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Interpolation methods for tracking spacecraft in ultratight formation

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Abstract. By measuring the centroid of a beam on a detector, one can track the movement of that beam across the detector. By tracking this movement, one can track the object encompassing the detector, for example, a spacecraft. A variety of system-specific performance inhibitors can make this a challenge, requiring a robust calibration method. The goal of this investigation is to model the true beam position of the instrument in terms of the measured beam position. For this, a mathematical model is created that interpolates and corrects the measured beam position using precollected position data—a “calibration model.” The real-world scenario for this investigation is the flight-representative model of the fine lateral and longitudinal sensor (FLLS) instrument, built by Neptec Design Group and Neptec UK for the European Space Agency mission PROBA-3. Performance inhibitors for FLLS are cropping of the beam, imperfect optics, and a varying distance the beam has traveled (up to 250 m). Using bivariate spline interpolation for the FLLS calibration model gives the best performance, achieving a measurement accuracy well within the mission requirement of <300 μm . © 2019 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JATIS.5.2.028003]

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1 Introduction

Calculating the position and movement of a beam centroid is a common requirement among many scientific disciplines and is a well-established practice.¹ Some wide-ranging applications of centroid measurement are star image analysis for star trackers and detectors,^{2,3} photon beam localization,⁴ tracking cloud movement,^{5,6} and monitoring plant health.⁷ In practice, centroid measurement is highly sensitive to the shape of the object being imaged, and the method must be tailored to the application.

This investigation concerns the tracking of a moving beam on a detector, in the presence of optical imperfections and beam obstructions. In such conditions, the shape of the imaged beam can be altered in ways that are difficult to model a priori, and a system that uses the beam’s centroid position must be able to compensate for these effects.

The focus of the investigation is on the performance of several methods for calibrating the measurement of a beam centroid under these conditions. A computationally simple method is sought while achieving the best measurement accuracy possible. These requirements are due to the use case of the investigation: the fine lateral and longitudinal sensor (FLLS) instrument, described in Sec. 4. The lateral system within FLLS must measure centroid displacement with an accuracy of 300 μm to satisfy the mission requirements. FLLS images a near-Gaussian beam to track the movement of one spacecraft relative to another. The beam, and thus centroid position, can be anywhere within a known field-of-view (FoV) of the instrument, outside which the beam is cropped. On-ground centroid (spacecraft) positions must be gathered and interpolated to calibrate and accurately measure the spacecraft’s movement in flight. Calculation of the beam centroid is the only image-processing possible in space.

Calibration of instrumentation is often the most time-consuming stage of building an instrument. Ground support equipment must be purchased and set up correctly, the calibration model/s must work seamlessly with the hardware, and all potential sources of error should have been predicted and understood. By modeling part of the calibration before carrying out the routine with hardware, the risk of uncovering unexpected performance errors is dramatically reduced. Internal sources of error within the instrument can be found, understood, and corrected. This allows for quicker calibration and successful, on-time delivery of instrumentation. The goal of this work is to test the efficacy of each of the proposed calibration methods in correcting the centroid position.

Within this paper, Sec. 2 describes the centroid calculation algorithm. Interpolation models investigated as calibration methods are described in Sec. 3. The use case for the calibration methods is explained in Sec. 4, with Sec. 5 describing the instrument performance inhibitors in greater detail. Section 6 presents the overall methodology of this investigation. The results of the calibration methods for this use case are discussed in Secs. 7 and 8. Conclusions of the investigation are drawn together in Sec. 9.

2 Calculating the Beam Centroid

When imaged by a sensor, the beam centroid is measured by summing the intensity-weighted pixel coordinates, as shown below:

$$\bar{i} = \frac{\sum_i \sum_j i I_{ij}}{\sum_i \sum_j I_{ij}},$$

$$\bar{j} = \frac{\sum_i \sum_j j I_{ij}}{\sum_i \sum_j I_{ij}}.$$

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where i and j denote pixel indices, and $\bar{i}$ and $\bar{j}$ are the centroid pixel indices. From this, the centroid in pixels, in the detector coordinate frame, is mapped to X , Y position of the beam in the instrument coordinate frame. This method will not be described here as it is considered outside the scope of this paper.

