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**THE EFFECT OF DEBONDING IN FIBRE-REINFORCED COMPOSITES
ON ULTRASONIC BACKSCATTERING**

by

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A thesis submitted to the
Faculty of Science
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"Keep banging the rocks together guys..."

Ford Prefect

(from The Restaurant At The End Of The Universe).

ABSTRACT

The work presented in this thesis concerns the problem of detecting and characterising the effect of fibre-matrix debonds in a fibre-reinforced composite, on ultrasonic backscattering. Theoretical and experimental investigations were conducted into this problem.

Three mathematical models were examined. The first assumed that the debond was a thin crack with non-contacting faces surrounding the fibre. The second modelled the debond by allowing tangential slip between the matrix and fibre defined by an effective shear modulus, K . For the third model, the debond was approximated by a thin visco-elastic layer separating the matrix and fibre. The results of the modelling suggested that for an incident longitudinal wave, the first model acts as an air-filled void with a sharp resonance present in the low ka region. The second and third models both show the backscattering to be attenuated.

Experimental investigations were carried out on scale models of a single fibre embedded in an araldite matrix. Steel or copper wires were used for the reinforcing fibre. The agreement between theory and experiment for a well-bonded wire was excellent. The effect on the longitudinal backscattered wave of the wire immediately after debonding was to attenuate heavily the resonances in the backscatter form function. However, after approximately an hour the scatter is seen to relax, closely resembling that from a well-bonded wire.

In view of the only partial ability of longitudinal incident

waves to detect debonds, shear wave (SH) incidence was investigated. It was shown that shear waves were far more sensitive to the presence of fibre-matrix debonds.

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CHAPTER 1

INTRODUCTION.

1.1 Fibre Reinforced Composites.

A composite material can be defined as a multi-phase material in which the properties of the continuous phase are enhanced by the inclusion of sheet-like, fibrous or particulate fillers. The continuous phase, which is known as the matrix, may be a polymer (i.e epoxy resin), metal, glass or ceramic. The reinforcement material can be equally as varied, some examples are glass or carbon fibres, thin sheets of wood, as in plywood, or Silicon Carbide (SiC) particles in modern metal matrix composites. Despite the wide variety of materials which are composites, the work presented here is concerned primarily with fibre reinforced composites, such as Glass Reinforced Plastics (GRP) where long stiff and strong glass fibres are used to stiffen and strengthen weak and compliant matrices. The matrices in turn provide protection for the fibres, and the pathway by which stresses are applied to the fibres. It should be noted that a composite can be designed to optimise almost any required property. The final properties of a composite structure depend on the characteristics and interaction of the two phases within the composite.

The reason for the increased use of these materials is that they can be designed to meet a specific need and often out-perform traditional or conventional materials. An obvious example of this is the increased use of advanced composites in the aerospace industry, where the combination of high strength with low weight is

particularly attractive.

A structure constructed of a long fibre composite is very often in the form of a laminate, each layer consisting of an array of fibres aligned along a particular direction. Fig 1.1 shows a typical fibre reinforced composite laminate, it is termed a " $0^{\circ}/90^{\circ}/0^{\circ}$ lay up". Owing to the various directions of laminars within a laminate e.g. $0^{\circ}/45^{\circ}/90^{\circ}/45^{\circ}/0^{\circ}$, the physical properties such as stiffness vary, depending upon the axis along which they are measured. This feature can be exploited, for example the German firm MBB uses a composite hub to attach the rotor blades of one of their latest helicopters to the output shaft of the engine. In a conventional approach this hub is a complicated mechanism, which allows the blade to pitch and roll corresponding to its position relative to the aircraft. By using a composite material for the hub all the movement can be accommodated by the flexing of the composite laminated structure, the relevant motions having been designed into the structure.

It can be seen that composites are generally used in areas where a high performance is expected of the material, e.g. highly stressed load bearing structures in aircraft. The consequences of failure, however, can be catastrophic, and for this reason an understanding of the processes leading to the degradation and ultimately the failure of such materials is essential.

1.2 Damage In Composite Materials.

A general composite laminate will have reinforced layers parallel, perpendicular and at some other angle(s) to the direction

Figure 1.1: Lay up of a $(0^\circ/90^\circ/0^\circ)$ composite laminate.

