min. The motor drove a Geneva mechanism and through a 3.1 gear ratio the scanning disc. The disc connected in turn each of twelve ports to a central port to which the transducer was connected. One complete revolution of the scanning disc occurred every $15\frac{1}{2}$ seconds, during this time each port was sensed for 0.97 seconds, and motion between one port and its adjacent one took 0.33 seconds. The rate of scanning and the response time of the transducer-hole combination was adequate for a steady state reading to be obtained. The seal on the scanning disc face was achieved with 'O' rings and no leakage was detected over the required operating range (+ 80 lbf/in to - 13. lbf/in). Tests were conducted on each port individually and with all ports in operation, in all cases the result were satisfactory. In addition tests were made to ensure that the pressures indicated were not falsified by either the length of the connecting tubes or by the process of scanning. A continuous reading using the shortest possible connection was compared with a scanning reading of two adjacent pressure holes previously found to indicate the same reading. Two tests were performed exchanging the connections after the first test, in neither case was any error discernible.

Scanning by means of the Drallim valves was performed manually and each port was connected in turn to the transducer for at least 2 seconds.

The remaining pressure tappings were continuously connected to a 35-tube multi-tube manometer, using mercury, which was equipped with a clamping mechanism for 'freezing' the displacement of the columns.

The galvanometer deflections induced by the pressure transducer signals were recorded on photo-sensitive paper in the recorder, together with these, further signals indicating the position of the scanning switch and the Drallin valves were recorded.

Prior to each test run all transducers were calibrated, by the static application of pressure, against a mercury U-tube manometer to atmosphere, or a Standard Test Gauge. Every record of a test run therefore displayed at its commencement a calibration of the transducers used.

With the range of pressure measuring devices used and their correct application to appropriate pressures it was estimated that the overall accuracy of the measurements was better than $\pm 2\%$.

3(vii) Optical System

Shadowgraph and Schlieren photographs were taken using the same basic system.

The apparatus was of the conventional single pass type based upon two 4 inch diameter concave mirrors of focal length 48 inches. The source unit was a 50 watt car headlamp bulb powered by a heavy-duty 12 volt battery. The light from the bulb was focused with a condenser lens and an adjustable slit unit placed at this focus. The effective source size was therefore adjustable and was sharply defined by the slit. The position of the source unit was arranged so that the effective source was at the focus of the first concave mirror. The light leaving this mirror was therefore formed into a parallel beam which passed perpendicularly through

the windows of the rig and the working section then falling on upon the second mirror. The image formed was recorded on 35 mm. Ilford H.P. 4 film held in a single lens reflex camera body with a focal plane shutter and using a separate simple focusing lens of 175 mm focal length. In most cases the exposure time used was nominally .002 seconds.

Schlieren photographs were obtained by placing a cut-off at the focus of the second mirror, thus blanking off part of the image of the source that was initially formed there. Initially a razor blade was used for the cut-off but latterly a graded-filter was used, this gave improved focusing of the final image. For the Schlieren work the source and cut-off units were arranged vertically or horizontally (parallel to or perpendicular to the impingement plate), and the images recorded at both of these settings.

The entire system was set in a horizontal plane which could be positioned at levels between one corresponding to the horizontal centre-line of the nozzle and one 12 inches higher.

The two windows forming the side-walls of the apparatus were 24 x 6 ins and could be set with the long axis vertical or horizontal, thus giving full coverage of the flow paths.

Some difficulty was experienced with light reflected from the impingement and nozzle plates, these were therefore painted black but in spite of this and very careful adjustment and setting of the optical units the reflections could not be eliminated completely.

Generally the optical work was conducted with no other lighting in the laboratory except for a single light illuminating the test gauge indicating settling chamber pressure to the operator of the control valve.

3(viii) Traversing Mechanisms

Total pressure traverses were made during the nozzle setting process and then in the far field to determine the properties of the jet.

A simple hand operated wormwheel traversing gear was mounted on appropriate parts of the main structure and the probe attached to it. The duration of a test run was insufficient for the probe to be set in predetermined positions and reading taken at each of these. A continuous traverse was therefore undertaken with the position of the probe simultaneously recorded. The record of the position was obtained from a resistance chain with a linear stroke potentiometer connected to the traversing arm. The stroke of the potentiometer was 5 inches thus allowing a complete traverse across the width of the jet between the windows, and the full height of the jet even at the extreme distances in the jet. The potentiometer signal and the signal from the pressure transducer monitoring the probe pressure were simultaneously recorded by the U.V. Galvanometer Recorder. The rate of traversing was determined by the operator but was generally arranged so that by steady rotation of the handle a traverse 'across and back' to the starting position could be made during a single test run of about 40 seconds duration. (A traverse rate of about 1 inch in 5 seconds.)

Two probes were used during the measurements, a square ended total pressure tube .065 inches outside diameter and an ellipsoidal nose pitot-static tube .096 inches outside diameter.

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PART III
THE FREE JET

Chapter 4

THE FREE JET RESULTS

It was decided at any early stage that a good knowledge of the properties of a free jet was required in order to study carefully the impinging jet phenomena. At the high velocities used for the jet it was felt that in spite of its deficiencies the best method to obtain the required information for the particular jet under investigation was by pressure traverses. In order to obtain some indications of the effects of jet aspect ratio two settings of the nozzle exit height were used. The initial nozzle setting was for an exit height of 1 inch and hence aspect ratio 4. The second setting for the same Mach Number (1.4) was an exit height of 0.5 inch (aspect ratio 8). The characteristics were required for both jets operating under the correctly expanded conditions. The jets were therefore operated at their design pressure ratios of 3.18 with exit at ambient pressure, and pressure traverses were performed.

4(i) Reduction of Pressure Results

Knowledge of the pitot pressure and the static pressure enable one, by use of the gas equations for adiabatic flow, to calculate the local Mach Number of a flow. The relationships so obtained are:

$$\frac{p}{p_0} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{-\frac{\gamma}{\gamma-1}} \dots \dots 4(i)1$$

(where p is the static pressure and p₀ is the total pressure)

for subsonic flow when $\frac{p}{p_0} > \left(\frac{\gamma+1}{2} \right)^{-\frac{\gamma}{\gamma-1}}$ and

$$\frac{p}{(p_2)_0} = \left[\frac{2\gamma M^2 - (\gamma-1)}{\gamma+1} \right]^{\frac{1}{\gamma-1}} \left[\frac{2}{(\gamma+1) M^2} \right]^{\frac{\gamma}{\gamma-1}} \dots \dots 4(i)2$$

(where (p₂)₀ is the pitot pressure) for supersonic flow.

Tabulated values of these functions may be found in reference 55.

For the purposes of this work it was felt adequate to assume that the static pressure within the jet was constant across the jet and equal to the local ambient value. This assumption is open to question but the means available at present for measuring the static pressure in a high speed jet, are not sufficiently accurate to give much greater total accuracy to the results.

4(ii) Pressure Probes

Two probes were used for the traverses as described in Section 3(viii). The smaller of the two was found to be liable to excessive vibration. A second tube of the same nominal design was therefore made and supported by a thin faired steel strip, this eliminated the vibration at the expense of a bulky support. However, when the problem of the indicated asymmetry of the jet was encountered, (see Section 4(iii) below) it was decided to try the larger tube which required low support. The results of a traverse in the same axial position with the two probes are shown in Figure 9. Since the traverses show good agreement the larger tube with the ellipsoidal nose was used for all subsequent tests. No further problems with vibrations were encountered.

4(iii) Traverse Details

Close to the nozzle exit it was possible to perform an axial traverse accurately along the jet horizontal centre line in the 1 inch jet. As the distance from the exit was increased it became difficult to set the probe on the centre-line to sufficient accuracy. Therefore traverses in a vertical direction across the jet at discrete distances from the jet exit were performed. These traverses supplemented the axial traverses in the initial region of the 1 inch jet and extended the exploration of this jet

to a maximum distance of $10\frac{1}{2}$ inches downstream from the nozzle exit. The $\frac{1}{2}$ inch jet was explored with vertical traverses only. The centre-line of the jet was then determined from considerations of maximum reading and symmetry. During early traverses some asymmetry was apparent in the Mach Number profiles obtained by these vertical investigations. It was found that this asymmetry was dependent upon the mounting of the traverse gear but was independent of the direction of the traverse. With the traverse gear and mounting arrangement for the probe above the jet, traverses in both directions showed the jet to be wider below the centre-line than above it. Mounting the traverse gear below the jet showed the reverse trend. It was concluded that the support for the probe was entering the jet and was deflecting it. Figure 10 illustrates the results obtained by vertical traverses from above and below the jet and with the downward traverse plotted in reverse about the indicated maximum reading. In view of the agreement obtained by this method of reverse plotting it was decided that the displacement was taking place beyond the jet centre-line only and that the traverse inwards towards the centre was unaffected. Since it was not possible to mount the probe in a more remote manner and retain the required rigidity and reliability of location, the traverses were continued as before. All of the results presented were therefore based upon the section of each traverse in the inward direction towards the centre for the side of the jet nearest to the mounting point of the traverse gear. In the 1 inch jet a single traverse was performed 1 inch east of the centre-line of the jet at a distance $\frac{x_j}{b} = 8.5$ downstream from the nozzle exit. The results are presented in Figure 11 where they are compared with those for a traverse on the centre-line at the same axial position. The differences between the results were very small indeed. No general conclusions could be drawn from this single traverse

but it did indicate some consistency in the jet profiles.

4 (iv) Discussion of the Results of the Jet Traverse

The general trend of the results was in agreement with the well established characteristics of free jet flows described above (Section 2 (i)). The variation of maximum Mach Number in the jets with distance from the nozzle exit is shown in Figure 12.

A constant Mach Number Core extended downstream from the nozzle exit followed by a far field region in which the Mach Number decayed as $x_j^{-\frac{1}{2}}$ approximately.

The composite Mach Number distribution curves for the 1 inch jet (Figure 13) confirmed this general description. The constant Mach Number region appeared to extend downstream to a position $\frac{x_j}{b} \approx 5$ at which some deviation from the jet Mach Number became apparent.

Since the experimental results fit closely into the well established divisions the results will be discussed below under the two usual headings.

4 (iv) a Jet Mixing Region

Several authors have considered the mixing region at the edges of a free jet issuing into still air. It is usual to present data concerning the width of the mixing region and the time mean velocity distribution within it. Similar data is presented here. The origin for the velocity profiles has previously been chosen as the point at which the local velocity is one half the maximum value. Here the position at which

the Mach Number ratio was one half has been used throughout. Various definitions of the width of the mixing region have been used and following Johannesen (34 & 35) the three used most frequently are quoted here

a) Δy_a = Distance between $\left(\frac{U}{U_m}\right) = 0.9$ & $\left(\frac{U}{U_m}\right) = 0.1$;

i.e. $\frac{M}{M_m} = 0.9$ & $\frac{M}{M_m} = 0.1$.

b) Δy_b = Distance between $\left(\frac{U}{U_m}\right)^2 = 0.9$ & $\left(\frac{U}{U_m}\right)^2 = 0.1$;

i.e. $\frac{M}{M_m} = 0.949$ & $\frac{M}{M_m} = 0.316$.

c) Δy_c = Distance between $\left(\frac{U}{U_m}\right) = 0.95$ & $\left(\frac{U}{U_m}\right) = 0.1$;

i.e. $\frac{M}{M_m} = 0.95$ & $\frac{M}{M_m} = 0.1$.

All of these definitions should produce the same virtual origin for the mixing region. Figures 14a) & b) show the results for each of these definitions of width at various positions in the two jets. Also shown are the best lines through the experimental points, determined by a least squares procedure. Both jets show a linear growth with distance in all of these cases. The origins of the mixing layer, extrapolated from the lines for each definition of width shows good agreement in both jets. The number of points in the narrower jet was rather small but served to illustrate the general agreement. In each jet the origin of the mixing layer was a virtual one lying upstream of the jet exit. The rate of increase of width of the region was slightly greater in the narrower jet for all definitions of the width. The numerical values for the rate of spread show similar values to those reported by

Johannesen. The results for the narrower jet were almost identical with the values reported whilst in the wide jet the values were approximately 15% lower for Δy_a and Δy_c

Goertler (24) indicated that the velocity distribution in the mixing region of a jet should take the form

$$\frac{U}{U_m} = \frac{1}{2} [1 + \operatorname{erf} \xi] \quad 4(1v)$$

where $\xi = \sigma \frac{y'}{x'}$, and σ is a free constant (see 2(i)). The best value of σ for each jet was determined as follows. For each definition of jet width a best line had been determined. The slopes of these lines were compared with the values of ξ at the appropriate values of $\frac{U}{U_m}$. A value of σ was thus obtained from each jet width definition and an average obtained for each jet. The results of this process yielded a value of $\sigma = 17.0$ for the 1 inch jet (aspect ratio 4) and a value of $\sigma = 16.9$ for the narrower jet. These values were in close agreement with the results of Gooderum et al (26) $\sigma = 15$ and Bershader & Pai (7) $\sigma = 17$ for plane jets. The results due to Johannesen (35) showed rather different values. Two types of mixing region were reported in one jet. Close to the nozzle exit a half jet flow existed for which $\sigma = 21.9$, beyond this region a partially developed axially symmetrical region was reported for which $\sigma = 11.9$. The second jet showed agreement with the latter region with a value $\sigma = 13.7$ throughout. The plane jet results therefore gave a higher value for σ than the axially symmetric region. The current results ($\sigma = 17.0$ and $\sigma = 16.9$) show agreement with this trend, i.e. a plane jet mixing region spreads more slowly than an axially one.

Figures 15 a & b show the experimental data for the Mach No.

distributions using the average value of σ for each jet as calculated above. The curve of equation 4 (iv) 1 is also shown for comparison. Good agreement is shown in both jets. The small divergence in the results at low values of Mach Number ratio in the narrowest jet near the nozzle could be attributed to experimental difficulties, the total width at this particular position was 0.11 inches only.

These results for the jet mixing region confirm the results of previous investigations in plane supersonic jets who used optical methods to determine the flow properties.

4 (iv) b. Fully Developed Jet Flow

The width of the jet in the fully developed region has been defined as follows:

$$\text{Jet width } k_x = 2 \delta_x = 2 \left[y_x (M = \frac{1}{2} M_m) - y_x (M = M_m) \right] \quad \dots \quad 4(iv)2$$

Figure 16 shows the variation of the jet width as defined above for the entire range tested in both jets. The results show the small rate of spread associated with the mixing region followed by the larger rate of spread experienced in the fully turbulent jet flow. Also shown on the diagram are straight lines drawn through the data for each jet. The lines were determined for the data $7 \leq \frac{x_j}{b} \leq 16$ by a least squares procedure in each case. The agreement of the data with these lines confirmed the expected linear growth with distance downstream. The resulting expressions for the jet width were as follows:

$$\begin{aligned} \frac{\delta}{b} &= 0.0953 \frac{x_j}{b} - 0.0073 \dots\dots\dots (1'' \text{ Jet}) \\ \& \frac{\delta}{b} &= 0.0796 \frac{x_j}{b} + 0.0876 \dots\dots\dots (\frac{1}{2}'' \text{ Jet}) \end{aligned} \quad \dots \quad 4(iv)3$$

These three relationships are plotted in Figure 18. The small differences only between them are mainly at the outer edges of the jet. The curve due to Bradbury was derived from comparison with several sets of experimental work in low speed jets. This curve was therefore drawn on Figures 17, for comparison with the currently reported data. The general agreement of the experimental data was good in each jet. The results for the 1 inch jet showed good agreement with the curve of equation 4(iv) 5 whilst the results for the $\frac{1}{2}$ inch jet showed better agreement with equation 4(iv) 7. The differences however were very small and in view of the experimental difficulties, not significant. The only comparable work in the fully developed region of a supersonic jet was that due to Johannesen (34 & 35). The characteristics of the axially symmetric jet used in that investigation were similar to those existing in low speed jets. The current results for the two plane surface supersonic jets tested show that the consideration adopted in low speed jets are similarly suitable to supersonic jets.

