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**Analysis of Outrigger-Braced RC Supertall Buildings:
Core-Supported & Tube-in-Tube Lateral Systems**

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Analysis of Outrigger-Braced RC Supertall Buildings: Core-Supported & Tube-in-Tube RC Lateral Systems

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Summary

This study is primarily focused on the approximate analysis of reinforced concrete outriggers which are commonly used in the design and construction of Supertall Buildings subject to distributed horizontal loads. Existing global analysis formulae that provide preliminary results for lateral deflections and moments are reviewed for two lateral load resisting systems, namely Core-Supported-with-Outrigger system (CSOR) and less frequent Tube-in-Tube-with-Outrigger system (TTOR). These formulae are only applicable for CSOR and neglect the reverse rotation of the outrigger actually suffered due to the propping action from the outer columns; and give rather high predictions of the deflections compared to advanced numerical FE models. An improved model is proposed which overcomes this issue, and provides more consistent results to FE predictions. The same can also be extended to TTOR. Several case studies are investigated to verify the accuracy of the proposed methodologies. The global analysis is followed by the local analysis of reinforced concrete outrigger beams using strut-and-tie modelling and nonlinear finite element analysis to obtain optimised reinforcement layouts (reduction of quantities of reinforcement). The results highlight the different challenges in detailing such structural members which are heavily loaded (high congestion of reinforcement) and the behaviour at failure can be brittle.

Keywords

Supertall building; reinforced concrete; outrigger; tube-in-tube; lateral systems; simplified model.

List of Symbols

d	Width of building
d_o	Height of outrigger
f_c	Specified compressive strength of concrete
h_o	Effective span of outrigger
k	Ratio between flexural stiffness of core and push-pull columns
t_o	Total thickness of outrigger walls on one side of core
w	Uniformly distributed lateral loading
x_i	Distance of outrigger i from top of the building
z	Distance of outrigger from bottom of the building
A_{COL}	Total cross sectional area of columns on one side
A_{OR}	Total cross sectional area of outrigger walls on one side
C	Summation of columns' inertia to height ratios at each level
CSOR	Core-Supported-with-Outrigger lateral system
$(EA)_{COL}$	Axial stiffness of external hold-down columns
$(EI)_{CORE}$	Flexural stiffness of core
$(EI)_{OR}$	Flexural stiffness of outrigger
$(EI)_{Transf.}$	Equivalent (transformed) flexural stiffness of TTOR system
G	Summation of beams' inertia to span ratios at each level
$(GA)_{FRAME}$	Shear stiffness of external moment frame (external tube)
$(GA)_{OR}$	Shear stiffness of outrigger
H	Height of building
I_{CORE}	Second moment of area of core
I_{OR}	Total second moment of area of outrigger walls on one side
$M/(bd^2)$	Flexural index in concrete beams
$M_{b,Transf.}(z)$	Bending moment of core in TTOR system, at level z
M_{ORi}	Moment transferred by outrigger set i to the centre of the core
$M_{CORE}(z)$	Moment taken by core at height z
$M_{FRAME}(z)$	Moment taken by shear frame at height z
S	Flexural stiffness parameter of core and pull-down columns
S_1	Flexural stiffness parameter of the outrigger
S_2	New index proposed accounting for reverse rotation of outrigger
TTOR	Tube-In-Tube-with-Outrigger lateral system

α	Square root of frame stiffness to shear wall stiffness (2D frame)
α^*	Revised α factor to account for 3D structure
β	Flexural stiffness ratio between core and outrigger
ε_1	Principal tensile strain in the concrete
η	Concrete compressive strength reduction parameter
θ_{CORE}	Rotation of outrigger-restrained core at level x_i for due to loads
θ_1	Rotation of outrigger, function to its bending stiffness
θ_2	Rotation of outrigger due to axial deformation of outer columns
θ_3	Rotation of outrigger due to reverse deformation
λ_{3D}	Transformation factor
ν	Flexural stiffness ratio between core and the outer columns
ω	Omega index for CSOR system (Smith & Coull 1991)
$\Delta_{\text{Dual}}(z)$	Deflection of wall-frame structure at level z (from base)
Δ	Deflection of the building at the top
Δ_2	Axial deformation of hold-down columns
Δ_3	Deflection of outrigger, function to flexural and shear stiffness

1 Introduction

The architectural design of all-concrete modern tall building typically comprises a central spine that encloses elevators, stairs, and service shafts throughout the height. The usable space between the central spine and outer perimeter provides high-rise views generally through enveloping glass façades. The use of Mechanical-Electrical-Plumbing (MEP) floors is almost inevitable for tall buildings to achieve an economical design and easier maintenance; this floor allocates all equipment required within reasonable distances from the higher units. Such advantageous setup has been exploited by structural engineers to fit in the components of appropriate-for-height lateral load resisting systems, including RC cores at the centre of the building, columns, and beams at the perimeter.

Core-supported and tubular lateral systems are generally recognised as most efficient for 40 to 80 storeys^[1]. However, if the heights exceed 300 m (around 90 storeys) when the building is classified as Supertall^[2], an additional key element called outrigger would be necessary to marry with the aforementioned systems and enhance the efficiency against lateral loads. Some examples of buildings with outrigger walls in the bracing system include Torrespacio (56 storeys), Jin Mao (88 storeys), Taipei (101 storeys) and Burj Khalifa (162 storeys).

The outriggers, which are basically one or two storeys deep beams cantilevering from central core and propped by perimeter columns, are most appropriately located within the intermediate MEP floors to minimize any loss of sellable spaces without disturbance to circulation or panoramic views. The fundamental role of outriggers is reducing the lateral sway of core-supported buildings by rigidly linking the central core to the external columns. An opposing concentrated moment is generated in the core upon

applying lateral loads as shown in Figure 1 and Figure 2. Recent research has been carried out on the seismic response of outrigger-braced buildings (e.g. [3, 4, 5]).

Structural analysis of a statically indeterminate outrigger-braced system is a task that many would argue is for computers to resolve, using 3D Finite Elements Analysis (FEA) as commonly adopted in current practice. However, using approximate methods based on mechanical models [6, 7, 8] can be very useful towards building initial structural design schemes (i.e. optimising the number and location of outriggers [9, 10, 11, 12], or deciding the type of outrigger-bracing system) as well as verifying results from complex computational models. Conceptual analytical models can provide confidence throughout the engineering design process although in some cases these models are not applicable to all types of outrigger-bracing systems and they can provide inconsistent results to FEA predictions. Guidelines for practitioners on FEA of concrete structures [13, 14] highlight the need of simplified conceptual models for the verification process. There is a strong international interest in developing robust guidelines and different organisations and committees (e.g. NAFEMS, ASME) have been formed.

