Performance Evaluation of a Particle Filter Framework for Respiratory Motion Estimation in Nuclear Medicine Imaging

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Abstract—With the continual improvement in spatial resolution of Nuclear Medicine (NM) scanners, it has become increasingly important to accurately compensate for patient motion during acquisition. Respiratory motion produced by lung ventilation is a major source of artefacts in NM that can affect large parts of the abdominal-thoracic cavity. As such, a particle filter (PF) is proposed as a powerful method for motion correction in NM imaging. This paper explores a basic PF approach and demonstrates that it is possible to estimate non-stationary motion using a single respiratory cycle as training data. Using the XCAT phantom, 7 test datasets that vary in depth and rate of respiration were generated. The results using these datasets show that the PF has an average Euclidean distance error over all voxels of only 1.7 mm, about half of the typical dimensions of an NM voxel for clinical applications. The conclusion is that use of the PF is promising, and can be adapted to handle more sophisticated data such as those that arise in clinical situations.

I. INTRODUCTION

NUCLEAR medicine (NM) is the most sensitive approach for imaging functional processes of the human body and is an important tool in oncology, particularly in the early diagnosis of cancerous lesions. There has been significant improvement in spatial image resolution and reduction in acquisition times with the clinical adoption of improved detector and software technologies. Typical scan times of around 5 minutes per bed position for PET are now common [1]. However, such scan times are still subject to significant patient motion. As spatial image resolution improves, it additionally adds more importance that the issue of patient motion during such scans is addressed adequately.

One source of motion which affects most of the torso is respiration. In this paper, the proposed framework continually estimates respiratory motion throughout the NM scan, to enable the use of all available data for image reconstruction. Respiratory motion has been shown to exhibit complex behaviour [2][3]. Therefore, it is postulated that a probabilistic methods are needed for motion estimation and this forms the basis of the particle filter (PF) approach. To the knowledge of the authors, this represent the first time a PF has been used in NM motion estimation.

In this approach, internal organ deformation is inferred from a stereo surface capture of the anterior portion of the external surface of the torso. The inference is based on a state

transition model, which describes how the configuration of the organs $\mathbf{X}_k$, change over a discrete time index k , and a measurement model, which describes how the state $\mathbf{X}_k$, relates to the external observation, $\mathbf{Z}_k$. The overarching framework has been described in [3] and this paper focuses on the PF and further preliminary evaluation of its performance on simulated data generated from the XCAT digital phantom [4].

In this paper Section (II) will introduce the background of the PF as a Bayesian tracking method. Section (III) then outlines the current implementation of the PF including application specific adaptations. Section (IV) outlines the tests used for evaluation followed by results and discussion in Section (V). Finally concluding remarks and notes on further work are presented in Section (VI).

II. BACKGROUND

A particle filter is based on a Bayesian tracking perspective of the motion estimation problem, wherein it is represented as a first-order hidden Markov model (HMM) where the state is a hidden variable $\mathbf{X}_k$, but can be related to a non-hidden observation $\mathbf{Z}_k$, and state evolution is also inferred to a degree. The HMM structure is shown in Figure 1 and this generic nonlinear dynamic system can be formally described in state-space form:

$$\begin{aligned} &\text{Transition probability density} \\ \mathbf{X}_k = \mathbf{a}(\mathbf{X}_{k-1}, \mathbf{v}_{k-1}) &\leftrightarrow \overbrace{f(\mathbf{X}_k | \mathbf{X}_{k-1})} \end{aligned} \quad (1)$$

$$\begin{aligned} &\text{Measurement probability density} \\ \mathbf{Z}_k = \mathbf{b}(\mathbf{X}_k, \mathbf{w}_k) &\leftrightarrow \overbrace{g(\mathbf{Z}_k | \mathbf{X}_k)}. \end{aligned} \quad (2)$$

where the models $\mathbf{a}$ and $\mathbf{b}$ have respective stochastic components $\mathbf{v}_{k-1}$ and $\mathbf{w}_k$ representing uncertainties. This gives rise to their respective probability densities f and g .