The time mean properties of the two jets used in the impingement studies were thus established and shown to conform to the usual pattern in low speed jets. The characteristics could therefore be used in consideration of the impingement phenomenon.

4 (v) Flow Visualisation of the Jet

A schlieren photograph of the jet expanding freely from the 1 inch nozzle is shown in Figure 19. The composite photograph covers the range $0 \leq \frac{x_j}{b} \leq 5$. A Shadowgraph is also shown which covers the range $0 \leq \frac{x_j}{b} \leq 3$.

In the schlieren photograph the disturbance arising from the nozzle surface can be seen only too clearly whilst disturbances occurring just

upstream of the lip of the nozzle can also be seen. Considering first the disturbances from the nozzle upstream, these propagate initially as straight lines at the Mach Angle to the flow. The angle that they make with the flow direction confirms the value of the initial Mach Number. ($M_j = 1.4$, $\mu = 45.5^\circ$; $44\frac{1}{2}^\circ \leq \theta \leq 46\frac{1}{2}^\circ$). Additionally the very small deviations as the disturbances from the upper surface interact with the ones from the lower surface indicates that they are in fact very weak. These disturbances were due to small irregularities in the nozzle surface which were caused by the method used to attach the nozzle contour plate to the main structure. With regard to the disturbances from the lip of the nozzle a study of the shadowgraph revealed that these gave rise to the classic type of regular reflection of weak shock waves. This was confirmed where the first waves reach the jet boundary and were reflected as expansion waves as can be seen in the schlieren photograph. The latter photograph showed these reflections recurring downstream to $\frac{x}{b} \approx 4$ at which stage the photograph became indistinct. Although schlieren studies were attempted further downstream no reliable results were obtainable. At pressure ratios other than the one chosen for this test, and used subsequently, the lip disturbances were very much more severe. Consideration of this flow pattern together with the readings of pressure tappings positioned at the nozzle exit established the best expansion pressure ratio for the nozzle. Ideally the shock reflection pattern would be reduced to Mach Wave intersections, however, unless the pressure ratio was very high a weak shock wave system was present.. This shock system arose from the joint of the nozzle contour plate with the nozzle exit plate. However, regular reflection on shock waves occurs only for weak shock waves, therefore the strength of these waves must have been very small and was therefore acceptable.

PART IV

THE IMPINGING JET

Chapter 5

THE GENERAL PROPERTIES OF THE IMPINGING JET

The flowfield of a low speed impinging jet has a relatively simple physical description, this is not true of a supersonic impinging jet. With supersonic flow a complicated pattern of expansion and weak compression waves together with shock-waves becomes apparent. The first aim of the investigation was to describe this pattern at various separation heights. from the results of flow visualisation studies. The description was then to be correlated with pressure measurements.

5 (i) The Description of the Flow Field

An overall description of the flowfield will be given first with detailed description following. It seems permissible to suggest that the conditions in a free jet will be important when that jet impinges upon a surface.

Consider then a supersonic jet with the characteristics described above in section 2 (i). At some distance downstream from the nozzle the flow in the jet will be subsonic entirely. If an impingement plate is placed normal to the axis of the jet in this region then the flow field is in general that of a subsonic sheathed jet impinging upon the plate. In this case the streamline curvature^{ure} will be continuous and there will be no discontinuities in the flow. The free jet studies indicated that for the particular jets of this investigation the entire flow was subsonic for $\frac{x_j}{b} > 15$ Upstream of the subsonic portions of the jet there is a region where the vast majority of the jet is subsonic with a small ^central section that is just supersonic. This latter region merges into one in which there is a considerable portion

of supersonic flow but with a highly sheared profile. These two regions appear to extend between $15 > \frac{x}{b} > 5$. The impingement phenomenon in these regions is rather obscure and no single description would appear to be possible. However to decelerate the supersonic flow it must pass through a shock wave or waves. It would seem likely that at the larger separation distances the deceleration takes place through oblique waves and then a small section of normal shock appears as the separation distance is decreased.

The regions described so far have been influenced considerably by viscous effects in that the sheared profile arises from the edge mixing region of the jet spreading inwards. At distances $\frac{x}{b} \leq 3$ the main portion of the jet was of nominally ^{uniform} supersonic velocity with a relatively small mixing layer surrounding it. In this region the impinging flow behaves substantially in the manner of the classical inviscid supersonic flow. Within the jet, at some distance upstream of the impingement plate, a shock wave is formed spanning the jet. At the intersection of the shock wave with the jet edge an expansion takes place turning the edge streamline. Further turning of the edge streamline takes place along a constant pressure boundary until the flow becomes a free boundary flow along the plate. The edge of the jet and the free boundary flow have mixing regions along them and the flow along the plate is subject to the growth of a boundary layer, a wall jet type of flow. However, the central portion of the flow is in the main inviscid. The experiments of Artingstall (5) demonstrated that in the outer region a counter flow takes place inwards towards the main jet along the surface of the nozzle plate. This induced flow supplies additional mass flow for the growth of the wall jet region. The general features described within the central impingement region hold for decreasing separation distances until at some configuration the

increasing width of the flow along the plate fills the space between the two boundary plates. The flow away from the axis between these plates is then a duct flow, this change in flow pattern would seem to occur at $\frac{h}{b} \approx 1$. Further decrease of the separation distance leads to the ultimate disappearance of the supersonic jet. Eventually the flow area in the impingement region decreases to a smaller area than the throat area of the nozzle. The sonic condition is therefore transferred from the throat of the nozzle to the new critical cross-section. The impingement region is then one of a subsonic turning flow with supersonic duct flow away between the plates. This final critical condition occurred at $\frac{h}{b} = 0.65$ in the 1" jet.

There are therefore five main types of impingement flow:

- a) entirely subsonic impingement at large separation distances;
- b) partial subsonic impingement at moderate separation distances;
- c) inviscid supersonic impingement in the nearfield of the nozzle;
- d) supersonic impingement with duct flow away from the impingement region;
- e) subsonic impingement with supersonic duct flow at very small separation distances.

Flow visualisation by shadowgraph and schlieren techniques has yielded useful results in categories (c) and (d) only. Some studies were made in region (b) but the pictures were obscure in the impingement region and no really clear features were apparent in the jet. At very small separation distances great difficulty was encountered with reflections from the plate surfaces. These reflections occurred because of the depth of the flow field (4 inch) and the severe density gradients encountered within it. The smallest separation distance at which reasonable photographs could be obtained was at $\frac{h}{b} = 0.8$ (in the 1 inch

jet and hence $0.8'$).

Since it was possible to produce flow visualisation data in the types c) and d) of the impingement flow detailed description of the flow field will be confirmed to this range of separation distances.

5 (ii) The Structure of the Jet Impingement Region

Figure 20 shows drawings of the impingement region for a correctly expanded uniform supersonic jet impinging normally upon a flat plate. The effects of mixing layers and boundary layers have been ignored. The phenomenon is shown in the physical plane in Figure 20 (a). A uniform parallel jet is spanned by the curved shock wave 1 - 2, this wave is drawn on the shock polar for the initial Mach Number (M) in Figure 20 b) with the corresponding notation $1' - 2'$. At the junction of the shock wave and the jet edge, 2, the flow downstream of the shock wave is sonic and a centred expansion wave ($2a' - 2'b$) expands the flow to the boundary pressure and a velocity slightly below the initial free stream velocity. The expansion fan spreads into the jet as far as the sonic line ($2b - 3a$) from which it is reflected as a compression wave ($3a' - 3b'$). The compression wave returns across the super sonic region to the constant pressure jet edge from which it is reflected as an expansion. Repeated reflections take place between the jet edge and the sonic line turning the entire flow until an expansion wave from the jet edge, reaches the impingement plate instead of the sonic line. The first expansion wave to reach the plate will be reflected as an expansion wave thus accelerating the flow along the plate to supersonic velocities. It should be noted that the final velocity possible along the impingement plate is less than the jet edge velocity because the stagnation pressure drop across the shock wave at (1) is greater than that at (2).

It is apparent from the above description for a completely parallel jet issuing freely, that the shape of the shock wave and its location relative to the impingement plates, should be unaffected by the separation distance. However, in practice the surfaces at the exit from the nozzle and the near proximity of the impingement plate modify the pressure surrounding the jet and give rise to disturbances within the jet. Also as distance from the nozzle is increased the mixing layers at the jet edge spread and these will affect the outer parts of the shock wave.

5 (iii) Shadowgraph and Schlieren Photograph

The foregoing sections described the general features of the flow field of a supersonic impinging jet. The results of the present series of tests will now be compared with that general description. The flow visualisation studies were performed in the 1 inch jet mainly although a few photographs were taken in the $\frac{1}{2}$ inch jet for checking the nozzle setting. Shadowgraph and schlieren photographs were taken at most settings and where it was appropriate the schlieren cut-off was rotated through 90° to illuminate further details of the flow pattern. (A description of the optical system was given in section 3 (vii)).

Commencing with the studies at the larger separation distances, the schlieren photograph for $\frac{h}{b} = 3$ (Figure 21) showed the impingement region as a rather indistinct compression area. This dark area had a relatively sharp upstream edge which could be described loosely as a shock wave spanning the jet. There is however, some doubt in this interpretation, inspired by the general dark (compression) area downstream. The exit from the nozzle showed the internal disturbances and a small expansion around the lip of the nozzle. With the separation distance reduced to $\frac{h}{b} = 2$ the schlieren photograph (Figure 22) showed the shock wave distinctly, the

approximate position of the free streamlines could also be seen. In this case the disturbances arising at the nozzle lip appeared stronger and the shock wave was located at approximately the same position as that at which the lip disturbances reached jet boundary. Downstream of the shock wave the free boundary of the deflected flow could be observed turning to a direction generally along the impingement plate. With further decrease in separation distance the lip expansions from the nozzle lip began to intersect the shock wave and modify its shape, (Figures 23-26).

In those photographs which showed the flow away from the centre-line a compression could be seen within the flow (Figures 22 & 36b). This compression system became apparent on the schlieren photographs at a separation distance $\frac{h}{b} = 2$ and became stronger with decreasing separation distance. Also the position of the system seemed to move inwards towards the axis with decreasing separation distances, but this movement was not sufficiently marked for definite conclusions to be drawn.

A composite photograph covering the entire flow on one side of the jet axis for a separation distance, $\frac{h}{b} = 1.0$ is included as Figure 37h. The flow visualised was in the category with supersonic impingement and supersonic duct flow (d) of section 5 (i)). The impingement region followed the description above but the free boundary could be seen, to reach the nozzle plate. A compression arose from the alteration of the free jet boundary, this compression reached the impingement plate and gave rise to a pair of oblique shock waves which were then reflected between the two plates gradually growing weaker until a shock expression pattern finally decelerated the flow. The intersection of the first compression with the impingement plate coincided with the compression system mentioned previously. The small disturbances arising from the nozzle plate in this photograph were due to the fact that some pressure

tappings on the nozzle plate were left open to atmosphere. At these locations the flow static pressure was sub-ambient and hence a small amount of air entered the flow via the tapping tubes. These disturbances were used to give an additional indication in the photographs of the distance along the plate. It can be seen that these extra disturbances did not interfere with the main features of the flow.

5 (iv) The Structure of a Slightly Underexpanded Jet

The schlieren and shadowgraph photographs of the previous section showed expansion waves emanating from the lip of the nozzle. That condition is described as the 'under-expanded' jet condition.

It is pertinent to describe the effects of these expansions on the flow within the jet. Figure 27 shows a sketch of the general pattern.

The lip expansions arose because the static pressure of the jet at the nozzle exit was higher than the local static pressure outside the nozzle. Since the flow was supersonic an expansion takes place which may be a centred expansion, located at the nozzle lip. Through this lip expansion the flow Mach Number was increased and the edge streamline underwent a sudden change of inclination, towards the region of lower pressure. The diagram (Figure 27) shows the first and last Mach Waves of the expansion and the free boundary of the jet. The Mach Waves (or characteristics) of the expansion are straight lines in regions of uniform flow where waves of the opposing family are absent. Where both families are present the characteristics are curved lines. The jet flow shows the two lip expansions crossing in region Z on the jet axis, this does not affect the sense of the waves and they continue as expansions until they reach the free jet boundary. At the free constant

pressure boundary the expansion waves are reflected as compression waves and this results in a change of direction of the streamlines. Beyond this region the compression waves continue and may coalesce to form oblique shock waves.

It is the initial expansive flow region which is of pertinence to the flow visualisation studies so no description of the flow further downstream will be offered.

The diagram shows also a typical streamline (cd) within the flow and the variation of Mach Number along the jet axis.

5 (v) The Modification of the Shock Wave by Expansion Waves

The presence of weak disturbances in the jet emerging from the nozzle modified the shape of the shock-wave (1-2 in Figure 20). As a result the magnitude of the jet edge expansion and the turning of the free streamlines would also be modified. The other main features of the flow field remain similar. The weak disturbances in the jet arose from two sources a) irregularities in the nozzle profile and surfaces, and b) lip expansions at the nozzle exit. The latter were more severe than the former but pressure measurements on the nozzle plates indicated that they were always expansion waves. The main characteristics of the jet flow were, therefore, similar to those of an under expanded jet as described in section 5(iv) above. At some separation distances the flow upstream of the shock wave was affected considerably by the disturbances but two cases presented situations in which the disturbances were limited. These two cases will be discussed first and a general discussion of the remaining results will follow later.

The two cases to be studied initially are shown in Figure 28. The effects of the expansion waves upon the shock wave will depend upon the nature of the intersections. Two general types of intersection occur.

- a) Expansion waves intersect the shock wave before they have crossed the jet axis (i.e. expansion waves and the shock wave are of opposite 'family').
- b) Expansion waves cross the jet axis before they intersect the shock wave (i.e. expansion waves and the shock waves are of the same 'family').

The velocity profiles for the two cases illustrate the difference between them. Consider the profile as the distance from the axis is increased. In the first case the disturbance becomes apparent as an increase in Mach Number whilst in case (b) a decrease occurs.

The effects of the disturbances are to change the pressure, Mach Number, and streamline inclination of the flow upstream of the shock wave. In turn these changes modify the properties of the shock wave in terms of its inclination and the downstream pressure, Mach Number and streamline inclination. In a flow with 'weak' shock waves (i.e. supersonic flow downstream of the waves) a model of the region of the intersection can be constructed similar to that of Figure 31. (A centred expansion and a curved shock wave can be treated as a series of single wave/oblique wave intersections). The Mach wave intersects the shock wave, the shock wave inclination is changed and a slip surface and compression wave propagate downstream from the point of intersection. Figure 31 a shows the physical plane and Figure 31 b shows the hodograph plane. If the shock wave is a strong shock wave (i.e. subsonic flow downstream) the model is no longer acceptable

since a subsonic flow cannot sustain the downstream wave necessary to the model. In the current examples the axis of symmetry requires that the shock wave is normal on the axis. Hence it is strong on the axis and the indications were that it remained strong as far as the jet edge layer. The weak shock model is therefore not applicable to the current cases and an alternative must be found. Consideration of the hodograph diagrams of the situation do not give any real clues as to the likely trends at the intersections, the flow downstream of the shock wave remains undefined. Therefore, one is forced to consider the shock wave properties alone, in the knowledge that this can give a partial indication only, of the likely occurrences.

