Link-Level Performance of Rate-Splitting based Downlink Multiuser MISO Systems

Hongzhi Chen, De Mi, Zheng Chu, Pei Xiao

Institute for Communication Systems,

University of Surrey, Guildford, UK

Email: {hongzhi.chen, d.mi, zheng.chu, p.xiao}@surrey.ac.uk

Yin Xu, Dazhi He

School of Electronic Information and Electrical Engineering,

Shanghai Jiao Tong University, Shanghai, China

Email: {xuyin, hedazhi}@sjtu.edu.cn

Abstract—This work provides the first link level performance evaluation of the Rate-Splitting (RS) based precoding scheme in a downlink multi-user multiple input single output (MU-MISO) system. Contrary to the existing works on the RS precoding that mostly focused on the sum rate or minimum rate maximization, this work bridges the optimization results with the bit error rate (BER) performance, initiating the RS software implementation. We demonstrate that, in an overloaded scenario, the conventional precoding schemes suffer from the BER error floor that corresponds to their rate saturation, which can be overcome by the RS-based strategy that adding the message decodability of certain users with the interference-limited message rate.

Index Terms—Rate-splitting, MISO-BC, link-level simulation, optimization, overloaded scenario, software implementation.

I. INTRODUCTION

Considering a multiuser (MU) multiple input single output broadcast (MISO-BC) system, the related works [1]–[3] have proved that with the insufficient number of transmit antennas, i.e., in an overloaded scenario, conventional linear precoding schemes that treats the inter-user interference (IUI) as noise works in a interference limited regime. A rate-splitting (RS) based linear precoding scheme has been proved to better manage the interference for the MISO-BC system [2]–[6]. The RS-based strategy, to be detailed later in this paper, involves message splitting, superposition coding and precoding optimization at the transmitter side, as well as successive interference cancellation (SIC) at the receiver side. The main focus in the literature is either maximizing the minimum rate under a given transmit power budget or minimizing the transmit power subject to a minimum Quality of Service requirement.

Despite huge potentials, it is essential to validate the performance gain of the RS-based strategy over the conventional schemes and the practicality of the strategy itself, via different evaluation methodologies. Therein has a link level simulation which is widely used for the software implementation of radio access technologies. Metrics output from the computationally intensive link level simulations, such as bit error rate (BER) or block error rate, provide insightful guidelines for the testing technology implementation in practice. Leveraged on our project work [7]–[9], a link level simulator compatible to the latest Long Term Evolution (LTE) and New Radio (NR) standardizations has been developed. In this work, we aim to introduce the optimized RS-based transmission strategy for the MU MISO-BC system into the simulator, initiating the

practical implementation of such strategy. To this end, we first study the optimization problem to obtain the optimal RS-based precoder and equalizer. Then, as the main concentration and contribution of this paper: a

- We adapt RS functionalities to the link level simulation diagram for a transceiver processing chain. This includes adding the message splitting, the superposition coding, the RS-based optimized precoding/equalizing and SIC, and designing the RS-based decoding that takes into account the message decodability of different users.
- We evaluate the BER performance of the RS-based system compared with the conventional precoding system, and verify the rate improvement of using the RS-based strategy. To the best of the authors' knowledge, this is the first work that bridges the RS-based optimization results with the BER performance.

Notations: we use lower case letters, boldface lower case letters and boldface upper case letters to denote scalars, vectors and matrices respectively. Blackboard bold letters are used to denote sets, $\mathbb{C}$ denotes the set of complex numbers, $\mathbb{C}^{A \times B}$ denotes the set of $A \times B$ matrices with complex entries. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate-transpose operation respectively. $\|\cdot\|$, $\mathbb{E}\{\cdot\}$ and $\mathbb{L}(\cdot)$ are the second order norm, statistical expectation operators and the size of a given vector, respectively. $\mathbf{A}_{i,j}$ denotes the entry on the i -th row and j -th column of matrix $\mathbf{A}$, while $\mathbf{A}_{:,j}$ denotes all the entries on the j -th column. $\Re\{\cdot\}$ denotes function that output the real part of a given number.