This paper is divided in two main parts. The first part (section 2) presents a modified mechanical model for the global analysis of tall and supertall buildings which provides more consistent results to FEA predictions compared to simplified existing classical mechanical models which can also be applied to different outrigger-braced systems. In the second part of the paper (sections 3 and 4), the local analysis for the design of a reinforced concrete outrigger element is presented using strut-and-tie modelling with the member forces obtained from the global analysis. The results from the local analysis (in terms of required reinforcement) are also compared to optimised solutions using design-orientated nonlinear FEA tools [15].

2 Global analysis

Some design formulae are available in the literature for the global analysis of Supertall structural systems to calculate lateral sway and moments/shears of the main structural components [1, 6, 16, 7, 17, 18]. The widely accepted approach proposed by Smith and Salim (1981) [6] is applicable to Core-supported-with-outrigger (CSOR) lateral systems shown in Figure 3a. This approach has limited accuracy and it is normally used to obtain preliminary results. An enhanced model is proposed in this section which considers the reverse rotation of the RC outriggers due to the propping forces of the outer columns, providing more accurate predictions of the global response. This approach is also extended to tube-in-tube-with-outrigger (TTOR) lateral systems (Figure 3b) by considering the bending stiffness of the external tube. TTOR systems are less frequent than CSOR in current practice due to cost and disruption to the outer perimeter, although in some cases (e.g. irregular building shapes) TTOR might be needed.

In this section, the review of the existing methods is followed by the derivation of the proposed enhanced formulae, and a comparison between the proposed and existing approach against FE simulations for a series of case studies, in which the rigidity of the outrigger was varied. Noting that the rigidity of outriggers is normally expressed by a numerical index proposed by Smith and Coull (1991) [7], called omega factor ω :

$$\omega = \frac{\beta}{12(1 + \nu)} \quad (1)$$

$$\beta = \frac{(EI)_{CORE} d}{(EI)_{OR} H} \quad (2)$$

$$\nu = \frac{(EI)_{CORE}}{(EA)_{COL} (d^2/2)} \quad (3)$$

Where H is the height of the building, d is the width, $(EA)_{COL}$ is the tie-down axial stiffness of the columns (on one side of core), $(EI)_{CORE}$ and $(EI)_{OR}$ are the flexural

stiffness of the core and outriggers respectively. Omega is an important index to quantify the structural performance of outriggers; very much like $M/(bd^2)$ in RC beams. Omega relates the core and outrigger inertias, slenderness of the building and axial area of the exterior columns. Zero value of omega corresponds to infinite rigidity and high moment transferred by the outrigger section, and any value above 1 reflects an extremely inefficient section. Typical values of omega ω should be between 0.1 and 0.4. Figure 4 shows omega values for different buildings slenderness, core-outrigger inertia ratios, and ratio of core to push-pull columns total inertia ν in [7].

2.1 Review of existing simplified approaches

2.1.1 CSOR system (Smith and Salim, 1981 [6])

Smith and Salim (1981) [6] proposed an approximated formula (Equation (4)) for the general solution of outriggers moments in CSOR systems. The formulae can be derived by equating the rotation of the outrigger beam with that of the core. They considered in their calculations the flexural rigidity of the core and outrigger, axial area of external tie-down columns, height of building, and its width (Figure 5). The stiffness of the outriggers were assumed equal throughout the height.

$$\frac{w}{6(EI)_{\text{CORE}}} \begin{bmatrix} H^3 - x_1^3 \\ H^3 - x_2^3 \\ \vdots \\ H^3 - x_n^3 \end{bmatrix} = \begin{bmatrix} S_1 + S(H - x_1) & S(H - x_2) & \dots & S(H - x_n) \\ S(H - x_2) & S_1 + S(H - x_2) & \dots & S(H - x_n) \\ \vdots & \dots & \ddots & S(H - x_n) \\ S(H - x_n) & S(H - x_n) & \dots & S_1 + S(H - x_n) \end{bmatrix} \times \begin{bmatrix} M_{\text{OR1}} \\ M_{\text{OR2}} \\ \vdots \\ M_{\text{ORn}} \end{bmatrix} \quad (4)$$

The parameters S and S_1 are given by $S = \left[\frac{1}{(EI)_{\text{CORE}}} \right] + \left[\frac{2}{d^2(EA)_{\text{COL}}} \right]$ and $S_1 = \left[\frac{d}{12(EI)_{\text{OR}}} \right]$ respectively, $M_{\text{OR}i}$ is the total moment of the outrigger set i at centre of the core (total push-pull columns reaction multiplied by width of building d), w is the lateral loading, x_i is the distance of outrigger i measured from the top of the building.

The maximum horizontal deflection at the top, Δ , can be obtained using first principles according to the following expression

$$\Delta = \frac{wH^4}{8(EI)_{\text{CORE}}} - \frac{1}{2(EI)_{\text{CORE}}} \sum_{i=1}^n M_{\text{Ori}}(H^2 - x_i^2) \quad (5)$$

2.1.2 Dual core-frame system without outrigger (Smith and Coull, 1991^[7]):

Smith and Coull (1991)^[7] provided an approximated formulae to calculate the moments taken by the core and the frames in 2D wall-frame (or dual) systems, by assuming the frames take the shear whilst the core takes the moments. This model is used in the proposed approach later in the paper, as an intermediate step in the derivation of the formulae for TTOR systems.