The upstream expansion waves are continuous and cause changes of Mach Number as streamline direction as described in 5 (iv) above. They may be described by the expression

$$\frac{d\theta}{dM} = + \frac{\sqrt{M^2 - 1}}{M \left(1 + \frac{\gamma-1}{2} M^2\right)} \quad 5(v)1$$

The inclination of the shock wave to the jet axis is given by:-

$$\delta = \theta + \zeta_1 \quad 5(v)2$$

The change of inclination resultant upon a Mach Wave intersection is given by:-

$$d\delta = d\theta + d\zeta_1 \quad 5(v)3$$

The change of upstream flow direction is given by equation 5 (V) 1, it remains to determine $d\zeta_1$. The curved shock wave may be related to the oblique shock wave by considering individual streamlines. The oblique shock wave equations may be written as:-

$$\zeta_1 = f(M_1, \bar{p}_0) \dots \dots 5(v)4$$

where $\bar{p}_0 = \frac{p_2}{p_{01}}$ This form of pressure ratio is used because the stagnation pressure does not change through a Mach Wave.

$$\therefore d\zeta_1 = \left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0} dM_1 + \left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1} d\bar{p}_0 \quad 5(v)5$$

The differentials take the following form:

$$\left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0} = \frac{(M_1^2 - 2) \cdot \sin^2 \zeta_1 \cdot -\frac{\gamma-1}{2}}{2 M_1 \sin \zeta_1 \cdot \cos \zeta_1 \cdot (1 + \frac{\gamma-1}{2} M_1^2)} \cdot 5(v)6$$

and

$$\left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1} = \frac{1}{2 M_1^2 \sin \zeta_1 \cdot \cos \zeta_1} \cdot \frac{\gamma+1}{2\gamma} \left[1 + \frac{\gamma-1}{2} M_1^2 \right]^{\frac{\gamma}{\gamma-1}} \cdot 5(v)7$$

These functions are shown in Figures 32 and 33 for air ($\gamma = 1.4$)

It can be seen that $\left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1}$ is always positive but that $\left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0}$ changes sign according as $(M_1^2 - 2) \sin^2 \zeta_1 \lessgtr \frac{\gamma-1}{2}$. At low Mach Numbers it is negative and at higher values it becomes positive. At this stage the information available is insufficient to predict the likely changes of shock angle. Further details are required and these can be obtained partly from consideration of the particular experimental results.

Case a) Separation Distance $\frac{h}{b} = 0.9$ Effective Free Stream Mach Number 1.4.
At this Mach Number {the term $\left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0}$ is always negative}.

From 5 (v) 3 and 5 (v) 5

$$d\sigma = \pm \left(\frac{d\theta}{dM_1} \right) dM_1 + \left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0} dM_1 + \left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1} d\bar{p}_0 \cdot 5(v)8$$

The position shown for the intersection of the expansion wave with the shock waves shows an increase in M and an increase in θ due to the wave. The first term in 5 (v) 8 is therefore positive and the second term is negative. Figure 33 shows that at this Mach Number $\left| \frac{\partial \xi_1}{\partial M_1} \right| > \left| \frac{d\theta}{dM_1} \right|$ for all shock angles greater than 83° approximately. Hence the first two terms in 5 (v) 8 yield a negative quantity for values of $\xi_1 > 83^\circ$ at this Mach Number. It remains to consider the nature of the third term, $\left| \frac{\partial \xi_1}{\partial \bar{p}_0} \right|_{M_1}$, is always positive therefore this term takes the sign of $d\bar{p}_0$. If $d\bar{p}_0$ is negative then the entire expression for $d\sigma$ is negative, for $M = 1.4$ & $\xi_1 > 83^\circ$. Alternatively if $d\bar{p}_0$ is positive then the relative magnitude of the terms becomes of great importance. Unfortunately the oblique shock equations give no guide to the related magnitudes and signs of $d\bar{p}_0$ and dM without reference to further unknown quantities. Some further assumptions are therefore required. The prevailing pressure gradient along the shock wave on its downstream side was negative prior to the intersection point. (The downstream Mach Number was tending towards the sonic condition). If this condition were to prevail through the intersection $d\bar{p}_0$ and hence $d\sigma$ would be negative. Alternatively if it were positive one returns to considerations of relative magnitudes. However in this particular range of conditions the difference in magnitudes between $\frac{d\theta}{dM_1}$ and $\left| \frac{\partial \xi_1}{\partial M_1} \right|_{\bar{p}_0}$ is very sensitive to small changes in shock angle and Mach Number. The change in shock angle due to a Mach Wave in this range is therefore extremely sensitive to changes in downstream pressure.

Consider now the experimental results of Figure 26. The pressure measurements on the nozzle plate near to the lip indicated a total expansion to a final Mach Number 1.8 approximately. The photograph showed the first Mach Wave of the expansion intersecting the shock wave at a shock wave

inclination of 85° approximately. The initial Mach Number at this position was 1.4. At the intersection of the expansion fan with the shock wave, the shock wave inclination decreases initially ($d\sigma < 0$), continuing along the shock the inclination finally increases ($d\sigma > 0$). Whilst detailed quantitative comparison was not possible this result is consistent with the argument given above.

Case (b) Separation Height $\frac{h}{b} = 1.6$

Effective Free Stream Mach Number 1.75

The free stream Mach Number was determined from the maximum impingement pressure on the plate and an examination of the included angles of the expansion fan.

The condition for the shock wave modification for this case becomes

$$d\sigma = - \left(\frac{d\theta}{dM_1} \right) dM_1 + \left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0} dM_1 + \left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1} d\bar{p}_0 \dots 5(v)9$$

In this case $\left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0}$ is always positive and also $\left| \frac{\partial \zeta_1}{\partial \bar{p}_0} \right|_{M_1}$ is positive, dM_1 through the expansion is in this case negative.

For $\zeta_1 > 75^\circ$ at $M_1 = 1.75$

$$\left| \left| \frac{\partial \zeta_1}{\partial M_1} \right|_{\bar{p}_0} \right| > \left| \frac{d\theta}{dM_1} \right| \quad \text{and hence}$$

the first two terms of 5 (v) 9 take the sign of $dM (< 0)$ ie.

$$d\sigma = A(< 0) + (> 0) d\bar{p}_0 \dots 5(v)10$$

Once again the sign of $d\bar{p}_0$ is critical but except for $\zeta_1 \approx 75^\circ$ the extreme sensitivity of the first term to small changes of Mach Number and Shock Angle has diminished. Also when the total expansion is

considered it is apparent that the region of intersection is spread much more widely than in the previous case. The photograph of Figure 23 shows the flow field which corresponds to the above case. The shock inclination shows a small but continuous decrease throughout the rather indistinct region of interactions. In this case however the mixing region at the jet edge was also interacting with the shock wave. It is suggested that for the two cases considered the arguments presented above give a partial explanation for the behaviour of the shock shape at the region of intersection.

The shock shape at the remaining separation distances will now be considered. For most of these cases the upstream conditions were most nonuniform. In some cases the lip expansions produced a flow upstream of the shock wave which was generally diverging from the axis. One example of this type of flow is shown in Figure 25 for a separation distance $\frac{h}{b} = 1.1$. In this case the shock wave was positioned just downstream of the intersection of the lip expansions. The shock wave shape was concave to the jet flow direction but still produced streamline deflections away from the axis. This type of shock wave has been reported many times in highly underexpanded jets (eg. Stitt 64) and is possible because of the divergence of the main flow. (In the terms of equation 5 (v) 2, Θ is large.

The shapes of the shock wave were the most obvious results from the flow visualisation photographs and some unusual changes of shape were observed. The ultimate shape of the shock wave represents a sensitive balance between the upstream and downstream properties of the flow. It is suggested that whilst a complete explanation for the changes in the shape has not been presented a satisfactory indication of the trends has been achieved.

5 (vi) The Maximum Impingement Pressure on the Plate

In low speed jet impingement it would be expected that the maximum impingement pressure would be almost equal to the local stagnation pressure of the free jet. In supersonic jet impingement this is true also, but with the possibility of additional shock waves being present within the flow the knowledge of the variation of the stagnation pressure is less precise. In general variation of the pressure on the plate showed different characteristics at small separation distances from those at larger separation distances. At small separation distances ($\frac{h}{b} < 5$) the results were dominated by the considerations of inviscid supersonic flow whilst at larger distances the results were clearly influenced by the turbulent mixing.

Figure 34 shows the variation of maximum impingement pressure with separation distance. At the small separation distances ($\frac{h}{b} < 5$) the results for the 1" jet were much more detailed than those for the $\frac{1}{2}$ " jet, therefore the former results will be discussed first.

At very small separation distances ($\frac{h}{b} \leq 0.65$ in this case) the flow through the nozzle and to the impingement plate was subsonic along the axis. The plate stagnation pressure was therefore equal to the nozzle stagnation pressure. As the separation distance was increased the nozzle attained supersonic flow and over a small range ($0.65 \leq \frac{h}{b} \leq 0.9$) the centre of the shock wave was unaffected by the lip expansions from the nozzle. The compression was thus achieved through a normal shock on the axis from the nominal nozzle Mach Number 1.4. Subsequent increases in separation distance showed the shock wave positioned downstream of the area where the lip expansions crossed the axis. The recompression thus took place through a normal shock wave from an increasing upstream Mach Number and led to a decreasing impingement pressure ($0.9 \leq \frac{h}{b} \leq 1.8$). Beyond this

section the lip expansions had been reflected from the free boundary of the jet as compression waves. In turn this led to a decreasing Mach Number on the axis and an increasing impingement pressure, $(1.8 \leq \frac{h}{b} \leq 3)$. With further increase of separation distance the photographs of the flow pattern became indistinct. Also the mixing regions at the jet edges became of appreciable width. However, it would appear that a shortened repetition of the flow structure between $0.9 \leq \frac{h}{b} \leq 3.0$ took place between $3.0 \leq \frac{h}{b} \leq 4.3$ since the pressure variation followed a similar pattern.

As the separation distance was increased further the jet flow began to be dominated by the mixing regions at its edges. Hence the plate pressure was dictated by the rate of decay of the jet. The free jet investigations established the rate of decay at correctly expanded conditions for both jets. The two curves on Figure 34 show the jet stagnation pressure variation for the free jets. The impingement pressure for both jets show a similar decay with separation distance however under all conditions the impingement pressure was considerably less than the free jet pressure. It is suggested that the difference between the magnitudes of the pressures could be described as an increase in the decay of the jet. The change in the decay rate could be attributed to the changes in the jet structure. The pressure field into which the jet issued became sub-ambient due to the induced flows about the closely confined jet. The nozzle flow was therefore under-expanded. Apart from any momentum exchanges between the jet and its surroundings the low pressure would give rise to lip expansions from the nozzle exit. These lip expansions would continue as disturbances within the jet. Johannesen (34) in comparing the results of two jets established that a jet with internal disturbances showed a greater rate of decay than one without them. The jet with internal disturbances showed a shorter 'potential' core and subsequently a more rapid decay. The differences between the two jets investigated by Johannesen were of similar order to the differences

between the current results for the free jet and the peak impingement pressures. Additionally the differences between the results for the 1 inch jet and those for the $\frac{1}{2}$ inch jet could also be attributed to the same source. The lip pressures in the $\frac{1}{2}$ inch jet were lower than those in the 1 inch jet (see Figure 57) and hence the lip expansions were more severe. If these differences were now attributed to wave phenomenon this in turn could account for non-uniformity in some of the results.

The importance of the nozzle lip conditions and the induced flow along the nozzle plate prompted a short series of tests in which the nozzle plates were removed. This was done as an attempt to reproduce free jet impingement but the presence of the supporting structure of the nozzle still exerted some constraint upon the flow pattern. Further details of these tests and the results are presented below (Chapter 7) where the pressure distribution results are discussed.

Previous work on this particular aspect of the phenomenon indicated similar results to those of the current work at small separation distance. The earlier investigations were with axisymmetric jets. The inviscid behaviour at these small separation distances was in close agreement in principle with the results and description by Artingstall (5) although of course the magnitudes were not comparable. The only work at large separation distances that was in any way comparable was that of Donaldson and Snedeker (15). The only point of significance to the current work was that the results for their impinging jet did not compare directly with those for the free jet until the jet flow was subsonic. In the current tests of the free jets the 1 '' jet was supersonic over the entire range tested and the sonic condition was attained in the $\frac{1}{2}$ '' jet at a distance $\frac{x_j}{b} \approx 15$. The results for the $\frac{1}{2}$ '' impinging jet at the largest separation distance show a trend towards the free jet results.

5 (vii) Centre-line Shock Wave Height

Blunt body theory indicates that the centre-line shock stand-off distance (height) for this class of body is a function of Mach Number only, and that it decreases as Mach Number increases. Therefore in a uniform impinging jet at a given Mach Number the centre-line shock height should be independent of separation distances. In the tests reported here the centre-line shock height varied with separation distance as is shown in Figure 35. However, the centre-line Mach Number upstream of the shock wave varied due to the influence of the lip expansions from the nozzle (see 5 (iv) and 5 (vi) above). The Mach Number upstream of the shock wave is also shown in Figure 35. (This Mach Number was calculated from the maximum impingement pressure assuming normal shock recovery). It can be seen that no general trend towards agreement with blunt body theory was demonstrated. However blunt body theory presupposes a uniform upstream flow and in the current experiments this was rarely the case. Indeed with a slightly underexpanded jet the flow would be uniform only at the nozzle exit and possibly over the central portion at the location where the expansion waves reach the jet boundary. This latter position was one of the two positions used in Chapter 6 later for comparison with the numerical prediction method.

It was apparent therefore that severe non-uniformities in the jet flow not only affect the shock shape as described previously but also affect its location.

5 (viii) Comparison Between Flow Visualisation and Pressure Distribution

Results

In order to confirm further the description of the flow field some flow visualisation photographs and pressure distributions are compared directly. ^{Two} The separation distances are studied one typical of each of the categories (c) and (d) of section 5 (i). The two types of result showed good agreement and supported fully the description advanced above. The agreement between the photographs and the pressure measurements also served to confirm the repeatability of the results since the two types of test were not made at the same time.

5 (viii) (a) Supersonic Impingement in the Near Field (Category c)

The separation distance chosen to illustrate this type of flow was $\frac{h}{b} = 1.5$. In this case the jet was able to develop before impinging upon the surface. Figure 36 a shows the pressure distributions on the nozzle plate and the impingement plate whilst Figure 36b shows two schlieren photographs for this case. The schlieren cut-off for these photographs was a graded filter perpendicular to the plates, arranged so that compression regions appeared dark. The general flow pattern can be seen to fit the descriptions advanced above, also the pressure distribution showed the expected trends. The impingement plate distributions indicated a region of high pressure on the axis with subsequent expansion and compression regions along the plate. The nozzle plate distribution showed a smooth profile with a region of low pressure adjacent to the nozzle exit.