II. SYSTEM MODEL

We consider a downlink system where the transmitter is a single base station equipped with N_t transmit antennas, serving K users with a single-antenna receiver. Here $N_t < K$ as in overloaded scenarios. We define the set of users as $\mathbb{K} = \{1, \dots, K\}$, and the original information as $\mathbf{W}^{og} = [\mathbf{w}_1^{og}, \dots, \mathbf{w}_k^{og}, \dots, \mathbf{w}_K^{og}]^T$, in which the row vector $\mathbf{w}_k^{og}$ denotes the message intended for the k -th user. We assume all the messages have the identical length equal to L , denoted as $\mathbb{L}(\mathbf{w}_k^{og}) = L$, $\forall k \in \mathbb{K}$. The RS-based strategy divides the messages into the common and private parts. This message splitting process is represented as $\mathbf{w}_k^{og} = [\mathbf{w}_{k0}, \mathbf{w}_{k1}]$, where $\mathbf{w}_{k0}$ and $\mathbf{w}_{k1}$ represent the common and private parts split

from the original message $\mathbf{w}_k^{og}$, respectively. All the common parts are packed into one super common message, as given by

$$\mathbf{w}_c = [\mathbf{w}_{10}, \dots, \mathbf{w}_{k0}, \dots, \mathbf{w}_{K0}]. \quad (1)$$

Thus, the message after RS-based splitting can be given in matrix form as:

$$\mathbf{W}^{RS} = \begin{bmatrix} \mathbf{w}_c & \mathbf{0}^{1 \times (L - \mathbb{L}(\mathbf{w}_c))} \\ \mathbf{w}_{11} & \mathbf{0}^{1 \times (L - \mathbb{L}(\mathbf{w}_{11}))} \\ \vdots & \vdots \\ \mathbf{w}_{K1} & \mathbf{0}^{1 \times (L - \mathbb{L}(\mathbf{w}_{K1}))} \end{bmatrix} \quad (2)$$

with zero padding on each individual message, i.e., each row of $\mathbf{W}^{RS}$, including the common message. Then each independent message is encoded and modulated, i.e., $\mathbf{w}_c \rightarrow \mathbf{s}_c$ and $\mathbf{w}_{k1} \rightarrow \mathbf{s}_k$, $\forall k \in \mathbb{K}$, forming $K + 1$ transmit streams. Assuming a code rate of 1 and modulation order of M , the number of symbol per stream is given as $J = L / (1 * \log_2 M)$. We represent the transmit symbol stream in matrix form as: $\mathbf{S} = [\mathbf{s}_c, \mathbf{s}_1, \dots, \mathbf{s}_K]^T \in \mathbb{C}^{(K+1) \times J}$, where $\mathbb{E}\{(\mathbf{S}|_{:,j})(\mathbf{S}|_{:,j})^H\} = \mathbf{I}$, $j \in \{1, \dots, J\}$. These symbol streams are then mapped onto the transmit antennas through a RS-based precoding matrix, denoted by $\mathbf{P} \triangleq [\mathbf{p}_c, \mathbf{p}_1, \dots, \mathbf{p}_K]$, where $\mathbf{p}_c \in \mathbb{C}^{N_t}$ and $\mathbf{p}_k \in \mathbb{C}^{N_t}$ are the precoding vectors for the common and private streams, respectively. Hence, we have the transmit signal, as given by

$$\mathbf{X} = \mathbf{P}\mathbf{S} = \mathbf{p}_c \mathbf{s}_c + \sum_{k=1}^K \mathbf{p}_k \mathbf{s}_k, \quad (3)$$

where $\mathbf{X} \in \mathbb{C}^{N_t \times J}$. The transmit signal is subject to an average power constraint $\mathbb{E}\{(\mathbf{X}|_{:,j})^H(\mathbf{X}|_{:,j})\} \leq P_t$. We use $\mathbf{h}_k \in \mathbb{C}^{1 \times N_t}$ to represent the channel between the transmitter and k -th user. Therefore, the symbol stream received by the k -th user is given as