For the modeling, the true centroid position is calculated using an optical simulation of FLS in Zemax. This represents the data that would be taken and verified by an independent method in the laboratory, such as a laser tracker.

3 Calibration Methods

3.1 Overview

Calibration quantifies measurement uncertainties in an instrument, corrects systematic errors, and establishes required reference frames. This section describes several calibration methods and how they correct systematic errors in the centroid measurement.

The difference between the corrected centroid position and the true centroid position quantifies the calibration method's performance. This is what is meant by the "centroid error" in this paper. Each satellite range (see Sec. 4.1) has its own corresponding training and test dataset, with which the calibration is carried out.

The results of applying these models to the use case are presented in Sec. 6.

3.2 Power Loss Method

This method attempts to compensate for beam cropping that occurs at the edges of the detector's FoV and correct the resulting centroid error, assuming an otherwise circular beam. A circular aperture is positioned concentrically over the detector or simulated by means of image masking, in order to ensure that the cropping is circularly symmetric about the center of the detector FoV.

At the peripheries of the detector's FoV, the beam is cropped by the aperture's edge, causing an inward bias in the measured beam centroid, as defined in Sec. 2, and a corresponding drop in the power measured by the detector. This relationship can be modeled by a radial correction to the measured centroid r_m , as a function Δr of the fractional power loss observed by the detector:

$$r_c = r_m - \Delta r \left(1 - \frac{P}{P_{\max}} \right),$$

where P is the measured power corresponding to r_m , and $P_{\max}$ is the total power measured by the detector for a centered/on-axis beam (where there is minimal cropping).

The calibration model then consists in mapping the measured power loss to the radial correction, Δr , to be applied to the measured centroid position. For a perfect Gaussian beam, the functional form of Δr can be modeled exactly and visualized as in Fig. 1.

When the shape of the beam is more complicated (but still circular), the relationship can be established by empirical measurements, with a 1-D interpolation being applied to construct a mapping from power loss to radial correction.

Fig. 1 Scatter plot showing the magnitude of the radial difference between true and measured centroid positions, against the associated power drop.⁸

3.3 Direct Interpolation

Rather than model the centroid correction in terms of power loss, this method uses a 2-D mapping directly from the measured centroid to the true centroid. This mapping is constructed by interpolation of the true centroid position with respect to a set of empirically measured centroid positions. Three different interpolation algorithms are discussed: nearest neighbor, natural neighbor, and bivariate spline. In all cases, two interpolated mappings are produced: one from the measured X and Y coordinates to the corrected X coordinate, and one from the measured X and Y coordinates to the corrected Y coordinate.

3.3.1 Nearest neighbor

Nearest neighbor interpolation⁹ is a common direct interpolation method used to interpolate scattered sample data points. It simply determines the value of the nearest sample data point and uses it for the point being interpolated. This results in step-like changes around sample points, which limits the accuracy of this method. However, nearest neighbor interpolation is computationally efficient, and so, this was investigated as part of this work.

3.3.2 Natural neighbor

Natural neighbor interpolation¹⁰ was also investigated due to its ability to interpolate scattered data. Instead of simply using the nearest value, the algorithm uses a distance-weighted average of neighboring values, based on their Voronoi tessellation. This technique results in a much smoother interpolation function at a greater computational cost. The smoothness of this interpolation greatly improves the accuracy of the resulting calibration model.

3.3.3 Bivariate spline

Multivariate spline interpolation fits piecewise polynomial functions to the sample data points,¹¹ an example of which is shown in Fig. 2. In the case of centroid measurement, the

Fig. 2 The “natural” cubic spline interpolant, which has the smallest second derivative when fitting the data.¹¹

SmoothBivariateSpline class provided by the scipy Python package¹² is used as it is able to cope with a nongridged arrangement of sample data points. This method was chosen for its heritage in the Hitomi mission instrumentation discussed in Sec. 4.3.

