Direct reaction spectroscopy of exotic nuclei

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The direct reaction spectroscopy of rare, exotic nuclei is discussed with specific reference to advances in applications and understanding of one- and two-nucleon removal processes for this purpose. Advances in absolute spectroscopic factor determinations from one-nucleon knockout are reviewed and non-spectator-core calculations for nucleon plus deformed core systems are outlined. A new eikonal plus shell model scheme for calculations of two-nucleon stripping is introduced and the first applications of the method are presented which show good agreement with data for both inclusive and partial cross sections.

1. INTRODUCTION

Since the RNB5 conference there have been very significant experimental and theoretical advances in our ability to undertake more systematic, precise and detailed *quantitative* analyses of exotic nuclei produced as fast and slow radioactive beams. New detection systems for the study of single-nucleon transfer reactions, in inverse kinematics, have advanced apace, for use with both fragmentation and ISOL beams. Several new results are discussed elsewhere in this volume [1]. Similarly, Coulomb excitation, nuclear inelastic excitation and elastic scattering data of high precision are now available more routinely from the GSI, MSU, RIKEN, GANIL and elsewhere [2]. Studies of breakup reactions are also yielding higher statistics data and more exclusive observables. This is allowing a more realistic assessment of our understanding and modelling of the role of the continuum and of the continuum response of weakly bound systems.

The emphasis in this short contribution is on the lessons that have been learned to date from the investment in an extended campaign of selected one-nucleon knockout experiments, with coincident gamma-ray detection, carried out at the National Superconducting Cyclotron Laboratory (NSCL). We discuss the theoretical tools that are being developed to assess their precision for detailed spectroscopy. We also consider a significant new development: the cross section for the knockout of two correlated nucleons. This reaction mechanism is of interest both as a route to populating excited states of very exotic nuclear species, and as a potential direct spectroscopic probe of two-nucleon correlations in such systems.

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2. SINGLE-NUCLEON KNOCKOUT REACTIONS

Since their first exploratory application to the phosphorus isotopes in 1998 [3], single-nucleon knockout reactions have been extensively tested and exploited to study the single-nucleon spectroscopy of exotic light [4–7] and medium mass nuclei [8,9]. The initial studies emphasized the understanding of the single-particle level ordering and ground state configurations of weakly-bound and halo-like states [5,10,7,11]. More recently, these have been supplemented by detailed analyses of more tightly bound systems [12,9].

It is now well established that the very strong surface dominance of the knockout reaction mechanism at fragmentation beam energies [6] allows quantitative estimates of absolute single-particle spectroscopic factors. These are deduced from comparisons of measured partial cross sections to discrete final states with eikonal model calculations. Considerations of uncertainties within the eikonal model [6], and the alternative use a fully quantum mechanical coupled-channels (CDCC) approach for the breakup component of the cross section [13], support this view. Two up-to-date reviews of the theoretical basis and status of such analyses can be found in references [14] and [15]. The earlier emphasis on weakly bound states found spectroscopic factors in broad agreement with the shell model within the statistics available [15].

The study of more bound systems started with an analysis of the inclusive cross sections for ^{12}C [12]. Here the deduced spectroscopy agreed well with that from electron-induced ($e, e'p$) knockout reactions. This required that the shell model spectroscopic factors be suppressed by a factor $R_s \approx 0.5 - 0.6$: attributed to a range of two- and multi-nucleon correlations within the nuclear volume, see [14]. The same phenomenon has been observed for the tightly-bound neutron removal from the proton-rich $N = 16$ isotones with deduced average values of $R_s(^{31}\text{S})=0.56(7)$, $R_s(^{33}\text{Ar})=0.41(7)$ and $R_s(^{32}\text{Cl})=0.53(7)$ [9]. On the other hand for the more weakly-bound neutron- and proton-rich systems ^8B , ^9C and ^{15}C [12,16–18,11,14] the deduced R_s are very much closer to unity with, for example, $R_s(^{15}\text{C})=0.898(39)(45)$, $R_s(^8\text{B})=0.88(4)$ and $R_s(^9\text{C})=0.82(6)$. These developments hold promise for our being able to use knockout reactions to map, as never before, the effects of short-range, tensor, cluster and other correlations from stable through dripline nuclei.