$$\mathbf{y}_k = \mathbf{h}_k^H \mathbf{X} + \mathbf{n}_k, \quad (4)$$

where $\mathbf{y}_k \in \mathbb{C}^{1 \times J}$, and $\mathbf{n}_k$ represents the receiver Additive White Gaussian Noise (AWGN), whose entries are Gaussian random variables with zero mean and variance $\sigma_{n,k}^2$. Without loss of generality, we assume that $\sigma_{n,k}^2 = \sigma_n^2$, $\forall k \in K$. We further assume that the transmitter has the perfect channel state information, and the channel remains constant in one transmission period. The average receive power at the k -th user, denoted by $T_{c,k}$, is independent from the length of symbol stream J and therefore, this power can be represented as a scalar, given as:

$$T_{c,k} = \underbrace{|\mathbf{h}_k^H \mathbf{p}_c|^2}_{S_{c,k}} + \underbrace{|\mathbf{h}_k^H \mathbf{p}_k|^2}_{S_k} + \underbrace{\sum_{l=1, l \neq k}^K |\mathbf{h}_k^H \mathbf{p}_l|^2 + \sigma_n^2}_{T_k}, \quad (5)$$

where, $S_{c,k}$, S_k , I_k denote the received power for the common part, the private part and the interference plus noise, respectively. T_k denotes the interference for the common part reception. At the receiver side, the common part is first decoded by treating all the interference as noise, thus

the k -th user estimates the common symbol $\mathbf{s}_c$ stream as: $\hat{\mathbf{s}}_{c,k} = g_{c,k} \mathbf{y}_k$ where $g_{c,k}$ represents the equalizer scalar for the common symbol stream. After decoding the common part, successive interference cancellation (SIC) is applied to remove the estimated common stream from $\mathbf{y}_k$ before decoding the private part. The estimation of $\mathbf{s}_k$ is provided as: $\hat{\mathbf{s}}_k = g_k(\mathbf{y}_k - \mathbf{h}_k^H \mathbf{p}_c \hat{\mathbf{s}}_{c,k})$, where g_k represents the equalizer scalar for the k -th private symbol stream. The mean square error (MSE) from the k -th user's perspective for its common and private part are defined as: $\varepsilon_{c,k|j} \triangleq \mathbb{E}\{|\hat{\mathbf{s}}_{c,k|j} - (\mathbf{s}_c|_j)|^2\}$ and $\varepsilon_{k|j} \triangleq \mathbb{E}\{|\hat{\mathbf{s}}_{k|j} - (\mathbf{s}_k|_j)|^2\}$, respectively. The MSE can be calculated as:

$$\varepsilon_{c,k|j} = |g_{c,k}|^2 T_{c,k} - 2\Re\{g_{c,k} \mathbf{h}_k^H \mathbf{p}_c\} + 1 \quad (6a)$$

$$\varepsilon_{k|j} = |g_k|^2 T_k - 2\Re\{g_k \mathbf{h}_k^H \mathbf{p}_k\} + 1. \quad (6b)$$

We can see both $\varepsilon_{c,k|j}$ and $\varepsilon_{k|j}$ are independent from the length of the symbol stream, thus we can drop off the subscript j . The optimum equalizer scalars can be calculated following the Minimum MSE (MMSE) equalization, as: $(g_{c,k}^{\text{MMSE}}, g_k^{\text{MMSE}})$, where $g_{c,k}^{\text{MMSE}} = \mathbf{p}_c^H \mathbf{h}_k (T_{c,k})^{-1}$ and $g_k^{\text{MMSE}} = \mathbf{p}_k^H \mathbf{h}_k (T_k)^{-1}$. Substituting the corresponding MMSE equalizer scalars back to (6a) and (6b) yields: $\varepsilon_{c,k}^{\text{MMSE}} = (T_{c,k})^{-1} T_k$ and $\varepsilon_k^{\text{MMSE}} = (T_k)^{-1} I_k$, respectively. Therefore, the k -th user experiences an average signal-to-interference-plus-noise ratio (SINR) as: $\gamma_{c,k} = S_{c,k}/T_k = (\varepsilon_{c,k}^{\text{MMSE}})^{-1} - 1$ and $\gamma_k = S_k/I_k = (\varepsilon_k^{\text{MMSE}})^{-1} - 1$. Then we can formulate the corresponding achievable rate as: $R_{c,k} = \log_2(1 + \gamma_{c,k}) = -\log_2(\varepsilon_{c,k}^{\text{MMSE}})$ and $R_k = \log_2(1 + \gamma_k) = -\log_2(\varepsilon_k^{\text{MMSE}})$. Considering that the common stream should be successfully decoded by all users, it should be satisfied that the common rate $R_c = \min_{k \in \mathbb{K}} R_{c,k}$.