4 Use Case: The FLLS Instrument for the PROBA-3 Mission

4.1 PROBA-3 Mission

The Project for On-Board Autonomy-3 (PROBA-3) is a European Space Agency mission, due to launch in 2020.¹³ The PROBA-3 mission has two goals: to demonstrate high-accuracy formation flying and, using this, to study the solar corona.

The spacecraft fly as a pair with the distance between them ranging from 25 to 250 m. A large disc is mounted on the back of the “occulter” spacecraft, to occlude the center of the Sun with respect to the “coronagraph” spacecraft at an intersatellite range (ISR) of 150 m. Figure 3 shows the two satellites in this formation. This design allows the coronagraph instrument carried on the coronagraph spacecraft to view the solar corona, creating a 150-m-long coronagraph. For the coronagraph

Fig. 3 Artist's impression of the coronagraph spacecraft (right) in formation within the shadow of the occulter spacecraft (left) [Credit: European Space Agency (ESA)].

instrument to function correctly, its position needs to be maintained to an accuracy of $\leq 300 \mu\text{m}$. The instrument measuring the spacecraft displacement to this accuracy is the FLLS instrument discussed in this paper.

4.2 FLLS Instrument

Using a retroreflected laser beam, the FLLS instrument provides three-axis displacement measurements of one spacecraft with respect to the other in the formation. FLLS comprises two separate sensor systems: the lateral and the longitudinal. The lateral system measures the displacement of the other satellite in the Y and Z axes by imaging the returning laser beam. The longitudinal system measures displacement along the X axis by synchronous demodulation of the modulated laser beam.¹⁴ The co-ordinate axes of FLLS are presented in Fig. 4. This paper covers the performance of FLLS's lateral system only.

FLLS is being built by Neptec UK (NUK) and Neptec Design Group (NDG) and comprises three subassemblies: the laser control electronics unit (LCEU), the optical head unit (OHU), and the cube corner retroreflector (CCRR). These subassemblies are shown in Fig. 5.

The LCEU contains a 980-nm continuous-wave laser diode and the signal processing electronics. It is coupled to the OHU, which is designed and built by Micos Engineering GmbH.

Fig. 4 PROBA-3 formation showing measurement axes (spacecraft image credit: ESA).⁸

Fig. 5 The main subassemblies of the FLLS instrument and the corresponding responsibility of each.

Fig. 6 Flight data from the CAMS unit on-board JAXA's Hitomi satellite, with the color scale showing the relative intensity of each beam.¹⁶ The same detector used in CAMS is being used in FLLS. (a) CAMS1 intensity map and (b) CAMS2 intensity map.

The LCEU and the OHU are mounted as one unit on the occulter spacecraft.

The OHU expands and collimates the beam emitted by the LCEU to ~ 30 mm in diameter and reflects it out of the occulter spacecraft to the CCRR. At the CCRR, the beam is retro-reflected back into the OHU, where it is reduced in size, and split into two beams by a beamsplitter. One beam travels to the longitudinal detector—an avalanche photodiode, and the other beam to the lateral detector—a CMOS sensor of 1024×1024 pixels.

Fig. 7 Sketch showing the returning beam entering the circular OHU window (turquoise) and being imaged onto the lateral detector (gray). The beam is cropped by the OHU window.⁸

The LCEU processes the detector signals to compute the spacecraft displacement.

Further information on the two systems within FLLS can be found in Refs. 8 and 15.

4.3 Previous Flight Data

The lateral system is based on previous instrument heritage from the Canadian Metrology System (CAMS), built by NDG for JAXA.¹⁶ CAMS flew on the Hitomi mission and successfully provided data before the mission was terminated. This data (shown in Fig. 6) was used to confirm that the modeling of the FLLS beam is representative of what can be expected in flight.

5 Performance Inhibitors within FLLS

The following inhibitors challenged the performance of the calibration algorithm when applied to the FLLS instrument. Some of the inhibitors arise from the optical design of the instrument, whereas others arise from operational mission requirements.