The schlieren photographs showed that the lip expansions had crossed the jet axis upstream of the shock wave and therefore the centre-line Mach Number had temporarily achieved a stable value. Comparisons between the lip pressure and the maximum impingement pressure, and the

inclinations of the first and last waves of the expansions fan together with the initial boundary inclination showed close agreement. Considering the flow along the impingement plate the photographs showed a series of compression regions within the flow. The extent of these regions corresponded very closely with the position of the recompression region on the pressure distribution $2.3 \leq \frac{x}{b} \leq 4$.

5 (viii) (b) Supersonic Impingement with Duct Flow (Category (d))

The separation distance was for this case chosen as $\frac{h}{b} = 1.0$.

The pressure distributions and the photographs are shown in Figures 37 a and b. The schlieren cut-off was again perpendicular to the plates with compression regions dark.

The pressure distribution results showed the impingement region followed by a duct flow at sub-ambient pressure with repeated compression and expansion regions. The number of pressure measurements towards the outer edges of the plate was insufficient to determine accurately the exact pattern of compressions and expansions. However, reference to the schlieren photographs indicated general agreement and in particular the final compression region was in close agreement. This result was gratifying since the supersonic flow in a parallel duct is very sensitive to the boundary layer effects upon the walls and the position of a final shock wave can vary considerably.

In general the flow visualisation studies and the pressure distribution measurement indicated close agreement with the description of the flow field advanced earlier in the chapter.

Chapter 6

A NUMERICAL TREATMENT BY THE METHOD OF INTEGRAL RELATIONS

The Method of Integral Relations for the numerical solution of problems in the flow of gas around a blunt body travelling at supersonic speed was proposed first by Dorodnitsyn. Surveys of the method have been published by Belotserkovskiy (6) and Hayes & Probst (30). In general for these types of flow, both axisymmetric and plane flows have been considered, scale is given to the flow field solution by a typical length in the body. This typical length usually arises from a change of cross-section in the body and/or the location of the sonic point on the body. To draw an analogy between the blunt body type of flow in an unbounded fluid and the impingement of a jet of fluid upon a plate a similar length scale must be found.

It is apparent that if the impingement plate is small in comparison with the jet width then the situation is exactly similar to the blunt body problems and then the plate size will determine the scale of the solution. Alternatively if the impingement plate is large in comparison with the jet width the plate cannot present scaling criteria. The only remaining source of a scaling parameter is the point of intersection between the shock wave and the edge of the jet. Hence the scale of the solution arises from the width of the jet. Since a scaling condition is available an analogy is possible between the two flow fields and a blunt body type can be performed. Gummer & Hunt (27) recently presented a solution to the problem of an axisymmetric jet using the method of integral relations and a polynomial approximation. A solution to the two-dimensional impinging jet is presented here using a similar approach.

6 (i) The Method of Integral Relations and the Polynomial Approximation

The equations of motion for a two-dimensional impinging jet flow written in the coordinate system shown in Figure 38 are as follows:

Continuity:
$$\frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial x} (\rho u) = 0 \dots 6(i)1$$

Momentum in the axial direction:

$$v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} = - \frac{1}{\rho} \frac{\partial p}{\partial y} \dots 6(i)2$$

These equations may be rewritten in the following non-dimensional form:

Continuity
$$\frac{\partial}{\partial \bar{x}} (Q_1) + \frac{\partial}{\partial \bar{y}} (Q_2) = 0 \dots 6(i)3$$

Axial Momentum:
$$\frac{\partial}{\partial \bar{x}} (P_1) + \frac{\partial}{\partial \bar{y}} (P_2) = 0 \dots 6(i)4$$

The variables of state were non-dimensional with respect to jet stagnation conditions, velocities with respect to adiabatic escape velocity, and cartesian coordinates with respect to the jet width.

The functions Q_n and P_n were defined as:-

$$\left. \begin{aligned} Q_1 &= (1 - \bar{q}^2)^{\frac{1}{\gamma-1}} \cdot \bar{u} \quad ; \quad Q_2 = (1 - \bar{q}^2)^{\frac{1}{\gamma-1}} \cdot \bar{v} ; \\ P_1 &= \bar{p} \bar{u} \bar{v} \quad ; \quad P_2 = \frac{\gamma-1}{2\gamma} \bar{p} + \bar{p} \bar{v}^2 \end{aligned} \right\} \dots 6(i)5$$

Implicit within the above rearrangement are the assumptions that the flow within the shock layer was the inviscid, adiabatic flow of a perfect gas, with constant specific heat ratio (γ). It was also assumed that the flow along streamlines was isentropic.

The procedure was then to integrate the equations (6(i) 3 & 4) across the shock layer at a particular value of x . For the purposes of the integration the functions $Q_n P_n$ were represented by polynomial approximations. When the integration had been performed the equations became ordinary differential ones. The simplest approximation for the functions $Q_n P_n$ was a linear relationship, and this was adopted in the current work i.e.

$$Q_n = [Q_n]_{\bar{y}=0} + \frac{\bar{y}}{\bar{y}_s} \left([Q_n]_{\bar{y}=\bar{y}_s} - [Q_n]_{\bar{y}=0} \right)$$

and a similar expression for P_n ... 6 (i) 6

When this ^{substitution} ~~solution~~ was made and the integration performed the equations took the following form:

$$\begin{aligned} \frac{d}{d\bar{x}} (Q_1)_s + \frac{d}{d\bar{x}} (Q_1)_p + \frac{1}{\bar{y}_s} \frac{d(\bar{y}_s)}{d\bar{x}} [(Q_1)_s + (Q_1)_p] \\ + \frac{2}{\bar{y}_s} (Q_2)_s = 0 \end{aligned}$$

... 6 (i) 7

and

$$\frac{d}{d\bar{x}} (P_1)_s + \frac{1}{\bar{y}_s} \frac{d(\bar{y}_s)}{d\bar{x}} (P_1)_s + \frac{2}{\bar{y}_s} [(P_2)_s - (P_2)_p] = 0$$

... 6 (i) 8

where subscript S indicates conditions at the shock wave and subscript p indicates conditions at the plate. The functions $Q_n P_n$ on the shock wave may, for a given Mach Number, be written in terms of the shock wave angle (σ) only. Hence derivatives of these functions may be rewritten as derivatives with respect to σ eg.

$$\frac{d}{d\bar{x}} (Q_n)_s = \frac{d(Q_n)_s}{d\sigma} \frac{d\sigma}{d\bar{x}}$$

... 6 (i) 9

Similarly along the plate:

$$\frac{d}{d\bar{x}} (Q_n)_p = \frac{d}{d\bar{u}_p} (Q_n)_p \frac{d\bar{u}_p}{d\bar{x}} \quad \dots 6 (i) 10$$

where $\bar{u}_p$ is the non-dimensional velocity along the plate.

Substituting 6(i) 9 & 10 into 6 (i) 7 & 8 we have:-

$$\frac{d}{d\sigma} (P_1)_s \frac{d\sigma}{d\bar{x}} + \frac{1}{\bar{y}_s} (P_1)_s \frac{d\bar{y}_s}{d\bar{x}} + \frac{2}{\bar{y}_s} [(P_2)_s - (P_2)_p] = 0 \quad \dots 6 (i) 11$$

and

$$\begin{aligned} & \frac{d}{d\sigma} (Q_1)_s \frac{d\sigma}{d\bar{x}} + \frac{d}{d\bar{u}_p} (Q_1)_p \frac{d\bar{u}_p}{d\bar{x}} \\ & + \frac{1}{\bar{y}_s} \frac{d\bar{y}_s}{d\bar{x}} [(Q_1)_s + (Q_1)_p] + \frac{2}{\bar{y}_s} (Q_2)_s = 0 \quad \dots 6 (i) 12 \end{aligned}$$

Consideration of the geometry of the shock wave closes the set of equations which may be written finally as follows:

$$\frac{d\bar{y}_s}{d\bar{x}} = - \cot \sigma \quad \dots 6 (i) 13$$

$$\frac{d\sigma}{d\bar{x}} = - \frac{1}{\frac{d(P_1)_s}{d\sigma}} \left[\frac{1}{\bar{y}_s} \frac{d\bar{y}_s}{d\bar{x}} (P_1)_s + \frac{2}{\bar{y}_s} ((P_2)_s - (P_2)_p) \right] \quad \dots 6 (i) 14$$

$$\begin{aligned} \frac{d\bar{u}_p}{d\bar{x}} = - \frac{1}{\frac{d(Q_1)_p}{d\bar{u}_p}} & \left[\frac{1}{\bar{y}_s} \frac{d\bar{y}_s}{d\bar{x}} ((Q_1)_s + (Q_1)_p) \right. \\ & \left. + \frac{2}{\bar{y}_s} (Q_2)_s + \frac{d(Q_1)_s}{d\sigma} \frac{d\sigma}{d\bar{x}} \right] \quad \dots 6 (i) 15 \end{aligned}$$

This set of equations was suitable for integration by stepping procedures. Commencing on the axis of the flow with initial conditions $\bar{x} = 0$, $\bar{y}_s = (\bar{y}_s)_0$, $\sigma = \frac{\pi}{2}$ & $\bar{u}_p = 0$ the problem becomes an initial

value one commencing from the assumed centre-line height $(\bar{y}_s)_0$. The integration could then proceed in the direction of increasing x until terminating conditions such as the edge of the jet or the sonic point on the plate were reached. A rescaling procedure could then be performed to give the required physical size to the solution.

6 (ii) Numerical Peculiarities within the solution

When general equations are written in difference form and numerical procedures adopted to solve them peculiarities often arise within the solution that are not present in the physical situation. This arises within the current solution. Equation 6 (i) 14 contains a term $\frac{d(P_i)}{d\sigma}$ as a general denominator. This term takes various values within the solution and for a given Mach Number there is a value of shock angle which makes the term equal to zero. (See Figure 39). This of course gives rise to an infinite value of $\frac{d\sigma}{dx}$. If this singularity occurs within the shock layer before the calculation is complete then no solution is possible by this method of solution. From scaling considerations outlined below (6(iii)) terminating conditions are concerned with the attainment of sonic conditions on the shock wave and/or the plate. Jet Mach Numbers of less than 1.4 approximately lead to the singularity occurring before sonic conditions have reached on the shock wave. At values of Mach Number less than 2.3 approximately the singularity occurred before the sonic point had been reached on the plate. Solutions for Mach Numbers less than 1.4 were therefore not possible.

6 (iii) The Scale of the Solution

It was suggested above that the scale of the solution was determined in the current case by the width of the jet. At the extremities of the jet the stand-off shock wave terminates and hence the shock layer solution must also terminate. Any solution therefore must be a free boundary problem with constant pressure boundary conditions. It is necessary to obtain criteria which will enable this break in the solution to be made. The traditional approach to this question in blunt body work is to consider the geometry of the sonic line. Gummer and Hunt followed this method for the axisymmetric jet impingement problem. Their arguments were conducted for plane flow and are outlined in Appendix I. Their conclusions will be used here without further discussion.

The result of the scaling argument was that the shock layer solution should be terminated either when sonic conditions are attained on the shock wave or when sonic conditions are attained on the plate. Generally the former will be attained before the latter. In axisymmetric flow there was little difference between the radius at which these conditions occurred. In plane flow the difference between the two conditions is shown to be larger and for the range of Mach Numbers used herein only the shock condition could be obtained.

In the case of an impinging jet, upon obtaining the chosen critical condition from an initial assumed value of the centre-line shock height a simple linear rescaling with respect to the jet width yields the final physical size of the solution.

6 (iv) Continuation of the Solution beyond the Shock Layer

Since the equations of motion beyond the shock layer may be written in a form similar to that of equations 6(i) 3 and 6 (i) 4, the solution may be continued beyond the critical points with new boundary conditions. In order to establish these conditions, consideration must be given to the flow at the intersection of the shock wave with the jet boundary.

Upon passing through the shock wave the flow is compressed and undergoes a change in direction towards the free boundary. Since this boundary must remain at constant pressure an expansion must take place. Sonic conditions (or supersonic ones) at this point enable this expansion to take place as a centred expansion wave. This expansion wave gives a further deflection to the flow together with acceleration to the following constant pressure and velocity boundary. These conditions form the initial conditions for the remainder of the solution. Unfortunately this set of equations also gives rise to a singularity in 6 (i) 11 in that $\frac{d(P_1)_s}{d\sigma} = 0$ when the inclination of the free streamlines is 45° . This condition is of course limiting if sonic conditions have not been attained along the plate.

6 (v) Results of the Computation and the Comparable Experimental Data

The set of equations 6 (i) 13, 14 & 15 were programmed and several cases calculated. The computation was arranged in such a form that the step sizes were easily modified to achieve the appropriate accuracy overall and in the location of critical points. The results for the shock wave shape and plate pressure distribution are shown in Figures 40 to 45 where they are compared with experimental results. Figure 46 shows the flow visualisation photographs for the similar conditions.

The experimental results used for comparison were for three nominal jet Mach Numbers $M_j = 1.4, 1.58$ and 1.8 . The latter two conditions were obtained from some limited tests performed for this particular purpose. At each jet Mach Number two separation distances were used, the smaller separation distance ($\frac{h}{b} \approx 1$) gave conditions at the nominal jet Mach Number. The larger separation distance ($\frac{h}{b} \approx 2$) enabled expansions in the jet to produce higher Mach Numbers and also provided a relatively uniform flow. For each of the latter cases an equivalent Mach Number was calculated which corresponded with normal shock recovery to the plate stagnation pressure. The Mach Numbers used for the numerical procedure were, therefore, 1.4 and 1.94 , 1.58 and 2.0 , 1.8 and 2.23 . In all of these cases the scale of the solution was taken from the sonic point on the shock wave. This range of Mach Number lies between the conditions where the sonic point on the shock wave was just obtainable and the conditions where the plate sonic point was obtainable. The calculation at $M = 1.4$ terminated when the Mach Number downstream of the shock wave was 0.988 . In this particular case this point was taken as the approximate sonic point. In all of the other cases the calculation was able to proceed beyond the shock sonic point and it was therefore possible to locate it accurately. Extension of the solution beyond the shock layer was attempted in two cases only, $M = 1.4$ & $M = 1.94$. In neither of these cases was it possible to continue the solution to sonic conditions upon the plate, even beyond the shock layer. In the first case the calculation proceeded for some distance beyond the shock layer but the singularity in 6 (i) 11 occurred before sonic conditions. At a Mach Number of 1.94 the singularity occurred almost immediately following the change of boundary conditions. (The free streamline inclination following the critical point was 36°).

A comparison between the predicted conditions and the experimental results showed differences that were not inconsiderable. A survey of the results is given here and a discussion of some possible reasons for the differences given later (6 (vi)).

Consider first the results for the three lower Mach Numbers and the experimental results at the smaller separation distances. In all of these cases the centre-line shock heights were considerably over-estimated, with the over-estimation greatest at the lowest Mach Numbers. The predicted shape of the shock wave was convex to the flow direction as was expected and in general the experimental shape was also. However, it was shown earlier that small disturbances in the upstream flow could have a profound effect upon the shock wave. Hence very close agreement could not be expected. The experimental and predicted pressures were in agreement on the jet axis since this was an initial condition in the calculation. The predicted pressure distribution showed a slower decrease with distance than did the experimental results, but on the whole the agreement was satisfactory within the jet region.