For the system without outriggers, Smith and Coull (1991)^[7] give the following equations for the moments at each height-station (z) from the bottom:

$$M_{\text{CORE}}(z) = wH^2 \left\{ \frac{1}{(\alpha H)^2} \left[\frac{\alpha H \cdot \sinh \alpha H + 1}{\cosh \alpha H} \cdot \cosh \alpha z - \alpha H \cdot \sinh \alpha z - 1 \right] \right\} \quad (6)$$

$$M_{\text{FRAME}}(z) = \frac{w(H-z)^2}{2} - M_{\text{CORE}}(z) \quad (7)$$

Where $M_{\text{CORE}}(z)$ and $M_{\text{FRAME}}(z)$ are the moments in the core and the frame respectively (at height z from the base, in buildings subjected to a horizontal uniformly distributed load w). All parameters are functions of α which relates to the shear force Q at the frame (Figure 6) and the core's flexural stiffness using a 2D model as follows:

$$\alpha = \sqrt{\frac{(GA)_{\text{FRAME}}}{(EI)_{\text{CORE}}}} \quad (8)$$

$$(GA)_{\text{FRAME}} = \frac{12E}{h \left(\frac{1}{C} + \frac{1}{G} \right)} \quad (9)$$

$$C = \sum (I/h)_{\text{COL}} \text{ per floor}, \quad G = \sum (I/L)_{\text{BEAM}} \text{ per floor} \quad (10)$$

The model described above is relatively simple to implement in a spreadsheet to consider the stiffness of all the height stations. The lateral deflection is in this case:

$$\Delta_{Dual}(z) = \frac{w}{\alpha^4 (EI)_{CORE}} \left[\frac{\alpha H \cdot \sinh \alpha H + 1}{\cosh \alpha H} \cdot (\cosh \alpha z - 1) - \alpha H \cdot \sinh \alpha z + (\alpha H)^2 \left(\frac{z}{H} - 0.5 \left(\frac{z}{h} \right)^2 \right) \right] \quad (11)$$

2.2 Proposed approaches for global analysis

2.2.1 Proposed enhanced model for CSOR systems:

A more accurate formula is proposed in this paper by following the same compatibility method of equating rotations of outrigger and core, but with additional term accounting for the outrigger's reverse rotation under the propping force of outer columns as shown in Figure 7. The rotation of the outrigger and the vertical deflection has three components, namely (1) rotation of the core, (2) rotation due to the shortening of the outer columns, and (3) rotation due to the reverse deformation of the outrigger. These rotations are labelled with subscripts 1, 2 and 3 respectively. These rotations are

$$\theta_1 = \frac{M_{OR} d}{12 (EI)_{OR}} \quad (12)$$

$$\theta_2 = \frac{\Delta_2}{(d/2)} = \frac{2M_{OR}(H-x)}{d^2(EA)_{COL}} \quad (13)$$

$$\theta_3 = \frac{\Delta_3}{(d/2)} = -\frac{2M_{OR}h_o^3}{3d^2(EI)_{OR}} - 2.4 \frac{M_{OR}h_o}{d^2(GA)_{OR}} \quad (14)$$

and the vertical deflections are $\Delta_2 = \frac{M_{OR}(H-x)}{d(EA)_{COL}}$ and $\Delta_3 = -\frac{M_{OR}h_o^3}{3d(EI)_{OR}} - 1.2 \frac{M_{OR}h_o}{d(GA)_{OR}}$

The rotation of the core for the uniformly distributed horizontal load is obtained from integrating the moment equation leading to the following expression:

$$\theta_{CORE} = \frac{w}{6 (EI)_{CORE}} (H^3 - x^3) - \frac{M_{OR}}{(EI)_{CORE}} (H - x) \quad (15)$$

From compatibility, $\theta_{CORE} = \theta_1 + \theta_2 + \theta_3$ and substituting each rotation gives:

$$\frac{w(H^3 - x^3)}{6 (EI)_{CORE}} = M_{OR} \times \left[\frac{d}{12 (EI)_{OR}} + \frac{2(H-x)}{d^2(EA)_{COL}} + \frac{(H-x)}{(EI)_{CORE}} - \frac{2h_o^3}{3d^2(EI)_{OR}} - \frac{2.4h_o}{d^2(GA)_{OR}} \right] \quad (16)$$

Equation (16) can be further simplified using similar parameters S and S_1 proposed by Smith and Salim (1981) ^[6] (refer to section 2.1.1) leading to:

$$\frac{w(H^3 - x^3)}{6(EI)_{\text{CORE}}} = M_{\text{OR}} \times [S_2 + S(H - x)] \quad (17)$$

Where M_{OR} is the moment of the outrigger which can be now obtained from equation (17). A new term is defined as $S_2 = S_1 - \frac{2h_o^3}{3d^2(EI)_{\text{OR}}} - \frac{2.4h_o}{d^2(GA)_{\text{OR}}}$ where $S_1 = \left[\frac{d}{12(EI)_{\text{OR}}} \right]$ as defined by Smith and Salim (1981) ^[6] and h_o is equal to $0.95(d/2)$ or $0.85(d/2)$ for outriggers with span-to-height ratio between 2 and 3 or 1 and 1.5 respectively. The general solution for the moments of n outriggers can be extended to:

$$\frac{w}{6(EI)_{\text{CORE}}} \begin{bmatrix} H^3 - x_1^3 \\ H^3 - x_2^3 \\ \vdots \\ H^3 - x_n^3 \end{bmatrix} = \begin{bmatrix} S_2 + S(H - x_1) & S(H - x_2) & \dots & S(H - x_n) \\ S(H - x_2) & S_2 + S(H - x_2) & \dots & S(H - x_n) \\ \vdots & \dots & \ddots & S(H - x_n) \\ S(H - x_n) & S(H - x_n) & \dots & S_2 + S(H - x_n) \end{bmatrix} \times \begin{bmatrix} M_{\text{OR}1} \\ M_{\text{OR}2} \\ \vdots \\ M_{\text{OR}n} \end{bmatrix} \quad (18)$$

Proposed Equation (18) follows an identical format to Equation (4) in ^[6] but with parameter S_2 instead of S_1 . The horizontal deflection Δ is obtained similarly in this case using Equation (5) with the moments of the outriggers from Equation (18).

2.2.2 Proposed enhanced model for TTOR systems:

The proposed model can be adapted to TTOR lateral systems; the main difference in the model in this case is that the outer frame takes moments due its bending capacity, in addition to the known push-pull couple generated by the outrigger link. This suggests the following procedure consisting of 5 steps to analyse a TTOR system:

Step 1. Analyse the ‘outriggerless’ system according to section 2.1.2 to obtain the moments of the core and frame, Equations (6) and (7) respectively, using a revised parameter α^* given by Equation (19) instead of α . The lambda factor in Equation (19)

transforms the 3D structure into an equivalent 2D model; this factor was derived empirically by the authors based on the results from FE simulations.

$$\alpha^* = \frac{1}{\lambda_{3D}} \cdot \sqrt{\frac{(GA)_{\text{FRAME}}}{(EI)_{\text{CORE}}}} \quad (19)$$

Where $\lambda_{3D} = (k)^{(1/k)^{0.27}}$ with $k = [(EI)_{\text{FLANGE}}/(EI)_{\text{CORE}}] > 1.2$ where $(EI)_{\text{FLANGE}}$ is the moment of inertia for push-pull columns about the centre of the building, and $(GA)_{\text{FRAME}}$ is the shear stiffness of the frames parallel to the loading direction given by Equations (9) and (10).