3. CORE DEFORMATION DEGREES OF FREEDOM

The calculations above make use of the spectator-core approximation to the knockout reaction in which the core, or residue, is assumed not to be excited dynamically during the reaction. The importance of core deformation effects on the spectator assumption remains to be clarified. Core rotational excitation was estimated earlier in the case of neutron knockout from ^{11}Be [10] where it was needed to explain an enhanced ^{10}Be 2^+ final state partial cross section. There the changes were expected to be due to an enhanced elastic breakup component. Core deformation effects on the inclusive stripping cross section were also calculated previously within a strong-coupling (Nilsson) approximation [19]. The effects in that case were found to be relatively small.

Here we present preliminary results for both the stripping and elastic breakup contributions to the inclusive single-nucleon knockout cross section in the case of weakly-bound systems where a weak-coupling deformed-core plus nucleon description is reasonable. For

example, the stripping component reads

$$\sigma_{str} = \frac{1}{2J+1} \sum_M \int d\vec{b} \int d\hat{\Omega} \langle \Psi_{JM} | |\mathcal{S}_c(\hat{\Omega})|^2 (1 - |\mathcal{S}_n|^2) | \Psi_{JM} \rangle , \quad (1)$$

where the deformed core has orientation $\hat{\Omega}$ and wave function Φ_I with spin I , and

$$\Psi_{JM}(\vec{r}, \hat{\Omega}) = \sum_{\ell j I} [\phi_{j\ell}(\vec{r}) \otimes \Phi_I(\hat{\Omega})]_{JM} . \quad (2)$$

The deformed-core-target $\mathcal{S}_c(\hat{\Omega})$ are calculated assuming a quadrupole deformed core density with deformation β .

Our preliminary calculations [20] for ^{11}Be are consistent with the earlier estimates made in [10]. For $\beta=0.67$ and a 0^+ core state spectroscopic factor of 0.85, the inclusive diffractive cross section is increased by 8 mb, which is expected to feed, predominantly, the 2^+ core state. The stripping cross section is found to be essentially unchanged, consistent with [19].

Another very interesting case is neutron removal from ^{17}C . Whereas the shell model predicts only a very small component of the $^{16}\text{C}(0^+)$ ground state configuration in the $^{17}\text{C}(3/2^+)$ ground state (with spectroscopic factor 0.03) and a theoretical neutron knockout partial cross section of only 2 mb, experimentally this cross section is 22(11) mb [7]. The major spectroscopic strength (1.44) is associated with a d -state neutron moving about a $^{16}\text{C}(2^+)$ core. Assuming this to be a deformed-core ground state configuration, with $\beta=0.55$, we find a very significant increase in the inclusive diffraction cross section compared to that for an assumed spherical spectator core: of 18 mb. Once again the stripping cross section is found to be essentially unchanged. While in this inclusive calculation we cannot apportion the additional cross section to a specific ^{16}C final state, the magnitude of the core deformation effect is of the same size as the missing 0^+ cross section observed experimentally [7]. A partial cross section calculation would therefore be very worthwhile.

4. TWO-NUCLEON KNOCKOUT REACTIONS

Direct two-proton knockout from neutron rich nuclei has recently been proposed as a direct reaction process [21,22] and analyzed approximately. Here we have space only to outline a new calculation scheme, able to combine the full shell model two nucleon configurations and their associated spectroscopic amplitudes, with eikonal reaction theory. We show that the calculated cross sections for two-proton removal from ^{28}Mg are in good agreement with the predictions of the shell model and with the available experimental data.