An estimate of the state can be deduced from the posterior probability density $p(\mathbf{X}_k | \mathbf{Z}_{1:k})$. This posterior is conditional on the set of all previous observations up until the present time, $\mathbf{Z}_{1:k} \equiv \{\mathbf{Z}_1, \dots, \mathbf{Z}_k\}$. The actual estimate of the state may be taken as some moment such as the expected value $E[\mathbf{X}_k]$. In a PF, the posterior p is approximated as

$$p(\mathbf{X}_k | \mathbf{Z}_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(\mathbf{X}_k - \mathbf{X}_k^i). \quad (3)$$

The approximation in (3) samples probability (indicated by the weighted impulse train) along the space of $\mathbf{X}_k$ (i.e. possible

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states) by a set of N point masses, $\mathbf{X}_k^i$. The point masses are called particles and constitute Monte Carlo samples of the state space. The probability of the posterior at the locations of these point masses are given by the weights, w_k^i .

Fig. 1. Diagram of the system in state-space form illustrating the structure as a first order hidden Markov model. The posterior of the current state is represented by the bolded arrows and circles showing the information available at time k .

III. PARTICLE FILTER IMPLEMENTATION AND ADAPTATIONS

A. Implementation as an SIR filter

At the present stage of development, the PF is a Sampling Importance Resampling (SIR) filter [5]. In this implementation, the particles, $\mathbf{X}_k^i$ are generated through Monte Carlo sampling of the transition density, $f(\mathbf{X}_k | \mathbf{X}_{k-1})$. Consequently, the weights are then proportional to the measurement density (see Fig. 2):

$$\omega_k^i \propto g(\mathbf{Z}_k | \mathbf{X}_k^i). \quad (4)$$

Equation (4) is implemented by first using an estimate of the measurement density, g as an initial value for the weights. The weights are then normalized so that they sum to unity.

Fig. 2. Illustration of an iteration of the SIR filter. Diagram is based on [5].

In an SIR filter, resampling from the posterior (3) is also performed at each time step k so that the particles are more concentrated at regions where the state is more probable to avoid degeneracy, i.e. where many particles have negligible weight. This is achieved by forming a new set of samples

$\{\mathbf{X}_k^{i*}, i=1, \dots, N\}$ which are related to the original samples, $\mathbf{X}_k^j$ such that

$$P\{\mathbf{X}_k^{i*} = \mathbf{X}_k^j\} = \omega_k^j. \quad (5)$$

As a consequence of (5), at each time step, particles with larger weights will have more copies that are propagated to the next time step. Particles with very small weights have a low probability of propagation. The resampling step in an SIR filter is thus analogous to how genetic algorithms work. An iteration of the SIR filter is illustrated in Figure 2 and further details can be found in [5].

B. Transition and Measurement Models

As a simple first assumption, a second-order autoregressive process, AR(2), has been chosen as the transition model **a**, as it postulated that this would reasonably represent the pseudo-oscillatory nature of respiratory motion. By having the AR(2) process describing the evolution of organ configurations in another variable, $\mathbf{x}_k$, the state $\mathbf{X}_k$ can be made to consist of organ configurations from two time points, $\mathbf{x}_k$ and $\mathbf{x}_{k-1}$. Hence in $\mathbf{X}_k$, the transition model **a** is a first-order autoregressive process. Consequently the transition density f will then be a Gaussian. The parameters for the AR(2) process is found from stepwise least squares (LS) estimation [6] on a training dataset.

Likewise, the measurement density g is also assumed to be Gaussian. Its generative form **b**, is thus a linear map of the state with a stochastic component:

$$\mathbf{Z}_k = \beta \mathbf{X}_k + \beta_0 + M \mathbf{w}_k. \quad (6)$$

$\mathbf{X}_k$ consists of organ configuration from two time points as previously defined. $\mathbf{w}_k$ in (6) has elements of independent standard normal variable resulting in the covariance of $\mathbf{Z}_k$, $\Sigma_{\mathbf{Z}} = M M^T$. This covariance accounts for estimated inaccuracy of the map constants (β and β_0) and observation noise. In this paper the noise is assumed to be isotropic with a root mean squared error (RMSE) of 0.25 mm (based on the Polaris¹ 3D optical stereo tracking system). This system has been used before for motion correction in NM imaging such as for brain imaging [7]. The parameters for (6) are also found from LS estimation on training data.

