Bayesian Reasoning: Evidence from the Field

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We conduct a field test of Bayesian reasoning, examining whether agents form expectations by placing a larger weight on cues that are more informative with lower process variance. To test this notion we analyse subjective probabilities inferred from odds on the outcomes of tennis matches, exploiting *exogenous* variation in process variance related to the format with which tennis matches are played. Our results are consistent with “*process variance neglect*”, i.e., agents are not adjusting their ex-ante probabilities sufficiently according to process variance, and thus violate Bayes Rule. This result is robust to inferring subjective probabilities from odds offered by professional bookmakers or odds achieved on a person-to-person betting exchange. The resulting biases in expectations are costly.

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1 Introduction

Tests of Bayes Rule have been mainly conducted in controlled laboratory experiments, where objective probabilities are set by the experimenter, and subjective probabilities are identified with high precision (e.g., Kahneman, Slovic and Tversky, 1982; Grether, 1980).¹ The results from these experiments suggest that errors in expectations are systematic, and have contributed significantly towards the advancement of behavioural economics.² However, because “real-life” economic decisions differ substantially from decisions in the laboratory (Harrison and List, 2004), it is important to complement the experimental literature with tests of Bayesian reasoning based on naturally occurring data from the field. In this study we take a step in this direction, utilizing a feature of professional tennis matches to test an aspect of Bayes Rule, analysing subjective probabilities inferred from the odds offered by professional bookmakers on the outcomes of these matches.

Bayes Rule requires that agents attach a larger weight on signals that are more informative with lower “*process variance*”. In a seminal paper on Bayesian Updating, Edwards (1968) shows that experimental subjects “*underreact*” to information with low process variance that implies a large update from the prior probability. We examine whether economic agents behave in a similar manner in the field, by analysing subjective probabilities inferred from odds offered by bookmakers on the outcomes of tennis matches, exploiting *exogenous* variation in process variance that is related to the format with which tennis matches are played.

Men’s singles tennis matches are played in two formats: A best- out-of-three set format (BO3), where a player must win two out of possible three sets to win a match,

¹For a review of the experimental literature on Bayesian Updating see Appendix B in Benjamin, Rabin and Raymond (2016).

²The experimental evidence of non-Bayesian reasoning have received applications in decision theory (e.g., Rabin and Vayanos, 2010; Gennaioli and Shleifer, 2010; Benjamin, Rabin and Raymond, 2016) and financial economics (e.g., Shefrin and Statman, 1994; Barberis, Shleifer and Vishny, 1998; Barberis, Greenwood, Jin and Shleifer, 2015).

and a best-out-of-five set format (BO5), where a player must win three out of possible five sets to win a match.³ In our sample, the BO5 matches are called “Grand Slams” (GS), and the BO3 matches are called “ATP World Tour Masters 1000” (MS). In each of these match types bookmakers observe signals that capture a player’s skill *on the day*, calculate the subjective probability that this player wins the match, and set their odds accordingly.

One such signal that is related to a player’s skill on the day is their official ranking, which is based on their cumulative performance during the previous 52 weeks.⁴ Higher ranked players are generally more skilful, and therefore more likely to win a match. If ranking is a perfect indicator of skill on the day, then process variance is zero and the probability that the higher-ranked player wins is one. However, because skill on the day is affected by random components, ranking is an imperfect indicator of skill on the day, so the probability that the higher-ranked player wins is less than one. How much less depends on the variance of the process, i.e., how likely it is that the lower-ranked player produces a surprise and wins the match. If a surprise is less likely, then the probability that the higher-ranked player wins the match should be adjusted upward.

A surprise is *less* likely in the GS format because the lower-ranked player must win *more* sets to win the match.⁵ To illustrate this idea with an analogue from coin spins, assume that we have a biased coin with probability of heads in a single toss equal to

³Each set is comprised by individual games, and to win one set a player must win at least 6 games. To win one game a player must win at least four points. For more information on the rules of tennis see <http://www.atpworldtour.com>.

⁴Rankings are based on the ammount of points accumulated by a player during their past 52 weeks performance from a total of 18 tournaments (or 19, if a player participates in the ATP World Tour Finals). A player’s total ranking points are calculated from the four Grand Slams, eight compulsory Masters 1000 tournaments (out of nine), and his best six results (in terms of points) from all other ATP and Futures tournaments that he participates. For more information see <http://www.atpworldtour.com/en/corporate/rulebook>. For each match that involves two players, we refer to the player with the highest ranking (i.e., a smaller ranking number), as the higher-ranked player.

⁵Klaassen and Magnus (2014) also make this point. They develop a model to estimate the probability of a player winning a specific point, and show that the higher skill player is more likely to win a longer, as opposed to a shorter, match.

60%. The probability that we receive two heads out of a possible three spins is 64.8%, and the probability that we receive three heads out of a possible five spins is 68.3%. The increase in the probability in each case reflects a reduction in process variance in the sense that it becomes more difficult for the low probability event (i.e., tails in this example or the lower-ranked player winning in our framework) to “win” when more successes are required. Indeed, in line with this logic, we find that higher-ranked players are 7.2% more likely to win a GS match, as opposed to an MS match, which implies that ranking is a lower process variance signal for GS. Therefore, Bayes Rule dictates that bookmaker’s probabilities for higher-ranked players winning must be adjusted upward for GS. Our objective is to test whether such an adjustment takes place.

For our main analysis we use subjective probabilities inferred from fixed decimal odds offered by several major betting houses on professional men’s tennis matches for the period 2005 to 2014. For each match we infer the subjective probability that the higher-ranked player wins the match from bookmaker odds, π , estimate the “objective” probability as the fitted value from a logit model, $\hat{p}$, and define *bias* as $\pi - \hat{p}$. We test our hypothesis based on the difference in the average *bias* between MS and GS.⁶ If bookmakers upwardly adjust π for GS to reflect the increase in $\hat{p}$ due to the longer match format (BO5), then this difference should be 0. Alternatively, under the alternative hypothesis of *underreaction* due to *process variance neglect*, *bias* should be lower for GS matches.

Our results show that *bias* is equal to 0.4% for MS matches and -3.8% for MS matches, for a statistically significant difference of -3.4%. Given that $\hat{p}$ is 7% higher for GS matches, this result shows that bookmakers are adjusting their probabilities in the correct direction, but stop roughly half-way from the level implied by full Bayesian

⁶By testing the hypothesis based on the difference in *bias* across MS and GS our conclusions are not affected by any systematic errors in the estimation of π or $\hat{p}$ that are constant across the two match formats. We discuss our methods in detail in Section 2.

Reasoning ($7.2\% - 3.4\% = 3.8\%$). This is the central finding of our paper, which suggests that deviations from Bayesian Reasoning matter for real-life economic decisions.

Could our results reflect strategic behavior by rational bookmakers to exploit “irrational” punters and not process variance neglect? In this setting bookmakers are in a position to act strategically because they set the odds on *both* players, thus could “salt” them in opportune directions. For example, Levitt (2004) finds that, in spread-betting markets, bookmakers sometimes offer biased prices to exploit certain tendencies among punters.

To examine whether strategic behavior is driving our results we re-do our tests using subjective probabilities inferred from odds which are set competitively in a person-to-person real-life market called Betfair. In this setting strategic incentives do not exist, since the odds for the two players are not set by the same agent but rather are determined competitively by different punters in two different markets.⁷ Analysing tennis matches for the period 2009-2014, we find that *bias* is lower for GS by -2.7%, and this difference is highly statistically significant and comparable in magnitude to that found with bookmaker data.

To further examine the role of strategic incentives we examine whether it is optimal for bookmakers to offer *excessively high* odds for the higher-ranked player for GS matches (which amounts to a negative *bias*). To put this in context, these excessively high odds on the higher-ranked player imply that bookmakers are collecting 4.5% less on average of every dollar bet on the higher-ranked player for GS matches. Therefore, for the negative *bias* to be optimal, bookmakers should be earning proportionately more from bets on the lower-ranked player, so that their total profitability per GS game increases. Contrary to this notion, we find that bookmaker’s total profits per match (as a proportion of the total volume staked) are *lower* for GS matches by an average of -2.5%, which suggests that

⁷In these markets prices converge to satisfy no arbitrage conditions, i.e., the sum of the implied probabilities for each player winning are close to 1.

underreaction is costly.

Overall, the analysis from the betting exchange data and bookmaker’s profits suggests that our results are unlikely to reflect the strategic behavior of rational bookmakers.

A belief that is held by some economists is that individual biases do not matter for the broader economy since they are wiped out when aggregated in a market setting (e.g., Fama, 1998). Even though the results with the betting exchange data do not support this notion,⁸ we conduct a sharper test for the “wisdom of crowds” hypothesis, by examining whether high-volume matches, which aggregate information from a larger number of punters, price the change in process variance in GS more accurately. We find that they do, with the difference in *bias* between MS and GS being closer to 0. This result provides some support to the idea that “deeper” markets are more efficient. However, we find that high-volume markets do not completely eliminate the bias of underreaction due to process variance neglect.

Our analysis implicitly assumes that MS and GS matches are the same in every respect, except from the length of the match. However, GS matches offer more ranking points and higher prize money to players, and attract more attention from punters. Could our findings reflect these differences? For example, it is possible that bookmakers are not be pricing correctly any “extra-motivation” of players to win the more lucrative GS matches.

To examine whether such differences are driving our results we conduct a placebo test using data for professional women’s tennis matches. For women, GS matches also offer more ranking points and prize money, and attract more betting volume but they *are played in a BO3 format, exactly like the MS tournaments*. Hence, women’s matches preserve key differences across the two match formats *except* for the change in process variance. If the bias we document is driven by factors other than process variance neglect,

⁸Camerer (1987) also shows that experimental asset markets do not eliminate biases.

bias should be lower for GS in the women’s data as well.

Logistic analysis confirms that the type of match (MS vs GS) *does not* affect the probability that the higher-ranked player wins for women’s matches, consistent with no difference in process variance across match formats. Therefore, bookmakers should not adjust π upward for GS. When we compare *bias* calculated from bookmaker odds for MS and GS we find that the difference is *insignificant*. When we do the same for *bias* calculated from betting exchange odds, we find that *bias* is *higher* in GS by 1.2%. Overall, the results obtained from the women’s data are starkly different to those obtained from the men data, which suggests that our baseline results more likely reflect underreaction due to process variance neglect.

We conduct various robustness checks. First, we show that our results continue to hold for different specifications for *bias*. Second, we consider alternative explanations for our findings, such as different levels of “noise trading”, changes in bookmaker preferences, or different levels of competition across MS and GS matches. We conclude that such alternative explanations cannot offer a parsimonious interpretation for all our results, across bookmaker and betting exchange markets, for both men’s and women’s matches.

However, with the field data we cannot rule out the possibility that our findings are influenced by an economic quantity that varies between MS and GS. Motivated by this limitation, our final robustness test is a laboratory experiment, which allows testing process variance neglect in a setting that removes the effect of any confounding economic factors. Even though the experiment is subject to the general methodological limitations of experimental methods, it is nevertheless a useful plausibility test, since our field analysis suggests a sharp hypothesis that can be tested easily in the laboratory.

We invited university students who were involved in tennis activities to participate in the experiment. These students were asked to consider upcoming matches from two tournaments, one MS and the other GS, and assign probabilities to each player winning.

Our objective was to test whether subjects adjust their probabilities for the effect of the length of the match on the probability that the higher-ranked players wins.

The results from the experiment reveal a significant bias of underreaction due to process variance neglect. Specifically, we find that for MS matches *bias* is equal to 0.00%, whereas for GS matches *bias* is equal to -3.30%, with the difference being statistically significant. This difference is driven by the fact that the subjective probabilities expressed by students show no adjustment to reflect the reduction in process variance in GS. Overall, the experimental analysis provides further support to the claim that our baseline findings from the field reflect process variance neglect.

Biases due to process variance neglect are likely to influence outcomes in equity markets, where investors set prices by continually responding to information that varies in process variance. If investors respond to signals according to their process variance, stock prices would respond immediately to these signals, thus future stock returns would be unpredictable. However, if investors are insensitive to process variance, low process variance signals would not move prices sufficiently, which will lead to future stock returns being predictable in the direction implied by the signal (i.e., rise after good news and fall after bad news).

For our final test we test this hypothesis using financial data. The signal we use is the median earnings forecast revision issued by professional sell-side analysts for a specific company prior to the actual earnings announcement. To measure the process variance of the signal we split the companies in two groups; high-coverage firms followed by many analysts, and low-coverage firms followed by few analysts. The signal is of lower process variance for high-coverage firms because it incorporates information from more analysts about future earnings. In line with this logic, we show that signals associated with high-coverage firms are *more* predictive of earnings shocks, which means that Bayesian investors should price them more strongly. We find that investors do this

partly, which makes future returns predictable. The economic effect associated with this predictability is not trivial, amounting to an annualized return differential between high and low coverage firms of around 2%.