Step 2. Calculate the deflection at the top of the dual outriggerless system Δ_{Dual} using Equation (11) and using α^* instead of α .

Step 3. Analyse the system as core-and-outrigger only, neglecting the flexural stiffness of outer frame, based on Equation (18) with fundamental difference as follows:

$$\frac{w}{6(EI)_{\text{CORE}}} \begin{bmatrix} H^3 - x_1^3 \\ H^3 - x_2^3 \\ \vdots \\ H^3 - x_n^3 \end{bmatrix} = \begin{bmatrix} S_2 + (H - x_1) & (H - x_2) & \dots & (H - x_n) \\ (H - x_2) & S_2 + (H - x_2) & \dots & (H - x_n) \\ \vdots & \dots & \ddots & (H - x_n) \\ (H - x_n) & (H - x_n) \dots & S_2 + (H - x_n) \end{bmatrix} \times \begin{bmatrix} M_{\text{OR}1} + M_{\text{FRAME}1} \\ M_{\text{OR}2} + M_{\text{FRAME}2} \\ \vdots \\ M_{\text{OR}n} + M_{\text{FRAME}n} \end{bmatrix} \quad (20)$$

Where $M_{\text{FRAME}i}$ are the moments taken by the frame alone from Equation (7).

Step 4. The total lateral deflection can be calculated from the following expression:

$$\Delta = \frac{wH^4}{8(EI)_{\text{Transf.}}} - \frac{1}{2(EI)_{\text{Transf.}}} \sum_{i=1}^n M_{\text{OR}i} (H^2 - x_i^2) \quad (21)$$

Where $(EI)_{\text{Transf.}} = wH^4 / (8\Delta_{\text{Dual}, z=H})$

Step 5. The moment diagram of the core is obtained as the summation of the basic moment and the moments of the outriggers at different levels using the following

$$M(z) = M_{b, \text{Transf.}}(z) + \sum_{i=1}^{\text{NO}} M_{\text{OR}i} \quad (22)$$

where NO is the number of outriggers above position z and $M_{b,Transf.}(z)$ is a proposed basic moment given by Equation (23) which is based on the Smith and Coull (1991) ^[7] expression (Equation(6)) with the variation in the coefficient 0.6.

$$M_{b,Transf.}(z) = wH^2 \left\{ \frac{1}{(\alpha H)^2} \left[\frac{\alpha H \cdot \sinh \alpha H + 0.6}{\cosh \alpha H} \cdot \cosh \alpha z - \alpha H \cdot \sinh \alpha z - 0.6 \right] \right\} \quad (23)$$

2.3 Comparison between existing and proposed approaches with FE results:

The proposed approaches were validated against the results given by existing formulae ^[6, 7] and the results from 2D and 3D FE models for eleven case studies of buildings with a CSOR system and two with a TTOR system. The main two variables changed were the value of omega and the number of outriggers. The plan and side view of the systems are shown in Figure 8, including the main geometry parameters adopted. The buildings considered had a total height between 200 m and 500 m (5 m height between storeys) with an aspect ratio ranging between 4 and 10 (anything above 5 is considered slender in ^[1] amongst others) and ω between around 0.8 and 0.1. The geometry of the case studies is summarized in Table 1.

The FE analyses were carried out using ETABS ^[15] finite element software with frame and shell elements and diaphragms as shown in Figure 9. Linear elastic FE analyses were carried out for consistency with existing and proposed formulae. The Young modulus and Poisson ratio adopted for all structural elements in global analysis were 28,000 MPa and 0.2 respectively which are typical values for normal strength concrete. A uniform lateral load w was applied in the FE models as assumed in the design models; the values adopted are given in Table 1. The FE results of the lateral deflection and moments of the outriggers obtained are shown in Table 2.

Figure 10a shows that existing approaches ^[6, 16, 7] provide global deflections which are larger than FE predictions; for values of omega ω near 1 the differences obtained were

around 40% larger whereas for values of omega of around 0.1 the predictions are closer to each other. The proposed approach provided consistent results of lateral deflections with FE analysis irrespectively of the value of omega. These results were similar for CSOR systems studied with one, two, and three outriggers (Figure 10a,b).

Regarding outrigger moment predictions (M_{ORi}), the proposed formulae provided consistent results to FE analysis for all cases considered as shown in Figure 11a-d. Significant differences were found between these analysis and those obtained using formulae in [6, 16, 7]. For the bottom outrigger, the previous approaches provided significantly lower values of moments. The differences were as large as 250% which is shown for M_{OR1} for one outrigger cases (Figure 11a), M_{OR2} for two outrigger cases (Figure 11c) and M_{OR3} for three outrigger cases (Figure 11d). On the contrary, for the top outriggers in cases with two or three outriggers, Figure 11b shows that the predicted moments were larger using existing approaches [6, 16, 7] than those obtained using FEA. The moments of the intermediate outrigger M_{OR2} in the case of three outriggers were similar to that obtained using FE and proposed formulae (Figure 11c).

The TTOR systems considered (OR1-3D-TinT and OR2-3D-TinT) are summarized in Table 1; in this case the shear stiffness of the side frames parallel to the load is taken into account as given by Equation (19). For the two TTOR cases considered, the two frames corresponding to the side columns and beams (Figure 8) consist of 7 columns (1.5 m square and 5 m height) and 6 beams (5 m length, 1.5 m deep and 0.5 m wide). These dimensions lead to a value of $(GA)_{FRAME}=17,640 \text{ MNm}^2$, $k=10.526$ and $\alpha^*H=1.84$ according to Equation (19). The maximum lateral deflection of the dual outriggerless system for the two TTOR cases studied is $\Delta_{DUAL}=46.52 \text{ mm}$ according to Equation (11). The deflections and moments of the outriggers were obtained according

to the proposed method described in 2.2.2 (Smith and Coull 1991 ^[7] formulae is not applicable to TTOR systems). In terms of deflections, the proposed formulae provided comparable results to FE predictions (between 15% and 30% differences) as shown in Figure 10 and the outrigger moments were also consistent with FE predictions for all cases considered. Figure 12a shows the basic moment predictions for the outriggerless system from the FE and the proposed equations (19) and (23). As shown in Figure 12b,c the distribution of bending moments given by Equation (22) gives comparable results to those obtained numerically with FE models.