Figure 1.2: Diagram of the types of damage that can occur in a composite laminate.

Fig. 1.1

Fig. 1.2

of loading (i.e. 45° in a $0^\circ/45^\circ/90^\circ/45^\circ/0^\circ$ laminate). The stresses in each ply can be calculated using laminated plate theory [Jones 1975]. This can be modified to include thermal stresses due to moisture absorption.

Unidirectional, continuous fibre reinforced composites are basically linearly elastic to failure. However, in general, in a composite laminate containing off-axis plies, various damage mechanisms occur under load which cause redistribution of the load, and consequently the stress-strain response is non-linear. Similar damage occurs under cyclic loading.

Damage to a composite material can be divided into three main categories. These are: matrix cracking, delamination, and fibre breakage (see Fig 1.2). In a composite laminate such as that shown in figure 1.2, under tension in the directions P, the first significant damage usually observed is **matrix cracking** of the transverse ply. In fact it is debonding of fibres at an angle to the loading direction which is the first damage [Bailey and Parvizi, 1981]. These debonds, under further loading, coalesce to form the transverse ply cracks. As the loading increases, the density of cracks also increases, appearing to stabilize at a unique value, characteristic of a particular composite.

Delamination occurs between layers of a laminate due to the interlaminar stresses which exist near a free edge [Composite Materials Technology, 1990]. These out of plane stresses arise because the transverse and shear stresses in each ply of the laminate must decay to zero at the free edge. This gives rise to the interlaminar stresses which can be tensile in nature.

Delaminations usually occur at the free edge and propagate inwards.

Fibre breakage under static loading prior to final failure is related to the statistical distribution of flaws along the length of the fibre, but a stress concentration due to a matrix crack in an adjacent ply can also cause fibre breakage. This has been seen under cyclic loading. Debonding of the broken fibres is then likely, due to the high shear stresses at the interface between the fibre and the matrix near to the break.

It is important to be able to detect damage to a composite structure at an early stage, especially in critical situations like the components of an air frame. As reported by Bailey and Parvizi [1981] debonding of fibres and matrix ^{cracking} is the earliest indication of damage to a composite laminate, and as such it is important to be able non-destructively to test structures for this type of weakness. It is the detection of the debonding of fibres with which the present thesis is primarily concerned.

1.3 Non Destructive Testing of Composites.

As outlined above the increased use of composite materials in applications where the failure of a component can be catastrophic requires the ability to be able to detect the onset of damage in a structure. It has been shown that a composite laminate has many potential forms of damage which can finally lead to the complete failure of a component, but detecting the onset of damage in a composite is not straight forward due to the very nature of the material.

In the testing of composites there are four main methods that

have been used, these are acoustic emission, wave propagation methods, X-Ray radiography and thermography. A brief review of each of these techniques will be given below.

1.3.1 Acoustic emission.

This particular type of testing differs fundamentally to the others that will be discussed since it is not strictly a non-destructive method of testing. Acoustic emission relies on the detection of damage events (i.e. fibre breakage) from the acoustic energy produced from each event. Typically a sample under test will be loaded, and the resulting acoustical signals monitored in terms of the amplitude and the number of events recorded at a receiving transducer.

As a method of testing composites, acoustic emission is particularly popular: Hamstad [1986] in a review article reports that over 500 papers describing acoustic emission applied to composites have been published since 1970. Typical types of damage that have been reported to have been detected are delaminations, [Selden and Gustafson 1985, K.Ono 1988] fibre breakage [Sato et al 1986, Kumosa et al 1987], matrix cracking [Favre and Laizet 1989, Sato et al 1988].