5.1 Cropping of Return Beam

When the CCRR is beyond 20 mm of travel in the YZ plane from the center of the detector, the returning beam is cropped by the instrument housing. The associated loss in optical power causes a significant inward bias in the centroid calculations described in Sec. 2. Due to the mass-volume requirements of the mission, the OHU aperture cannot be enlarged, and so the cropping is unavoidable and must be corrected. Figure 7 shows a simplified sketch of the returning beam being cropped as it passes through the circular OHU window to be imaged on the detector.

A simulated image of the cropped beam at an ISR of 250 m is given in Fig. 8. The true, measured, and corrected centroid positions are marked on Fig. 8 to show the inward bias.

Fig. 8 Simulated image of the (a) cropped return beam on the lateral detector, with a (b) close-up showing the associated diffraction pattern.⁸

5.2 Imperfect CCRR

A hollow CCRR formed from three optical surfaces bonded together, known as “petals,” is used in FLLS. The angle between each petal—the dihedral angle—should be 90 deg for a “perfect” CCRR. Any error in this angle results in deviation of the outgoing beam from the incoming beam.

The investigation began using a stock imperfect CCRR supplied by Micos Engineering GmbH, as this allowed the work to progress faster. This “coarse” CCRR had a beam deviation of ~ 2.2 arc sec, so would exaggerate the effects expected for the flight version of the CCRR, allowing a very worst-case environment to be analyzed.

The flight model CCRR for FLLS, manufactured by PLX Inc, has a beam deviation of 0.5 arc sec. A clear aperture of 90 mm enables the full beam to be captured by the CCRR and returned to the OHU. Each petal of the CCRR is gold-coated, with the coating thickness adapted for maximum reflectivity at 980 nm. Applying the coating to the front of the petals limits the effect of ghosting. Figure 9 shows an image of the FLLS flight CCRR and the corresponding Zemax model.

5.3 Optical Distortions

Due to the mass-volume constraints imposed on the design, the OHU optics introduce distortions in the returning beam that make it non-Gaussian in shape and difficult to model. These

Fig. 9 (a) Photo of FLLS CCRR and (b) corresponding Zemax model with the retroreflected beam.

Fig. 10 (a)–(c) Simulation of the beam imaged on the FLLS lateral detector at the three ISRs. Note the change in the shape of the imaged beam as the ISR increases.

effects interfere with the centroid calculations, particularly in the case of the power loss method described in Sec. 3.2.

5.4 Varying Inter-Satellite Range

Performance effects of the beam cropping and optical imperfections vary in their severity with ISR, and there is, therefore, a range-dependent error on the measured YZ position. The PROBA-3 mission requirement of longitudinal displacement between 25 and 250 m makes compensating for these effects more challenging since the corrections must work at any range. Figure 10 shows one example of the variation between ISR: beam distortion.

6 Calibration Methodology of FLLS

By simulating the performance of the calibration model of FLLS, the risk of delays and unexpected performance errors of the traditional hardware-only approach is reduced.

Figure 11 shows the course of the investigation carried out to determine the optimum calibration model for FLLS. Each interpolation method investigated is shown in Fig. 11, as well as the decisions, is taken for each set of results. The rest of this paper details the stages of this process.

For the first step in assessing the calibration method, a simulated beam is translated in both axes across the full FoV of the detector while the detector remains stationary. At each set of predetermined positions within the FoV, the beam is imaged and the centroid position is calculated. This “training data” set is accompanied by the true centroid positions, as defined in the simulation (also known as “truth data”), and together these form the basis of the calibration model. In practice, the true centroid positions will be measured by separate instrumentation, e.g., a laser tracker, that will have a negligible error with respect to that of FLLS. A separate “test data” set tests the ability of the calibration model to correct the centroid position. This is a denser set of beam positions than the training dataset, to sufficiently test the robustness of the calibration model.

7 Using Coarse Retroreflector

7.1 Sample Size

Three different-sized training datasets were investigated through the course of this work to determine the required number of

Fig. 11 Flow diagram of the methodology of the calibration model for the lateral system of FLLS.

interpolation points (beam images) to construct the calibration model. These training sets were of 39 points, 145 points, and 231 points. A larger test dataset of 900 points was used to validate the resulting calibration model by testing its performance in the space between interpolation points.