A. Precoder and Equalizer Acquisition for Rate-Splitting

In this paper, we consider the max-min fairness (MMF) optimization. The optimal precoder is obtained by solving the MMF problem, as follows:

$$\begin{aligned} \max_{\mathbf{C}, \mathbf{P}} \quad & \min_{k \in \mathbb{K}} (C_k + R_k) \\ \text{s.t.} \quad & R_{c,k} \geq \sum_{k=1}^K C_k, \quad \forall k \in \mathbb{K} \end{aligned} \quad (7a)$$

$$C_k \geq 0, \quad \forall k \in \mathbb{K} \quad (7b)$$

$$\|\mathbf{p}_c\|^2 + \sum_{k=1}^K \|\mathbf{p}_k\|^2 \leq P_t \quad (7c)$$

where, $\mathbf{C} \triangleq [C_1, \dots, C_K]$ is the concatenated vector of the common rate portion for each user. The portion is given as $C_k = \frac{\mathbb{L}(\mathbf{w}_{k0})}{\sum_{k=1}^K \mathbb{L}(\mathbf{w}_{k0})} R_c$. The constraints shown in (7a) and (7b) guarantee the decodability and non-negative rate of the common stream for each user. (7c) is the total power constraint. We use the rate-to-weighted MMSE (Rate-WMMSE) relationship to reformulate the problem [4], for acquiring the optimum precoder and equalizer. We define the augmented weighted-MSE for the k -th user as:

$$\zeta_{c,k} = u_{c,k} \varepsilon_{c,k} - \log_2(u_{c,k}) \quad (8)$$

$$\zeta_k = u_k \varepsilon_k - \log_2(u_k) \quad (9)$$

Fig. 1: Adapted transmitter side diagram of the RS-based precoding system

where $u_{c,k}$ and u_k are the non-negative valued weights for the common and private streams. With a given set of precoders, the MSEs $\varepsilon_{c,k}$ and ε_k are functions of $g_{c,k}$ and g_k , respectively. The Rate-WMMSE relationship can be established with the optimized equalizer and weights, such that:

$$\zeta_{c,k}^{opt} = \min_{u_{c,k}, g_{c,k}} \zeta_{c,k} = 1 - R_{c,k} \quad (10)$$

$$\zeta_k^{opt} = \min_{u_k, g_k} \zeta_{c,k} = 1 - R_k \quad (11)$$

The optimum equalizer and weights are obtained by the first order optimality conditions. More specifically, $g_{c,k}^{opt} = g_{c,k}^{MMSE}$, $g_k^{opt} = g_k^{MMSE}$, $u_{c,k}^{opt} = (\varepsilon_{c,k}^{MMSE})^{-1}$, and $u_k^{opt} = (\varepsilon_k^{MMSE})^{-1}$. Therefore, the problem (7) is equivalent to:

$$\begin{aligned} & \max_{\mathbf{C}, \mathbf{P}, \mathbf{g}, \mathbf{u}, r} r \\ & \text{s.t. (7b), (7c)} \end{aligned} \quad (12a)$$

$$C_k + R_k \geq r, \forall k \in \mathbb{K} \quad (12a)$$

$$1 - \zeta_{c,k} \geq \sum_{k=1}^K C_k, \forall k \in \mathbb{K} \quad (12b)$$

$$1 - \zeta_k \geq R_k, \forall k \in \mathbb{K} \quad (12c)$$

where r is an auxiliary variable, denoting the lowest user experienced rate which is the object to be maximized. $\mathbf{g} \triangleq \{g_{c,k}, g_k | k \in \mathbb{K}\}$ and $\mathbf{u} \triangleq \{u_{c,k}, u_k | k \in \mathbb{K}\}$ are the sets of equalizer and weights. Then the optimum precoders and equalizers can be obtained via, e.g., the alternating optimization described in Algorithm 1 in [4], which are used for the link-level simulations in next section.

III. LINK-LEVEL SIMULATIONS

Based on the output/solution of the RS optimization, the transmitter can find the optimum precoder and equalizer for a given channel.