Acoustic emission suffers a major drawback as a non-destructive testing or evaluation tool in that in carrying out the test, damage will be introduced into the structure and it is not apparent what effect this damage may eventually have on the integrity of the composite. In addition there is the problem of attributing the signals recorded to particular damage modes. Hamstad [1986] reported on this problem that although acoustic

emission has many potential applications in the testing of composites there appeared, in the literature, to be several fundamental errors in the application of this technique. Firstly there was a lack of understanding of how wave propagation affects the acoustic emission signals; a particularly important point in view of the anisotropy and large number of scatterers in a fibre-reinforced composite material. Secondly insufficient attention was paid to the effect of extraneous sources of noise, for example the grips used to hold a specimen under test. Thirdly there tended to be some very questionable guessing as to what were the actual sources of the acoustic emission signals. These points are of particular concern if this technique is to be considered as a possible method of quantitative non-destructive evaluation of a composite structure. Several authors have published work attempting to quantify the results of acoustic emission [Guild et al 1985, J.Yuan et al 1986, Choi and Takahashi 1990], but to quote Siegmann and Kander [1991]:

"unambiguous assignment of acoustic emission signals to specific damage events is still impossible unless the event responsible for the acoustic emission signal is directly and simultaneously observed."

In general acoustic emission is a useful tool for the study of the propagation of damage in composites [Eckles and Awerbuch 1988, Caldwell et al 1987], and can still be applied in limited cases to damaged specimens where the acoustic emission signal is assumed to be due to friction between damaged areas [Sato et al, 1988]. With regards to the particular problem of detection of fibre debonding,

there are conflicting reports in the literature. Yuan et al [1986] report that in PVC reinforced with short glass fibres, if the fibres are poorly bonded the majority of the acoustic emission observed was due to fibre-matrix debonding. However, Netravali et al [1989] report that for push-out tests carried out on single glass fibres in an epoxy matrix there was no observed signal due to debonding. These two cases serve to highlight the difficulty of making a quantitative evaluation of damage using acoustic emission.

1.3.2 Wave propagation methods.

Wave propagation methods include all those methods where the ultrasonic or acoustic wave energy is introduced externally, and the effect of defects in the material under test is detected in the received wave after it has propagated some distance through the structure. Included in this category are pulse-echo techniques using a single transducer as the source and receiver, and transmission techniques where separate transmitting and receiving transducers are used.

With pulse-echo techniques the transducer is excited by a short pulse which then propagates through the composite, reflections from the front and back faces of the structure, as well as from any internal structures (e.g. flaws) are then detected by the transducer. The data recorded in this manner is commonly displayed in one of three ways: an A-scan, a B-scan or a C-scan. An A-scan displays the amplitude of the received signal against time. A B-scan is basically a series of A-scans taken at subsequent positions over the sample under investigation as the transducer is swept over the surface. The amplitude of the echoes are often

represented in terms of a grey scale and the time^{delay} of the echoes is proportional to the depth of the structure within the sample. The image produced using a B-scan is a cross section parallel to the incident beam. The C-scan is similar to the B-scan except that the incident beam is swept in 2-dimensions over the surface of the specimen under test and the amplitude information at a particular time (effectively the depth), is displayed in terms of a grey scale. Thus the image is a cross-section perpendicular to the incident beam. Typical defects that are readily detected by this method are delaminations [Lloyd 1989, Lustiger et al 1990, Cousins and Markham 1977] and subsurface flaws such as transverse cracks [Raju 1986, Schuster and Steiner 1990].

Pulse-echo techniques have good spatial resolution and are used in general to detect flaws and damage sites over relatively small areas of a composite. To test a large area can be difficult since complicated scanning rigs may be needed [see Schuster and Steiner 1990]. There are several methods which can be used to test the global integrity of composite structures. One of the simplest methods is the tap test [Cawley et al 1985, Cawley and Adams 1987] where the response of the composite under test is compared with the response for a known "good" area. Velocity measurements have also been used to characterise composites [Gazanhes et al 1986]; later work has used velocity measurements to obtain the elastic constants of a composite laminar [Sachse et al 1990, Veidtland and Sayers 1990]. In addition, velocity and attenuation measurements have been made to determine fibre volume fraction [Kim and Park 1987]

allowing resin rich areas to be detected. This approach has also been applied to the detection of porosity in composite materials [Martin 1977, Hsu and Jeong 1989]. Velocity measurements are particularly important with regards to making an accurate assessment of material thickness, Markham [1981] has made a study of the errors introduced when using thickness meters on composite materials. In recent years the possibility of using leaky Lamb waves to detect matrix cracks has received some attention [Dayal and Kinra, 1987] as well as their application to the detection of delaminations [Datta et al 1990, Bar-Cohen and Mal 1990, Chimenti and Martin 1991].