The 95th percentile of centroid error was used as a figure of merit to determine whether each interpolation dataset produced adequate corrections to centroid measurements. It was decided to use 231 interpolation points to test the calibration models, as this dataset gave a centroid error within the mission requirement of 300 μm for the key interpolation models. Future work could investigate how much further larger datasets could improve the centroid error. However, this investigation limited the number of data points to reduce model run time (for 231 points: ~ 27 h using an Intel Core i7-6700HQ processor).

The 39-point dataset was used for faster analysis where appropriate, for example, comparing bivariate spline and natural neighbor interpolation in Sec. 7.3.2.

7.2 Sampling Pattern

It was observed during the investigation that the arrangement of the interpolation points (i.e., training data) influenced the results of the interpolation. Two patterns were used during this investigation: gridded and sunflower seed. The results are presented in the relevant sections.

7.2.1 Gridded pattern

A gridded collection pattern is the simplest choice for taking data for FLLS, because the translation stages used

Fig. 12 Extreme example of spline interpolation for 39 gridded interpolation points. Note the magnitude of the errors (shown as blue arrows) at the edges of the sampling region, where the spline cannot cope with the cropping effects due to lack of data on the cropping boundary.

to gather the instrument data naturally step horizontally and vertically.

However, it became clear during the investigation that the direct interpolation models require that the interpolation points enclose the entire range of CCRR movement. Figure 12 gives an extreme example of what is observed for a gridded dataset when attempting a spline interpolation to the FoV edges with no data available in that region.

Boundary points were therefore added on the circumference of the FoV with spacing as close to the gridded spacing as achievable. Figure 13 shows the gridded pattern (green crosses) with the additional boundary points added to cover the edge of the CCRR range (blue asterisks). This pattern is known in this paper as the “gridded-plus-boundary” pattern.

7.2.2 Sunflower seed pattern

Due to requiring data points around the edge of the FoV, patterns were investigated that already allowed for this. The “sunflower seed” pattern¹⁷ shown in Fig. 14 has evenly distributed points around the FoV and around the circumference. Manipulating the number of points on the boundary improved the performance of the spline interpolation. For consistency, the number of points used to create the sunflower seed pattern was the same as used for the gridded pattern, with an equivalent number of boundary points.

7.3 Interpolation Method

7.3.1 Power loss method

The accuracy of the mapping described in Sec. 3.2 in correcting for beam cropping was assessed in terms of the centroid error over FLLS’s measurement range. Thirteen images per ISR were

Fig. 13 The gridded-plus-boundary training data collection pattern across the full CCRR range. The green crosses represent the usual gridded pattern for a 231-point dataset; the blue points around the circumference are the added boundary points. The corresponding radial movement of the CCRR is shown on the diagram.⁸

Fig. 14 The sunflower seed training data collection pattern across for the full CCRR range. The boundary points have been modified to be distributed evenly around the circumference, as for the gridded pattern.⁸

taken for the test dataset, against which centroid errors were calculated, as shown in Fig. 15. The cumulative distribution of the centroid error for each ISR is presented in Fig. 16. For the nominal ISR of 150 m, the median error is 105 μm , and the maximum is 148 μm .

During this modeling, certain instrument configurations produced a simulated total optical power of 5% to 10% greater than that of the centered CCRR at that ISR. This seemed unphysical, given the on-axis beam is not cropped and therefore should produce maximum power.

After some investigation, it was noticed that at the extreme edges of CCRR movement (i.e., beyond $\pm 20\text{ mm}$), the returning beam passes through the OHU beamsplitter subassembly at a more acute angle. Thus, for this configuration, the optical transmission is greater and the losses are fewer than for the on-axis beam.

This demonstrates that the optical system’s characteristics can cause spurious changes in the measured power and that the power loss method, therefore, cannot reliably predict centroid

Fig. 15 True and corresponding corrected centroid positions of the perfect CCRR when using the power loss method.⁸

Fig. 16 Cumulative fraction plot showing the observed centroid errors for each ISR when using power loss interpolation on a dataset of 13 points.⁸ The discrete steps are due to the small number of data points.

error. Care must be taken to analyze all scenarios of the system if the power drop method is used, to ensure all such cases are fully understood and compensated.