The results for the higher Mach Numbers showed better agreement than those for the lower Mach Numbers. In each of these cases the experimental results showed that the jet had spread by about 10% at the shock wave location. However, no allowance was made for this in the prediction method. In this series of tests the predicted and experimental values of the centre-line shock height agreed quite closely and as a result the shock wave shapes show comparatively good agreement. The pressure distribution results showed very good agreement. This latter set of results was therefore relatively satisfactory.

Comparison with the earlier work of Gummer and Hunt (27) would be appropriate at this stage. In general their results showed the same trends as the current work, namely a gross over-estimation of the shock height at low Mach Numbers with improving agreement at higher values. Good agreement in the predicted and experimental pressure distributions was also demonstrated in their results for all Mach Numbers. Two significant differences were apparent between the experimental work by the above authors and that reported here. The first difference arose from the much smaller separation distances used by Gummer and Hunt. Since they used a free axisymmetric jet they were able to use separation distances, $0.8 \leq \frac{h}{\tau_N} \leq 1.05$. In the current work the jet became subsonic at $\frac{h}{b} = 0.65$ and reliable photographs were not possible for $\frac{h}{b} < 0.8$. As a result the mixing layers around the jet had greater effect in the current work than in the other work. The second difference arose from the comparison between an axisymmetric case and a two dimensional one. In the axisymmetric work the experimental results showed that the sonic condition on the plate occurred at $\tau_N < \tau_{SON} < 1.2 \tau_N$ that is almost at the jet edge. The current results indicated that the sonic condition was attained at $1.6 \frac{b}{2} < x_{SON} < 2.4 \frac{b}{2}$ hence considerably further away from the nominal jet edge. (See Figure 47). Whilst Gummer and Hunt were able to consider sonic conditions on the plate as an alternative scaling point for the shock layer calculation a similar consideration was inappropriate for the current work.

6 (vi) Discussion

The Method of Integral Relations has been used by many authors for predicting the flow field about blunt bodies in free flight. Comparisons between the predictions and the experimental data have on the whole been very successful. The accuracy of the prediction has been best at high Mach Numbers and differences of a few per cent only have been common. (See for example Hayes and Probst (30)). This accuracy has not been sustained at lower Mach Numbers.

The current work and also that of Gummer and Hunt concentrated at this lower end of the Mach Number range and showed the decrease in accuracy. Some possible reasons for this deficiency are suggested below.

The prime assumption for the polynomial approximation was that the distance from the shock wave to the body was small in comparison with the width of the jet. This assumption allowed the approximation to supplant momentum considerations along the plate. It is apparent in the current case where the shock layer thickness was of the same order as the jet width that this initial assumption could well be in some doubt. Close to the jet axis at high Mach Numbers where rates of change in the shock layer are large in the axial direction and small along the plate the solution is adequate. However, with lower Mach Numbers the axial gradients decrease and with stepping out along the plate the 'radial' gradients increase. Both of these conditions lead to inadequacies in the approximation and hence the predictions. Various authors have used higher polynomial approximations and achieved some improvement in agreement with experimental data at higher Mach Numbers. However, it was felt that the small improvement did not warrant the added complication for this case.

On the whole the numerical procedure adopted was capable of predicting the shock wave shape and plate pressure distributions adequately at higher Mach Numbers. The solution was confined to a region equal to the width of the jets but was nevertheless a useful prediction.

Chapter 7

THE PRESSURE MEASUREMENTS

Measurements were made of the pressure distribution on the plates with several jet impingement configurations all at a nominal jet Mach Number of 1.4.

The configurations were as follows.

Group I

- a) 1" Nozzle exit height ($b = 1"$) with nozzle plates and impingement plate 24" overall for $0.45 \leq \frac{h}{b} \leq 11$.
- b) $\frac{1}{2}"$ Nozzle exit height ($b = \frac{1}{2}"$) with nozzle plates and impingement plate 24" overall for $1 \leq \frac{h}{b} \leq 18$.
- c) $\frac{1}{2}"$ Nozzle exit height with nozzle plates and impingement plate 12" overall for $2 \leq \frac{h}{b} \leq 10$.

Group II

- d) 1" Nozzle exit height, nozzle plates removed and with 24" impingement plate for $1.5 \leq \frac{h}{b} \leq 10$.
- e) $\frac{1}{2}"$ Nozzle exit height, nozzle plates removed and with 24" impingement plate for $1.0 \leq \frac{h}{b} \leq 14$.

In the tests of Group II the supporting structure for the nozzle near the nozzle exit provided an obstruction equivalent to nozzle plates $\pm 2\frac{1}{4}"$ either side of the jet axis.

The first set of tests (a) was the most comprehensive and corresponded exactly with the flow visualisation photographs. The second set of tests (b) was performed in order to illustrate some of the effects of the aspect ratio of the jet. [(a) Aspect Ratio 4:1 and b) Aspect Ratio 8:1 ($4:\frac{1}{2}$)]. The third set of tests (c) was a brief set with the same ratio of plate height to nozzle exit height as that used in tests (a).

The tests of Group II were performed in order to view the situation when the jet emerged almost freely.

The positions of the pressure tappings were described in Section 3 (V) and shown in Figure 7. The same plates and pressure tappings were used throughout and in the case of the 12" plates they were simply curtailed at ± 6 " either side of the centre-line. Since there was a small clearance around the mounting bolts for the impingement plate the central pressure tapping was not always exactly on the centre-line of the jet. However, it had been shown that the free jet emerged perpendicularly from the nozzle plates and the space between the plates was carefully adjusted at each setting. It was assumed therefore, that any 'off-axis' tendency in the central impingement pressure distribution was due solely to the non-coincidence of the central pressure tapping and the jet axis. The axis of the pressure distribution was therefore, determined by considering the symmetry of the central pressure distribution profile. The resulting off-set was small in all cases, the maximum value was .04" which is of similar order to the $\frac{1}{32}$ " nominal clearance in the mounting slots.

In the early exploratory tests the impingement plate was arranged

so that for the central portion of the plate the pressures indicated by pressure tapings on the vertical centre-line could be compared with those off-set by 0.75". The tests were conducted with the off-set first on one side of the centre-line and then on the other. The results for one separation distance are shown in Figure 49. It can be seen that there were no notable differences between the results. Subsequently pressure measurements were performed on the vertical centre-line only and treated as being representative of the general distribution. The pressure tapings on the nozzle plate were equispaced about the vertical centre-line and these were also treated as representative of the general distribution although no check measurements were made in this case.

7 (i) The Results for the Pressure Distributions

The initial use for the pressure measurements was to describe the pressure distributions along the plates and to give further indications of the flow patterns. In particular the measurements were used to indicate the transition points between the types of flow described in Section 5 (i). The location of these points will be described first and consideration of the pressure distributions will follow.

The transition from subsonic to supersonic flow through the nozzle was observed from measurements of the pressure inside the lip of the nozzle. Under all circumstances of supersonic flow in the nozzle the pressure inside the nozzle was equal to the ambient pressure. When the plates were closed up so that the nozzle flow was subsonic the pressure inside the lip rose above the ambient pressure. The measurements indicated that at $\frac{h}{b} = 0.65$ in the 1" jet the flow was subsonic whilst at $\frac{h}{b} = 0.70$ the flow was supersonic. Duct flow away from the axis existed for these and larger separation distances. The next change of

flow pattern was the change from duct flow between the plates to free boundary impingement flow. Throughout the duct flow patterns some positive pressures were apparent on the plates, when the free boundary impinging flow appeared these were no longer present. In the 1" jet this latter transition took place at a separation distance $1.0 < \frac{h}{b} < 1.1$. It would seem that this transition took place quite rapidly and that there was either a large extent of duct flow or the free boundary flow prevailed. It was not possible to locate this transition point with equal accuracy in the experiment with the $\frac{1}{2}$ " jet. It was possible, however, to state that at $\frac{h}{b} = 1.25$ free boundary flow appeared to have prevailed, in tests (b). The agreement between the two results was a little surprising since the length of the plates, in the direction of flow, for the $\frac{1}{2}$ " jet was the same as for the 1" jet. If it is argued that duct flow occurs when the wall jet type flow along the impingement plate fills the gap then it would be expected that the agreement would be good only between tests (a) and (c). (The growth of a wall jet is directly proportional to the distance from the origin (23)). It would seem from these results that there were two possible explanations for the agreement.

- (1) The transition was dominated by the initial compression system along the impingement plate and its reflections from the nozzle plate. Under these conditions the scale of the system would be dictated by the non-dimensional separation distance $(\frac{h}{b})$.
- or (2) The two results were inadequate for any conclusion to be drawn and thus the agreement was fortuitous or false.

Following the change from duct flow the pressure measurements indicated the type of flow system described by Artingstall (5). The impinging jet flowed outwards along the plate whilst an inward counter-flow was established along the nozzle plate.

The pressure distributions were of the expected pattern and indicated that the flow accelerated away from the stagnation point on the axis to finally achieve ambient pressure at the plate edge. The acceleration was, however, not always continuous and at smaller separation distances an intermediate recompression process was required because of an initial over expansion. This recompression system has been mentioned above but the reasons for this process will be described. Figure 20a shows a drawing of the suggested flow pattern, 1-2 is the shock wave, 2-12 is the free boundary. At the junction point 2 of the shock wave with the jet boundary the sonic line 2,3,5,7 originates together with a centred expansion wave. The expansion waves extend towards the sonic line from which they are reflected as compression waves. The compression waves are reflected from the free boundary as expansion waves. This process is continued until an expansion wave from the free boundary reaches the impingement plate (8-9). The reflection at the impingement plate is as an expansion wave (9-10) thus accelerating the plate flow. This expansion wave is then reflected from the free boundary as a compression wave (10-11) which in turn is reflected from the plate as a further compression wave (11-12). At the free boundary 11-12 becomes an expansion and the process repeats until the plate edge is reached or the flow becomes subsonic. This flow pattern is manifested upon the impingement plate as an initial expansion followed by cyclic compression and expansions. The pressure measurements for all tests at appropriate separation distances indicated this type of pattern. In order to specify the range of separation distances over which this type of flow was present it is pertinent to consider the location of the sonic point on the plate. Figure 47 shows the variation with separation distance of the distance from the jet centre-line

to the sonic point on the plate. It is apparent that the position of the sonic point was sensibly constant at $\frac{x}{b} \approx 1$ for separation distances $1 \leq \frac{h}{b} \leq 4$ but then showed a systematic outward movement with further increases in separation distance. The commencement of the outward movement coincided approximately with the disappearances of the compression-expansion system. At the smaller separation distances the system led to subsonic flow along the plate against a severe adverse pressure gradient (Fig 50b). As the separation distance was increased the recompression system became less severe and the flow along the plate remained supersonic. Figure 48 shows the ratio of minimum pressure on the plate to the plate stagnation pressure for various separation distances. This ratio gives an indication of the maximum Mach Number achieved along the plate. (The Mach Number achieved if isentropic flow is assumed is shown on the diagram). The experimental data shows two types of relationship, for $\frac{h}{b} < 10$ approximately the pressure ratio is susceptible to the variation of both the maximum and minimum pressures upon the plate. At larger separation distances ($\frac{h}{b} > 10$) supersonic flow was not apparent along the plates. The minimum pressure was almost constant and equal to the ambient pressure whilst the maximum pressure decreased with increase in the separation distance.

As a passing reference Figure 47 which shows the sonic position on the plate could be compared with the similar results of Gummer & Hunt as suggested in section 6 (v), and also with those of Artingstall (5). Good agreement was demonstrated between the two axisymmetric tests.

Artingstall for $0.38 \leq \frac{h}{d_N} \leq 3.38$	$0.61 - \frac{r_{son}}{d} - 0.69$
Gummer & Hunt for $\frac{h}{d_N} \approx 0.5$	$0.6 - \frac{r_{son}}{d} - 0.7$

The larger values for the current results were consistent with the differences that would be expected between an axially symmetric field

and a two-dimensional one.

Some typical pressure distributions are shown in Figure 50 , they have been chosen to illustrate the various modes of flow categorised in section 5 (i) and are from each of the jet configurations.

A further step in the consideration of the pressure distribution is to correlate the distributions at various separation heights. Considerations of the properties of a fully turbulent jet were used by Bradbury (11) to correlate the pressure distribution under a low speed impinging jet at various separation heights. A similar type of correlation is presented below for the current experiments at the larger separation heights. At smaller separation distances a less general form of collapse is used. The arguments used by Bradbury were applicable only where similarity of mean velocity profile had been established (i.e. beyond the potential core of the jet). The reduction of the experimental data will therefore be presented and discussed in two regions, firstly at large separations ($\frac{h}{b} \geq 6$) and then for smaller separations ($\frac{h}{b} \leq 5$). In each case the central impingement section of the results will be considered only, since a similar approach to the flow along the plates would not be appropriate.

7 (ii) Pressure Distributions for $\frac{h}{b} \geq 6$

The reduction of experimental data for pressure distributions requires a characteristic length and a characteristic pressure. In low speed jet impingement several authors have attempted to reduce pressure distributions, the most successful method was the one adopted by Bradbury (11). The general trend of the argument was that the conditions in the free jet at the impingement location controlled the impingement process. Also it was argued that in the immediate impingement region the inertia

terms were considerably larger than the turbulent stress terms. It was concluded that the characteristic properties that should be used to correlate results at various separation distances were the dynamic pressure on the jet centre-line and the width of the jet at the appropriate location in the free jet. The width of the jet was defined as the distance from the jet axis to the point at which the local velocity was one half the maximum velocity. Similar characteristics have been adopted in the current work. The distance to the point at which the local Mach Number was one half the local Mach Number on the axis was chosen as the characteristic jet width (δ in section 4 (iv) b). The characteristic pressure was not quite so self evident. The final correlation was attempted on the basis of ^{au} gauge pressures with the stagnation pressure on the axis of the free jet at the impingement location. In the case of supersonic flow, normal shock recovery was used from the jet Mach Number (i.e. pilot pressure). The results of this reduction of data are shown in Figures 51-52. Figures 51 (a), (b), (c) and (d) show the results for the jet impingement region when the nozzle plates were in position at the nozzle exit (i.e. Group I tests). The results for the 1" jet (Figure 51 (a)) showed good correlation over the range of separation distances used $6 \leq \frac{h}{b} \leq 11$. Figure 51 (b) shows the results for the $\frac{1}{2}$ " jet which similarly showed good correlation for $6 \leq \frac{h}{b} \leq 11$ but showed some variation at larger separation distances. Comparisons between the results for $\frac{h}{b} = 6$ and $\frac{h}{b} = 10$ for all three jet configurations are shown in Figures 51 (c) and (d). These results showed varying quality of agreement. The results for the $\frac{1}{2}$ " jet at a separation distance $\frac{h}{b} = 18$ (Figure 50h) (with the large plates) showed distinct differences from the other tests. The pressure distribution on both the impingement plate and on the nozzle plate showed large regions of negative pressure. It is suggested that this was due to non-uniformity

in the jet and that at this distance the jet was no longer a simple plane jet. Tests at larger separation distances were also most inconsistent and were not repeatable. The test at $\frac{h}{b} = 18$ must therefore be regarded as the limit of the reliability of the results.