The results shown in this section confirm that the proposed approaches provide comparable predictions of the global response of the buildings to FE analyses. The proposed formulae can be easily incorporated in optimisation techniques similar to those in ^[17, 1, 19, 12] amongst others, to assess the optimal number of outriggers and their location, although this is not the main focus of this work. The outrigger bending moments obtained with the proposed approach are more consistent with FE analyses than using existing formulae as it takes into account the reverse rotation introduced at the cantilever end of the outrigger by the outer columns or belt. Therefore, it can be concluded that the proposed formulae can be a very useful tool for the preliminary stages of the design as well as for subsequent stages in the design during the verification of more complex analysis considering the final outrigger-bracing scheme adopted.

3 Local analysis of outrigger beams

Outriggers are essentially heavily loaded deep structures with typical span-to-depth ratios between 1:1 and 3:1, such aspect ratio is necessary to achieve minimum values of omega ω ^[7] resulting into a high efficiency of the outrigger-bracing system. Outriggers can consist of steel trusses (e.g. 2IFC Building in Hong Kong or New York Times Tower in the USA) or reinforced concrete walls (e.g. Trump Tower in the USA or Waterfront Place in Australia). This paper focuses in the latter case; different designing and construction challenges are discussed in sections 3 and 4. Reinforced concrete outrigger beams are treated in design as “discontinuity regions” which are generally defined as areas where beam theory is not applicable ^[20]; in this case beam theory is not valid due to the low span-to-depth ratio. International design codes for concrete structures such as Eurocode 2 ^[21] and ACI 318-14 ^[22] or basis for future codes such as Model Code 2010 ^[23] recommend discontinuity regions to be designed using strut-and-tie modelling (STM) or stress field analysis (e.g. ^[24]).

Some debate has taken place over the last 20 years, with some contradicting views (e.g. ^[25, 26, 27]) on whether size effect is significant in deep and short span beams. This can be a point of concern in shear critical thick concrete members such as outriggers (member depths can easily exceed 10 m). Recent large scale test in Toronto ^[27] on a 4-m deep shear reinforced deep beam showed that STM was able to predict the strength reasonably well, which supported that size effect was not an issue in this particular case. In this work the STM was applied for the design of outrigger beams (refer to section 3.1). A preliminary FE plane stress analysis was performed to investigate the elastic stress fields for conventional outrigger dimensions. The reinforcement obtained from the STM approach was then used in a nonlinear FEA to assess the level of conservatism

provided by the STM approach and to optimize the design (quantities of reinforcement) using an iterative manual approach. STM was also used for the design and analysis of the deep link beams in Burj Khalifa Tower in ^[28]. However, the deep beams in ^[28] are fully fixed at both ends as part of a buttress core system whereas in the outrigger investigated in this paper one end is partially fixed (CSOR system).

3.1 Strut-and-Tie Modelling (STM)

STM involves establishing different load transfer paths within the concrete section by drawing the appropriate notional strut and ties following the principles of plasticity. Early work on STM ^[20] recommended the use of elastic analysis to define different strut-and-tie systems satisfying equilibrium. However, nowadays it is widely accepted that nonlinear approaches (considering the influence of the discrete location of the reinforcement) provide more reasonable predictions of the stress fields which can be used more confidently towards developing consistent strut-and-tie models ^[29, 30, 19]. After the strut-and-tie system is defined, the forces of each member is assessed in order to calculate the reinforcement required in each tie and to check the nodal regions and verify that local failure is avoided. Detailing is critical in order to provide ties which are well anchored and to avoid localization of strains leading to the development of critical cracks. The STM method is based on the lower bound theory of plasticity and therefore it results into conservative designs, although the level of conservatism is uncertain and this varies depending on the governing mode of failure. This was investigated in section 3.3 with a case study using plausible dimensions for an outrigger beam with moments obtained from the global analysis discussed in previous sections.

In order to develop a reasonable strut-and-tie model for the outrigger, the structural member was considered as cantilever deep beam with a concentrated load applied at the outer push-pull column at the end of the span resulting in the elastic stress field shown in Figure 13a. The lateral-load bending moment diagram of the shear wall is reversed (or reduced in other cases) below the outrigger level as demonstrated by the principal tension and compression lines which travel diagonally to the opposite side of the wall as they spread down. The elastic stress field shown in Figure 13 suggests a classical diagonal configuration of strut and ties with some level of fixity provided by the outer columns (Figure 14). Such load transfer arrangement (statically indeterminate system) includes two possible load paths; namely a top-loaded and a bottom-loaded beam as shown in Figure 13b. The model also simulates the fixity of the outrigger at the top and bottom by the monolithically concreted diaphragm floors. The fraction of the total load that is transferred by each load path depends on the amount of hanging reinforcement provided in the design used to transfer the load into the top of the beam. The final strut-and-tie model adopted is shown in Figure 13b and Figure 15.

The diagonal strut-and-tie model adopted is commonly used in shear-dominated deep coupling beams in seismic design in American and European codes^[22, 31] as recognised in^[32]. Eurocode 8^[31] also requires an additional column-like reinforcement with hoops to prevent buckling, as well as uniform reinforcement meshes on the faces of the beam that should not be anchored to the diagonal elements but extended into the coupled shear walls by no more than 150 mm. The reinforcement meshes can contribute towards resisting vertical loads and also towards providing additional load paths for stress redistribution. Therefore, in this work, a “X” type of reinforcement cage was finally adopted as shown in Figure 16 which is further discussed in section 4. This

reinforcement arrangement was chosen based on strength considerations whereas stiffness considerations were addressed in the global analysis (section 2) for sizing the outrigger elements. Alternative reinforcement layouts (e.g. orthogonal) and strut-and-tie models can be considered towards optimising the member stiffness instead. Early work on STM^[20] recommended using strut-and-tie models with short length ties in order to minimise the strain energy and deformations; a “X” type arrangement would not be the optimal solution in this case. Research on optimisation techniques for the development of strut-and-tie models has looked at different optimisation criteria including minimizing global energy^[20], maximizing stiffness^[33], as well as minimizing quantities of reinforcement^[29, 34] or crack widths^[35].