The errors from this calibration method are significant (Fig. 17). This is due to beam distortions caused by the OHU optics, violating the assumption of a Gaussian return beam. Moreover, this calibration method performs worse (i.e., largest error/worst fit) at greater ISR. While it is not entirely understood, it is possible that this is because the beam is not perfectly collimated. Using this calibration method, the centroid

Fig. 17 Median centroid errors produced when using the power loss interpolation method as applied to the coarse CCRR. The error for each ISR is shown. These results include the modification necessary to deal with the measured powers greater than on-axis power.

error is not reliably within the required 300 μm accuracy for all ISRs.

7.3.2 Direct interpolation

Direct interpolation, as described in Sec. 3.3, maps measured centroid directly to true centroid. The first such interpolation method tested was the nearest neighbor interpolation. While this method is simple to implement, it leads to large step changes in the corrected centroid position, increasing the centroid error. The median error was beyond the requirement value, as shown in Fig. 18, so this method was discarded, and the investigation moved on to natural neighbor interpolation.

Natural neighbor interpolation yielded much smaller centroid errors, with the medians more than an order of magnitude smaller than those of nearest neighbor, as shown in Fig. 18. A visual comparison of the fit of the two neighbor interpolations is shown in Fig. 19.

However, the maximum errors were much larger than expected, given the low median errors: 3066 μm at 25 m; 2238 μm at 150 m; and 2877 μm at 250 m. When inspecting the results, it was observed that the errors were larger at the peripheries of the FoV, due to insufficient coverage of the sampling pattern, leading to extrapolation beyond the interpolation domain. An extreme example is shown in Fig. 12. Henceforth, the sampling pattern was augmented with boundary points, as described in Sec. 7.2, in order to overcome this deficiency.

The third and final interpolation method tested was a bivariate spline. Other spline interpolations exist, such as a constrained cubic spline,¹⁸ which may reduce the effects of numerical ringing. However, this is not expected to be an issue in this case because of the smoothly varying nature of the interpolated function.

To save computation time, a comparison between the bivariate spline and natural neighbor was carried out using a 39-point dataset. The results (Fig. 20) showed that the natural neighbor interpolation produced slightly lower centroid errors than the bivariate spline for the shorter distances, although they became comparable for the full ISR. In the same test, the sunflower seed

Fig. 18 Median centroid errors produced when using the nearest neighbor and natural neighbor interpolation as applied to the coarse CCRR. The error for each ISR is shown.

pattern gave slightly lower values than the gridded-plus-boundary pattern.

However, as mentioned in Sec. 3.3.2, when implementing such an interpolation on an field-programmable gate array, natural neighbor interpolation demands considerably more computational outlay than bivariate spline interpolation. Given the computational limitations of FLLS, the spline was chosen to be taken forward in this investigation.

A 231-point dataset was used in the final performance analysis of FLLS. Figure 22 shows the median centroid errors for the spline interpolation of the coarse CCRR. Again, the sunflower seed pattern yields marginally lower centroid errors (see Fig. 21). The centroid error for all three ISRs is well within

Fig. 20 Comparison between natural neighbor and bivariate spline interpolation using the 39-point dataset, investigating both sunflower seed (SF) and gridded-plus-boundary (GB) sampling patterns.

the required 300 μm requirement, which is further improved upon in Sec. 8, as predicted.

7.4 Short-Range Calibration

Calibrating a high-accuracy instrument such as FLLS at 250 m presents practical challenges. As far as the authors are aware, there are no 250-m-long vacuum facilities available in the United Kingdom, so a multipass system of mirrors is necessary. A multipass system allows the separation between the CCRR and OHU to be any distance up to the full ISR.

Multipass systems can add translation errors caused by the mirror mounting, optical beam degradation from the mirror

Fig. 19 (a) Nearest neighbor interpolation and (b) natural neighbor interpolation of the same 231-point grid of data at an ISR of 150 m. Each blue arrow indicates the magnitude of the error between the true and corrected centroid position at that point.