The results for the tests conducted with the nozzle plates removed are shown in Figures 52 a and b. These results showed good correlation for $\frac{x}{\delta} = 1.5$ at all separation distances but some disagreement was apparent at smaller distances from the axis. The results for the Group I tests are shown in Figure 53. The hatched area denotes the area within which lie the results for the $\frac{1}{2}$ " jet (tests (b)). The two lines show the results for the 1" jet and the results for the $\frac{1}{2}$ " jet over a similar range of separation distances $6 \leq \frac{h}{b} \leq 11$. Figure 54 shows in a similar manner the results of the Group II tests (with the nozzle plates removed). In this case single curves were not appropriate and two curves were drawn enclosing the test data in each case. A superior form of correlation for these latter tests is suggested later.

In all of the tests the 1" jet showed a better collapse of the data than did the $\frac{1}{2}$ " jet. Generally the agreement was most satisfactory at all separation distances tested for $\frac{x}{\delta} > 1.5$ but the agreement was not so good nearer to the axis.

Further comment is not necessary in the outer region of the distribution where the agreement was good but some remarks concerning the central portion are required. The differences between the results could be typified by the differences in the maximum values of the pressure ratio $\frac{R}{R_j}$. Figures 55 and 56 show the variation of this ratio $\frac{R_m}{R_j}$ with separation distance. It is apparent that the behaviour was completely different when the nozzle plates were removed from when they were present. (Group I different from Group II).

The tests with the 1" jet with the nozzle plate in position showed a constant relationship between the maximum impingement pressure and the free jet pitot pressure. The results for the $\frac{1}{2}$ " jet over the same range of separation distance ($6 \leq \frac{h}{b} \leq 11$) showed a similar trend but were not of the same high uniformity as the former jet. The mean value of the ratio of pressures was 0.5 approximately in the $\frac{1}{2}$ " jet and 0.6 approximately for the 1" jet. At a larger separation distances which were possible only in the narrow jet, the relationship between the pressure ratio and the separation distance appeared to change. The ratio $\frac{R_m}{R_j}$ showed an increase with separation distance for $\frac{h}{b} \geq 12$.

It would seem therefore that within the range of separation distances $6 \leq \frac{h}{b} \leq 11$ a reasonable prediction of the pressure distribution could be obtained from free jet data, either measured or assumed, together with a factor (0.5 $\rightarrow$ 0.6) for the magnitude of the peak pressure. However some justification would be necessary in respect of the 'factor' (0.5 $\rightarrow$ 0.6).

In section 5 (vi) above it was suggested that the differences between the maximum impingement pressure and the free jet pressure could be attributed to disturbances in the jet originating at the nozzle lip. This idea requires a little elaboration. Consider first the overall difference between the free jet results and the impingement results, the differences between the 1" jet and the $\frac{1}{2}$ " will then follow.

Johannesen (34, figure 20) demonstrated very large differences in the decay rate between the two jets for which comprehensive results were presented. An example of the differences obtained is that at a position 10 nozzle diameters downstream from the nozzle exit the velocity ratios $\frac{U_m}{U_j}$ were 0.91 in the uniform jet and 0.53 in the disturbed jet. (The pressure ratios equivalent to these velocities were $\frac{P}{P_0} = 0.83$ and

0.45 respectively). It is possible therefore that disturbances within a jet could modify the jet structure upstream of the impingement area sufficiently to account for the differences between the impingement pressures and the jet pressures. A study of the lip pressures on the nozzle plate (Figure 57) showed that for separation distances $6 \leq \frac{h}{b}$ the lip pressures were below the jet exit pressure and hence lip expansions were present at the nozzle exit. (The reasons that the pressures were different between the 1" jet and the $\frac{1}{2}$ " jet will be discussed later). This lip pressure deficit decreased with separation distance and hence at large separation distances the disturbances should be small and the impingement pressure should tend towards the free jet pressure. This trend was certainly present in the results for the $\frac{1}{2}$ " jet. In fact a force balance (see Figure 58) of the impingement region including the induced flow properties and the shear stresses indicate that the maximum impingement pressure should be slightly higher than the jet stagnation pressure. This force balance introduces some of the properties that were different between the 1" jet and the $\frac{1}{2}$ " jet. There were two major differences between the test configurations. The first difference was the aspect ratio of the jet itself, the aspect ratio of the 1" jet (tests (a)) was 4 whilst that for the $\frac{1}{2}$ " jet was 8. The second difference was in the size of the impingement plate and nozzle plates relative to the nozzle exit height. The plates for the tests (b) were effectively twice the size for those of tests (a). This latter difference should contribute to differences in the induced flow along the nozzle plate and to the flow along the impingement plate at large distances from the axis ($\frac{x}{b}$ large). This proved to be the case, the lip pressures of tests (b) were lower than those of tests (a) and the impingement plate loads discussed later (Section 7 (iv)) confirmed the second effect. The lower lip pressures of tests (b) inferred

larger lip expansions and hence jet disturbances and then a greater decay rate in the jet. The impingement pressures in tests (b) were lower than those in tests (a) and confirmed this suggestion. The effects of the jet aspect ratio should manifest themselves as changes in the 'end' or 'edge effects'. The only demonstration of any such effects, except as mentioned previously at $\frac{h}{b} = 18$, was at $\frac{h}{b} \approx 8$ in tests (b). At this position a distinct sensitivity to the separation distance was noted. It is at this position that any transverse disturbance originating at the nozzle exit would reach the opposite side wall. In tests (a) this position was at $\frac{h}{b} \approx 4$ and these effects could not be separated from others that were present. Alternatively this occurrence could have been similar to the sensitivity noted by Henderson (31), and attributed to a Hartmann generator phenomenon, since the general noise level of the flow was very high at this particular setting.

The tests (c) were intended to give further information on the effects of the two geometric variables. Only two separation distances within the range of interest of this section were used $\frac{h}{b} = 10$ and 6. At $\frac{h}{b} = 10$ the results of tests (a) and (c) showed extremely close agreement in all respects as was expected. At $\frac{h}{b} = 6$ the agreement was not so good and it was possible that the end effects observed at around $\frac{h}{b} = 8$ were of some significance here. At larger separation distances the effects of the nozzle plates in constraining the induced flow into the jet and producing low lip pressures decreased. The flow conditions will then be comparable with a free jet impingement process. The induced flow will still be present but the velocities along the nozzle plate will be lower and hence the lip pressure deficit will be less. Therefore the effects of the induced flow upon the impingement pressure will be reduced. This last step takes one logically to consideration

of the results of the tests when the nozzle plates were removed from the nozzle exit. The removal of these plates should lead to a similar situation to that existing at the larger separation distances in the previous tests. It must be pointed out that the remaining support structure for the nozzle still presented some obstruction to the induced flow. The effective size of this obstruction was at least $\pm 2\frac{1}{4} b$ for the 1" jet and $\pm 4\frac{1}{2} b$ in the $\frac{1}{2}$ " jet.

The maximum impingement pressure will once again be used as the criterion for studying the agreement within the tests. Figure 56 shows the variation of pressure ratio with separation distances for the tests of Group II. The results show an increase in pressure ratio $\frac{R_m}{R_j}$ with separation distance for both jets as did the tests (b) at the larger separation distances. This relationship with separation distance can be shown to correspond roughly to a movement of the effective origin of the jets, backwards along the jet axis. Two curves are shown on Figure 56 in which the origins of the jets have been moved to give agreement with the experimental results. The required movement was larger for the narrow jet as would be expected from the larger effective size of the support structure. This movement of the origin also effects the width of the jet and Figure 59 shows the pressure distributions of tests (e) on the basis of the new jet origin. The collapse of the profiles was superior to the previous method and also produced a maximum value of unity approximately.

It has been possible to reduce the experimental data for pressure distributions to single non-dimensional curves for each of the test configurations. It would have been desirable to predict the magnitudes of the movements of the origins in the latter tests (Group II) and to predict the magnitude of the initial pressure ratio in the previous

cases (Group I). The current data was insufficient for any general prediction since a detailed knowledge of the jet behaviour with internal disturbances would be required.

As a final step in the consideration of the results of the tests (e) it is appropriate to compare them with those of previous investigations. The particular previous work with two-dimensional or rectangular jets was that by Milne-Thomson (41) and Cartwright and Russell (13). The inviscid incompressible low speed jet was studied by Milne-Thomson and a pressure distribution computed from his solution is shown in Figure 59. The relationship between the uniform jet width (k) of the original solution and the characteristic jet width δ was calculated from assumptions of a typical turbulent jet velocity profile and an equality of jet thrust. From a velocity distribution

$$\frac{U}{U_m} = \exp(-0.6932 \eta^2) \quad (\text{where } \eta = \frac{y}{\delta})$$

the calculated relationship was

$$\delta = 0.665 k.$$

The experimental results of Cartwright and Russell were summarised by an exponential pressure distribution related to the separation distance

$$\frac{R}{R_m} = \exp(-48.2 (\frac{x}{h})^2). \quad \text{This expression was}$$

rewritten in the form

$\frac{R}{R_m} = \exp(-A(\frac{x}{\delta})^2)$ and has been plotted in Figure 59 as the two curves a) and b). Curve a) is the result of assuming a typical rate of spread for the free jet $\frac{\delta}{x_j} = \frac{\delta}{h} = 0.109$. Curve b) is the result of assuming a thrust from the profile equal to the thrust from the potential flow profile. The rate of spread for this result was $\frac{\delta}{h} = 0.085$ which is a rather low value for a low speed high aspect

ratio jet.

The experimental results for tests (e) with the modified origin showed good agreement with the results from these two references.

7 (iii) Pressure Distributions for $\frac{h}{b} \leq 5$

The correlation of the far field region pressure measurements was performed on the basis of previously established characteristics of the jets. A similar type of correlation was not possible in the near field region. Furthermore the lip effects at the nozzle exit were sufficiently severe to make a correlation on the basis of the nozzle exit height and the nozzle stagnation pressure inappropriate. Since the characteristic parameters previously established were not appropriate it was decided to correlate the pressure distributions on the basis of the maximum impingement pressure and the sonic position on the plate. Both of these latter characteristics were determined from the experimental results. Figures 60 show the results of correlating the experimental data on the basis proposed for separation distances $0.45 \leq \frac{h}{b} \leq 5$. The distributions at pressures above the ambient value all showed excellent agreement. Some divergence in the results was apparent which changes in separation distance for the sub-ambient pressure section of the distribution.

Once again correlation was not attempted beyond the central impingement region. In this case the reason for restricting the comparison was that the alternating compression and expansion region was sufficiently unpredictable to render pointless any comparison in that region. Unfortunately this type of reduction of pressure distribution is suitable only as a means of presenting data and does not really assist in predictions. Whilst it would be possible to determine

empirical rules from the current tests to enable one to 'predict' the lip pressure and hence the centre-line Mach Number distribution and hence the maximum impingement pressure for identical situations this would not be especially useful. Too many arbitrary constants would be required and the geometry of the tests was not sufficiently specific for any general application of the results to be successful.

It still remains therefore to determine a comprehensive knowledge of the induced flow phenomenon and hence a means of predicting the impingement region pressure distribution.

7 (iv) Plate Loads

The final step in the reduction of the experimental data on pressure distribution was the computation of the total load upon the impingement plate, and nozzle plate where appropriate, at each separation distance. It is well known that when a low speed jet impinges upon a plane surface and the emergent air flows away between two plates there occurs a reversal of the force on the plate at certain separation distances. Artingstall (5) showed that this was also true for supersonic axisymmetric flow and the tests reported here showed that it can also occur in two-dimensional flow.

The thrust of a supersonic jet at a particular Mach Number discharging at the 'correct expansion' pressure can be related uniquely to the stagnation pressure of the flow. The Mach Number chosen for these tests was 1.4 at which value the relationship between ρ_1^2 and P_0 attains a maximum. (For variations of $\pm 10\%$ in Mach Number the variations in the ratio $\frac{\rho_1^2}{P_0}$ amount to $\approx 2\%$ only). The thrust of the jet could therefore be predicted accurately and this was the characteristic force in the phenomenon. All estimated forces were therefore related to the thrust of the jet (J').

The detailed pressure measurements enabled a close examination of the plate loads to be made. The computation of these loads was made by using a trapezoidal rule treatment between the experimental points. The results of the calculations are shown in Figures 61 a) b) and c) where forces in the direction of the jet were deemed positive.

Figures 61 a) and b) relate to the tests of Group I with the nozzle plates in position and Figures 61 c) relate to the Group II tests.

Considering first the tests of Group I, it will be seen that the relationship between the force on the impingement plate $\frac{J}{J_1}$ and the separation distance was different in each case. It is necessary therefore to explore the reasons for these differences. The considerations of the previous sections (7 (ii) and (iii)) showed good agreement for the results in the immediate impingement region. In each case the impingement region of the plate carried a force, in the direction of the jet, of the same order of magnitude as the jet thrust. The direction of the total force on the impingement plate was therefore determined mainly by the pressure distribution outside the impingement region. (This latter region gave rise to a force in the direction opposite to that of the jet efflux). Figures 62 show the rate of growth of the plate forces with outward movement from the jet axis. The results for the jets are compared at two separation distances. At $\frac{h}{b} = 10$ and $\frac{h}{b} = 6$ the tests for a) and c) showed extremely close agreement both in the ultimate forces and in their distributions on the impingement plate. The general characteristic was that of the jet thrust being concentrated in the impingement region and an increasing negative force as the distance from the axis increased. The force on the nozzle plates showed an almost linear

increase with distance from the nozzle lip. The results for tests (b) at these two separation distances showed the effects of the relatively larger plates. The negative pressures on the plates were larger and led to a more rapid growth of force on the nozzle plate and a more rapid decrease of force on the impingement plate.

At smaller separation distances the main characteristics of the results were similar to those described but with larger forces carried on the outer parts of the plates. Some agreement was shown between the tests (b) and (c) but this was not systematic.

A study of the overall results for the 1" jet (tests (a)) showed rapid changes in the forces at the transition to duct flow between the plates ($\frac{h}{b} = 1.1$) and also at the transition to subsonic flow through the nozzle $\frac{h}{b} = 0.65$. These results confirmed the divisions made previously on the basis of inspection of the pressure distribution. The change of direction of the force on the plates was not accompanied by any changes in the general flow pattern. The sequence of the variation of the impingement plate load with separation distance was precisely similar to that described by Willis (68) for low speed jets. The results for the tests (b) showed similar tendencies but were insufficient at these small separation distances to warrant such close scrutiny.

In all of the tests in Group I the total force was greater than the nominal jet thrust by 10% approximately. This was unexpected and no single explanation is offered for this result. Instead it is suggested that it can be due only to an accumulation of several effects. The possible sources for this result were as follows.

- (i) Errors in the pressure measurements.
- (ii) Errors in the computation of the load.

- (iii) Small excess pressure at the jet exit.
- (iv) Non-uniformities across the width of the jet at the nozzle exit.
- (v) Uneven ambient pressure on the reverse sides of the impingement plate and the back plate.
- (vi) Axial components of the momentum of the outward impinging flow at the plate edge, due to the growth along the plate.
- (vii) Axial components of the momentum of the induced air at the plate edge.
- (viii) Three-dimensional effects resulting in the centre-line pressures being higher than the mean.