Tests of Bayes Rule have been mainly conducted in controlled laboratory experiments (Grether, 1980, 1992; Camerer, 1987; Griffin and Tversky, 1992; Holt and Smith, 2009; Antoniou, Harrison, Lu and Read, 2017). However, because the extent to which laboratory findings carry over into the field is unclear, it is important to analyse both laboratory *and* field generated data to properly understand the process of belief formation. Along these lines, DeBondt and Thaler (1990) present evidence that the earnings forecasts of professional security analysts are affected by representativeness, and Chen, Moskowitz and Shue (2016) show that the decisions of asylum judges, loan officers and baseball umpires are affected by the gambler’s fallacy.

We contribute to this literature conducting a field test of Bayesian reasoning with three attractive features: Firstly, subjective probabilities are inferred from the decisions of expert agents who are pricing securities in their natural habitat with significant monetary consequences. Secondly, uncertainty is fully resolved when the match is finished, which allows us to test for a bias in subjective probabilities with relatively weak assumptions. Thirdly, and most importantly, in the tennis data the variation in process variance that allows us to test for Bayesian reasoning is completely exogenous, in the sense that it is governed solely by the rules of the game. Our findings suggest that decision makers do not combine cues optimally when forming expectations and support the claim that violations of Bayes Rule affect real-life economic decisions.

Our analysis also contributes to the behavioral finance literature, which has attributed various asset pricing anomalies to investor underreaction (i.e., Ball and Brown, 1968; Jegadeesh and Titman, 1993; Loughran and Ritter, 1995; Ikenberry, Lakonishok and Vermaelen, 1995; Michaely, Thaler and Womack, 1995; Zhang, 2006; Jiang and Zhu,

2016). According to this explanation, investors fail to recognize that certain information signals are good indicators of fundamentals, and therefore do not price them as strongly as they should. With the passage of time, as the fundamentals are slowly revealed and expectations are adjusted, prices drift toward their equilibrium values. A caveat of these interpretations (including our own interpretation of the results from the aforementioned asset pricing test) is that they rely on assumptions related to *expected* equity returns, or what Fama (1998) calls the *joint hypothesis problem*. An interesting exception in this literature is Moskowitz (2015), who tests for the existence of asset pricing anomalies in sports betting markets where risks are completely idiosyncratic, and therefore Fama’s (1998) critique does not apply. In a similar spirit, we contribute by testing underreaction in a setting that requires relatively weak assumptions. Our results support the claim that underreaction can cause stock prices to adjust slowly to certain information.

Finally, our work makes a more general contribution to the literature that discusses whether behavior in the laboratory captures behavior in the field (e.g., Levitt and List, 2007; Al-Ubaydli, List and Suskind, 2017). Our results show that the biases in the experimental data are qualitatively similar (i.e., of the same sign) to those in the field data, which suggests that behavior is broadly similar in the two domains. However, the results also show that the biases in the laboratory are larger, since the subjective probabilities of students show no adjustment to the reduction in process variance for GS matches. This is an important difference in behavior, which reflects the fact that conditions in the two domains diverge in significant ways.⁹ These findings support the notion that the two approaches are complementary, as concluded by Harrison and List (2004). Laboratory

⁹As discussed in Harrison and List (2004) the field differs from the laboratory in at least two fundamental ways: Firstly, the field can help create heuristics that affect decision making, which are probably not present in the “sterile” environment of the laboratory where subjects encounter artificial tasks for the first time (e.g., List, 2004 and Haigh and List, 2005). And secondly, incentives in the laboratory tend to be weak, therefore behavior inferred from this domain may not be directly applicable to real-life economic decisions where the stakes are significantly higher (e.g., Andersen, Ertac, Gneezy, Hoffman and List, 2011). Other studies that point to differences between lab and field behavior include Levitt and List (2007a,b) and Levitt, List and Reiley (2010).

experiments, with their flexibility and control, are useful in capturing broad patterns in behavior and generate hypotheses, but field tests are required to properly test the robustness of an experimental phenomenon, and recover the magnitude of the underlying parameters.

The next section describes our data, methods and hypothesis. The third section presents and discusses the results, and the fourth section concludes the paper.

2 Data and Methods

2.1 Data

For our baseline analysis we obtain data from www.tennis-data.co.uk.¹⁰ For every match this database contains data the name of the tournament, the date of the match, the names of the two competing players, their official ATP rankings, the winner of the match, as well as fixed decimal odds from various international betting houses on both players.¹¹ In our analysis, we average the odds offered by the various bookmakers on the two players and then use these average odds to infer subjective probabilities.

We include in our sample Grand Slam (GS) matches, which are played in a BO5 format, and ATP World Tour Masters 1000 matches (MS), which are played in a BO3 format. GS tournaments are the most prestigious, with the winner receiving 2,000 ranking points, and on average collecting 2.5 million dollars (in 2015). For comparison, the winner of an MS tournament earns 1,000 ranking points and, on average, 0.8 million dollars (in 2015). There are other tournaments that are played in a BO3 format, which yield, for example, 500 or 250 ranking points to the winner of the tournament, and offer less prize money. Such tournaments are significantly less prestigious, involving on average lower-

¹⁰Data from this database has been used previously by academic work on tennis matches (e.g., Forrest and McHale, 2007; Del Corral and Prieto-Rodriguez, 2010).

¹¹This dataset contains odds from eight different bookmakers; Bet365, Centrebet, Expekt, Ladbrokes, Interwetten, Pinnacles Sports, Stan & James, and Unibet.

ranked players, and attracting less attention from punters. In our analysis, to ensure that the BO3 matches are as similar as possible to the BO5 GS matches, we focus on the more prestigious tournaments from the BO3 class.

We apply the following criteria to the initial dataset ($n=10,790$) to create our final sample: (i) we drop matches that were not completed, matches with missing rankings information, matches with no odds for either player, and matches that entail a negative house take or “Vig”,¹² ($n=10,266$); (ii) we drop matches where the higher-ranked player is indicated as an outsider by bookmakers even though he is ranked by at least 15 places higher than his opponent at the beginning of the tournament ($n=9,230$). Such cases are likely to reflect recent developments like injuries, which are not yet incorporated in the rankings, thus making them outdated indicators of skill.¹³

Our final sample consists of 9,230 tennis matches from 2005-2014. Table 1 breaks down the matches by year and tournament. Overall, we have data for 4 GS tournaments, and for 12 MS tournaments. Some MS tournaments are discontinued and others are introduced at various point in time.

[Insert Table 1 here]

2.2 Methods

Assume a tennis match between players X and Y . The bookmaker offers fixed decimal odds for player X to win equal to d_X , and for player Y equal to d_Y , where d_X and d_Y are greater than 1. To obtain subjective probabilities, we first invert the quoted odds for X , $O_X = \frac{1}{d_X}$, and for Y , $O_Y = \frac{1}{d_Y}$. In a perfectly competitive and frictionless market with

¹²Vig stands for “vigorish”, and reflects the fact that odds, when inverted and added together, typically sum to more than 1. This reflects a form of commission collected by the bookmaker. Matches with negative vig are likely to be data errors.

¹³Since our analysis examines how rankings, as signals about skill on the day, are priced across MS and GS, filter (ii) eliminates cases where rankings are outdated, thus less informative. However, our conclusions remain the same if we do not impose this filter. This is discussed in more detail in Section 3.6 of the paper.

a risk-neutral bookmaker, O_X and O_Y correspond to true subjective beliefs. However, typically $O_X + O_Y > 1$, which reflects the vigorish or “vig”, a form of commission collected by the bookie. To obtain subjective probabilities we normalize O_X and O_Y to sum to 1, using $\pi_X = \frac{O_X}{O_X + O_Y}$ and $\pi_Y = \frac{O_Y}{O_X + O_Y}$. The vig is thus split proportionally between the two players, depending on their relative odds.¹⁴ Throughout the analysis we refer to π_i as the subjective probability that the higher-ranked player wins match i .

To examine whether process variance changes across match format (MS vs. GS) we use the logistic model, shown below:

$$Pr(Y_i = 1 | GS_i, Rskill_i) = F(\alpha + \beta_1 GS_i + \beta_2 Rskill_i) \quad (1)$$

The dependent variable, Y_i is a binary indicator taking the value of 1 if the higher-ranked player wins match i , and 0 otherwise. GS_i is a dummy variable that equals 1 if match i is Grand Slam, and 0 otherwise. $Rskill_i$ captures differences in player rankings for match i and is calculated as $\log(\text{lower-ranked player ranking}) - \log(\text{higher-ranked player})$, following the specification in Klaasen and Magnus (2001).¹⁵ F is the logistic distribution.

As we discuss in more detail in the next section, GS matches entail more players in the draw, and therefore entail higher average ranking differences between the two players. The model in Equation (1) captures the effect of the change in process variance across MS and GS, whilst controlling for these ranking differences. That is, the coefficient on GS should be positive and significant, reflecting the increase in the probability that the higher-ranked player wins a GS match due to the longer match format (BO5).

¹⁴This method of recovering beliefs from odds is typical in the literature (e.g., Croxson and Reade, 2014; Smith, Paton and Williams, 2009).

¹⁵As discussed in Klaasen and Magnus (2001) player skill in tennis resembles a pyramid (i.e., the difference in skill between players ranked #1 and #10 is higher than the skill difference between players ranked #80 and #90), therefore a logarithmic transformation is appropriate. Note also that the ranking of the lower-ranked player is a larger number than the ranking of the higher-ranked player. Our results are the same if we calculate $Rskill$ on the basis of differences in the ranking points of the two competing players.

To estimate the bias in subjective probabilities we use $bias_i = \pi_i - \hat{p}_i$, where $\hat{p}_i$ is the fitted value from the logit model. In this setting $\hat{p}_i$ acts as a benchmark, which for each match takes into account the format (MS vs. GS) and $RSkill$, and on average reflects the actual frequency that higher-ranked players win MS and GS matches in our sample. We test our hypothesis based on the difference in average $bias$ between MS and GS matches, $\Delta_{bias} = \overline{bias_{GS}} - \overline{bias_{MS}}$. With this formulation our conclusions are not affected by any systematic errors in π_i or $\hat{p}_i$ that are symmetric across the two match formats.¹⁶

Under the null hypothesis of Bayesian reasoning, where subjective probabilities are properly adjusted to changes in process variance between MS and GS, $\Delta_{bias} = 0$. The alternative hypothesis of underestimation due to process variance neglect is $\Delta_{bias} < 0$. We test these hypotheses by examining the sign and statistical significance (using two-sided tests) of β_1 in the ordinary least square regression shown below:

$$bias_i = \alpha + \beta_1 GS_i + \beta_2 RSkill_i + \epsilon_i \quad (2)$$

2.3 Psychological Foundations

In a seminal laboratory-based experiment on Bayesian updating Edwards (1968) showed that subjects underreact to information that implies a large update to the prior probability. Similar results are shown by Griffin and Tversky (1992), and more recently by Antoniou, Harrison, Lau and Read (2017). Such behaviors could reflect what Benjamin et al. (2016) call a *non-belief in the law of large numbers* (NBLLN).¹⁷ This phenomenon was demonstrated in an early study by Kahneman and Tversky (1972), who compared responses to questions like “*What is the probability that I obtain between 4 and 6 heads*

¹⁶However, it is possible that the errors are asymmetric and influence our results, e.g., the logit model may omit factors that vary across MS and GS matches. In Sections 3.6 and 3.7 we discuss in detail potential sources of asymmetric errors, and when possible address them econometrically.

¹⁷Benjamin et al. (2016) show that a decision-maker who displays NBLLN will always underreact to information from large samples. Moreover, they show that NBLLN influences key decisions such as information acquisition, optimal stopping and the valuation of risky prospects.

if I toss a fair coin 10 times?” and *“What is the probability that I obtain between 4,000 and 6,000 heads if I toss a fair coin 10,000 times?”* The law of large numbers implies that the probability is much higher in the latter question. However, subjects expressed similar probabilities in these questions, which suggests an insensitivity to the effect of sample size on sampling errors.

We can draw a parallel between sample size in these experiments and match length in our real-world analysis. In our setting a sampling error occurs when the player with the highest skill on the day, which we proxy using rankings, loses the match. A sampling error is less likely in the longer GS matches, thus Bayesian agents should assign higher probabilities to higher-ranked players for these matches. However, if due to NBLLN agents do not appropriately account for the effect of match length on sampling error, they would underestimate the likelihood that the higher-ranked player wins a GS match.