The two removed nucleons are assumed to be stripped from a set of active occupied single-particle orbitals ϕ_j with quantum numbers $(n\ell jm)$. The isospin and angular momenta of the two active, removed nucleons couple to I and isospin T . The shell model wave function of these two nucleons in the projectile ground state relative to a specified residue or core state f is therefore a coherent sum over the many contributing two-particle configurations, as

$$\Psi_{J_i M_i}^{(f)}(1, 2) \equiv \langle \Phi_{J_f M_f}(A) | \Psi_{J_i M_i}(A, 1, 2) \rangle$$

$$= \sum_{I\mu\alpha} C_{\alpha}^{J_i J_f I} (I\mu J_f M_f | J_i M_i) [\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I\mu} , \quad (3)$$

where α denotes each available orbital pair. Here

$$[\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I\mu} = -N_{12} \langle 1, 2 | [a_{j_1}^{\dagger} \otimes a_{j_2}^{\dagger}]_{I\mu} | 0 \rangle \quad (4)$$

is a normalized, antisymmetrized pair wave function with $N_{12} = 1/\sqrt{1 + \delta_{12}}$. The $C_{\alpha}^{J_i J_f I}$ are the signed two-nucleon amplitudes which carry the phase and parentage information of each two-nucleon configuration in the projectile ground state with respect to the final states f of the residue.

We calculate the dominant stripping contribution to the two-nucleon removal cross section. This is the projectile ground state average

$$\sigma_{str} = \frac{1}{2J_i + 1} \sum_{M_i} \int d\vec{b} \langle \Psi_{J_i M_i} | |\mathcal{S}_f|^2 (1 - |\mathcal{S}_1|^2) (1 - |\mathcal{S}_2|^2) | \Psi_{J_i M_i} \rangle , \quad (5)$$

and an integral over all projectile center-of-mass (cm) impact parameters b . The $\mathcal{S}_i$ are the eikonal S -matrices [6] for the scattering of the two nucleons (1,2) and the A-body residue f with the target, each a function of the impact parameter of that constituent. It is important to note that the cross section now involves a coherent sum of contributions from the many contributing two-nucleon amplitudes. As a result, unlike the one-nucleon knockout situation, there is no factorisation of the theoretical cross section into a structure (spectroscopic factor) and a dynamical cross section for the reaction. The shell model structure and eikonal theory are now inextricably linked.

We make the following reasonable, simplifying approximations. We assume that the residue-target S -matrix is diagonal, and, for all final states f , equal to that for the residue ground state $\mathcal{S}_c$. This is the spectator-core approximation used in single-nucleon knockout [14], which assumes that dynamical excitation of the core in the collision is small. We also neglect recoil effects associated with the heavy residue.

Given the spectator-core and no-recoil approximations the inclusive stripping cross section is the sum of partial cross sections from each residue final state

$$\sigma_{str} = \sum_f \sigma_{str}^{(f)} = \sum_f \frac{1}{(2J_i + 1)} \sum_{M_i} \left[\int d\vec{b} |\mathcal{S}_c|^2 \langle \Psi_{J_i M_i}^{(f)} | (1 - |\mathcal{S}_1|^2) (1 - |\mathcal{S}_2|^2) | \Psi_{J_i M_i}^{(f)} \rangle \right] . \quad (6)$$

We present below calculations of both the inclusive and partial cross sections.

4.1. Calculations for ^{12}C

As an orientation to the proposed method we consider the inclusive cross sections for both two-neutron and two-proton stripping from a ^{12}C projectile. Such data are available for collisions on a ^{12}C target at 250, 1050 and 2100 MeV/nucleon [23,24]. The shell model predicts that the spectroscopic strength for single-nucleon removal from ^{12}C is exhausted by knockout to just three bound states in the $A = 11$ systems [12]. Hence the (non-direct) cross sections for populating the $A = 10$ residues via nucleon stripping into the $A = 11$ continuum, followed by nucleon evaporation, are expected to be small. The required residue- and nucleon-target S -matrices in Eq. (6) are calculated as was described in the single-nucleon knockout calculations from ^{12}C [12]. Knockout of an np -pair is not

considered here because (a) of the complexity of the ^{10}B final state level scheme below the nucleon thresholds, and (b) since many of these observed excited states are absent from the truncated shell model space calculation. The results of the $2p$ and $2n$ stripping calculations are collected in Table 1. The overall agreement with the available data is excellent. We note that the two-nucleon cross sections do not require any overall scaling or suppression factor, in contrast to the earlier observations in single-nucleon knockout [12]. This very interesting result will need to be explored and understood.