Fig. 21 Cumulative distribution function for the spline interpolation of the coarse CCRR using both the gridded-plus-boundary and sunflower seed pattern datasets.⁸

surfaces, and the multiplication of environmental errors if the beam passes through the same physical space multiple times. It can be difficult to separate out instrumentation errors from equipment-induced errors, so to already be able to predict what the instrument will achieve is vital for such a procedure.

To avoid these drawbacks, the system was instead calibrated at 25 m and the resulting model tested at larger distances. The 25 m is the minimum ISR requirement of the PROBA-3 mission and is practical to test in a laboratory setting. The latter two sets of results in Fig. 22, “spline—25-m interpolation,” are the resulting centroid errors when only calibrating FLLS at 25 m and applying it at greater ISRs. While at the 25-m ISR, the centroid error remains the same, as expected, the errors at 250-m ISR almost double with respect to a system calibrated at the correct

Fig. 22 Median centroid errors of the calibration model for the coarse CCRR, when interpolating with the full ISR dataset and with only the 25-m dataset.

range. Figure 22 also shows that the choice of the sampling pattern does not improve this result.

8 Using Flight Retroreflector

The final stage of the investigation was to replace the coarse CCRR with the flight CCRR, to mimic the expected performance of FLLS, minus any external errors such as from the calibration facility.

Applying the bivariate spline interpolation method to the instrument training data, Fig. 23 shows the true and corrected centroid positions at an ISR of 150 m.

Figure 24 shows the cumulative distribution of centroid error when using bivariate spline interpolation method. During the investigation, a discrepancy in the 25-m ISR data was observed,

Fig. 23 The corrected centroid positions (red) plotted against the original true centroid positions (blue) for 150-m ISR using bivariate spline interpolation on flight CCRR data. The arrows in the legend indicate the magnitude of the errors between each of these pairs, but these are small enough not to be visible at this scale.

Fig. 24 Cumulative fraction plot showing the observed centroid errors at each ISR when using bivariate spline interpolation on a set of 231 measurements. The centroid errors at 25-m ISR have been extrapolated from current and previous work.

caused by limitations of the modeling software. This created spuriously large centroid errors not representative of the actual performance of the calibration model. Extrapolating from previous work⁸ and from the flight-representative results for 150 and 250 m, the performance of the calibration model at an ISR of 25 m was estimated. The result is comparable with that of the 150-m ISR and should yield slightly lower centroid errors (median $\leq 40 \mu\text{m}$).

Figure 24 shows that, for the nominal ISR of 150 m, the median centroid error is $41 \mu\text{m}$ and the maximum is $84 \mu\text{m}$. Figure 25 shows the median centroid errors for all ISRs for both sampling patterns. Again, while the sunflower seed pattern presents slightly lower centroid errors, the difference is negligible.

9 Conclusions

Direct interpolation by way of bivariate spline yielded the best performance results for calibrating the FLLS instrument. The errors presented in Fig. 25 are lower than those achieved for the power loss method (see Fig. 17), showing that direct interpolation, specifically spline interpolation, is more successful in overcoming the performance inhibitors of this system. Generally, a sunflower seed sampling pattern gave lower centroid errors, though the difference was marginal.

The maximum error on the centroid position at the nominal spacecraft range of 150 m is $84 \mu\text{m}$ when calibrating at 150 m. This result is well below the measurement accuracy requirement of $300 \mu\text{m}$ for the PROBA-3 mission. When calibrating at a shorter distance (i.e., only at 25 m), the centroid error increases by approximately a factor of 2, due to the cropping of the returning beam and the imperfections of the CCRR at longer ISRs.

For a more general case, this result shows that the use of direct interpolation, particularly multivariate splines, can be used with and implemented on irregularly spaced data to achieve respectable performance results. The sampling pattern should be decided by the user as a trade-off between what is most practical and the desired performance. The most appropriate interpolation

Fig. 25 Median centroid errors for FLLS calibration model when using bivariate spline interpolation on the flight CCRR. The results for both sampling patterns are shown.

method to use depends on the system's optical characteristics, particularly in the presence of cropping.

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