C. Tracked State and Observable

Unlike other PF applications which track particular objects, the organ configurations that are tracked in this work $\mathbf{x}_k$, are affine transformation parameters for each organ o at each time point k . These are registered back to a baseline configuration θ , selected from training data. The parameters are obtained from iterative closest point (ICP) registration on training data as mentioned in [8]. This simulates actual training which will be achieved using, for example a series of a low dose dynamic CT scans acquired separately from the NM emission scan. The

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organs chosen for the current evaluation are the heart, liver, spleen, kidneys, lungs and ribcage. Surface renders of these organs generated from XCAT are shown in Figure 3. For the purpose of this paper, the baseline configuration θ is taken to be the respiratory rest phase in XCAT, which is at maximum exhalation.

Fig. 3. Left: Surface render of the chosen organs. Right: Surface render including an anterior portion of the external surface of the torso. The points chosen as the observable are marked in red (*).

The chosen observable, $\mathbf{Z}_k$ are the 3D coordinates of a set of 24 points on the anterior portion of the surface of the torso. The locations of these points are chosen so that their projections onto the coronal plane are spaced 7.8 cm apart during the rest phase. Their locations are also shown in Figure 3. In practice, the observable could correspond to physical markers or other representations of the anterior torso surface.

D. Application Specific Adaptations

As the SIR uses a sub-optimal importance density i.e. the propagation density f , and uses simplistic transition and measurement models, exploration of the state space may be deficient. Furthermore, as the training stage of the particle filter framework is assumed to consist of known organ configurations representing only one normal respiratory cycle, the transition and measurement models have a tendency to be over fitted to training data and thus their stochastic components will not reflect the actual variability seen during the NM imaging process. Therefore, three methods have been employed so that the models used in the PF framework can adapt to the expected variability in natural respiration. These methods are:

- 1) Dimensionality Reduction,
- 2) Incorporation of estimated respiratory parameters, and
- 3) Planned sampling of particles.

In method (1), the actual state and observable used for the transition and measurement models are PCA (principle component analysis) projections of the actual state estimate and stereo camera observation data. The respective PCAs are performed initially on training data for the respective models. The number of principle components (PCs) that are kept are based on the rank of the data matrix [9] and PCs that are too

small are also excluded. Additionally, the variance of the first PC of the projected state, is also increased by the amount given by an AR(2) approximation using only the first PC.

In method (2), the most significant PC of the observable is obtained from the PCA projection used in method (1). The amplitude ratio of each respiratory cycle is then estimated from this PC. This ratio is defined as the amplitude of a cycle relative to that of the training data as measured along the PC vector. Cycle periods are also estimated from the PC. The two parameters are then used respectively in the following manner:

- a) Time-warping observation data so that each cycle matches the period of the training data. This is accomplished using cubic spline interpolation.
- b) The estimated amplitudes of each cycle are used as scaling factors in planned sampling. This is designated as adaptation method (3).

In method (3), the samples or particles $\mathbf{X}_k^i$ are modified at every time point when respiration is estimated to be at the rest phase. This method of planned sampling is similar to that introduced in [10]. However in the PF framework of this paper, the sampling of all particles is planned. This is in contrast of performing planned sampling in just a proportion of them. In this paper the planned samples are also derived from training data.

The planned samples use a stochastic scalar scale factor based on the estimated amplitude ratio $\tilde{r}_k$, of the current respiratory cycle. The ratio is calculated from the estimated amplitude of the respiratory cycles as in method (2). This stochastic scale factor effectively prohibits the use of a Kalman Filter for tracking as the transition model is *not* linear-Gaussian with this modification.