2.4 Descriptive Statistics

Table 2 presents descriptive statistics for our main sample, separately for GS (Panel A) and MS (Panel B) matches. The average posted odds offered by bookmakers that the higher-ranked player wins the match (HR_{Odds}) are much lower than those for the lower-ranked player (LR_{Odds}) (1.35 vs. 6.05 for GS and 1.48 vs. 4.10 for MS), which shows that player ranking is indeed a metric for skill on the day that is used by bookmakers. The *vig* is roughly equal in the two match formats (0.05 in GS and 0.06 in MS) and has low volatility, as noted by other authors (Forrest and McHale, 2007). For GS the average ranking of high- and lower-ranked players (HR_{Rank} and LR_{Rank}) is 28 and 99 respectively, whereas for MS the corresponding rankings are 22 and 67. GS matches entail higher ranking differences because GS tournaments allow more players in the draw.¹⁸ The

¹⁸GS matches involve 128 players, whereas MS matches involve 64 players on average (96 players: Indian Wells and Miami; 56 players: Monte Carlo, Madrid, Rome, Montreal, Toronto, Cincinnati, Shanghai; 48 players: Paris). In all tournaments the “seeded” players are allowed to compete automatically in the tournament, for example the top 32 players in the world. The remaining positions are filled by lower

higher ranking differences in GS imply that the probability of the higher-ranked player winning a GS match is higher, therefore we test the hypothesis using the model in (2), which includes *RSkill* as a control variable.

The next two rows show the average estimate of the objective probability obtained as the fitted value from the model in Equation (1), ($\hat{p}$). As it can be seen, $\hat{p}$ is higher for GS than MS (0.78 vs 0.69), consistent with the view that higher-ranked players are more likely to win a GS match (we present the full results from the logit model in the next section). The penultimate row in the table shows the average subjective probability that the higher-ranked player wins the match (π), which is higher for GS compared to MS (0.74 vs. 0.68). This means that bookmakers are adjusting their subjective probabilities for GS matches relative to MS matches in the direction predicted by Bayes Rule. However, this adjustment seems insufficient, i.e., it does not completely reflect the increase $\hat{p}$ for GS. This result, shown in Figure 1, provides some early support to the process variance neglect hypothesis, which we formally test in the next section.

[Insert Table 2 and Figure 1 here]

3 Analysis

3.1 Changes in Process Variance

We start our analysis by examining whether the higher-ranked player is more likely to win a GS as opposed to an MS match using the model shown in Equation (1).

The marginal effects associated with each variable are shown in Table 3. We run specifications with and without *RSkill*, and a specification that additionally includes ranked players who earn their position by qualifying in a single elimination tournament prior to the main event. The number of seeded players varies by tournament, and is higher in GS than in MS.

fixed effects related to the surface that the match is played on (clay vs. hard),¹⁹ the round of the match (1 to 7), and the year that it takes place.²⁰

From Column (1) we observe that the marginal effect associated with *GS* is 9.5% and highly statistically significant, indicating that higher-ranked players are more likely to win a *GS* match. Once we control for *RSkill* in Column (2) the marginal effect associated with *GS* reduces to 7.2%, but remains highly statistically significant. *RSkill* is also positive and highly significant. From Column (3), we observe that the fixed effects do not materially influence the results, with the coefficient of *GS* being equal to 6.9%.

Overall, this analysis shows that higher-ranked players are more likely to win *GS* matches that are played in a BO5 format.

[Insert Table 3 here]

3.2 Biases in Subjective Probabilities

Table 4 shows our main results. In a univariate setting in Panel A, we find that $\overline{bias_{GS}}$ is equal to -3.8% and $\overline{bias_{MS}}$ is equal to 0.4%, making Δ_{bias} equal to -3.4%, and highly statistically significant.

In Panel B, we test the hypothesis in a multivariate setting. We find that the coefficient on *GS* is essentially the same, equal to -3.4% and highly statistically significant. This result is not sensitive to surface, round and year fixed effects (Column 2).²¹ The

¹⁹The type of surface influences the speed that the match is played. The fastest surface is grass, followed by hard courts, then carpets, then clay. One *GS* match is played on grass, one on clay, and two on hard courts. In terms of *MS* tournaments, seven are played on hard courts and four are played on clay courts. In our sample, the only tournament played on grass is the Wimbledon (*GS*), and the only tournament played on carpet is Paris (*MS*) between 2005 and 2008. Because in terms of speed, grass and carpet surfaces are more similar to hard than clay surfaces, we include them in the hard court category. Surface dummies absorb any affects on the probability that the higher-ranked player wins related to match tempo.

²⁰Because prize money increase with the round of the match, round dummies absorb the effect that such variations in prizes may have on the probability that the higher-ranked player wins. Year dummies absorb any time-variation on the probability that the higher-ranked player wins.

²¹In this model, the fixed effects additionally capture any variations in the betting market that are related to these dimensions.

coefficient of $RSkill$ is also negative and significant.

In untabulated analysis we replace $bias$ with π_{HR} as the dependent variable and re-run the same model. In this specification the coefficient on GS is equal to 4.1% and highly statistically significant. This shows that, to large extent, the increase in π_{HR} for GS (from 68.1% to 74.3%) reflects an adjustment according to match format (BO3 vs. BO5) in the direction implied by Bayesian reasoning. However, the adjustment is insufficient, equal to roughly 59% of the change in $\hat{p}$ that is attributed to match format (4.1%/6.9%).

Overall, the results in Table 4 show that bookmakers are systematically underestimating the probability that the higher-ranked player wins a GS match, consistent with the hypothesis of process variance neglect.

[Insert Table 4 here]

3.3 Betting Exchange Data

Could our results in the previous section reflect the strategic incentives of rational bookmakers to exploit punters? In this section we test our hypothesis using subjective probabilities inferred from odds achieved on a person-to-person betting exchange called Betfair,²² where the capacity to manipulate odds does not exist. Moreover, this data set contains information on the volume bet on each player, which we utilize to test the wisdom of crowds hypothesis (WCH), (e.g., Wolfers and Zitzewitz, 2004). Our sample contains 4,893 observations for the period 2009-2014. Descriptive statistics for this sample are shown in Section 2 of the Appendix.

²²Betfair is the largest person-to-person betting exchange, with almost one million active users (Croxson and Reade, 2014). The Betfair data set was purchased from Fracsoft, available at <http://www.fracsoft.com/>. The dataset has incomplete coverage of the Australian Open (GS tournament), with no observations for 2009 and 2010. Moreover, it does not include data for matches that were completed in more than one day (for example due to rain delays). We use “back” odds on the two players to calculate subjective probabilities (i.e., odds available to punters who want to bet that a specific player wins).

[Insert Table 5 here]

The results are shown in Table 5. In a univariate setting in Panel A, we find that $\overline{bias_{GS}}$ is equal to -1.8% and $\overline{bias_{MS}}$ is equal to 0.5%, making Δ_{bias} equal to -2.3%, and highly statistically significant.

In Panel B, we test the hypothesis using multivariate analysis. Similar to the analysis in Table 4 we observe that controlling for $RSkill$ does not change our findings, as the coefficient on the GS dummy is equal to -2.3% and statistically significant. Adding surface, year and round fixed effects does not materially influence the result.

The point estimate of Δ_{bias} is considerably smaller in the Betfair data than in the bookmaker data (-2.3% vs -3.4%). However, we cannot reject the hypothesis that the two are equal at the 5% level of confidence. Thus, the evidence in support of WCH from this test is rather weak.

For our next test we conduct a sharper test of WCH by examining whether Δ_{bias} is closer to zero for high-volume matches, which incorporate information from a larger number of traders. We use data on the volume that backs the higher-ranked player (Vol_{HR}) to test this hypothesis. Specifically, for each match type (MS vs. GS) and round of match (1 to 7), we rank according to Vol_{HR} , noting the median of the distribution. If, for a specific match, Vol_{HR} is higher than the corresponding median then $VolD$ equals 1, otherwise it is 0.²³ We re-estimate the model in Equation (2), by including $VolD$ and the interaction between $VolD \times GS$ as additional regressors. The coefficient on $VolD \times GS$ tests whether the bias related to process variance neglect is different for high-volume matches.

The results are shown in Column (3) of Table 5. We find that the coefficient on GS (indicating Δ_{bias} for low-volume matches) is equal to -3.0% and highly significant. The

²³We note the median after we sort on the basis of match type and round of match because GS matches attract higher volume, as do later rounds in a tournament. In this way, the dummy $VolD$ is not capturing mechanically such cases.

coefficient on the interaction $VolD \times GS$ is *positive* and significant at 1.6%, indicating that Δ_{bias} is closer to zero for high-volume matches ($-3.0\% + 1.6\% = -1.4\%$), which suggests that “deeper” markets are more efficient, supporting WCH. However, the Δ_{bias} of -1.4% for high-volume matches is statistically significant,²⁴ which suggests that these markets do not completely eliminate the bias.

Overall, this analysis shows that a bias due to process variance neglect also exists in the betting exchange data, where strategic incentives do not exist. Moreover, the bias is smaller in matches that attract higher betting volume.

[Insert Table 5 here]

3.4 Profits

In this section, we examine the economic implications of the biases documented in Section 3.2. We derive various indicators of bookmaker’s profitability, using both bookmaker (Panel A) and betting exchange odds (Panel B). The analysis in Panel B corresponds to the profits collected by a hypothetical bookmaker, who offers odds on both players that equal those achieved on the betting exchange.

The first profitability indicator we analyse is how much of each unit bet on the higher-ranked player is collected by the bookmaker, r_{HR} , using the expression below. D_{HR} is an indicator variable, equal to 1 if the higher-ranked player has won the match, and 0 otherwise.

$$r_{HR} = \begin{cases} 1 & \text{if } D_{HR} = 0 \\ -(O_{HR} - 1) & \text{if } D_{HR} = 1 \end{cases} \quad (3)$$

²⁴We obtain this result in untabulated analysis, where we re-run the model by specifying the volume dummy based on low-volume matches.

We can define an analogous metric for the lower-ranked player, r_{LR} . We calculate r_{HR} and r_{LR} for each match, and then compare the averages across MS and GS matches.

The results are shown in Table 6. From Panel A we see that r_{HR} for MS is equal to 5% and highly significant, whereas for GS it is 0.5% and insignificant. The difference of -4.5% is highly significant. From the second column, r_{LR} is 9.4% for MS and 29.8% in GS, for a significant difference of 20.4%. Similar relationships are found in Panel B, although the profitability is generally lower, and the differences between MS and GS less significant.²⁵ Overall, these figures are in line with the results in Tables 4 and 5. Bookmakers are offering excessively high odds for the higher-ranked player in GS ($bias < 0$), and therefore on average earn less per unit staked for the corresponding bets. And, by symmetry, bookmakers are on average offering excessively low odds for the lower-ranked player in GS, thus earn more per-unit staked for those bets.

These figures are indicative, as the overall profitability of bookmakers depends on how the betting volume is split across the two players. Specifically, as we show in Section 1 of the Appendix, it is optimal to “salt” the odds to imply a negative $bias$ in GS (as shown in Tables 4 and 5) *only* if the probability that the higher-ranked player wins is *higher* than the proportion of volume that backs the higher-ranked player. This condition, however, does not seem to hold in the data, since higher-ranked players win 78% of GS matches, whereas they are on average backed by 86% of the volume. This suggests that bookmakers are actually earning less in GS by offering odds that imply a negative $bias$.

To estimate this more concretely we estimate bookmaker’s actual profits, as a proportion of the total volume staked for match i , Π_i , using the volume information from the betting exchange as below:²⁶

$$\Pi_i = 1 - RVol_{HR} \times d_{HR} \times D_{HR} - RVol_{LR} \times d_{LR} \times D_{LR} \quad (4)$$

$RVol_{HR}$ is the proportion of the total volume that backs the high ranked player, d_{HR}

²⁵The profits to the hypothetical bookmaker who offers betting exchange odds are lower because these odds do not contain a vig (see Table 1 in Section 2 of the Appendix).

²⁶This test assumes that volume for bookmakers behaves similarly to volume on the betting exchange.

are the fixed decimal odds offered on the higher-ranked player and D_{HR} is an indicator that equals to one if the higher-ranked player won the match. $RVol_{LR}$, d_{LR} and D_{LR} are analogously defined for the lower-ranked player. From the third column of Panel A we observe that Π_i is equal to 2.5% for GS matches, and 5.00% for MS matches, for a significant difference of -2.5%. The corresponding figure in Panel B is -1.9%.²⁷ Thus, bookmakers are earning a smaller proportion of the volume staked in GS relative to MS matches.