A. RS-based transmission processing chain

We first adapt the transmission processing chain to introduce the RS functionality. Leveraged on our prior works [8], [9] where a link-level simulator equipping the LTE/NR compatible transmission chain has been developed, we propose a modified transmission chain as shown in Fig. 1. The modifications include:

- Additional block Message Splitter to transform the original message $\mathbf{W}^{og}$ to the RS-based message $\mathbf{W}^{RS}$.
- Additional block Rate-Splitting Precoder to precode the transmit symbol stream $\mathbf{S}$ to the RS-based transmit signal $\mathbf{X}$.
- Additional block Message Complementing between Message Splitter and modulator. The function of this block

Fig. 2: SIC Receiver Diagram

is to complement all the messages (include the common message) to match their length. More specifically, if each of the original messages is assumed to have a finite length, then after the message splitting, the $K+1$ streams most likely have unequal length. As a result, at certain channel conditions, the requirement $\mathbb{E}\{(\mathbf{S}_{:,j})(\mathbf{S}_{:,j})^H\} = \mathbf{I}$, $j \in \{1, \dots, J\}$ cannot be fulfilled, which may lead to non-optimal precoders and/or equalizers for that given channels.

- Channel state information at the transmitter side is available. Further investigations can be carried out by using pilot-based channel estimators to evaluate the robustness of RS to the imperfect channel state information.
- No channel coding is performed. We explicitly focus on the link-level performance comparison among different precoder schemes, thus using the uncoded BER instead of the coded BER.

B. SIC receiver

The receiver block diagram with SIC for the RS-based system is shown in Fig. 2. The received signal at the k -th user $\mathbf{y}_k$ is equalized symbol-by-symbol by the obtained MMSE equalizer $g_{c,k}^{MMSE}$, then passed through a soft de-modulator to estimate the entire common message i.e., $\hat{\mathbf{w}}_{c,k} \in \mathbb{R}^{1 \times L}$. The corresponding common message $\hat{\mathbf{w}}_{k0}$ for the user k can be extracted from $\hat{\mathbf{w}}_{c,k}$. For the private message, after substituting the estimated common symbol $\hat{s}_{c,k}$, successive interference cancellation is operated and the estimated private symbol $\hat{s}_k$ is obtained by applying equalizer g_k^{MMSE} symbol-by-symbol. Finally, the common and private parts are combined to form the estimated message $\hat{\mathbf{w}}_k$.

C. Fixed-rate transmission

Following the RS-based optimization, we can establish an equality relationship:

$$\frac{\mathbb{L}(\mathbf{w}_{k0})}{\mathbb{L}(\mathbf{w}_c)} = \frac{C_k}{\sum_{k=1}^K C_k}, \quad (13)$$

which involves the length of the common message, $\mathbb{L}(\mathbf{w}_c)$, and the length of the contributed portion from each original message, $\{\mathbb{L}(\mathbf{w}_{k0})|k \in \mathbb{K}\}$. These lengths indicate the optimal number of bits that each stream can carry within a given channel. However, in order to implement the link level simulator, instead of having different/variable rates (modulation orders) for common and private messages per given channel, the transmission rate is fixed at $\log_2(M)$ bits for both private and common messages, for every channel realization during the link level simulation.

D. Bit Error Rate Calculation

1) *Algorithm 1*: The optimization results or the optimum precoder and equalizer sets, i.e., $\mathbf{P}^{opt}$ and $\mathbf{g}^{MMSE} \triangleq \{g_{c,k}^{MMSE}, g_k^{MMSE}|k \in \mathbb{K}\}$, are directly input to the Rate-Splitting Precoder. The received signal can be simply obtained as: $\mathbf{y}_k = \mathbf{h}_k^H \mathbf{P}^{opt} \mathbf{S} + \mathbf{n}_k$. Then the estimated common and private message bits are obtained following the SIC operation in III.B. To calculate the BER, it can be emulated such that all users decode $\log_2(M)$ common bits and every user decode $\log_2(M)$ private bits, then the BER per channel realization is given as: $BER = \frac{e_c + e_p}{K * \log_2(M) * 2}$, where e_c and e_p denote the number of error for common and private message bits per channel realization.