Care had been taken to reduce the first three possibilities to very small magnitudes and it is felt that this had been achieved. The fourth possibility had been investigated with the free jet before any tests were performed and was shown to be immeasurable except for the thin wall boundary layer. The uneven ambient pressure (v) was of unknown magnitude but was certainly not sufficient to contribute all of the augmentation. The two possibilities (vi) and (vii) were rather problematical and it would seem that these were perhaps the most likely sources for small amounts of augmentation. The final possibility (viii) must also be considered and if the effects were severe it could easily account for the variation. However, it is unlikely that these three dimensional affects would be independent of separation distance, and the augmentation was present at small and large separation distances. Also the tests quoted earlier showed that the effect was quite small at the particular distances tested. Therefore no single one of these possibilities was thought to be large enough alone to yield the total amount of the augmentation.

The results of the tests with the back plates removed (Group II)

showed that at the smaller separation distances ($\frac{h}{b} \leq 6$) some load was carried by the supporting structure (Figure 61 (c)). This result was consistent with the earlier results for the impingement region (see Section 7 (ii)) which suggested that at $\frac{h}{b} > 6$ the phenomenon was one of free jet impingement but with the origin of the jet changed. Only in this last category was it possible to predict the force on the impingement plate. With the closer proximity of the impingement plate to the nozzle and the installation of back plates, the effects of the size of the plates and the complexity of the flow within the gap made general prediction impossible.

In general the results of the calculation of the plate loads were remarkably consistent. It can only be remarked from these tests that the occurrence of any reversal of load will be dictated by the shape and size of the duct surrounding the impingement region. The nature of the jet whilst important for the detail of the results, can have a small effect only upon this parameter.

Chapter 8

A TIME-DEPENDENT NUMERICAL SOLUTION TO THE FLOW-FIELD OF AN IMPINGING SUPERSONIC JET

Several authors have used time-dependent methods to predict steady-state solutions to mixed subsonic supersonic two dimensional flow fields. Sinha, Zakkay and Erdos (62) reported a solution of this type to the problem of a supersonic jet impinging upon a plate. After some correspondence, Professor Zakkay kindly ran a brief solution to a case comparable with the current work. The results from this computation are presented later and compared with the experimental data. A brief outline of the principles of the time dependent method of solution is given first together with some comments upon its associated merits and problems.

The particular procedure used by Zakkay was that the equations of unsteady two-dimensional inviscid fluid motion, were written in finite difference form. These finite difference equations were modified to include a stability term often called a 'pseudo-viscosity' term. The flow field was described by a close net mesh in x and y (square mesh) and initial conditions at time $t = t_0$ were prescribed. Steady state boundary conditions were also established. Commencing from the initial conditions a two step Lax-Wendroff method in time was used to determine conditions at time $t_0 + \Delta t$ from which the solution marched onwards to an asymptotic state at large t which was deemed to be the steady state solution. Details of the general Lax-Wendroff method are given in reference 50 and the origins of the method were reported in references 38 and 39.

Whilst the eventual results of this type of computation have often been most impressive there are many problems associated with the procedure. In the main these problems are posed by the stability of the solution, the boundary conditions of the flow field and the capacity of the computer. The stability of the solution is often determined by the accumulation of errors from previous steps in the solution and also the ability to converge onto a particular solution. This stability is notoriously sensitive to the size of the net and to the size of the time step. The parameter $\bar{\alpha}$ in the method of reference 62 was included to damp out oscillations in time. Guidance can be obtained as to the order of magnitude of the step sizes but eventually a trial of the solution must be made. The statement of the boundary conditions often poses another problem. The well known reflection techniques were used by Sinha, ^rExdos and Zakkey in their case but these are not universally applicable. Also on occasions the problem can become over prescribed by too many boundary conditions. Finally the computer capacity required for the solutions is always large and is often beyond the scope of the more usual installations. To give an idea of the scale of the capacity required by solutions of this type the requirements of the solution quoted by Sinha et al could be taken as typical. The net in x and y in the rectangular flow field was 60 x 56 and to attain the asymptotic solution a computing time of $3\frac{1}{2}$ hours approximately was required on a CDC 6600 computer. Steady state conditions in that solution were attained after 1500 steps in time. The more limited solution quoted below was performed on the same computer and with a 26 x 42 net in x and y ran for $5\frac{1}{2}$ minutes approximately, for 400 steps in time.

It is apparent therefore that a successful solution by a time dependent method requires considerable computer capacity and a long programme development time.

The particular case for which Professor Zakkay produced a limited solution was for a jet Mach Number $M_j = 1.4$ at a separation distance $\frac{h}{b} = 1.857$. This separation distance was chosen for convenience in the computation. The domain of the computation reached $\frac{x}{b} = 3$ and the physical mesh size was $\frac{b}{14}$. Two calculations were performed using different values for the stability parameter δ (0.25 & 0.80). The results from the computation were predictions for the pressure distribution on the impingement plate and the Mach number distribution along the jet axis. These results are reproduced in Figures 63 & 64. The results for the Mach Number distribution show the shock wave spread over several mesh points with a larger spread associated with the higher value of the stability parameter. In general the centre line shock height prediction ($\frac{y_s}{b} = 0.75 \rightarrow 1.05$ say 0.9 mean) showed good agreement with the experimental results ($\frac{y_s}{b} = 0.85$ at $\frac{h}{b} = 1.8$ and $\frac{y_s}{b} = 0.91$ at $\frac{h}{b} = 1.9$). Figure 65 shows the results for the impingement plate pressure distribution and also shows the experimental points for $\frac{h}{b} = 1.8$ and $\frac{h}{b} = 1.9$. It will be noticed that the initial agreement was very good, however, the computed results did not show the recompression at $\frac{x}{b} \approx 2.5$. This highlights one of the problems of the present solution in terms of the boundary condition. It was necessary in this limited solution to specify zero pressure at the jet boundary, therefore no recompression was required along the impingement plate. The reason for this requirement was not explained and a study of the initial equations and conditions did not indicate the necessity for such a condition.

In conclusion the results of this type of numerical approach to the flow field prediction were most encouraging.

PART V

CONCLUSION

CHAPTER 9.

CONCLUDING REMARKS

The characteristics of a correctly expanded rectangular jet of air at a Mach Number of 1.4 have been investigated as a preliminary stage to a study of jet impingement phenomena. Two aspect ratios have been used for the jet and both cases exhibited similar characteristics to those of low speed jets. The mixing region and the fully developed region were investigated. The results for the mixing region showed close agreement with published results and predictions for the Mach Number profile when a value $\sigma = 17$ was used for the free scaling constant. This value was almost identical with the value obtained by previous investigations in plane high speed jets and indicated a slower rate of spread than is usual for low speed jets ($11 \leq \sigma \leq 12$ for plane low speed jets). The results in the fully turbulent region also showed good agreement with the results of other investigations in plane low speed jets, no comparable results in plane high speed jets are available. Again the rates of spread of the jet and of the decay of the centre-line Mach Number were slightly lower than the low speed results.

The studies of the impinging jet showed the expected characteristics and that the shape of the shock wave at small separation distances was very sensitive to jet disturbances. A qualitative study of the shock wave equations presented an explanation for this sensitivity which was in agreement with the experimental shock wave shapes. A further investigation into the shape of the shock wave was performed using 'blunt body' numerical procedure. Some spot experiments with

different jet Mach Numbers were performed to give data over a range of Mach Numbers. At the higher values the numerical treatment gave a good prediction of the centre-line shock-height and the impingement plate pressure distribution in the central area. The prediction at the lower Mach Numbers was not as good and a lower Mach Number limit of 2 approximately would seem to be necessary for worthwhile predictions. The variations in shock wave shape were due mainly to disturbances in the jet and the effects of the mixing layers at the jet edges. Neither of these two effects were present in the numerical model. The lower Mach Number limit in this case was below that required by another author in the axisymmetric case. An accuracy of prediction similar to that obtained here at $M = 2.0$ was not obtained until a Mach Number of 2.4 was used.

The pressure distributions on the impingement plate in the immediate impingement region at large separation distances ($\frac{h}{b} \geq 6$) were shown to correlate on the basis of conditions in the free jet. This method had been shown to apply to low speed jets by Bradbury. In the current tests some variation was noted in the peak impingement pressures which was ascribed to changes in the decay rate of the jet. A special series of tests with only the supporting structure surrounding the jet showed good correlation. This series of tests indicated that the effect of small structural members around the nozzle could be represented by a movement of the effective origin of the jet. A comparison between these correlated results and the potential flow analysis of Milne-Thomson showed close agreement. At small separation distances ($\frac{h}{b} \leq 5$) the impingement region pressure distribution results were shown to correlate well on the basis of the maximum pressure and the plate sonic point.

The total loads on the impingement plate and on the nozzle plate were also investigated over the entire range of the tests. The tests showed that when the flow through the nozzle was supersonic the force on the nozzle plate was in the direction of the jet efflux. The force on the impingement plate, however, showed the reversal of direction that is well known in low speed flows. The separation distance at which this load reversal occurred was shown to be dependent upon the geometry of the plates surrounding the jet although no statement of the form of the dependency was possible.

Finally it must be recorded that three-dimensional or edge-effects were present in all of these tests and consequently the results of the work must be treated with due caution. However, in some sections of the investigations it was clear that these effects were small. In other cases where the effects were not quantifiable the tests at the two jet aspect ratios gave some indications of the trends due to these effects.

Suggestions for Future Work

In the course of the investigations several aspects became apparent which warrant further investigation.

A full understanding of the induced flow mechanism and its effects is required. Two forms of investigation would be appropriate. Firstly a close investigation of the effects of internal disturbances in the jet should be made. A very careful nozzle design to produce a jet without disturbances should be undertaken. Then the nozzle should be

tested at various 'off design' pressure ratios to establish the effects upon the rate of spread and the decay rate of the jet. The range of pressure ratios to be tested could be suggested from the results of the current work (e.g. nozzle exit pressure from ambient to -0.2 atmos.).

A second investigation using a large aspect ratio jet and with varying sizes of nozzle plates would also provide useful information. With small nozzle plates and a detailed study of the induced flow some rationalisation of the numerous effects present might be possible.

These two types of investigation would throw further light on a complex phenomenon.

There remains one obvious extension of the current work. It has been demonstrated that a time-dependent numerical solution to the problem is possible. The development of a computer programme of this type would be most useful.

APPENDIX I

THE SCALE OF THE BLUNT BODY SOLUTION TO THE IMPINGING JET

a) The Shape of the Sonic Line

In the general set of coordinates shown the slope of the sonic line is given by:

$$\tan \omega_s = \frac{dy}{dx} \quad \dots\dots 1$$

Now for a perfect gas in adiabatic flow:

$$\frac{dq}{q} = \frac{1}{1 + \frac{\gamma-1}{2} M^2} \cdot \frac{dM}{M} \quad \dots\dots 2$$

Hence along the sonic line since $dM = 0$ then:

$$dq = 0 \quad \dots\dots 3$$

In general

$$dq = \frac{\partial q}{\partial x} dx + \frac{\partial q}{\partial y} dy$$

$$\therefore \frac{\partial q}{\partial x} dx = - \frac{\partial q}{\partial y} dy$$

$$\left[\frac{dy}{dx} \right]_{SON} = - \left[\frac{\frac{\partial q}{\partial x}}{\frac{\partial q}{\partial y}} \right]_{SON} \dots\dots 4$$

$$\therefore \tan \omega_s = - \left[\frac{\frac{\partial q}{\partial x}}{\frac{\partial q}{\partial y}} \right] \dots\dots 5$$

Consider now the sonic line at the impingement surface.

At this point $\frac{\partial q}{\partial x} > 0 \dots\dots 6$

To evaluate $\frac{\partial q}{\partial y}$ we must consider the vorticity at the surface.

In general $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

On the surface $v = 0 \therefore \frac{\partial v}{\partial x} = 0$

and $u = q \therefore \frac{\partial u}{\partial y} = \frac{\partial q}{\partial y}$

$$\therefore \zeta = - \frac{\partial q}{\partial y} \dots\dots 7$$

Crocco's vorticity law states that for a streamline in the x direction:

$$\zeta = \frac{1}{q} \left[T \frac{\partial S}{\partial y} - \frac{\partial h_o}{\partial y} \right] \dots\dots 8$$

For adiabatic flow

$$dh_o = 0$$

$$\therefore \frac{\partial h_o}{\partial y} dy + \frac{\partial h_o}{\partial x} dx = 0$$

Along the surface $\frac{\partial h_o}{\partial x} = 0 \quad \therefore \frac{\partial h_o}{\partial y} = 0$

$$\therefore S = \frac{1}{\rho} \left[T \frac{\partial S}{\partial y} \right] \quad \dots\dots 9$$

Within the impingement region we assume constant entropy along the streamlines. Since for a given upstream Mach Number the largest change of entropy occurs under normal shock conditions the centre-line entropy will be larger than the next stream line outwards.

This condition will be continued to and along the impingement ^{ing} surface: i.e..

$$S_{y=0} \geq S_{y=\Delta y} \quad \dots\dots 10$$

$$\therefore \frac{\partial S}{\partial y} \leq 0$$

$$\text{from 9 \& 7 } \frac{\partial q}{\partial y} = - S_{y=0} = - \frac{1}{\rho} \left[T (\leq 0) \right]$$

$$\therefore \frac{\partial q}{\partial y} \geq 0 \quad \dots\dots 11$$

Hence from 5, 6, & 11

$$\tan \omega_s = - \left[\frac{\geq 0}{\geq 0} \right] \leq 0$$

$\dots\dots 12$

$$\therefore \underline{\underline{\omega_s \geq \frac{\pi}{2}}}$$

Hence on the impingement surface the sonic line is either perpendicular to the surface or inclined towards the subsonic region (NB. As $M \rightarrow 1$, $\frac{\partial S}{\partial y} \rightarrow 0$ & $\omega_s \rightarrow \frac{\pi}{2}$)

Consider now the sonic line at the shock wave, the slope will again be given by equation 5, However, an alternative approach is possible. The curves of Figure A1 obtained from data presented by Hayes & Probst (30 page 396) show the various angles associated with sonic conditions on an oblique shock wave. It is clear that for Mach Numbers less than 2.9 approximately in air the sonic line leaves the shock in the direction of the subsonic region and for higher Mach Numbers the sonic line is inclined initially towards the supersonic region.

Hence for the Mach Number range of the current work the inclination of the sonic line at its extreme ends are as follows:

a) At the impingement surface:

Perpendicular to the surface or inclined towards the subsonic region.

b) At the shock wave:

Inclined towards the subsonic regions.

b Location of the Shock Sonic Point and the Width of the Jet. (After Gummer and Hunt (27))

Since there is a shock wave spanning the jet this shock wave will either weaken from a normal shock on the axis to a Mach Wave at the edge of the jet, or the shock will intersect the jet edge. If the former were to occur then the jet width would not impart a scale to the problem. Since a scale is essential it may be concluded that the shock wave intersects the jet edge.

It is apparent that at the junction of the shock wave and the jet edge an instantaneous expansion must take place to attain the free boundary pressure on the downstream jet edge. Such an expansion cannot take place at a free point in subsonic flow. However, a centred expansion in supersonic flow produces that effect. The sonic point on the shock wave must therefore lie at either jet edge or within the jet.