As a final test we calculate the *hypothetical* profit $H\Pi_i$ collected by bookmakers in a counterfactual world where they increased their subjective probabilities for the higher-ranked player for each GS match by an amount equal to the average *bias* shown in Tables 4 and 5 (-3.41% and -2.74%, respectively). For each GS match we calculate the hypothetical odds that correspond to these probabilities, which we use to calculate $H\Pi_i$ according to Equation 3.²⁸ As shown in Column 5 of Table 6 the average value of $H\Pi_i$ for GS matches is equal to 5.5%, significantly different from the average Π_i of 2.5% . Similar findings are found when using betting exchange odds, as shown in Panel B. This exercise suggests that bookmaker's profit as a proportion of volume would be higher in the absence of a negative *bias* for GS.

Overall the analysis in this section is consistent with the notion that the biases due to process variance neglect are costly to bookmakers.

[Insert Table 6 here]

3.5 A Placebo Test

Our tests implicitly assume that the only difference across MS and GS matches is match format, i.e., BO3 vs. BO5. However, as mentioned, GS tournaments are more prestigious than MS tournaments, offering more prize money and ranking points, and attracting higher betting volumes. To examine whether our findings are affected by these differences

²⁷Our results hold in a multivariate setting, when we regress Π_i on a *GS* dummy and *RSkill*, including round, time and surface fixed effects.

²⁸This procedure is explained in detail in Section 1 of the Appendix.

we conduct a placebo test using data for women’s tennis matches. This data provide an ideal setting for such a test as they preserve the key differences across MS and GS that are observed in the men’s sample (i.e., GS are more prestigious, offer more prize money and ranking points, and attract more betting volume), but for women there is *no change in process variance because both MS and GS are played in a BO3 format*. Therefore, if our results are not driven by changes in process variance, Δ_{bias} should continue to be negative and significant in the women data.

We construct the women’s sample using an approach similar to that used in our baseline analysis with the men’s data. GS tournaments are the same for women as for the men, and for MS tournaments we again focus on the more prestigious tournaments.²⁹ After applying the same filters to the initial sample as those for the men we end up with 2,527 MS matches and 3,624 GS matches from 2007-2014 for the bookmaker sample, and 1,425 MS matches and 2,259 GS matches from 2009-2014 for the betting exchange sample.

Tables 2 and 3 in Section 2 of the Appendix present descriptive statistics for the women samples for MS and GS, and show that the *vig* and ranking differentials are very similar as those found in the men data. From Table 3 we observe that *TotVol* is lower for women than for men, whereas *RVol* is the same (85% in GS and 82% in MS). Overall, the women samples are broadly similar to the men samples.

In Table 7 we present results from the logistic regressions shown in Equation (1) using the women data (Panel A bookmaker, Panel B betting exchange). As shown in Column (2), the coefficient on the *GS* dummy is insignificant once we control for *RSkill*, consistent with no change in process variance in GS matches. Thus, bookmakers *should not* upwardly adjust their subjective probabilities of the higher-ranked player winning a GS match.

²⁹Pre-2009, the top tier masters tournaments were called Tier-I and post-2009 they are called Premier Mandatory. For the bookmaker sample we have 12 MS tournaments prior to 2009 (Berlin, Charleston, Doha, Indian Wells, Miami, Montreal, Moscow, Rome, San Diego, Tokyo, Toronto, and Zurich) and the 4 Premier Mandatory tournaments after 2009 (Beijing, Indian Wells, Madrid, Miami). The betting exchange sample starts in 2009 so we have data on the 4 Premier Mandatory tournaments. The GS matches for women are the same as for the men (Australian Open, French Open, Wimbledon, and US Open).

[Insert Table 7 here]

The results from the model in Equation (2) with women data are shown in Table 8. From Column (2) in Panel A (bookmaker), we observe that the coefficient on GS is negative but statistically *insignificant*. The corresponding results in Panel B (betting exchange) show that the coefficient on GS is *positive* and significant, equal to 1.2%, reflecting the fact that punters are increasing their subjective probabilities for the higher-ranked player winning a GS match, even though this increase is unwarranted.

Overall, the placebo test with the women’s data produces results that are in stark contrast from those obtained with the men’s data. This suggests that our baseline results in Tables 4 and 5 more likely reflect biases due to process variance neglect.

[Insert Table 8 here]

3.6 Robustness Checks

In this section, we conduct various tests of robustness, presenting the results in Table 9. In Panel A, we define *bias* using subjective probabilities inferred from bookmaker odds and in Panel B using betting exchange odds.

In our baseline analysis *bias* depends on the specification of the logit model used to calculate $\hat{p}$. In Column (1) we test the hypothesis using a non-parametric specification for the objective probability. We first sort our sample according to match format (MS and GS) and then sort again within each format into deciles based on $RSkill$. In each decile we calculate the proportion of matches won by the high-rank player, p_{HR} , which we use as the estimate of the objective probability for all the matches that belong in this specific decile. We define bias for each match as $\pi - p_{HR}$, and estimate Equation (2). The results are robust in this specification, as the coefficient of GS is equal to -3.5% in Panel A and -2.6% in Panel B, both statistically significant.

In Column (2) we avoid altogether estimating an objective probability, and define *bias* as $\pi - D_{HR}$, where D_{HR} is a dummy that equals 1 if the higher-ranked player won the match, and 0 otherwise. The coefficient on GS continues to be negative and significant.

Our procedure of estimating $\hat{p}$ uses full sample information, which introduces some look-ahead bias. To make sure that such a bias does not influence our findings in Column (3), we estimate the logit model using only backward looking information. For example, for matches played in 2006 (2007) we calculate $\hat{p}$ estimated from a logit model that only uses data from 2005 (2005 and 2006), etc. The coefficient of *GS* is equal to -4.3% in Panel A and -3.2% in Panel B, both statistically significant.

Because *GS* matches offer more prize money it is possible that higher-ranked players “time” their form to peak at *GS* matches. Such timing effects could influence our findings. To address this issue in Column (4) we add an additional regressor in our models (including the logit model), *RStreak*, calculated as the difference in the proportion of games won by the high- and lower-ranked player in the two previous tournaments. The results show that the coefficient on *GS* remains negative and significant in this specification, equalling -2.7% in Panel A and -2.6% in Panel B.³⁰

The information signal we use is the ranking of the players at the start of the tournament. However, bookmakers are probably observing other information besides rankings before setting their odds. We examine whether our findings hold when we define as the high-skilled player the one who is favoured by the bookmakers (as shown by the odds). The results from this test are shown in Column (5), whereby the coefficient on *GS* is negative and significant (-2.6% in Panel A and -2.5% in Panel B).

For our baseline results we control for *RSkill* and test our hypothesis by observing the coefficient on the intercept dummy *GS*. An alternative method is to use a slope dummy, interacting *RSkill* with *GS*,³¹ expecting that the coefficient on the interaction is negative and significant. The results shown in Column (6) show that this coefficient is indeed negative and significant, and that the total effect of *RSkill* on *bias* is negative for *GS* matches ($0.000 - 0.013 = -0.013$ in Panel A and $0.05 - 0.009 = -0.004$ in Panel B).

³⁰The results from the logit model show that the marginal effect associated with *RStreak* is positive and significant. In addition, *RStreak* is higher for *GS* by roughly 2%. The results in Column (4) show that agents are not properly pricing the effects of *RStreak* on the probability that the higher-ranked player wins, as its coefficient is negative and significant. Note that this model entails a smaller number of observations due to missing values in *RStreak*, which occur when a player did not compete in the two previous tournaments.

³¹In this specification $\hat{p}_i = Pr(Y_i = 1 | RSkill_i, RSkill_i \times GS_i) = F(\alpha + \beta_1 RSkill_i + \beta_2 RSkill_i \times GS_i)$.

In untabulated analysis we find that our findings continue to hold when we estimate the hypothesis using a quantile regression model, which controls for the confounding effect of any outliers.³²

Moreover, our results hold when we do not impose filter (ii) from Section 2.1, which eliminates matches where the higher-ranked player is indicated as an outsider by bookmakers even though he is ranked by at least 15 places higher than the lower-ranked player. In this sample, the coefficient of *GS* is -2.46% for bookmaker data, and -2.02% for betting exchange data, significant at the 1% level. The point estimates are slightly lower in this setting, since rankings become noisier indicators of skill.³³

Overall, the results in this section show that our conclusions are robust to different *bias* and sample specifications.

[Insert Table 9 here]

3.7 Alternative Explanations

In this section, we discuss alternative explanations for our findings.

Our analysis shows that bookmakers are offering overly attractive odds to the higher-ranked player winning a *GS* match, thus earn proportionately less. However, their average profitability for *GS* matches remains positive. Could competition between rational bookmakers to attract bets push them toward this equilibrium where they offer overly attractive odds to the higher-ranked player for *GS*? The betting exchange data show that volumes are larger for *GS*, which suggests that betting capital is not scarcer for *GS*. So, it is not immediately clear why bookmakers face higher competition for *GS*.

³²Specifically, instead of using the least squares regression in Equation (2), we estimate a model where we express the median of the conditional distribution of *bias* as a linear function of *GS* and *RSkill*. The coefficient of *GS* is -2.30% for bookmaker data and -1.50% for betting exchange data, significant at the 1% level.

³³We conduct an additional test using a sample where, in addition to filter (ii) from Section 2.1, we also drop matches that involve players ranked outside the top 100 in the world. Since player skill is a pyramid, rankings are less informative for lower ranked players. Thus, in this sample, rankings are better indicators of skill, so our results should be stronger (comparing with Tables 4 and 5). Indeed, in these samples, the coefficient on *GS* is larger, equal to -4.0% in the bookmaker data and -3.02% in the betting exchange data.

Moreover, even if competition is higher for men’s GS matches, these competitive forces should be absent for women’s matches, since $\Delta_{bias} = 0$ in this sample. Finally, competition in this sense does not apply to the betting exchange data, where the odds on the two players are set through the trades of many different punters, who only enter the market if the prevailing odds imply an “arbitrage opportunity” in relation to their own subjective belief.³⁴

GS matches are more prestigious events, thus more highly publicized than MS matches. Could biases in GS matches be larger ($\Delta_{bias} < 0$) because these matches attract relatively more unsophisticated bettors who hold more biased expectations? This “noise-trader” explanation could explain the findings with the betting exchange data for men, but it cannot explain the findings with the bookmaker data where the same agents are setting odds for both MS and GS matches. In addition, the biases found in the women’s betting exchange data, where GS matches are also more prestigious and more publicized, are opposite to those found in the men data ($\Delta_{bias} > 0$). Therefore, under this explanation, the increased noise trading in GS should produce opposite results in men’s and women’s betting exchange markets.

Another alternative explanation is that there is a systematic error in π that is related to agent’s preferences, which is asymmetric across MS and GS. For example, if bookmaker’s risk attitude changes for GS matches due to higher exposure coming from heavier betting volumes, they will distort the odds offered for these matches further from risk-neutral probabilities. This explanation could explain our findings for the men’s bookmaker data, however it cannot explain the findings with the betting exchange data, where the exposure of individual punters is the same across MS and GS. Moreover, for bookmaker data, this explanation requires that these shifts in preferences do not occur for women’s GS matches, since $\Delta_{bias} = 0$ in this sample.

Overall, these alternative explanations do not seem to be able to offer a parsimonious

³⁴For example, suppose that the latest odds that player X wins are 1.50, indicating a probability of roughly 67%. A punter who believes that this probability is in fact 60% will choose to improve on those odds in order to take bets, and will offering odds of, say, 1.6 (implying a probability of 63%). If then another punter improves further on these odds and offers 1.7 (implying a probability of 59%) the original punter will choose to stay out of the market.

explanation for all our results.

3.8 A Laboratory Experiment

Because our framework does not constitute a natural field experiment, we cannot conclusively rule out the possibility that our results are driven by an economic quantity that varies between MS and GS. Motivated by this limitation our final robustness test is a laboratory experiment, in which students were invited to predict the outcomes of tennis matches.

To make sure that our subjects knew about tennis, and therefore could appreciate the effect of match length on the probability that the higher-ranked player wins, we only invited students that were involved in the Tennis Society of the University of Surrey to participate. Our experiment attracted 17 students, who participated in two different sessions lasting roughly an hour each. Each student received on average £22 for participating in each session.