2) *Algorithm 2*: Considering that a zero-forcing (ZF) precoder can be calculated by finding the null-space of each user's sub-channel (denoted as $\bar{\mathbf{H}}_k \triangleq [\mathbf{h}_1, \dots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \dots, \mathbf{h}_K]$), if the condition $N_t \geq K$ is satisfied, we propose this algorithm where, with RS, $K_c = K - N_t$ users' messages (with a set $\mathbb{K}_c$) can be merged into a common message. Hence, instead of having $\mathbf{p}_k = \text{null}(\bar{\mathbf{H}}_k)$ for zero-forcing, we have $\mathbf{p}_k = \text{null}(\bar{\mathbf{H}}_{k, \mathbb{K}_c})$ for rate-splitting. $\bar{\mathbf{H}}_{k, \mathbb{K}_c}$ denotes that the null-space of the sub-channel for only the k -th users, but also all the users whose message are mainly contributed to the common message. We perform a ZF-like precoding for those users not contributing to the common message, i.e., $k \notin \mathbb{K}_c$. Theoretically, we can find K_c non-zero values within the optimum common rate set $\mathbf{C}^{opt}$, meaning that totally K_c users have their original message contributed the common message and they are intentionally designed to receive IUI when transmitting the private message. So with this algorithm, we do not take into account these K_c users' private messages error rating after decoding.

IV. SIMULATION RESULTS

The performance of the considered RS-based precoding strategy with different system configurations is simulated. The baseline model is the classic linear precoding [10], denoted as NoRS. The simulation covers two aspects for both RS and NoRS systems:

- Achievable MMF rate,
- Average Bit Error Rate.

Results for both simulations are averaged over 100 independent and identically distributed Rayleigh channel realizations with the same power constraint. All the convex optimization

Fig. 3: MMF rate vs. SNR, $N_t = 4$, $K = \{5, 6\}$.

problems are solved by using the MATLAB based CVX toolbox [11]. Also, the uncoded BER performances are averagely measured across all channel realizations via bit level simulations. Quadrature Phase Shift Keying (QPSK) modulation is employed since the optimization does not set any limitation on the modulation order. Also, in both simulation aspects, we use the number of the transmit antennas $N_t = 4$ and the number of users $K = \{5, 6\}$ to emulate the overloaded scenario ($N_t < K$).

A. Achievable MMF rate

The results of the available MMF rate for the considered testing scenario are shown in Fig. 3. Due to the lack of transmit degree of freedom, the NoRS system in both configurations suffers rate saturation, with an achievable rate around 1.5bps/Hz and 2.25 bps/Hz at 30dB signal-to-noise ratio (SNR) for $K = 5$ and 6, respectively. With RS, the number of the required transmitter antennas is reduced by $K - N_t$, which, therefore, avoids the rate saturation. The gain of using RS over NoRS scheme is pronounced as shown in Fig. 3. On the other hand, with the same amount of the total transmission power, the less number of users being served, the more power being allocated to each user as well as the less inter-user interference being experienced by each user. Therefore the strictly higher MMF rate is achieved for the $K = 5$ than the $K = 6$ across all tested SNR values for both RS and NoRS schemes. The scheme is considered as the main

B. Simulation of average BER

In Fig. 4, the BER vs. SNR result for the RS-based system is presented, including both algorithms discussed in Sec.III.D. Here $K = 5$, and the QPSK modulation applies. In Fig. 4(a), an error floor appears in the high SNR regimes in the case of the RS with Algorithm 1. We take another look at the BER performance of the common and private parts separately, shown with dashed lines in Fig. 4(a). It can be observed that with Algorithm 1, the error floor appears due to the poor BER performance of the private part. Intuitively, with $K = 5$ and QPSK modulation, 10 bits are transmitted per channel

Fig. 4: BER vs. SNR, $N_t = 4$, $K = 5$ (a): Alg.1, (b): Alg.2.