Occasionally measurements made using a separate transmitter and receiver are termed acousto-ultrasonics [Hsu and Margetan, 1992]. This method has been used to determine the effects of ageing as well as typical strengths of composite components [Phani and Bose 1986, Srivastava 1986]. The technique generally involves detecting damaged or low strength regions of material in terms of a Stress Wave Factor. This is basically the number of oscillations in the received signal that are above a pre-set threshold limit [Williams and Lampert 1980]. The stress wave factor effectively measures the efficiency of the material to propagate the wave. There appears to be little consideration on the type of wave generated by the transmitter / ^{or of the mechanisms which cause} changes occur in the received waveform [Cawley and Adams, 1989].

In general pulse-echo and transmission measurements using ultrasonic and acoustic signals has been used with great success to detect large flaws, and gross damage such as delaminations or

matrix cracking. The problem of detecting fibre debonding appears not to have been solved using any of the methods outlined above [Cawley and Adams, 1989].

1.3.3. X-Ray radiography.

X-Ray radiography has been used with little success on composites, since the contrast between the fibre and matrix materials is generally not particularly large. Defects and damage such as delaminations are difficult to spot since they are usually perpendicular to the beam and have little attenuative effect. X-Ray radiography has been used with some success to detect transverse ply cracks in specimens under test [Favre and Laizet 1989, Selden and Gustafsen 1985]. To improve contrast high contrast dye penetrants have been used in order to follow damage in composites during fatigue testing [Soutis et al, 1991]. Russell and Sutton [1989] made strain field measurements on composite laminates using X-Ray radiography to determine the effect of damage on the strain fields. As an NDT technique for composites X-Ray radiography does not appear to be particularly promising.

1.3.4. Thermography.

Thermography techniques involve monitoring the surface temperature of the composite structure under test. Firstly the structure is heated either by cyclic loading [Jones et al, 1989] or by externally heating with for example a focused laser [Inglehart et al, 1986]. Various methods have been used to detect the temperature changes, usually an infra-red camera is used [McLaughlin et al 1987] although Inglehart et al [1986] used the deflection of a probe laser beam by the heated air above the test

sample to monitor the temperature changes.

Zhang et al [1990] used thermography to make a thermal stress analysis of a composite laminate under cyclic loading. McLaughlin et al [1987] reported that thermography was able to detect delaminations equally as well as an ultrasonic C-scan, while Jones et al [1989] have used thermography to detect impact damage in composite coupons. This shows that thermography has potential as an NDE method as applied to composites, although it is not as sensitive as ultrasonics when detecting defects such as matrix cracking. Furthermore it is reported [Cawley and Adams, 1989] that equipment costs are high.

1.3.5 Other techniques.

In addition to the techniques outlined above other less well known methods have been applied to composite testing. For example Jackson et al [1990] have used magnetic resonance imaging (MRI) to detect the ingress of moisture into a composite structure. Eddy currents have also received some attention [Lane et al, 1991] but are limited to composites with a conducting phase.

1.3.6 Summary.

Although the literature on the non-destructive testing of composite materials is large and many of the possible defects in a composite have been shown to be detectable, the problem of detecting matrix-fibre debonding appears not to have been addressed. There is, however, some evidence to suggest that it maybe possible to detect such a defect. Hollis et al [1984] used an acoustic microscope to image samples of low strength and high

strength reinforced polycarbonate (the reinforcements were glass fibres). In the low strength samples (these were samples that failed prematurely under loading) the fibres were clearly imaged, but in the high strength samples the fibres were particularly difficult to image. On examining the samples after failure with a scanning electron microscope (SEM), the fibres in the low strength samples were seen to be debonded from the matrix with no signs of the matrix and fibres having been in contact. It was suggested that the debonded fibres were acting as voids.

From this it would appear that backscattered ultrasound may well be sensitive to the presence of debonded fibres. The work presented in this thesis is thus concerned with the effect of fibre debonds on ultrasonic backscattering.