In each session subjects were asked to provide a probability that each player wins for several upcoming matches. In the first session the 33 matches were from the Rome ATP Masters (MS, BO3) and in the second the 64 matches were from the French Open (GS, BO5). Both tournaments are played on the same surface (clay). For each match we provided to subjects the name and ranking of each player at the start of the tournament. Moreover, our instructions in the beginning of each session explicitly mentioned the format that these matches were played (BO3 or BO5). For both sessions students received a show-up fee (£10) and a performance-related payment which depended on the accuracy of their predictions. We used the quadratic scoring rule to incentivise subjects choices, and the random lottery procedure to determine their performance-related payment in each session. Our full instructions, additional details about our experimental protocol and various descriptive statistics are provided in Section 3 of the Appendix.

In the first session, subjects were firstly asked to consider the extent to which the expression “*I follow tennis closely*” applies to them (1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly agree). The mean response to this question was 3.64, in-

dicating that on average our subjects did indeed follow tennis. To confirm this statement, we also gave students 8 tennis-related quiz questions (4 in each session), and found that on average students answered 5.4 of those questions correctly. The correlation between the response to the first question and the number of correctly answered quiz questions was roughly 0.7. These findings suggest that our subjects were on average knowledgeable about tennis.

The results are shown in Table 9. We used the same procedure as with the field data to calculate *bias* for MS and GS. In Panel A, in a univariate setting, we see that π and $\hat{p}$ for the MS matches are equal to 66.6%, and *bias* is equal to 0. For GS, we observe that $\hat{p}$ increases to 73%, but π only increases to 69.8%, resulting to a significant *bias* of -3.2%. Δ_{bias} is negative and significant, equal to -3.2% consistent with our field results from bookmakers and the betting exchange. In Panel B we conduct a regression that controls for *RSkill*. The coefficient on *GS* remains negative at -3.3% and statistically significant.

Although the magnitude of Δ_{bias} in the experiment is very similar to that found in the field data, closer inspection reveals that the extent of the bias is potentially larger in the laboratory. In unreported analysis, when we regressed π on *GS* and *RSkill*, we found that the coefficient on *GS* is insignificant. This means that subjects do not consider the effect of match length, and only increase their π 's for GS matches because *RSkill* is higher for these matches (see Table 2).³⁵ Perhaps this difference arises because field agents are more sophisticated than students.

Overall, the results from the experiment document the existence of biases due to process variance neglect. This finding provides further support to the notion that our baseline findings from the field reflect such biases.

³⁵At the end of the second session, after all the data were collected, we asked students to list the major factors that influenced their responses. Players rankings was the most important factor by nearly all the students, whereas no student mentioned match format (MS vs. GS). This finding corroborates the result that subjective probabilities are strongly linked *RSkill* but not the *GS* dummy.

3.9 Process Variance Neglect in Financial Markets

In this section, we conduct a test of the underreaction due to process variance neglect hypothesis in financial markets, examining how investors price earnings related information that varies in process variance.

The signal we use is the median earnings forecast revision issued by professional sell-side analysts for a specific company prior to the actual earnings announcement. To measure the process variance of the signal we split the companies in two groups: high-coverage firms followed by many analysts, and low-coverage firms followed by few analysts. The signal is of lower process variance for high-coverage firms because it incorporates information from more analysts about future earnings. The hypothesis under process variance neglect is that future stock returns are more predictable after low process variance signals, in the direction predicted by the signal (i.e., rise after positive revisions and fall after negative revisions). To conduct the test we follow a standard approach used in the empirical finance literature, which is explained in detail in Section 4 of the Appendix.

The results are shown in Table 11. We split our sample according to whether the median revision is positive (Panel A) or negative (Panel B), and estimate three separate models. In model (1) the dependent variable is a measure of the shock to earnings for firm i in quarter t . *Rev* is the median forecast revision, and *High Cov* is a dummy that flags firms that are followed by many analysts. The interaction *Rev* x *High Cov* is the variable of interest, which tests whether earnings shocks are more predictable from lower process variance signals. The results in Panel A for positive signals (Model 1) are supportive of this notion, since *Rev* x *High Cov* is positive and significant, which implies that Bayesian investors should respond stronger to these signals.³⁶

³⁶From Model 1 in Panel B we observe that negative signals are not more informative for higher coverage firms. Although we do not have a conclusive answer as to why this occurs, the results from previous studies are instructive. Specifically, a well-known finding in the analyst literature is that analysts tend to be overly optimistic (e.g., Jackson, 2005), perhaps due to their career concerns (i.e., Hong and Kubik, 2003). However, analysts also have the incentive to be accurate, which suggests that downward forecasts that are issued closer to the earnings announcement, such as the ones we include in our sample, are more likely to reflect a correction to previously optimistic forecasts as opposed to new information (this notion is supported by the fact we have roughly 55% more downward revisions than upward revisions in our sample). Given this evidence, a plausible explanation for the result from Model 1 in Panel B is that the information in negative signals from high coverage firms is diluted due to optimistic analysts.

This notion is tested by Model (2) in Panel A, where the dependent variable is the cumulative, market-adjusted return for company i measured during the same period as the signal. The results indeed confirm that investors are responding more strongly to these signals, since $Rev \times High\ Cov$ is positive and significant.

To test the underreaction hypothesis we examine whether post-announcement returns are more predictable from positive revisions for high-coverage firms. Under the null of efficient markets future returns should be unpredictable (Samuelson, 1965). However, as shown by Model 3 of Panel A future returns are strongly predictable in the direction predicted by the hypothesis of process variance neglect.³⁷

Although several patterns of underreaction in financial markets can be explained by process variance neglect (e.g., the stock price response to earnings shocks and various corporate events), the study by Dichev and Tang (2009) highlights this phenomenon clearly. These authors first illustrate that the earnings of firms with lower earnings variability are more predictable. They then examine whether sell-side analysts take into account earnings variability when they issue their forecasts in response to earnings shocks. In this setting, earnings variability captures the process variance of these signals. For high-earnings volatility firms, strong earnings shocks in quarter t are less persistent and therefore less predictive for earnings in quarter $t+1$. Thus, Bayesian analysts should revise their forecasts toward the mean more strongly for these firms. The results of Dichev and Tang (2009) show that analysts do consider earnings variability when forecasting, but by only 50% of the amount required by full Bayesian reasoning. This is consistent with process variance neglect, which is comparable in magnitude to our tennis-based results.

Overall, the results in this section are consistent with the notion that biases due to process variance neglect operate in financial markets.

³⁷A “rational” story for this finding is not straight forward, since it requires that higher coverage firms with positive revisions are riskier, and thus command a higher risk premium.

4 Conclusion

We conduct a field test of Bayesian reasoning by examining whether agents form expectations by placing a larger weight on cues that are more informative with lower process variance. We use subjective probabilities inferred from odds on the outcomes of tennis matches, exploiting *exogenous* variation in process variance related to whether these matches are played in a short or a long format. Under the null hypothesis of Bayesian reasoning bookmakers should assign higher probabilities to the high-skilled players winning the longer matches.

Our findings are consistent with underreaction due to “*process variance neglect*,” i.e., bookmakers are not adjusting their subjective probabilities sufficiently to reflect changes in process variance, and thus underestimate the probability that higher-skill players win the longer matches. This result is robust to inferring subjective probabilities from odds offered by professional bookmakers or odds achieved on a person-to-person betting exchange. Moreover, the bias is costly as bookmakers are earning less for longer matches. Overall, these findings support the notion that deviations from Bayes Rule influence real-life decisions with considerable economic consequences.

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Figure 1: Probabilities in MS vs GS

This figure depicts average objective probabilities (blue bars) and average subjective probabilities (red bars) for MS and GS matches.

Table 1: Observations by Tournament and Year

This table shows a breakdown of the data used in the analysis sorted by tournament and year. The data are retrieved from www.tennis-data.co.uk. Panel A contains data for Grand Slam (GS) matches which are played in a best-out-of-five format, and Panel B for ATP World Tour Masters 1000 (MS) matches, which are played in a best-out-of-three format. We drop non-completed matches, matches with missing information or negative vig, and matches where the higher-ranked player is indicated as an outsider by the odds even though he is ranked by at least 15 places higher than then lower-ranked player. Our final sample contains 9,230 matches from 2005-2014.

Tournament	Year										Total
	05	06	07	08	09	10	11	12	13	14	
Panel A: GS											
Australian Open	111	112	111	114	112	112	108	108	117	99	1,104
French Open	107	102	101	111	109	115	114	114	112	116	1,101
US Open	109	108	107	117	109	109	104	114	108	109	1,094
Wimbledon	105	107	98	101	103	107	107	108	105	108	1,049
Other	6	6	2	0	0	0	0	0	0	0	14
Total	438	435	419	443	433	443	433	444	442	432	4,362
Panel B: MS											
Cincinatti	57	59	47	43	46	45	47	51	46	51	492
Hamburg	53	53	48	47	0	0	0	0	0	0	201
Indian Wells	81	78	84	78	80	79	72	80	74	85	791
Madrid	40	36	43	41	43	45	49	50	53	50	450
Miami	75	69	77	79	81	79	77	82	73	76	768
Monte Carlo	50	59	46	45	50	48	48	50	47	48	491
Montreal	55	0	49	0	50	0	50	0	49	0	253
Paris	37	38	43	38	41	43	41	42	43	40	406
Rome	54	46	46	42	51	49	45	49	51	48	481
Shanghai	0	0	0	0	44	47	48	52	49	50	290
Toronto	0	51	0	52	0	47	0	44	0	51	245
Total	502	489	483	465	486	482	477	500	485	499	4,868

Table 2: Descriptive Statistics

This table shows descriptive statistics for the main variables used in our analysis. HR_{Odds} and LR_{Odds} are the average decimal odds offered by the betting houses that the higher-ranked (lower-ranked) player wins a match. Vig is the house take, which is obtained by summing the inverse of the odds for the higher- and lower-ranked players and subtracting one. HR_{Rank} and LR_{Rank} are the rankings for the high and low ranked player, respectively. $\hat{p}$ is the estimated "objective" probability, obtained from averaging the predicted values from the logit model shown in Equation (1) for MS and GS. π_{HR} is the subjective probability that the higher-ranked player wins the match, as derived from the bookmaker odds. The sample consists of 9,230 matches that satisfy the criteria outlined in Table 1. The last row in the table shows the average prize money collected by the winner of an *MS* and a *GS* tournament using 2015 prize money.

Variable	Mean	σ	Min	Q1	Median	Q3	Max
Panel A: GS							
HR_{Odds}	1.35	0.42	1.00	1.10	1.24	1.48	6.79
LR_{Odds}	6.05	5.73	1.09	2.62	4.01	7.17	60.00
Vig	0.05	0.01	0.00	0.05	0.06	0.06	0.11
HR_{Rank}	28	29	1	7	19	38	279
LR_{Rank}	99	96	2	48	80	119	1,370
$\hat{p}$	0.78	0.10	0.60	0.70	0.78	0.86	0.99
π_{HR}	0.74	0.15	0.14	0.64	0.76	0.87	0.98
Prize Money (\$)	2,525,000						
Panel B: MS							
HR_{Odds}	1.48	0.46	1.01	1.19	1.38	1.61	7.45
LR_{Odds}	4.10	3.25	1.09	2.27	3.01	4.58	32.80
Vig	0.06	0.01	0.01	0.05	0.06	0.06	0.09
HR_{Rank}	22	22	1	6	15	30	414
LR_{Rank}	67	83	2	29	49	78	1,517
$\hat{p}$	0.69	0.12	0.50	0.58	0.67	0.78	0.98
π_{HR}	0.68	0.15	0.13	0.59	0.69	0.79	0.97
Prize Money (\$)	790,000						

Table 3: Likelihood that higher-ranked player wins a Match

This table presents results from logit models, where the dependent variable takes the value of 1 if the higher-ranked player has won the match, and 0 otherwise. *GS* is a dummy variable that equals 1 if the match is GS, and 0 otherwise. *RSkill* is calculated as the log (lower-ranked player ranking) - log (higher-ranked player ranking) for MS (GS) matches. The table reports marginal effects associated with each of the independent variables. The sample consists of 9,230 matches that satisfy the criteria outlined in Table 1. The robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Variable	(1)	(2)	(3)
<i>GS</i>	0.095*** [0.009]	0.072*** [0.009]	0.069*** [0.009]
<i>RSkill</i>		0.122*** [0.005]	0.124*** [0.005]
Surface F.E.	NO	NO	YES
Year F.E.	NO	NO	YES
Round F.E.	NO	NO	YES
N	9,230	9,230	9,230
pseudo- R^2	0.010	0.071	0.073