Fig. 5: BER vs. SNR, $N_t = 4$, $K = \{5, 6\}$.

use. Applying RS, $K - N_t = 1$ user has its message split to form the common message. To ensure the decodability of the common message, RS precoding forces a dominant part of the available power for this user to deliver the common message, leaving the private part for this user not decodable, which results in the expected private part BER for the system is exactly $1/10 = 0.1$. On the contrary, using Algorithm 2 in Sec. III.D as shown in Fig. 4(b) significantly improves the BER performance, because the impact of the poorly decodable private message for the aforementioned user is removed. One commonality for both algorithms is that, at lower SNRs, the decodability of the private part is better than that of the common part. This exposes the benefit of multiplexing different private messages compared with transmitting one common message. In Fig. 5, the BER vs. SNR results for NoRS and RS systems with $K = 5$ and 6 are presented. Only the Algorithms 2 is considered. For both system configurations, the BER for the NoRS system declines slowly with the increased SNR, which can eventually reach an error floor. This error floor corresponds to the rate saturation. For the RS-based system, when $K = 6$, $K - N_t = 2$ users' messages should be merged

into the common message. Results show that the RS-based system is unlikely to experience a BER error floor. Also, together with the rate results Fig. 3, it can be stated that the increased number of users does decrease the performance of both rate and BER comparably. In low SNR region, NoRS scheme has a better BER performance than RS, which also proves the benefit brought by multiplexing different private messages than that comes from the common message, in noise-limited cases.

V. CONCLUSION

In this paper, we have first derived the optimization problem of the RS-based transmitter precoding in typical overloaded scenarios. Simulation results illustrated that the RS-based approach exhibits strictly higher and non-saturated MMF-rates compared to the NoRS method. Using the obtained optimum precoder and equalizer, we have then investigated the uncoded link level performance of the RS-based system, as the first step for the RS software implementation. From the BER performance, we have shown that the rate saturation in the NoRS system can be reflected by the BER error floor. For the RS-based system, we have considered different decoding algorithms, in the sense that the BER performance is limited by the user(s) with the interference-limited private rate, which can cause an error floor, especially with a fixed rate transmission. We have proposed a new decoding algorithm to address the issue, by dropping off the poorly decodable private messages.

REFERENCES

- [1] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Transactions on Signal Processing*, vol. 54, no. 6, pp. 2239–2251, June 2006.
- [2] H. Joudé and B. Clerckx, "Rate-splitting for max-min fair multigroup multicast beamforming in overloaded systems," *IEEE Transactions on Wireless Communications*, vol. 16, no. 11, pp. 7276–7289, Nov. 2017.
- [3] H. Chen *et al.*, "Rate-splitting for multigroup multicast beamforming in multicarrier systems," in *IEEE 19th International Workshop on SPAWC*, June 2018, pp. 1–5.
- [4] H. Joudé and B. Clerckx, "A rate-splitting strategy for max-min fair multigroup multicasting," in *IEEE 17th International Workshop on SPAWC*, July 2016, pp. 1–5.
- [5] H. Chen *et al.*, "Joint power and subcarrier allocation optimization for multigroup multicast systems with rate splitting," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 2, pp. 2306–2310, Feb. 2020.
- [6] B. Clerckx *et al.*, "Rate splitting for MIMO wireless networks: a promising PHY-layer strategy for LTE evolution," *IEEE Communications Magazine*, vol. 54, no. 5, pp. 98–105, May 2016.
- [7] D. Vargas and D. Mi (Eds), "LTE-Advanced Pro Broadcast Radio Access Network Benchmark," *5G-PPP project 5G-Xcast Deliverable D3.1*, Valencia, Spain, Nov. 2017.
- [8] H. Chen *et al.*, "Pioneering studies on LTE eMBMS: Towards 5G point-to-multipoint transmissions," in *IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, Aug. 2018, pp. 1–4.
- [9] M. Fuentes *et al.*, "Physical layer performance evaluation of LTE-Advanced Pro broadcast and ATSC 3.0 systems," *IEEE Transactions on Broadcasting*, vol. 65, no. 3, pp. 477–488, Sep. 2019.
- [10] D. Mi *et al.*, "Massive MIMO performance with imperfect channel reciprocity and channel estimation error," *IEEE Transactions on Communications*, vol. 65, no. 9, pp. 3734–3749, Sep. 2017.
- [11] M. Grant, S. Boyd, and Y. Ye, "CVX: MATLAB software for disciplined convex programming," 2008. [Online]. Available: www.stanford.edu/boyd/cvx/