For supersonic Mach Numbers near unity the sonic line is inclined at a small angle to the shock wave, and is directed towards the subsonic region. At these Mach Numbers the shock curvature is relatively small and therefore the shock layer is almost homentropic. The edge expansion fan intersects the sonic line, and in homentropic flow, will be reflected as compression waves. Repeated reflections will then take place between the free boundary and the sonic line until an expansion wave reaches the impingement surface. If the sonic point on the shock waves were to terminate on the shock wave inboard of the jet edge the shock wave would weaken towards the jet edge. The weakening could only be caused by expansion waves which in turn could only arise from the sonic line. In homentropic flow only compression waves may travel away from the sonic line into the supersonic region. The sonic line and the shock wave must intersect at the jet edge for Mach Numbers near to unity. The curves of Figure A1 show that in air for $M \leq 1.7$ the sonic lines leaves the sonic point on the shock wave at an inclination greater than that of the first characteristics from that point. Hence for $1.0 \leq M \leq 1.7$ the sonic point on the shock wave will be coincident with the intersection of the shock wave with the jet edge.

With increasing Mach Number the shock layer will depart from homentropic flow conditions and then the sonic line can give rise to both compression and expansion waves (See Hunt, Aero Quarterly pp Vol 1972).

When the Mach Number is larger than 1.7 the first characteristics lies in advance of sonic line and will therefore meet it at the sonic point on the shock wave.

If the edge expansion were still to originate at the sonic point, it and a number of the subsequent reflections would be compressed into a single point. Gummer & Hunt suggest that this is rather unlikely and that the sonic point moves inwards along the shock wave.

For Mach Numbers greater than 1.7 approximately the sonic point on the shock-wave lies within the jet. An alternative definition for the scale of the problem is required.

The equation for velocity along the impingement plate becomes singular at the sonic point. Blunt Body solutions for smooth body contours use a regularity condition at this point to determine the centre-line shock height. If however the blunt body has a sharp corner the velocity will change discontinuously at the corner and the singularity will indicate that the corner has been reached. This would be the case for a flat disc or plate. Returning to the impinging jet, the velocity on the surface must be continuous, also the sonic point on the surface must lie at or outside the line of the jet otherwise a sonic point might occur on a flat disc before the corner. (The disc would then lose its scale). Gummer & Hunt suggested therefore that the disc width and the jet width were analogous and that the sonic point on the impingement surface could be placed at the jet edge at these higher Mach Numbers.

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(N.B. Initial Jet Width = Nozzle Exit Height)

. Figure 1 The General Features of the Impingement
 Flow Field.

Figure 2. General Features of a Free Jet Flow.

Figure 3 General Arrangement of the Apparatus.

Figure 4. General Arrangement of the Nozzle and Working Section.

(a) A General View of the Apparatus.

(b) A View with the Window and the Side Plates removed and showing the 12" Nozzle Plates.

Figure 5. Photographs of the Apparatus.

(c) A Side View showing the Nozzle Profile and the Plates (overall Plate Height 12").

(d) A View showing the Nozzle Exit with the Nozzle Plates removed and with the 24" Impingement Plate.

Figure 6. The Dimensions of the Throat Block and the Nozzle

Hole No.	Position
0	Centre-Line
1 & 1'	+ 0.1
2 & 2'	- 0.1
3 & 3'	+ 0.2
4 & 4'	- 0.3
5 & 5'	+ 0.4
6 & 6'	- 0.5
7 & 7'	+ 0.6
8 & 8'	- 0.7
9 & 9'	+ 0.8
10 & 10'	- 0.9
11	+ 1.0
12	- 1.1
13	+ 1.2
14	- 1.3
15	+ 1.5
16	- 1.75
17	+ 2.0
18	- 2.25
18A	- 2.75
19	+ 2.5
20	- 3.5
21	+ 3.0
21A	+ 3.25
22	- 5.0
23	+ 4.0
24	- 7.0
25	+ 6.0
26	- 9.0
27	+ 8.0
28	-11.0
29	+10.0

Figure 7a. Pressure Tapping Positions on the Impingement Plate

Hole No.	Position, inches above/below centre	
	1" jet	1/2" jet
1 & 2	1.1	0.85
3 & 4	1.25	1.00
5 & 6	1.5	1.25
7 & 8	1.75	1.50
9 & 10	2.0	1.75
11 & 12	2.25	2.00
13 & 14	2.5	2.25
15 & 16	3.0	2.75
17 & 18	3.5	3.25
19 & 20	4.25	4.00
21 & 22	5.5	5.25
23 & 24	6.5	6.25
25 & 26	8.5	8.25
27 & 28	10.5	10.25

Figure 7 b.

Pressure Tapping Positions
on the Nozzle Plates.

Figure 8. The Pressure Switch.

Figure 9. Free Jet Mach Number Traverse, Comparison of Probes.

Figure 10. Free Jet Mach Number Traverse, Asymmetry of the Profile. (At position $\frac{x}{b}j = 8.5$)

Figure 11. Free Jet Mach Number Traverse, Centre-line and Off-centre-line Results at $\frac{x}{b} = 8.5$

Figure 12. Free Jet Centre-line Mach Number Distribution

Figure 13. Free Jet Composite
Mach Number Distribution
Diagram 1'' Jet.

Figure 14a. Free Jet, Mixing Region Width 1" Jet

Figure 14 b. Free Jet, Mixing Region Width $\frac{1}{2}$ " Jet

Figure 15 a. Free Jet Mixing Region, Mach Number Profiles across the Region 1'' Jet.

Figure 15 b. Free Jet Mixing Region, Mach Number Profiles across the Region $\frac{1}{2}$ '' Jet

Figure 16. Free Jet, Fully Developed Jet Width

Figure 17 a. Free Jet Fully Developed Region, Mach Number Profiles across the Jet 1'' Jet

Figure 17 b. Free Jet Fully Developed Region, Mach Number Profiles across the Jet, $\frac{1}{2}$ '' Jet

Figure 18. Free Jet Fully Developed Region, Comparison of Mach Number Profiles.

(a)
Schlieren Photographs
with a Knife-edge
cut-off.

(b)
Shadowgraph Photograph.

Figure 19. Flow Visualisation of the Free Jet.

(a) Physical Plane.

(b) Hodograph Plane.

Figure 20 The Structure of the Impingement Region.

Figure 21.

Schlieren Photograph
with a Graded Filter.

Separation Distance

$$\frac{h}{b} = 3.0 .$$

Cut-off parallel to
the plates .

Figure 22.

Schlieren Photographs
with a Graded Filter.

Separation Distance

$$\frac{h}{b} = 2.0 .$$

Cut-off parallel to the
plates .

Figure 23 .

Schlieren Photograph
with a Graded Filter.
Separation Distance

$$\frac{h}{b} = 1.6 .$$

Cut-off perpendicular
to the plates .

Figure 24 .

Shadowgraph Photograph.

Separation Distance

$$\frac{h}{b} = 1.3 .$$

Figure 25 .

Shadowgraph Photograph.

Separation Distance

$$\frac{h}{b} = 1.1 .$$

Figure 26 .

Shadowgraph Photograph.

Separation Distance

$$\frac{h}{b} = 0.9 .$$

Figure 27. Idealised Wave Diagram for a Slightly Under-expanded Supersonic Jet

Figure 28. Jet Disturbances due to the Nozzle Lip Expansions.

Figure 29. Hodograph Diagrams of the Shock Modification
by the Nozzle Lip Expansions.

p_{o1} = Isentropic stagnation pressure upstream of the wave

p_{o2} = Isentropic stagnation pressure downstream of the wave

Figure 30. Geometry of the Oblique Shock Wave.

(b) Hodograph Plane.

Figure 31 . Weak Shock Interactions.

Figure 32 a. Oblique Shock Wave Derivative for various Mach Numbers

$$\left| \frac{d\delta_1}{dM_1} \right|_{\bar{p}} \quad \text{v} \quad \delta_1 \text{ (degrees)}$$

Figure 32 b. Oblique Shock Wave Derivative

$$\left| \frac{\partial \zeta_1}{\partial M_1} \right| \text{ v } \zeta_1 \text{ (degrees)}$$

Figure 33. Oblique Shock Wave Derivative for various Wave Angles
 $\left| \frac{dS_1}{dM_1} \right|$ and Mach Wave Derivative $\frac{d\theta}{dM_1}$ v Mach Number M_1

Figure 34. Variation of the Maximum Impingement Pressure with Separation Distance

Figure 35. Variation of Centre-line Shock Height with Separation Distance.

Figure 36 a. Plate Pressure Distributions at Separation Distance $\frac{h}{b} = 1.5$

Impingement Plate

Nozzle Plate

Figure 36 b. Schlieren Photographs at a Separation Distance $\frac{h}{b} = 1.5$
(Graded Filter perpendicular to the Plates)

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Figure 37 a. Plate Pressure Distributions at Separation Distance $\frac{h}{b} = 1.0$

Figure 37 b. Schlieren Photographs at a Separation Distance $\frac{h}{b} = 1.0$

(Graded Filter perpendicular to the Plates)

Figure 38. Co-ordinate System for Chapter 6.

Figure 39. Numerical Procedure Critical Derivative $\frac{\partial(\bar{p}_u \bar{v})}{\partial \sigma}$ v Shock Angle σ for Several Mach Numbers.

(a)
Shock Shape

(b) Plate Pressure Distribution

Figure 40. Numerical Procedure Predictions $M_\infty = 1.4$

(b) Plate Pressure Distribution

Figure 41. Numerical Procedure Predictions $M_\infty = 1.58$

Figure 42. Numerical Procedure Predictions $M_\infty = 1.8$

(b) Plate Pressure Distributions

Figure 43. Numerical Procedure Predictions $M_\infty = 1.94$

(a) Shock Shape

(b) Plate Pressure Distributions

Figure 44. Numerical Procedure Predictions $M_\infty = 2.0$

(a)
Shock Shape

Figure 45. Numerical Procedure Predictions $M_\infty = 2.23$

Shadowgraph Photograph.

Nominal Jet Mach Number

$$M_j = 1.6 .$$

Separation Distance

$$\frac{h}{b} = 1.0 .$$

Shadowgraph Photograph.

Nominal Jet Mach Number

$$M_j = 1.8 .$$

Separation Distance

$$\frac{h}{b} = 1.0 .$$

Figure 46 a. Flow Visualisation Photographs for the cases used
in the Numerical Prediction Procedure.

Schlieren Photograph
with a Graded Filter.
Nominal Jet Mach Number

$$M_j = 1.4 .$$

Separation Distance

$$\frac{h}{b} = 1.8 .$$

Schlieren Photograph
with a Graded Filter.
Nominal Jet Mach Number

$$M_j = 1.6 .$$

Separation Distance

$$\frac{h}{b} = 2.0 .$$

Schlieren Photograph
with a Graded Filter.
Nominal Jet Mach Number

$$M_j = 1.8 .$$

Separation Distance

$$\frac{h}{b} = 2.0 .$$

Figure 46 b. Flow Visualisation Photographs for the cases used
in the Numerical Prediction Procedure .

Figure 47. Variation of the Impingement Plate Sonic Point with Separation Distance

Figure 48. Variation of the Impingement Plate Pressure Ratio ($\frac{P_{min}}{P_{max}}$) with Separation Distance

Figure 49. Pressure Distribution at a Separation distance $\frac{h}{b} = 2.9$

Centre- line and 'Off-axis' Comparison

Figure 50 a. Pressure Distribution 1" Jet at a Separation Distance $\frac{h}{b} = 0.45$.

Figure 50. b. Pressure Distribution upon the Plates 1" Jet at Separation Distance $\frac{h}{b} = 1.8$

Figure 50c. Pressure Distribution upon the Plates, 1" Jet at a Separation Distance $\frac{h}{b} = 2.4$

Figure 50 d. Pressure Distribution upon the Plates, 1" Jet at a Separation Distance $\frac{h}{b} = 8$

Figure 50.e. Pressure Distribution upon the Plates, $\frac{1}{2}$ " Jet at a Separation Distance $\frac{h}{b} = 8$

Figure 50 f. Pressure Distribution upon the Impingement Plate, with the Nozzle Plate removed, at $\frac{h}{b} = 8$

Figure 50 g. Pressure Distribution upon the Impingement Plate, with the Nozzle Plate removed, at $\frac{h}{b} = 8$

Figure 50 h. Pressure Distribution upon the Plates, $\frac{1}{2}$ " Jet at a Separation Distance

Figure 51 a. Impingement Plate Pressure Distributions
 Jet Width Correlated Distributions Group I 1" Jet

Figure 51 b. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Group I $\frac{1}{2}$ " Jet

Figure 51 c. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Group I at $\frac{h}{b} = 6$

Figure 51 d. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Group I at $\frac{h}{b} = 10$

Figure 52 a. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Group II 1.1 Jet

Figure 52 b. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Group II $\frac{1}{2}$ " Jet

Figure 53. Impingement Plate Pressure Distributions

Jet Width Correlated Distributions Summary of the Results

Group I

Figure 54. . Impingement Plate Pressure Distributions
 Jet Width Correlated Distributions Summary of the Results
 Group II

Figure 55. Variation of the Maximum Impingement Pressure Ratio with Separation Distance Group I

Figure 56. Variation of the Maximum Impingement Pressure Ratio with Separation Distance

Figure 57. Variation of the Nozzle Lip Pressure (Nozzle Plate Minimum Pressure) with Separation Distance

$$\sum_0^h p_i dy - \sum_0^h p_a dy = \sum_0^S \rho_i q_i^2 dy + \sum_S^h \rho_n q_n^2 dy$$

$$\sum_0^h R_L dy = \sum_0^S \rho_i q_i^2 dy + \sum_S^h \rho_n q_n^2 dy + \sum_0^x p \gamma_i dx - \sum_{\sum b}^x p \gamma_n dx.$$

Force Balance Perpendicular to the Jet Axis.

Figure 59. Impingement Plate Pressure Distributions

Jet Width Modified Correlated Distributions

Group II $\frac{1}{2}$ " Jet

Figure 60a. Impingement Plate Pressure Distributions
Sonic Width Correlated Distributions Group I
1" Jet at $0.45 \leq \frac{h}{b} \leq 1.1$

Figure 60 b. Impingement Plate Pressure Distributions

Sonic Width Correlated Distributions Group I

1" Jet at $1.5 \leq \frac{h}{b} \leq 5.0$

Figure 60 c. Impingement Plate Pressure Distributions
Sonic Width Correlated Distributions Group I

$$\frac{1}{2}'' \text{ Jet at } 1.25 \leq \frac{h}{b} \leq 5.0$$

Figure 60 d. Impingement Plate Pressure Distributions

Sonic Width Correlated Distributions Group I

at a Separation Distance $\frac{h}{b} = 2$

Figure 60e. Impingement Plate Pressure Distributions

Sonic Width Correlated Distributions Group I

at a Separation Distance $\frac{h}{b} = 4$

Figure 61 a. Total Plate Loads, 1'' Jet Tests (a).

Figure 61 b. Total Load upon the Impingement Plate, Group I Tests.

Figure 61 c. Total Load upon the Impingement Plate, Group II Tests.

Figure 62 a . Growth of the Impingement Plate Load along the Plate, Group I Tests at $\frac{h}{b} = 10$

Figure 62 b . Growth of the Impingement Plate Load along the Plate, Group I Tests at $\frac{h}{b} = 6$

Figure 63 . Time Dependent Method, Prediction of the Mach Number Distribution along the Jet Axis,

$$M_j = 1.4 \quad \frac{h}{b} = 1.857$$

Figure 64 • Time Dependent Method, Prediction of the Pressure Distribution upon the Impingement Plate.

Appendix I Figure A1. Shock Wave Angles for Sonic Conditions Downstream for Air ($\gamma = 1.4$)