Table 4: Biases in Subjective Probabilities

This table reports biases in subjective probabilities for GS and MS tennis matches. In Panel A, we present univariate analysis, and in Panel B multivariate analysis. In Panel A, π and $\hat{p}$ denote the average subjective and the average (estimated) objective probabilities that the higher-ranked player wins an MS or a GS match. $\hat{p}$ is obtained by averaging the predicted values from the logit model shown in Equation (1) for MS and GS, and π is derived from the average odds on the higher-ranked player to win the match offered by the bookmakers. *bias* is the average difference between π and $\hat{p}$ for MS and GS. In Panel B, we present results from OLS regressions with an intercept. The dependent variable is *Bias* and the independent variables are *GS* and *RSkill*, defined as in Table 3. The sample consists of 9,230 matches from 2005 to 2014 that satisfy the criteria outlined in Table 1. In Panel B the robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: Univariate			
	MS	GS	GS - MS
π	0.681	0.743	
$\hat{p}$	0.685	0.781	
<i>bias</i>	0.004	-0.038	-0.034*** [0.002]
N	4,868	4,368	
Panel B: Multivariate			
Variable	(1)	(2)	
<i>GS</i>	-0.032*** [0.002]	-0.034*** [0.002]	
<i>RSkill</i>	-0.007*** [0.001]	-0.005*** [0.001]	
Surface F.E.	NO	YES	
Year F.E.	NO	YES	
Round F.E.	NO	YES	
N	9,230	9,230	
R^2	0.028	0.051	

Table 5: Biases in Subjective Probabilities - Betting Exchange Data

This table reports biases in subjective probabilities for GS and MS tennis matches. The analysis in Panel A and in Panel B Columns (1) and (2) is the same as in Table 4. For the analysis in Column (3) we rank all matches according to type of match (MS vs. GS) and match round (1 to 7), and note the median of each distribution. *HighVol* equals 1 if a match is above this median, and 0 otherwise. We drop non-completed matches, matches with missing information or vig greater than 0.05, and matches which do not satisfy criterion (ii) in Section 2.1 of the paper. The sample consists of 2,751 MS matches and 2,142 GS matches from 2009-2014. In Panel B the robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: Univariate			
	MS	GS	GS - MS
π	0.701	0.764	
$\hat{p}$	0.696	0.782	
<i>bias</i>	0.005	-0.018	-0.023*** [0.003]
N	2,751	2,142	
Panel B: Multivariate			
Variable	(1)	(2)	(3)
<i>GS</i>	-0.023*** [0.003]	-0.027*** [0.003]	-0.030*** [0.005]
<i>RSkill</i>	-0.001 [0.001]	0.002 [0.002]	-0.015*** [0.002]
<i>HighVol</i>			0.067*** [0.004]
<i>HighVol</i> $\times$ <i>GS</i>			0.016** [0.006]
Surface F.E.	NO	YES	YES
Year F.E.	NO	YES	YES
Round F.E.	NO	YES	YES
N	4,893	4,893	4,893
R^2	0.010	0.024	0.107

Table 6: Bookmaker Profits

This table reports average values for different metrics of bookmaker's profitability. In Panel A, we use bookmaker odds and in Panel B betting exchange odds. The top row shows the variable analyzed. r_{HR} and r_{LR} are the per unit profit earned by the bookmaker for the high and low ranked player respectively, and is calculated using Equation (2). Π is an estimate of the actual profit earned by the bookmakers, using volume information from the betting exchange, and is calculated using Equation (3). $H\Pi$ is the average hypothetical profit earned by bookmakers for GS matches, in a world where they raised for each GS match their subjective probability on the higher-ranked player by an ammount equal to the average bias (from Tables 4 and 5). The procedure used for this test is explained in Section 1 of the Appendix. The final column presents the difference between $H\Pi$ and Π for GS matches. The third row in each panel shows the difference between MS and GS. The standard errors are shown in brackets. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A:Bookmaker					
Variable	(1: r_{HR})	(2: r_{LR})	(3: Π)	(4: $H\Pi$)	(5: $H\Pi - \Pi$)
<i>MS</i>	0.050*** [0.010]	0.094*** [0.023]	0.050*** [0.007]		
<i>GS</i>	0.005 [0.009]	0.298*** [0.025]	0.025*** [0.007]	0.055*** [0.007]	0.030***
$\Delta(GS - MS)$	-0.045*** [0.013]	0.204*** [0.034]	-0.025** [0.010]	[0.010]	
Panel B:Betting Exchange					
<i>MS</i>	-0.010 [0.014]	-0.015 [0.039]	0.015** [0.007]		
<i>GS</i>	-0.016 [0.013]	0.114* [0.069]	-0.005 [0.007]	0.015** [0.006]	0.020***
$\Delta(GS - MS)$	-0.026 [0.020]	0.129* [0.076]	-0.019* [0.010]		

Table 7: Likelihood that higher-ranked player wins a Match - Women's Data

This table presents results from logit models, where the dependent variable takes the value of 1 if the higher-ranked player has won the match, and 0 otherwise. All variables are defined as in Tables 3 and 4. In Columns 1 and 2 we estimate subjective probabilities using bookmaker data, and in Columns 3 and 4 using betting exchange odds. The robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

	A:Bookmaker		B: Betting Exchange	
	(1)	(2)	(3)	(4)
<i>GS</i>	0.042*** [0.012]	0.017 [0.011]	0.028* [0.015]	0.004 [0.015]
<i>RSkill</i>		0.128*** [0.006]		0.123*** [0.008]
Surface F.E.	NO	YES	NO	YES
Year F.E.	NO	YES	NO	YES
Round F.E.	NO	YES	NO	YES
N	6,151	6,151	3,684	3,684
R^2	0.002	0.065	0.001	0.060

Table 8: Biases in Subjective Probabilities - Women's data

This table reports biases in subjective probabilities for GS and MS women's tennis matches, presenting results from OLS regressions that include an intercept. The dependent variable is *bias* and the independent variables are *GS* and *RSkill*, defined as in Table 4. In Columns (1) and (2) we present results when subjective probabilities are derived from bookmaker odds, and in Columns (3) and (4) Panel B when subjective probabilities are derived from betting exchange odds. The robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Variable	A:Bookmaker		B: Betting Exchange	
	(1)	(2)	(3)	(4)
<i>GS</i>	0.001 [0.003]	-0.002 [0.003]	0.016*** [0.004]	0.012*** [0.004]
<i>RSkill</i>	-0.008*** [0.001]	-0.006*** [0.001]	-0.000 [0.002]	0.002 [0.002]
Surface F.E.	NO	YES	NO	YES
Year F.E.	NO	YES	NO	YES
Round F.E.	NO	YES	NO	YES
N	6,151	6,151	3,684	3,684
R^2	0.007	0.062	0.005	0.047

Table 9: Robustness Checks

In this table we conduct various robustness checks. In (1) and (2) we estimate the hypothesis without using parametric methods to estimate the objective probability, in (3) we estimate the logit model using only backward looking information, in (4) we add an additional control variable, in (5) we define the high-skilled player based on odds, and in (6) we test our hypothesis by interacting *GS* with *RSkill*. More details for these tests are provided in Section 3.6 of the paper. All models include an intercept term and surface, year and round fixed effects. In Panel A (B) we derive subjective probabilities using bookmaker odds and in Panel B using betting exchange odds. The robust standard errors shown in brackets are calculated using the Huber-White estimator. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: Bookmaker						
Variable	(1)	(2)	(3)	(4)	(5)	(6)
<i>GS</i>	-0.035*** [0.002]	-0.030*** [0.009]	-0.043*** [0.002]	-0.027*** [0.003]	-0.026*** [0.002]	
<i>RSkill</i>	-0.002* [0.001]	-0.007* [0.004]	-0.001 [0.001]	-0.003** [0.001]		0.000 [0.001]
<i>RStreak</i>				-0.022*** [0.004]		
<i>ROdds</i>					0.001*** [0.000]	
<i>RSkill</i> $\times$ <i>GS</i>						-0.013*** [0.001]
N	9,230	9,230	8,290	6,362	9,230	9,230
<i>R</i> ²	0.049	0.004	0.052	0.050	0.040	0.037
Panel B: Betting Exchange						
<i>GS</i>	-0.026*** [0.003]	-0.022* [0.012]	-0.032*** [0.004]	-0.026*** [0.004]	-0.025*** [0.003]	
<i>RSkill</i>	0.003* [0.002]	0.002 [0.005]	-0.000 [0.001]	0.005** [0.002]		0.005*** [0.002]
<i>RStreak</i>				-0.024*** [0.006]		
<i>ROdds</i>					0.001*** [0.000]	
<i>RSkill</i> $\times$ <i>GS</i>						-0.009*** [0.001]
N	4,893	4,893	4,164	3,524	4,893	4,893
<i>R</i> ²	0.023	0.005	0.039	0.029	0.047	0.016

Table 10: A Laboratory Experiment

This table reports biases in subjective probabilities for GS and MS women's tennis matches, presenting results from OLS regressions that include an intercept. The dependent variable is *bias* and the independent variables are *GS* and *RSkill*, defined as in Table 3. Subjective probabilities were obtained from a laboratory experiment, where 17 subjects expressed probabilities for 97 professional men's tennis matches, 33 played in an MS format and 64 played in a GS format. The experiments were conducted at the University of Surrey, and included students who are registered in the University's Tennis society. We incentivised students choices using the quadratic scoring rule, and used the random lottery procedure to determine the payment. Instructions, details on our experimental procedures and descriptive statistics are shown in Section 3 of the Appendix. In both Panels the standard errors shown in brackets are clustered on the subject level. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: Univariate			
	MS	GS	GS - MS
π	0.666	0.698	
$\hat{p}$	0.666	0.730	
<i>bias</i>	0.000	-0.032	-0.032** [0.012]
N	459	1,071	
Panel B: Multivariate			
Variable	(1)	(2)	
<i>GS</i>	-0.032*** [0.012]	-0.033** [0.012]	
<i>RSkill</i>		0.003 [0.007]	
N	1,530	7,569	
R^2	0.012	0.013	

Table 11: Process Variance Neglect in Financial Markets

This table reports results from OLS regressions with an intercept. Earnings data are from IBES, and data on stock prices, shares outstanding and stock returns are from CRSP. The sample period is from March 1994 to June 2016. In Columns 1 and 4 the dependent variable is the earnings shock for company i at quarter t (SUE). In Columns 2 and 5 (3 and 6) the dependent variable is cumulative abnormal return for firm i in the period -30,-4 (-1,65) relative to the announcement date (CAR). $Revision(Rev)$ is the median analyst forecast revision for company i at quarter t issued in the period -30,-4 relative to the earnings announcement date. $High Cov$ is a dummy that equals 1 if company i is at the top 30% in terms of analyst coverage at time t . $Dispersion$ is the coefficient of variation of these forecasts. $LogMV$ is the natural logarithm of firm size (price x shares outstanding). $EP(+)$ is the earnings to price ratio for firm i , and ED is a dummy that flags companies with negative earnings. $Lag(SUE)$ is the most recent available earnings shock for company i prior to quarter t . In Panel A (B) we perform a regression using firm-quarter observations where $Revision > 0 (< 0)$. All earnings-related variables are winzorized at the 0.5 and 99.5 percentile. Details on variable definitions and methodology can be found in Section 3 in the Appendix. We cluster standard errors at the firm level. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively. Coefficients and standard errors are multiplied by 100.

Variable	A:Revision>0			B:Revision<0		
	1: SUE	2: $ret_{-30,-4}$	3: $ret_{-1,65}$	4: SUE	5: $ret_{-30,-4}$	6: $ret_{-1,65}$
Rev	59.00** [29.56]	232.30*** [78.76]	46.97 [118.56]	-137.26*** [11.37]	-253.89*** [44.26]	88.95 [71.54]
$Rev. \times High Cov.$	113.62*** [36.43]	277.62*** [119.26]	506.74*** [191.71]	4.38 [15.93]	15.34 [69.09]	-128.96 [139.17]
$High Cov.$	-0.18*** [0.06]	-0.14 [0.36]	-0.68 [0.54]	-0.10* [0.05]	-1.57*** [0.38]	0.07 [0.63]
$Dispersion$	-0.17** [0.07]	-0.61* [0.38]	-0.40 [0.48]	-0.15*** [0.03]	0.24 [0.23]	-0.04 [0.38]
$LogMV$	0.08*** [0.02]	-0.43** [0.11]	-0.06 [0.20]	0.11*** [0.02]	1.12*** [0.11]	0.07 [0.18]
$EP(+)$	-36.64*** [3.84]	-24.73* [13.95]	82.21*** [25.80]	-47.50*** [3.57]	79.62*** [15.70]	84.82*** [24.66]
ED	1.21*** [0.17]	0.30 [0.54]	-1.99 [1.52]	0.57*** [0.14]	-2.08*** [0.49]	-1.99 [1.32]
SUE			59.22** [25.44]			36.44 [24.08]
$Lag(SUE)$	42.87*** [3.54]			40.16*** [2.43]		
N	5,184	6,379	5,675	7,612	10,941	8,769
R^2	0.477	0.101	0.080	0.571	0.136	0.074
Year-Quarter F.E.	YES	YES	YES	YES	YES	YES

Appendix

1. Expected Profits in GS

A. Bias and Expected Profits

The overall profitability of bookmakers depends on how the objective probability of the higher-ranked player winning compares to the relative volume that backs this player. To illustrate, assume that the objective probability that the higher-ranked player wins a match is p_{HR} , and correspondingly for the low ranked player is $p_{LR} = 1 - p_{HR}$. The bookmaker starts from this probability, adjusts to reflect a vigorish, v , and a bias b , arriving at p_{HR}^* :

$$p_{HR}^* = (p_{HR} + b) \times (1 + v) \quad (1)$$

Correspondingly, the adjusted probability for the low ranked player is:

$$p_{LR}^* = ((1 - p_{HR}) - b) \times (1 + v) \quad (2)$$

The odds offered for the high- and lower-ranked player are thus $d_{HR} = \frac{1}{p_{HR}^*}$ and $d_{LR} = \frac{1}{p_{LR}^*}$, respectively. Furthermore, Vol_{HR} and Vol_{LR} are the volumes that back the high and lower-ranked players respectively, and $TotVol = Vol_{HR} + Vol_{LR}$ is the total volume staked.