3.2 Nonlinear Finite Elements Analysis (NLFEA)

The behaviour of the outrigger beams at failure can be assessed through a nonlinear (material) FEA accounting for concrete cracking, reinforcement yielding and softening of concrete strength due to tensile transverse strains. Numerous studies have proposed different advanced optimisation techniques for obtaining the location and amount of reinforcement in deep beams and discontinuity regions combining different nonlinear analysis and assumptions^[29, 34, 36]. Although some of these approaches offer different levels of simplification, this study intends to build upon a fixed reinforcement arrangement (Figure 16) and therefore the optimisation focused only on the area of steel and not on its location. The purpose of this analysis was to verify the capacity of the outrigger designed using STM and subsequently optimize the design.

A design-orientated software package was adopted (ETABS^[15]) for the local analysis which is commonly used in tall building design, although some validation was required beforehand as the nonlinear capabilities of this tool are not normally used in complex

stress field analysis. The nonlinear shell-layered elements in ^[15] were adopted. This type of element, originally conceived for shear wall seismic analysis, includes several layers of materials (steel reinforcement and concrete) acting as membranes satisfying compatibility at a sectional level ^[15]. For simplicity, the constitutive stress-strain relationship for concrete was adjusted to neglect the tension capacity of concrete and using the default parabolic relationship for compression. A concrete strength reduction factor for cracked concrete η equal to $0.6(1 - f_c/250)$ was adopted from ^[21] for consistency with the STM; this assumption is further discussed in sections 3.3.1 and 3.3.2. The steel reinforcement was considered as a uniaxial membrane stress element with a linear perfectly-plastic behaviour. The concrete layer was modelled as a membrane with nonlinear stresses in the two orthogonal local directions and shear, whereas the reinforcement layers adopted nonlinear stresses in one direction and shear. The shell-layered element was validated using experimental data available ^[37] of reinforced concrete panels subjected to pure shear. The stiffness and predicted failure load compared well with test data for both cases where the predominant mode of failure was concrete crushing or rebar yielding as shown in Figure 17. The results were also comparable to predictions using the widely accepted Modified Compression Field Theory MFCT ^[38] as shown in Figure 17. The validation process showed that more accurate shear stiffness predictions were obtained when the layers in the element were orientated along the principal strain directions. Therefore, in the outrigger model in Figure 18, the shell-element layers were orientated along the principal strains which in this case coincided reasonably with the direction of the predominant reinforcement.

3.3 Case Study (STM and NLFEA)

This section includes the optimised design and verification of an outrigger using STM and extra-meshed ETABS model OR1-3D-04 summarized in Table 1. The acting forces in the member were obtained from the global analysis using FEA results shown in Table 2, which provided accurate results (within 7%) to the proposed model as shown in Figure 11a. The factored design moment (at face of the core) and shear were 344.625 MN.m and 22.974 MN respectively which were calculated assuming the following

1. The belt walls have been provided at push-pull sides only (not all around the perimeter as the usual case is) to eliminate participation of side columns on axis parallel to lateral loading, this is for simplicity and ease of interpreting the results. In typical cases the belt walls act as additional outriggers with aid of membrane forces of top and bottom slabs tending to perform as flanges. They also help activating the external columns to contribute to the lateral system.
2. The slab membranes/flanges above the outrigger reduce the horizontal differential movements between top and bottom of the outrigger at columns side to give considerable fixity against rotation. This is demonstrated in the moment diagram obtained from FEM analysis which shows a value for the moment at column side (see Figure 14) unlike the expected zero moment at the outer edge. The full static moment is taken in this study as a conservative approach, and to relate to the manual calculations as in Equation (18).
3. The core is subjected to sustained gravity loads and remains under certain compression stresses; it is assumed that the core is not a critical component in the system (core is not part of this study). This work focuses on the outrigger element and the outrigger-core interaction could be subject to future research.

3.3.1 STM Model

The strut-and-tie models developed in this work were based on Eurocode ^[21] recommendations using partial safety factors for the materials and loads. A characteristic concrete strength of 50 MPa was adopted with reinforcement yield strength of 500 MPa; the design values were 33.33 MPa and 435 MPa for the concrete and steel strengths (partial factors of 1.5 and 1.15 respectively). For simplicity, and more conservative approach, all struts were assumed to be cracked; i.e. stress limit of 16 MPa according to ^[21] adopting a concrete strength reduction factor η equal to $0.6(1 - f_c/250)$. The stress capacity at the nodes was also taken as 16 MPa. Figure 15 shows the geometry of the STM adopted and Table 3 summarises the results from the member forces obtained and the check of the stresses in the struts. The widths of the ties were chosen in order to establish the geometries of the nodal regions and struts widths, then the stresses were checked (Figure 15). It is noted that the outrigger receives moments in two opposite directions, depending upon the lateral loading; therefore struts will become ties and vice versa with reverse of loading, hence reinforcement will be mirrored accordingly. The uniform mesh on each face was taken as T16-200 (Figure 16), which is higher than the minimum value of 0.1% recommended in ^[21]. The reinforcement layout is summarised in Table 4 (reference model SL-01).

3.3.2 Comparison with NLFEA

The nonlinear static push-over curve for the outrigger is shown in Figure 19 from the shell-layered NLFEA using the reinforcement layout obtained from the STM method (SL-01 in Table 4). The predicted failure load for this case was 10% higher than the design load indicating that the amount of reinforcement could be optimised. This result

is fairly similar to that obtained in [8] for the deep beams for Burj Khalifa, where the failure load obtained from the NLFEA was 30% higher than the design load.

Subsequently, the amount of reinforcement was reduced iteratively in the top/bottom chords as well as in the diagonals to find the optimised value which was about 10% lower than the one initially obtained from the STM as shown in Table 4. According to the NLFEA the designed member did not fail in shear; failure was governed by the flexural response at the connection between the outrigger and the core as shown in Figure 20. Figure 20a shows the principal compressive stresses reaching values near the ultimate strength at the bottom chord near the core. Figure 20b shows the effective concrete strength reduction factor η from the MCFT ^[38] obtained from the principal tensile strains in the NLFEA; these results showed that (i) most cracking takes place due to flexure near the core and (ii) the constant effective strength reduction factor of 0.6v in Eurocode ^[21] gives in this case reasonable values of the strength of the cracked struts. The overall behaviour of the designed member was satisfactory as it provided some ductility, which is a point of concern in outrigger design as raised in ^[32]. The methodology shown above was relatively simple to implement and it allowed exploring alternative reinforcement layouts and verify the design from the STM. Reinforcement arrangements departing from the traditional “X” configuration, aiming to increase the member stiffness (i.e. using orthogonal meshes only), provided failure loads significantly lower than the design load. Alternative solutions could be further explored using post-tensioning solutions for example. Relevant aspects on detailing of outriggers are discussed in the following section.