Ultrasonic backscattering depends on the elastic parameters of the matrix material and the inclusion (the longitudinal and shear wavespeeds and the densities). In addition the scattering also depends upon the boundary conditions existing at the interface of the matrix and inclusion. Consequently any change at the interface due to debonding should have an effect on the backscattered ultrasonic wave. Thus although the conditions existing at the interface will have an effect on the attenuation and speeds of waves propagating through the bulk material, this thesis is concerned primarily with the basic mechanics of how the scattering of ultrasonic waves is influenced by the fibre-matrix interface.

1.4 Outline of the thesis.

For a continuous fibre-reinforced composite the geometry is

ideal for the analysis that is presented in this thesis. Firstly the fibres themselves are near perfect cylinders, and with a very large length to breadth ratio they are effectively infinite in length. Consequently the approach of modelling the scattering from a fibre, assuming the fibre is an infinite cylinder, is completely justified. In addition the experimental measurements can be expected to give good agreement with the theory.

Chapter 2 is concerned with a preliminary study of the potential effects of debonding of the scatterer from the matrix on ultrasonic backscattering. This was carried out using a computational model developed by Anson and Chivers [1993] to calculate the scattering from spherical scatterers allowing for the inclusion of a surrounding boundary layer. The debond was modelled as a continuous thin air shell separating the scatterer and matrix. The results of this study were encouraging suggesting that a thin air shell surrounding the inclusion caused the resulting system to act as an air-filled void with a sharp resonance in the low ka region. In view of these encouraging indications a cylindrical scattering model was developed.

In Chapter 3 the scattering theory used to calculate the scattering from cylindrical inclusions embedded in a solid elastic matrix is presented. The numerical approach used to solve the resulting equations is discussed, and the results of numerical testing of the computer code which was written are presented.

Chapter 4 is concerned with the different models of debonding as applied to the cylindrical scattering model discussed in Chapter 3. These models are the thin air shell proposed in the

preliminary study (chapter 2), the inclusion of an effective modulus characterising the amount of bonding present at an imperfect interface, and modelling a debond by a thin visco-elastic layer.

The experimental work and results are described in chapters 5 to 7. Chapter 5 outlines the experimental procedure and the reasons for using a pulse-echo technique for the measurements. Chapter 6 presents the results of longitudinal wave backscattering measurements made on scale model samples of a single wire embedded in an epoxy resin. The results of these experiments suggest that it is indeed possible to detect the prescence of a debonded fibre initially after it had been debonded. The general effect being a very large attenuation of the resonances in the backscattering. However the fibre and matrix appear to relax back into an intimate contact. The longitudinal wave backscattering from this damaged system then looks very similar to that for a well-bonded fibre.

Since longitudinal waves are only sensitive to debonds immediately after debonding it was decided to investigate the possibility of using shear wave incidence to detect debonding. Intuition suggests that a wave with its displacement parallel to the axis of the fibre (SH wave) would be sensitive to a weak or damaged interface. The problem of shear wave scattering is discussed in Chapter 7 with both the theoretical problem and experimental investigations carried out. As for the longitudinal scattering measurements in Chapter 6, good agreement is seen between theory and experiment for well-bonded scatterers and significant differences seen for the debonded samples.

Chapter 8 discusses the conclusions drawn from the work presented in the thesis and proposes further work which should be carried out in order to provide a new test instrument or technique.

CHAPTER 2.

PRELIMINARY STUDY ON SPHERICAL INCLUSIONS.

Before the model for cylinders was developed by the present author, a model for calculating the ultrasonic scattering from spherical objects had already been produced within the research group [Anson and Chivers 1990]. In its latest form it is a shell model permitting the inclusion of viscosity, thermal waves and viscoelastic effects in the calculations. The basic equations used are those developed by Hay and Burling [1982], the approach used in the calculations being effectively the same as described in the following chapter (concerned with scattering from cylindrical objects), but using a spherical coordinate system rather than a cylindrical one. It was felt that using this model would be helpful since it could provide an indication of the type of results that might be expected for scattering from cylindrical scatterers.

This model needs several input parameters: the longitudinal and shear wave speeds with their corresponding attenuations are needed for the solid phases; whilst for liquids the shear wave is set to zero with viscosity replacing the shear wave attenuation (see equation 3.25). In addition the density, thermal conductivity, specific heat capacity at constant pressure, and the coefficient of thermal expansion are needed for both liquid and solid phases.