IV. EVALUATION

The application of the PF framework to respiratory motion correction in NM has been evaluated as follows: seven XCAT respiratory cycles have been generated, each representing a dataset with unique set of respiratory parameters. The first dataset is considered to be a training set used to estimate model parameters. In practice this training set might, for example, be derived from low-dose dynamic CT. The remaining datasets have varied respiratory parameters to represent some aspect of natural variation in respiratory motion. The respiratory parameters for the training dataset is also used for an extra test dataset which includes a different realization of observation noise. In all tests, the respiratory cycles are set to begin at the rest phase i.e. at the end of exhalation. In reality the phase can be estimated from adaptation method 2 as described in subsection III(D).

Evaluation was performed by comparison with XCAT ground truth displacement data, which provides the known position of a 3D grid of organ-labelled points throughout a respiratory cycle. The chosen frame rate is 2 frames/sec with voxel dimensions of 3.25 mm. This is then interpolated in time using cubic splines to simulate an observation rate of 6 frames/sec. White Gaussian noise is added to the observation

data with an isotropic standard deviation of 0.25 mm to represent uncertainty in the simulated stereo camera observation.

On the other hand, to actually perform registration of organ voxels to a reference frame θ for evaluation of accuracy, another grid is initialized to voxel coordinates of the chosen organs in that reference frame. The position of these points on the grid during test sequences are then transformed in an affine manner using parameters estimated by the particle filter approach for the selected organ and compared to the positions in the reference frame. The particle filter (PF) framework is applied to the test datasets using the application specific adaptations, as described in subsection III(D). The parameters for the training and test datasets are shown in Table I. The results and discussion now follow in section (V).

TABLE I
RESPIRATORY PARAMETERS FOR TRAINING AND TEST DATASETS

Dataset	Motion Amplitude (cm)		Cycle Period (s)
	Diaphragm (SI)	Chest (AP)	
Training	2	1.2	5
Test 1	1	0.6	5
Test 2	3	1.8	5
Test 3	4	2.4	5
Test 4	1	0.6	3
Test 5	2	1.2	3
Test 6	3	1.8	3

V. RESULTS AND DISCUSSION

Using the test datasets outlined in Table III the Euclidean distance error averaged over organ voxels is shown in Figure 4. There error bars show the standard deviation over all test datasets. The comparison with the mean errors when there is no motion estimation show that the PF framework has greatly reduced the effect of respiratory motion. The overall mean error has been reduced from 12.1 mm to only 1.7 mm which is almost half the voxel dimension.

Fig. 4. Mean error caused by respiratory motion, with and without correction by the PF framework.

On the other hand, Figure 5 compares the mean errors of the PF framework with the mean errors of ICP registration using

assumed knowledge of organ configurations. The plot shows that the PF framework only adds a small amount of additional error to that of ICP registration, which can only be used on the test datasets if the configurations of internal organs is known. In postulating the required registration parameters, the PF framework thus is able to estimate the state of internal organs accurately. On average the best that ICP registration can perform is to only slightly reduce the errors to 1.6 mm.

Fig. 5. Mean error after motion estimation using the PF framework and ICP registration.

VI. CONCLUSION AND FURTHER WORK

The results discussed in Section V show that the PF framework as implemented in this work is a promising method of estimating internal organ state for motion correction in nuclear medicine imaging. The current implementation approximates the transformation needed to account for motion to a sufficient degree of accuracy for use in functional imaging. However, there is still scope for improvement based on further development of the piecewise-affine transformation for organ deformation.

In evaluating the PF framework itself, it is acknowledged that the simulation in this paper uses a somewhat simplified representation of clinical respiratory motion. As has been reported in [3], motion of the anterior portion of the external torso surface itself is not simple but can be categorized according to how the motion of the upper torso relates to motion of the lower torso. Internal motion is also expected to show a similarly complex pattern [2]. Future work will vary AP and SI motion amplitude independently and use XCAT 2.0 [11]. These will be compared with tests derived from real data that is nearer to clinical conditions. The incorporation of observables based on volunteer data in [12] is also under development.

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