4 Discussion on detailing of RC outriggers and core

The extensive use of intersecting diagonal reinforcement of normal yield strength of around 500 MPa introduces reinforcement congestions which in turn results into several challenges including difficulties in the logistics of lifting, access and assembly, and temporary stabilization to hold the reinforcement cage in the required place without failure or reaching excessive deformations. In addition, it could lead to poor conditions for vibration of the concrete and the formation of honeycombs. Providing high yield reinforcement (yield strengths around 1,000 MPa and 1,500 MPa) in diagonals and/or top/bottom chords is a practical solution which is commonly adopted in practice. In addition, post-tensioning is frequently adopted in outrigger construction as recognised in ^[39] to counteract the large load effects that outriggers are subjected to.

A difficulty in the detailing of reinforced concrete outriggers is anchoring the reinforcement (Figure 16) due to the use of large diameters, reinforcement congestion and the need to use basic meshes all around to avoid cracking or spalling. Codes such as Eurocode ^[21] allow using mechanical devices for anchoring or straight anchorages with links (stirrups) to achieve confinement. As shown in Figure 16, the area near the columns requires a bespoke solution for anchorage, as providing straight bars is not normally achievable. A potential solution is providing anchor steel plates, to which the reinforcement can be welded or secured by threads and nuts. These plates should be placed at the support between column rebars. This detailing solution is similar to solutions in post-tensioned construction at stressing points or end wedges and it also resembles to structural steelwork joints to some extent. Overlapping reinforcement is also challenging due to the large diameters generally required; Eurocode ^[21] states that

overlapping of large diameters should be avoided. Couplers can be used to avoid overlapping by threading the rebar and connecting them with nut-like accessory.

Overall, a solution to ensure the quality of the final detailing solution could include assembling the reinforcement at the workshop, then the full skeleton or reinforcement cage to be lifted using winches or mobile cranes which can handle large weights. For outrigger construction, novel prefabrication techniques of the reinforcement cages could provide significant savings in construction as recognised in ^[32].

5 Conclusions

Reinforced concrete outriggers are commonly used in design of Supertall buildings exceeding 300 m as part of different bracing systems to reduce lateral sway. This paper covers critical aspects regarding global analysis of different bracing systems (CSOR and TTOR) including a proposed mechanical model which can be used in design. The paper then moves into the local analysis of outrigger beams where physical strut-and-tie models are presented and compared with the predictions from nonlinear FEA. The reinforcement obtained in the outrigger members in the case study is further discussed in terms of the detailing challenges. The following conclusions can be drawn:

- 1- Mechanical models are still useful in practice for the global analysis of outrigger-braced systems to assess lateral deflections and moments in the outriggers. Mechanical models, as the one proposed in this paper, can provide quick and reasonably accurate results which can be easily applied to study different structural schemes at different stages in the design. They can also be used to verify the results from computer models of supertall buildings.

- 2- The proposed formulae for the global analysis are based on classical mechanical models based on first principles of equilibrium and compatibility. As opposed to existing formulae, the proposed method considers the reverse rotation in the outriggers due to the propping forces of the outer columns, providing more accurate predictions of the global response which is shown to be more consistent with the results from numerical FE models. This consideration also allows to extend its application to TTOR lateral bracing systems.
- 3- In general, the study of the case studies of different outrigger-bracing systems with different number of outriggers and values of omega ω showed that existing formulae (neglecting the reverse rotation of outriggers) provide significantly larger lateral deflections and smaller moments in the outriggers compared to the predictions from the FE models and proposed approach, except in the second outrigger from the top (in buildings with more than one outrigger) where the moments were higher. In general, the differences between the existing methods and the proposed formulae were larger for increasing values of omega ω . The accuracy of the proposed method was similar for both CSOR and TTOR.
- 4- The local analysis of an outrigger beam showed that the load can be transferred through different load paths depending on the reinforcement layout provided. A traditional “X” reinforcement configuration was demonstrated to provide a good overall performance (i.e. ductility and failure load above the design load).
- 5- The strut-and-tie model presented in this work, which was consistent with the reinforcement layout considered, provided a conservative design as expected. The assessment of the capacity of the designed member using nonlinear shell-layered finite elements demonstrated that the ultimate load was 10% higher than

the design load. This allowed to optimise the reinforcement rates in some areas of the outrigger which is helpful towards reducing congestion.

- 6- The design example and the following discussion highlight some of the challenges in detailing reinforced concrete outrigger elements which in some cases forces the designer to introduce bespoke solutions for anchorage in the outer columns connected with the outrigger. It is recognised that prefabrication and the use of high strength steel is needed in constructions of outrigger members to avoid some of the buildability issues raised.

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Appendix A -Tables

Table 1–Geometry and loading of case studies considered

	CSOR											TTOR	
	OR1-3D-01	OR1-3D-02	OR1-3D-03	OR1-3D-04 <i>Case Study</i>	OR1-2D-01	OR1-2D-02	OR1-2D-03	OR2-3D-01	OR2-2D-01	OR3-3D-01	OR3-2D-01	OR1-3D-TinT	OR2-3D-TinT
H (m)	200	300	400	400	200	300	400	379	250	487.5	404.5	210	210
d (m)	50	50	50	50	50	50	50	50	30	50	50	30	30
Aspect Ratio	4	6	8	8	4	6	8	7.6	8.3	9.7	8.1	7	7
ω	0.81	0.54	0.41	0.14	0.21	0.14	0.11	0.73	0.07	0.71	0.24	0.35	0.35
x_1 (m)	97.5	151.0	197.5	197.5	97.5	150.0	197.5	3.5	2.5	3.25	3.2	82.5	12.5
x_2 (m)	-	-	-	-	-	-	-	180.5	97.5	113.7	104.7	-	82.5
x_3 (m)	-	-	-	-	-	-	-	-	-	230.7	206.2	-	-
I_{CORE} (m ⁴)	5347	5347	5347	5347	5347	5347	5347	8045	58	8045	667	673	673
A_{COL} (m ²)	7	7	7	7	1	1	1	16	1	16	2	15.75	15.75
I_{OR} (m ⁴)	85	85	85	250	43	43	43	86	7	69	23	21	21
A_{OR} (m ²)	16	16	16	30	8	8	8	21	3	19	6.5	10	10
t_0 (m)	2	2	2	3	1	1	1	3	0.7	3	1	2	2
w (kN/m)	58	58	58	58	8	8	8	28	10	22	10	8.2	8.2