Table 1

Calculated and measured two-proton and two-neutron knockout inclusive cross sections (in mb) from ^{12}C on a ^{12}C target at 250, 1050 and 2100 MeV/nucleon. The calculations use the shell model two-nucleon amplitudes of Brown [25].

Energy/nucleon	250 MeV	1.05 GeV	2.1 GeV
^{10}Be Theoretical	5.82	5.33	5.15
^{10}Be Experimental	5.88	5.30(30)	5.81(29)
^{10}C Theoretical	4.26	3.91	3.84
^{10}C Experimental	5.33(81)	4.44(24)	4.11(22)

The experimental values are taken from from refs. [23,24].

4.2. Calculations for neutron-rich ^{28}Mg

The two-proton removal reaction from a ^{28}Mg radioactive beam, measured at MSU at 82 MeV/nucleon on a ^9Be target, is discussed in refs. [21,22]. The energetics of this two-proton removal reaction are such that the cross section for producing ^{26}Ne indirectly, via proton evaporation from intermediate excited states of ^{27}Na , is expected to be negligible. The corresponding neutron evaporation threshold is significantly below that for the proton. The reaction is therefore expected to proceed only by the direct removal of a pair of protons.

The reaction is analyzed in reference [21] using simplified structure models. It was shown that, assuming the most simple, uncorrelated extreme single-particle model for ^{28}Mg , yields an integrated $2p$ removal cross section of approximately the right magnitude but with the wrong distribution of spectroscopic strength between the measured partial cross sections to the 0^+ , 2^+ , 4^+ and 2_2^+ ^{26}Ne final states. The basic unit of cross section is found to be 0.29 mb per proton pair removed. Thus, assuming a $(1d_{5/2})^4$ ground state configuration, this yields an integrated cross section of 1.8 mb compared to the experimental value, Table 2, of 1.5(1) mb. Assuming instead that the two nucleons are removed through that part of the shell model wave function in which they are found as a correlated spin-singlet, isospin-triplet pair in a relative s -state, as arises for example in the (t, p) reaction vertex, gives a better partial cross section distribution, but without an absolute cross section prediction.

The approach followed here makes no such approximations, including as it does all terms in the full shell model two-nucleon wave function. The stripping cross section does not

select particular spin, isospin or relative angular momenta of the pair. The results of the present, fully-correlated wave function analysis are compared with the earlier uncorrelated results of [21] in Table 2. Results are shown both as absolute cross sections, in mb, and as ratios to the $2p$ unit cross section, defined above, which then yields an effective spectroscopic weight for each final state [26].

Table 2

Two-proton knockout cross sections from a ^{28}Mg projectile at 82 MeV/nucleon, to different final states of the ^{26}Ne residue. The spectroscopic weights S_{exp} and S_{th} are the corresponding cross sections divided by the cross section $\sigma_{22} = 0.29$ mb, calculated as the stripping cross section for removal of two uncorrelated d -state protons.

	S_{th} uncorr	S_{exp}	S_{th} corr	σ_{exp} (mb)	σ_{th} (mb)
0^+	1.33	2.4(5)	1.83	0.70(15)	0.53
2^+	1.67	0.3(5)	0.55	0.09(15)	0.16
4^+	3.00	2.0(3)	1.79	0.58(9)	0.52
2_2^+	—	0.5(3)	0.76	0.15(9)	0.22
Inclusive				1.50(10)	1.43

The experimental values and the uncorrelated theoretical cross sections are taken from ref. [21].

The theoretical spectroscopic weights in the very simple uncorrelated model do not predict the reduced cross section to the first 2^+ final state. Calculations with correlated protons, developed here, show good overall agreement with the experimental values. The calculated inclusive cross section is now 1.43 mb, in good agreement with the measured value, 1.50(10) mb [21].