The profit for the bookmaker for each match is therefore:

$$\Pi(\$) = TotVol - Vol_{HR} \times d_{HR} \times D_{HR} - (Vol_{LR}) \times d_{LR} \times D_{LR} \quad (3)$$

Dividing through by $TotVol$ leads to Equation 3 in the paper, where $RVol_{HR} = \frac{Vol_{HR}}{TotVol}$ and $RVol_{LR} = \frac{Vol_{LR}}{TotVol}$

$$\Pi(\%) = 1 - RVol_{HR} \times d_{HR} \times D_{HR} - RVol_{LR} \times d_{LR} \times D_{LR} \quad (4)$$

To illustrate the conditions that a negative bias ($b < 0$) is optimal it is useful to express the above equation in expectation form, replacing the ex-post indicators D_{HR} and D_{LR} with the corresponding ex-ante probabilities:

$$E(\Pi) = 1 - p_{HR} \times RVol_{HR} \times d_{HR} - p_{LR} \times RVol_{LR} \times d_{LR} \quad (5)$$

Expressing the odds as a function of p_{HR} , b and v :

$$E(R\Pi) = 1 - \frac{p_{HR} \times RVol_{HR}}{(p_{HR} + b) \times (1 + v)} - \frac{(1 - p_{HR}) \times (1 - RVol_{HR})}{((1 - p_{HR}) - b) \times (1 + v)} \quad (6)$$

We conduct a simple calibration to show how $E(\Pi)$ varies with b and $RVol_{HR}$, setting $p_{HR} = 0.79$ and $v = 0.05$. The results are shown in Figure 1 in this section of the Appendix. From the top panel of Figure 1, starting with the black solid line where $RVol_{HR} = p_{HR}$, we observe that $E(\Pi)$ is inverse U-shaped, with a unique maximum at $b = 0$. When $RVol_{HR} < p_{HR}$ $E(\Pi)$ increases as b decreases, and this relationship is steeper as $RVol_{HR}$ becomes smaller. Conversely, when $RVol_{HR} > p_{HR}$ $E(\Pi)$ decreases as b becomes smaller, and this relationship is steeper as $RVol_{HR}$ becomes larger.

In the bottom Panel of Figure 1 we plot the corresponding standard deviation of these profits, $\sigma(\Pi)$. If $RVol_{HR} = p_{HR}$, and $b = 0$ the bookmaker faces no risk. When $RVol_{HR} < p_{HR}$ a negative bias lowers $\sigma(\Pi)$, whereas the opposite is true when $RVol_{HR} > p_{HR}$.

Thus, whether a negative bias is suboptimal, crucially depends on how $RVol_{HR}$ compares with p_{HR} for GS. If, on average, $RVol_{HR} > p_{HR}$, then the bookmaker is better off by increasing the bias, receiving higher expected profits with lower standard deviation.

[Insert Figures 2 and 3 here]

A. Hypothetical Odds

The volume information from the betting exchange suggests that the negative bias in GS is suboptimal (since on average $RVol_{HR} > \sum \frac{D_{HR}}{N}$). In Section 3.4 of the paper we illustrate this by calculating the hypothetical profit of bookmakers in a scenario where they increase their probabilities for the higher-ranked player for each match by an amount equal to the average bias.

To calculate the numbers in Column 4 of Table 6 in the paper, we increase (decrease) π_{HR} (π_{LR}) in each GS match by an amount equal to the average *bias* (3.41% in Panel A and 2.71% in Panel B), derive the new hypothetical odds as per Equations 1 and 2 from Section A above, $HO_{HR,i}$ and $HO_{LR,i}$, and calculate the hypothetical profits, $H\Pi$.

Before we use the hypothetical odds to determine profits we need to consider how changing the odds in this manner would influence $RVol$ in this counterfactual scenario. To estimate the

hypothetical $RVol_{HR}$ we use the model below, estimated using the betting exchange data for GS matches:

$$RVol_{HR,i} = \alpha + \beta \times O_{HR,i} + \epsilon_i$$

We find that α is equal to 1.36 and β is equal to -0.36, both highly statistically significant. This shows that when the odds on the higher-ranked player increase (i.e., when he becomes less of a favourite), $RVol_{HR}$ decreases. This result reflects the preference of punters to bet on the favourite (e.g., Levitt, 2004). We obtain the hypothetical $RVol_{HR}$ for each GS match by calculating $\hat{\alpha} + \hat{\beta} \times HO_{HR,i}$. We use hypothetical volumes and odds, and the actual game outcomes (as indicated by D_{HR}) to calculate profits in this counterfactual scenario using equation 4 from section A.

Figure 1: Expected Profits, and Bias

This figure in the top Panel depicts Expected Profits (Y-axis) for different levels of bias (X-axis), following the procedure explained in Section 1 of the Appendix. The figure in the bottom Panel depicts the corresponding standard deviations.

2. Descriptive Statistics for Betting Exchange and Women Data

Table 1: Descriptive Statistics - Betting Exchange

This table shows descriptive statistics for the main variables used in the analysis with the Betting Exchange data. The variables are defined as in Table 1. *Totvol* is the total volume that backs the higher-ranked player (\$000's), and *Rvol* is the proportion of the total volume bet in the match on both players that backs the higher-ranked player. π_{HR} is the subjective probability that the higher-ranked player wins the match, as derived from the betting exchange odds. We drop non-completed matches, matches with missing information or vig greater than 0.05, and matches which do not satisfy criterion (ii) in Section 2.1 of the paper. The sample consists of 2,751 MS matches and 2,142 GS matches from 2009-2014.

Variable	Mean	σ	Min	Q1	Median	Q3	Max
Panel A: GS							
<i>HR_{Odds}</i>	1.39	0.49	1.01	1.09	1.26	1.52	8.60
<i>LR_{Odds}</i>	10.06	14.25	1.12	2.84	4.70	10.50	100.00
<i>Vig</i>	0.01	0.01	0.00	0.00	0.01	0.01	0.05
<i>HR_{Rank}</i>	27	29	1	7	18	36	285
<i>LR_{Rank}</i>	96	90	2	45	78	118	1,120
<i>TotVol</i>	240	450	4	44	86	210	6,200
<i>RVol</i>	0.86	0.20	0.02	0.85	0.94	0.97	1.00
$\hat{p}$	0.78	0.11	0.58	0.70	0.79	0.87	0.99
π_{HR}	0.76	0.17	0.12	0.65	0.79	0.91	0.99
Panel B: MS							
<i>HR_{Odds}</i>	1.52	0.54	1.01	1.21	1.40	1.66	9.00
<i>LR_{Odds}</i>	5.69	7.37	1.12	2.46	3.40	5.60	90.00
<i>Vig</i>	0.01	0.01	0.00	0.00	0.01	0.01	0.05
<i>HR_{Rank}</i>	21	23	1	5	14	29	414
<i>LR_{Rank}</i>	64	74	2	27	47	77	1,171
<i>TotVol</i>	170	220	2.3	39	84	200	2,200
<i>RVol</i>	0.82	0.23	0.02	0.81	0.91	0.96	1.00
$\hat{p}$	0.70	0.13	0.50	0.59	0.69	0.80	0.99
π_{HR}	0.70	0.16	0.11	0.60	0.71	0.82	0.99

Table 2: Descriptive Statistics - Women's Bookmaker Data

This table shows descriptive statistics for the main variables used in the analysis of the women bookmaker data. The variables and the sample construction criteria are defined as in Table 2 in the paper. The sample consists of 2,527 MS matches and 3,624 GS matches from 2007 to 2014.

Variable	Mean	σ	Min	Q1	Median	Q3	Max
Panel 1: GS							
HR_{Odds}	1.38	0.46	1.01	1.12	1.27	1.51	9.40
LR_{Odds}	5.17	4.05	1.06	2.55	3.71	6.34	35.40
Vig	0.06	0.01	0.00	0.05	0.06	0.06	0.08
$Rank_H$	28	27	1	8	19	38	252
$Rank_L$	94	84	2	49	80	113	1,208
$\hat{p}$	0.75	0.11	0.55	0.65	0.74	0.83	0.98
π_{HR}	0.73	0.15	0.10	0.63	0.74	0.85	0.97
Prize Money (\$)	2,525,000						
Panel B: MS							
HR_{Odds}	1.46	0.42	1.01	1.19	1.36	1.59	5.47
LR_{Odds}	3.91	2.52	1.14	2.31	3.13	4.51	22.00
Vig	0.06	0.01	0.02	0.05	0.06	0.06	0.09
$Rank_H$	24	25	1	7	16	35	248
$Rank_L$	73	83	2	32	54	86	1,132
$\hat{p}$	0.70	0.11	0.53	0.60	0.69	0.79	0.98
π_{HR}	0.69	0.14	0.17	0.59	0.70	0.79	0.98
Prize Money (\$)	665,000						

Table 3: Descriptive Statistics - Women's Betting Exchange Data

This table shows descriptive statistics for the main variables used in the analysis of the women bookmaker data. The variables and the sample construction criteria are defined as in Table 5 in the paper. The sample consists of 1,425 MS matches and 2,259 GS matches from 2009 to 2014.

Variable	Mean	σ	Min	Q1	Median	Q3	Max
Panel A: GS							
HR_{Odds}	1.44	0.54	1.01	1.15	1.33	1.57	11.50
LR_{Odds}	6.88	8.62	1.09	2.68	3.90	7.20	100
Vig	0.01	0.01	0.00	0.00	0.01	0.01	0.05
$Rank_H$	27	27	1	8	19	37	252
$Rank_L$	92	82	2	48	79	113	1,208
$TotVol$	120	250	0.36	19.09	42.96	110	2,900
$RVol$	0.85	0.21	0.01	0.84	0.93	0.97	1.00
$\hat{p}$	0.74	0.11	0.55	0.65	0.73	0.82	0.97
π_{HR}	0.73	0.16	0.09	0.63	0.75	0.86	0.99
Panel B: MS							
HR_{Odds}	1.52	0.57	1.02	1.24	1.41	1.67	6.20
LR_{Odds}	4.56	4.18	1.17	2.44	3.30	4.80	46.00
Vig	0.01	0.01	0.00	0.01	0.01	0.01	0.05
$Rank_H$	24	24	1	7	16	35	217
$Rank_L$	71	77	2	32	53	82	1,004
$TotVol$	55.52	82.48	0.41	12.76	25.51	58.56	690
$RVol$	0.82	0.23	0.02	0.79	0.91	0.96	1.00
$\hat{p}$	0.71	0.11	0.54	0.61	0.70	0.79	0.97
π_{HR}	0.69	0.15	0.16	0.59	0.70	0.80	0.98

3. Laboratory Experiment

Instructions and Procedures

In this section we include the full-instructions seen by the students, shown below. The statements in brackets in italics are clarifying additions, not seen by the students.

Welcome to the experiment!

This experiment is about predicting the outcome of professional tennis matches. The matches that you will be predicting in this session are matches that will be played at the ATP 1000 Masters event that is happening right now in Rome. The winner is decided in a best-of-three-set match. Before moving to the predictions please answer the questions below. For each of the quiz questions in Section 3 that you answer correctly you will receive an additional 50 pence (£0.50) at the end of the session.