2.1 Glass Reinforced Perspex.

As an approximation to a glass reinforced plastic, a system of a glass sphere in a perspex matrix was studied computationally using

the spherical shell model mentioned above. The reasons for using perspex are discussed in chapter 4 (4.2.1). The physical parameters input are given below in table 1 in appendix 2.

The longitudinal wave backscattering is calculated as a function of the acoustic radius (ka). $k = 2\pi/\lambda$ is the wave number, and a is the radius of the scatterer. However since the attenuation depends on the frequency, the radius of the inclusion must also be included in the calculation. In general fibres used in composites are of a very small diameter (approximately $10\text{ }\mu\text{m}$ [Composite materials Technology, 1990]). The radius of the inclusion used for these calculations was set to $30\text{ }\mu\text{m}$. For the ka range

$$0 \leq ka \leq 30$$

in steps of $0.1\text{ }ka$, this is equivalent to a frequency range of

$$2.14\text{ MHz} \leq f \leq 642.14\text{ MHz}$$

assuming the first calculation starts at $ka=0.1$. The calculations were programmed with a ka step size of 0.1 . The backscattered form function* for a glass sphere in a perspex matrix is shown in figure.2.1. The resulting graph shows a very resonant structure above $ka \approx 3$. This resonant structure depends upon many factors including the boundary conditions existing at the interface of matrix and inclusion and the parameters listed in table 1 (appendix 2).

To model a debonded sphere a thin air shell was introduced, the thickness was varied from 0.1 nm up to $0.01\text{ }\mu\text{m}$ and the effect on the backscattering calculated. The parameters for air are given in table 2 in appendix 2.

This time the calculations were only run for the range

$$0 \leq ka \leq 3.$$

* see p.52

Figure 2.1: Backscatter form function for a $30\mu\text{m}$ glass sphere embedded in a perspex matrix.

Figure 2.2: Backscatter form function for a glass sphere surrounded by a thin air shell in a perspex matrix. The scattering is calculated for air shell thicknesses of 0.1nm , $0.001\mu\text{m}$ and $0.01\mu\text{m}$.

Glass Sphere in Perspex Matrix

Fig. 2.2

Glass Sphere in Perspex Matrix

This gives a maximum frequency of approximately 60 MHz which was considered the maximum realisable frequency for experimental work with the equipment available. In addition the backscattering from an air filled spherical void was also calculated.

Figure 2.2 shows the results of the calculations of the backscattering from a glass sphere surrounded by a thin air-filled shell in a perspex matrix. The curves are for different shell thicknesses. The first and most striking feature is that the inclusion of a thin air shell (even at a physically unrealistic 0.1 nm) makes a great difference to the scattering as can be seen by comparison with the form function for the "bonded" case of a glass sphere in the perspex matrix. As the gap is increased, the form function approaches that for an air-filled spherical void.

Also of note is the sharp resonance that occurs for the 0.01 μm gap at ka 0.15 and also at $ka \approx 0.25$ for the 0.001 μm shell. This resonance appears to be shifting to higher frequencies as the thickness of the shell is decreased, while at the same time it is also broadening. Possibly the peak around ka 0.55 for the 0.1 nm shell is a continuation of this frequency shift and broadening. It is interesting that in a recent paper by Yang and Norris [1991] on shear wave scattering from a partially debonded fibre, a sharp resonance was noted in the low ka range ($0.1 \leq ka \leq 0.6$) in their cross-section calculations. However this was performed for a crack that was not continuous around the circumference of the fibre, although they assumed that the faces of the crack are non-contacting as is assumed in this thin air shell model. Yang and Norris attribute the low ka resonance to a translational mode of

the fibre inside the partial void created by the crack.

In figure 2.3 the first four partial waves of the backscatter form function are displayed for the ka range $0 < ka < 2$. The $n=0$ partial wave is associated with a breathing motion of the cavity around the inclusion [Flax et al, 1981] the effect of this mode is much larger than the $n=2$ and $n=3$ partial waves (quadrupole mode and hexapole modes respectively.) Of particular interest is the $n=1$ partial wave mode since this contains the sharp resonance for the 1nm air shell. The $n=1$ partial wave is the mode associated with the translational mode of a movable scatterer. The presence of this resonance in this mode suggests that the low ka resonance may well be due to some motion of the inclusion inside the cavity as is suggested by Yang and Norris [1991] for their calculations.