5. CONCLUDING REMARKS

There has been considerable progress in developing techniques for the spectroscopy of exotic nuclei. One-nucleon knockout reactions are now an established technique for single-particle spectroscopy of both weakly-bound and deep-hole states, revealing the role of correlations in and beyond the shell model. For hole states, nuclear knockout has been shown to yield results equivalent to those of the $(e, e'p)$ reaction - but with the ability to extend its scope to exotic and near-dripline systems. The systematics are confirming the importance and effectiveness of the method towards this goal. We have also presented exploratory calculations that show that, in projectile ground states with a significant deformed core component, the inclusive diffractive breakup cross section can be enhanced. In the two cases considered this enhancement accounts for cross section which is missing from the spherical spectator-core calculations for the same systems.

We have also demonstrated a comprehensive calculation scheme for the direct two-nucleon stripping reaction. The inclusive cross sections for $2p$ and $2n$ removal from ^{12}C

are consistent with measurements. For neutron rich ^{28}Mg we obtain absolute predictions of the partial cross sections to different ^{26}Ne final states. These are in agreement with available experimental data, and show considerable improvements compared to a simple uncorrelated structure approximation [21]. Our calculations provide further evidence of the direct nature of two-proton knockout from neutron-rich nuclei and of its potential for the study of two-nucleon correlations and spectroscopy.

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REFERENCES

1. J.S. Thomas *et al.*, contribution to this volume.
2. W. Korten *et al.*, contribution to this volume.
3. A. Navin *et al.*, Phys. Rev. Lett. **81**, 5089 (1998).
4. J.A. Tostevin, J. Phys. G **25**, 735 (1999).
5. A. Navin *et al.*, Phys. Rev. Lett. **85**, 266 (2000).
6. J.A. Tostevin, Nucl. Phys. A **682**, 320c (2001).
7. V. Maddalena *et al.*, Phys. Rev. C **63**, 024613 (2001).
8. J. Enders *et al.*, Phys. Rev. C **65**, 034318 (2002).
9. A. Gade *et al.*, *One-neutron knockout reactions on proton-rich nuclei with $N = 16$* , Phys. Rev. C, submitted.
10. T. Aumann *et al.* Phys. Rev. Lett. **84**, 35 (2000).
11. D. Cortina-Gil *et al.*, Phys. Lett. B **529**, 36 (2002).
12. B.A. Brown, P.G. Hansen, B.M. Sherrill and J.A. Tostevin, Phys. Rev. C **65**, 061601 (2002).
13. J.A. Tostevin *et al.*, Phys. Rev. C **66**, 024607 (2002).
14. P.G. Hansen and J.A. Tostevin, Ann. Rev. Nucl. Part. Sci. **53**, 219 (2003).
15. P.G. Hansen and B.M. Sherrill, Nucl. Phys. **A693**, 133 (2001).
16. J. Enders *et al.*, Phys. Rev. C **67**, 064301 (2003).
17. R. Terry *et al.*, *Absolute spectroscopic factors in neutron knockout on the halo nucleus ^{15}C* , Phys. Rev. C, submitted.
18. L. Trache, F. Carstoiu, A.M. Mukhamedzhanov, and R.J. Tribble, Phys. Rev. C **66**, 035801 (2002).
19. A. Sakharuk and V. Zelevinsky, Phys. Rev. C **61**, 014609 (1999).
20. P. Batham, PhD thesis, University of Surrey, in preparation.
21. D. Bazin *et al.*, Phys. Rev. Lett. **91**, 012501 (2003).
22. D. Bazin *et al.*, contribution to this volume.
23. J.M. Kidd, P.J. Lindstrom, H.J. Crawford and G. Woods, Phys. Rev. C **37**, 2613 (1988).
24. D.L. Olson *et al.*, Phys. Rev. C **28**, 1602 (1983).
25. B.A. Brown, <http://www.nscl.msu.edu/~brown/sde.htm>
26. J.A. Tostevin and G. Podolyák, in preparation.