Table 2- Results of case studies considered

	CSOR											TTOR	
	OR1-3D-01	OR1-3D-02	OR1-3D-03	OR1-3D-04 <i>Case Study</i>	OR1-2D-01	OR1-2D-02	OR1-2D-03	OR2-3D-01	OR2-2D-01	OR3-3D-01	OR3-2D-01	OR1-3D-TinT	OR2-3D-TinT
Δ_{FEM} [mm] ($\Delta_{FEM}/\Delta_{CALC.}$)	42 (1.05)	199 (1.02)	595 (0.99)	591 (1.04)	40 (1.00)	193 (0.98)	600 (0.97)	123 (0.98)	653 (1.00)	249 (0.97)	478 (0.97)	21 (0.87)	19 (0.77)
$M_{OR1,FEM}$ [MNm] ($M_{OR1,FEM}/M_{OR1,CALC.}$)	382 (1.03)	857 (0.98)	1553 (0.99)	1554 (0.97)	60 (1.00)	136 (0.98)	242 (0.99)	157 (0.94)	16 (1.04)	70 (0.82)	18 (0.86)	54.8 (1.10)	9.9 (1.06)
$M_{OR2,FEM}$ [MNm] ($M_{OR2,FEM}/M_{OR2,CALC.}$)	-	-	-	-	-	-	-	553 (0.97)	120 (0.96)	236 (0.92)	89 (0.97)	-	51.8 (1.37)
$M_{OR3,FEM}$ [MNm] ($M_{OR3,FEM}/M_{OR3,CALC.}$)	-	-	-	-	-	-	-	-	-	628 (1.01)	266 (1.04)	-	-

Table 3 – STM results (see Figure 15 and Figure 16)

Element	width x depth [m]	strut or tie force[kN]	Reinforcement	Stress [MPa]
T1	1.5 × 1.00	±22,787	42H 40 (3.5%)	Tension
T2	1.5 × 1.69	±26,600	50H 40 (2.5%)	Tension
T3	1.5 × 2.00	±11,207	20 H 40(0.8%)	Tension
S1	1.5 × 1.00	±22,787	Compression	15.2 (95%)
S2	1.5 × 1.69	±26,600	Compression	10.5(66%)

Table 4 – Optimization of steel quantities

ModelRef.	Failure Load [kN]	Δ [mm]	T1 reinf.	T2 reinf.	T3 reinf.	Mesh	Links	Steel [kg]	Volume[m ³]	Reinf.rate [kg/m ³]	Remarks
SL-01	25,300	193	42H40	50H40	20H40	H16-200	H16-200	42,045	225	187	Initial reinf. From STM
SL-04	23,135	175	36H40	44H40	20H40	H16-200	H16-200	37,785	225	168	90% of STM
SL-03	21,000	200	32H40	40H40	20H40	H16-200	H16-200	34,946	225	155	83% / fails
SL-02	17,915	235	40H32	48H32	20H40	H16-200	H16-200	29,327	225	130	70% / fails
Design load	22,975										

Appendix B - Figures

Figure 1– The concept of outrigger [based on [1]]

Figure 2– 3D View of direct outriggers (red) with belt walls (brown)

Figure 3- Lateral systems- (a) Core-Supported-with-outrigger-CSOR, (b) Tube-in-Tube-with-outrigger-TTOR

Figure 4– Omega of tall building (based on [7]). Values taken from curves should be divided by (1+v), where v is the ratio of core's inertia to push-pull columns' total inertia about the centre

Figure 5- Schematics of outrigger supported structure – (based on [6])

Figure 6- Typical storey of rigid frame subject to shear (based on [7])

Figure 7– Notation for the proposed formula

Figure 8– Key dimensions of 2D and 3D models studied

Figure 9- Examples of FE models developed for case studies with one outrigger: (a) 2D model (OR1-2D-xx) and (b) 3D model (OR1-3D-xx); refer to Table 1 for dimensions

Figure 10– Comparison of lateral deflection predicted using existing and proposed formulae with FE results: (a) buildings with one outrigger and (b) buildings with two and three outriggers (refer to Table 1 for geometry)

Figure 11– Comparison of outrigger moment predictions using proposed and existing formulae with FE results: (a) buildings with one outrigger and (b) moments taken by 1st outrigger (from top) for buildings with two and three outriggers (c) moments taken by outrigger 2 (from the top) in buildings with two & three outriggers and (d) outrigger 3 (from top) in buildings with three outriggers.

Note:

M_{OR1} = moment of first outrigger from top

M_{OR2} = moment of 2nd outrigger

M_{OR3} = moment of 3rd outrigger

(a)

(b)

(c)

Figure 12- Moment predictions along height in buildings with TTOR system: (a) outriggerless system, (b) and (c) comparison between proposed approach and FE results for one and two outrigger respectively.

Figure 13- Establishing STM Model (a) elastic stress field, (b) STM based on diagonal truss model

Figure 14- Moment diagram, showing fixity of outrigger at outer end as obtained from FEA

Figure 15- STM of numerical example: (a) STM Model (results in Table 3), (b) check of nodal regions and struts according to Eurocode 2 [21].

Figure 16- Reinforcement layout of outrigger beam: (a) diagonal reinforcement and orthogonal grid based on STM findings, (b) three dimensional views

Figure 17- Validation of nonlinear shell-layered element: (a) modelling shear panel test in pure shear, (b) results for specimen PV20 [38] with more reinforcement in one direction, (c) results for specimen PV6 [38] with same reinforcement in both directions and yielding at failure and (d) results for specimen PV27 [38] with same reinforcement in both directions and failure due to concrete crushing.

Figure 18- Finite element mesh of outrigger model: (a) geometry of STM model vs. shell-layered FE mesh, (b) shell-layered model with rotated local axes (for reinforcement rates refer to Table 4)

Figure 19- Nonlinear static push-over curves for the outrigger investigated with different amounts of reinforcement (Table 4)

Figure 20- NLFEA results from numerical example: (a) principal compression stresses in the concrete layer (units in MPa), (b) effective concrete strength reduction factor η based on principal transverse strains