[For the second session, the second and third sentence where replaced with: The matches that you will be predicting in this session are matches that will be played at the French Open that is happening right now in Paris. The winner is decided in a best-of-five-set match.]

[Students then provided the following information: Name, student ID, and Degree title.]

[Students then were asked to circle the answer that most applied to them in relation to the statement “I follow tennis closely”, from “strongly agree” to “strongly disagree” (5 levels)]

[Students then answered 4 tennis related trivia questions, like “How many Grand Slam titles has Roger Federer won?”]

[Students then proceeded to the next part of the instructions]

About probabilities

For each match we will tell you the players involved and their corresponding rankings. Your task is to think how likely each player is to win the match, and express this likelihood as a probability between 0% and 100%. Of course, we do not know yet the outcome of the match, so there is *uncertainty*.

To provide you with some background about probabilities, suppose we have a normal six-sided die, and you want to calculate the probability of rolling the die and getting a three (3). To calculate this probability you need to count how many outcomes lead to a “win”, and divide that by how many outcomes are possible. In this case, a win occurs with only one outcome,

which is to roll the die and get a three (3). There are six (6) possible outcomes (numbers 1 to 6) and, therefore, the probability of rolling a three (3) in one roll is $1/6$ (or roughly 17%). If instead you want to calculate the probability of rolling the die and getting an even number (2, 4, 6), then this probability is higher because you win with three outcomes instead of one, therefore the probability of winning in one roll is $3/6$ (or 50%).

A similar logic can be applied when thinking about probabilities of players winning a tennis match. For example, suppose that we have a match between players A and B. To calculate the probability of each player winning this match you need to estimate how many matches each player would win, if they hypothetically played 100 matches in total. For example, suppose that you think that B is a better player than A, so B would win 70 matches out of 100 (and correspondingly A would win 30 matches out of 100). This means that your estimated probability of B winning the match is $70/100$ (or 70%) and of A $30/100$ (or 30%).

Your task is to think along these lines for each match and provide your best estimate of the probability of each player winning the match. Note that the probabilities you express for the two players in each match must equal to 100% when added together! (in the above example $70\%+30\%=100\%$)

Please note that in our analysis we will use your predictions in the experiment **anonymously**.

[Students first read the above silently, and then it is read aloud by the experiment asking for clarifying questions.]

[Students then proceeded to the next part of the instructions]

How you will get paid for your predictions.

Apart from the money that you will be paid for showing up and for answering correctly the quiz questions, you will also be paid a **performance-based payment**. This payment will be based on the accuracy of your predictions using a procedure that is commonly used in economics experiments, known as the *quadratic scoring rule*.

[Students received £0.5 for each quiz question answered correctly]

To illustrate how this works, let's return to our hypothetical example between players A and B where your best guess of the probability that B wins is 70% (or 0.7). In this case, your payment would be calculated using the formula below:

$$\text{Payment} = £13 - £13 \times (D - p)^2$$

The symbol p in the equation stands for your estimate of the probability that player B wins, in this case 0.7. D is an indicator, which takes the value of 1 if B has won the match, or 0 if A has won the match. Of course you do not know D when you make your choices, this will be revealed after the match is played and we know the winner.

Assume, for example, that the match is now finished and player B has won. This means your prediction in this case was good in the sense that you assigned a higher probability to B winning. Your payment in this case would be:

$$\text{Payment} = £13 - £13 \times (1 - 0.7)^2 = £13 - £13 \times (0.3)^2 = £13 - £1.17 = £11.83$$

[This equation is written on the board and explained by the experimenter, when he read the instructions aloud]

If you had assigned a higher probability to B winning, say 0.8, your payment would be even higher and equal to:

$$£13 - £13 \times (1 - 0.8)^2 = £12.48.$$

However, you need to be careful, because the higher the probability you give for B winning, the lower your payment will be if A wins.

For example, when your probability of B winning is 0.7, your payment if A wins is:

$$£13 - £13 \times (0 - 0.7)^2 = £13 - £13 \times (-0.7)^2 = £13 - £6.37 = £6.63.$$

When your probability of B winning is 0.8, your payment if A wins is even lower:

$$£13 - £13 \times (0 - 0.8)^2 = £13 - £13 \times (-0.8)^2 = £13 - £8.32 = £4.68.$$

In general, with this payoff scheme, the largest possible payment is £13, which you will receive if you assign a 100% chance to B winning, and B wins. However, if you choose to assign 100% probability to B but A wins, your payment will be the lowest possible, equal to £0.

Since your prediction is made before the match is played and therefore you do not know what will actually happen, **the best thing you can do to maximize the expected payoff**

is to simply state what is your best guess for the probability that B wins the match. Any other prediction will *decrease* the amount you can expect to earn.

You will assign probabilities to players for several matches, and at the end we **will randomly choose one of these matches** to calculate your performance payment. This random choice will be made in your presence, at the end of this session. Of course we won't know the outcome of the match at the end of the session, so we will calculate your payment in a few days after the winner is announced. You will collect your performance-payment for this session after the second session of this experiment is completed. The details on how to collect your performance-based payment will be given to you at the end of the second session.

The second session will be held on Sunday 28th May at 10:00 a.m.

[Students first read the above silently, and then it is read aloud by the experimenter asking for clarifying questions.]

[Students then proceeded to provide their choices for the different matches]

[When all the students finished, each student approached the experimenter individually to receive the show-up fee and to select the match that would determine their performance-related payoff. The performance-related payoff for the first session was paid at the end of the second session, and the performance related payment for the second session was paid a few days after the end of that session when the games were completed.]

[In our sessions we also asked students to make predictions for the women's matches in the corresponding tournaments, and found results that are in line with those in the paper for the betting exchange (i.e., $\Delta_{\text{bias}} > 0$). To conserve space we do not report these results here, but they are available from the authors on request. The performance-based payment for each student in each session was calculated on the basis of one randomly selected match].

Table 4: Descriptive Statistics - Laboratory Experiment

This table shows descriptive statistics for the sample generated in the laboratory experiment. 17 students participated, who made choices for 33 MS matches and 64 GS matches. The GS matches were all first-round matches. The MS matches contained 7 qualifying matches, 17 first round matches and 9 second round matches. For each of the second round matches, we asked students to make assessments for each of the 4 possible combinations that could have resulted from the first round matches, which were unknown at the time that the MS session was held. The qualifying matches and the second round matches were included to increase the MS sample.

Variable	Mean	σ	Min	Q1	Median	Q3	Max
Panel A: GS							
$Rank_H$	41	31	1	17	19	33	129
$Rank_L$	125	82	37	70	98	153	463
$\hat{p}$	0.73	0.10	0.59	0.63	0.75	0.79	0.95
π_{HR}	0.70	0.17	0.20	0.60	0.70	0.80	1.00
Panel B: MS							
$Rank_H$	31	21	1	14	29	50	76
$Rank_L$	83	86	21	32	52	87	403
$\hat{p}$	0.67	0.10	0.56	0.58	0.63	0.72	0.93
π_{HR}	0.67	0.17	0.10	0.55	0.65	0.80	1.00

4. Process Variance Neglect in Financial Markets

This analysis uses earnings data from the IBES detail files that are adjusted for stock splits. We start the analysis in 1994 because prior to this year forecast release dates are inaccurate (Hilary and Hsu, 2013). From the initial file we drop observations with missing analyst codes and retain only one-quarter ahead earnings per share forecasts. If an analyst has made more than one forecast on the same day for the same company and quarter, we retain the forecast with the smallest absolute forecast error, defined as $|\frac{Forecast - Actual}{Actual}|$. To obtain the revision in analyst j 's forecast we calculate the difference between the last forecast made by analyst j for company i at quarter t with the penultimate forecast made by the same analyst for the same firm and quarter. In our sample we include revisions issued in the window between -30 and -4 relative to the earnings announcement date. It is commonly known that forecasts issued closer to the quarter end are more informative (Clement and Tse, 2005). Since we require a signal that is relevant when predicting earnings to test for process variance neglect, we focus on forecasts issued closer to the quarter end.

To this dataset we merge data on stock prices and shares outstanding from CRSP at the end of the previous quarter $t-1$. To be included in our sample a firm must be followed by at least 5 analysts in quarter t and have a stock price greater than 5 dollars at the end of quarter $t-1$. These filters are commonly used in the analyst literature to reduce the effect of outliers due to particularly low analyst coverage and/or small and illiquid firms (Chen and Jiang, 2006; Hilary and Hsu, 2013; Malmendier and Shanthikumar, 2014).

To calculate analyst coverage we follow a procedure similar to Hong and Stein (2000), who note that analyst coverage is strongly and positively related to firm size. Thus, if one sorts only on "raw" analyst coverage, there is a danger that any findings are reflecting variations on firm size. To eliminate this possibility we regress the natural logarithm of analyst coverage for firm i at quarter t on the natural logarithm of its market value at the end of quarter $t-1$, and use the residual from this regression as our measure of analyst coverage. Although this approach uses forward looking information, i.e., we

run one regression using all our data from 1994-2016 , the results are unlikely to change materially if we use backward looking information with rolling regressions because, as noted by Hong and Stein (2000), the relationship between analyst coverage and firm size is quite stable over time. In each quarter we then sort firms into deciles according to residual analyst coverage, and if a firm belongs in the top 30% of this distribution the dummy *High Coverage* is set to 1, otherwise it is 0.³⁸

The cumulative abnormal returns for firm i are calculated from the day -1 to +65, relative to the earnings announcement date. To obtain abnormal returns we subtract from the returns of stock i at day t the corresponding returns of the market portfolio on the same day, measured using the value-weighted average of the returns of all stocks included in the CRSP database. We allow a gap of 2 days between the calculation of *Signal* (ending in day -4) and the cumulation of returns (starting in day -1) to ensure that the signal is known to investors. Table 1 in this section defines all the variables used in the regressions in Table 11 of the main paper.

References

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2. M. Clement and S. Tse, "Financial analyst characteristics and herding behavior in forecasting," *Journal of Finance* vol. 60, no. 1, 307-341.
3. G. Hilary G and C. Hsu, "Analyst forecast consistency," *Journal of Finance*, vol. 68, no. 1, 271-297, 2013.
4. H. Hong and J. C. Stein, "Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies," *Journal of Finance*, vol. 55, No. 1, 265-295,

³⁸A very small number of firms have quarter-end months other than those most commonly used (March, June, September and December), which makes rankings in such "non-standard" quarter-end months inaccurate. To avoid this problem we shift the quarter-ends for these firms to the closest "standard" quarter-end month, i.e., if a firm has a quarter-end in April 2010, then, for the purpose of sorting according to analyst coverage, its quarter for that year is shifted to March.

2000.

5. U. Malmendier and D. Shanthikumar, “Do security analysts speak in two tongues?”
Review of Financial Studies, vol. 27, no. 5, 1287-1322, 2014.

Table 5: Variable Definitions

This table provides definitions for all the variables used in the regressions in Table 11 in the paper.

Variable	Description
<i>SUE</i>	Earnings of firm i during quarter t minus the earnings of the same firm for the same quarter in the previous year, scaled by stock price measured at the end of quarter $t-1$.
<i>CAR</i>	The cumulative, market-adjusted return for firm i in the period $[-30,-4]$ and $[-1$ to $+65]$ relative to the earnings announcement date.
<i>Rev</i>	The median analyst forecast revision for firm i during quarter t using all revisions issued in the period -30 to -4 relative to the earnings announcement date, scaled by stock price at the end of quarter $t-1$. For the models that includes the observations where <i>Signal</i> is less than 0 (A1 and B4 in Table 10) we use its absolute value in the regression.
<i>High Cov</i>	A dummy variable that equals 1 if firm i during quarter t is at the top 30% in analyst coverage that is orthogonal to firm size.
<i>Dispersion</i>	The coefficient of variation using all earnings forecasts for company i and quarter t issued in the period -30 to -4 relative to the earnings announcement data. We first calculate the standard deviation of these earnings forecasts, and scale by the absolute value of the mean forecast.
<i>LogMV</i>	The natural logarithm of firm value for company i at the end of the previous quarter, measured as price x shares outstanding.
<i>EP(+)</i>	The earnings-to-price ratio for firm i , using earnings and stock price in the previous quarter. If the earnings of the company were negative <i>EP(+)</i> is set to 0.
<i>ED</i>	A dummy variable that equals 1 if earnings for firm i in the previous quarter were negative, and 0 otherwise.
<i>Lag(SUE)</i>	The most recent available earnings shock for company i in our sample prior to quarter t , weighted by W . To calculate W we count the number of days between the end of quarter t and the previous quarter for which a SUE observation is available for company i in our sample (<i>Days</i>). W is equal to $90/Days$, so that SUE's that are further back in time receive a lower weight.