If such a resonance can be shown to occur for cylindrical inclusions then it may provide the basis of a useful experimental technique for determining whether or not there is any debonding of fibre and matrix in a sample of a fibre reinforced composite material. This work is discussed later in chapter 4.

2.2 The effects of surface layers on the ultrasonic scattering from spheres.

After some discussion it was decided to carry out an extended study on the effects of thin shells on the ultrasonic scattering of longitudinal waves from spheres. Previous work had been carried out in the group on the ultrasonic scattering from spheres of different materials immersed in water [Anson and

Figure 2.3: The first four partial waves for the backscattering from a glass sphere surrounded by a 1nm thick air shell in a perspex matrix.

Figure 2.4: The effect of thermal waves on the backscatter form function from a glass sphere in a perspex matrix, shown as the difference between the scattering with and without thermal waves included in the calculation.

Fig. 2.3

Fig. 2.4

Chivers, 1982], this work was later extended to the calculation of attenuation for a variety of liquid and solid inclusions suspended in liquids [Anson and Chivers, 1990]. The aim of the current work was to gain an idea of the general effect of the physical parameters such as the longitudinal and shear wavespeeds as well the density on the backscattered longitudinal wave. This may be contrasted with the approach of analysing the structure of the spectra for a particular system in order to identify the physical mechanisms which contribute to the overall structure (i.e Resonant Scattering Theory ,Flax et al [1981]).

Two extreme cases were examined, the first was with a thin gaseous (air) shell but varying the matrix and inclusion materials, the second had the properties of the shell only slightly different from those of either the inclusion material or the matrix. The latter system is, incidentally, of potential interest in the food industry as is detailed in section 2.2.2.

2.2.1 Gaseous Shell.

During the course of the study the gaseous shell systems became known as the gross shell model and this is the name with which it will be referred to here. Three different combinations of gross shell model were investigated, these are listed below;

- i) SOLID/AIR/SOLID
- ii) SOLID/AIR/LIQUID
- iii) LIQUID/AIR/SOLID

The format medium 1/shell/medium 3 refers to the matrix, shell and inclusion materials respectively, the shell is of course medium 2. Each of the three systems will be discussed separately.

2.2.1.1 Solid/air/solid.

The specific systems examined were Cu/air/glass, Al/air/glass, Mg/air/glass, Polyethylene/air/glass, Cu/air/Cu, Mg/air/Al. Physical parameters for these materials are listed in tables 1 and 2 (see appendix 2). The metals listed have quite a large range of wave speeds and densities [Anson and Chivers, 1982] ^{and references therein} (4759 ms^{-1} to 6374 ms^{-1} for c_c ^{the longitudinal wave velocity}, and 1738 to 8933 kgm^{-3} for the density) and for this reason it was felt they would offer a diverse range of effects in the backscattered spectra. To gain an idea of the general effect of non-metallic solids, glass and polyethylene were chosen for inclusion in the study.

It should be noted that the frequency dependence for the attenuation of glass has the same form as that for many fluids (i.e $\text{coefficient} \times f^2$) [Anson and Chivers, 1990], Polyethylene is quite different, with a markedly low shear wave speed (950 ms^{-1}) and density (960 kgm^{-3}), in addition it also exhibits a large attenuation ($20.5 \times 10^{-6} \text{ f np m}^{-1}$), especially for shear waves ($146 \times 10^{-6} \text{ f np m}^{-1}$) [Anson and Chivers, 1990] .

For all materials the effects of thermal waves were included. However their contribution to the backscattering is very small and is usually observed as nothing more than a small attenuation of some of the sharper resonances. Figure 2.4 displays the effect of

including thermal waves in the analysis has on the backscattering from a glass sphere in a perspex matrix. As can be seen the thermal effects are four orders of magnitude smaller than the typical features seen in the backscatter form function (c.f. fig. 2.4 and fig. 2.1. Thermal effects are usually only significant at low ka values [Anson and Chivers, 1990].

All spectra were calculated up to a maximum of $ka=6$ with a step size of $\Delta ka=0.1$. For an Aluminium matrix and a $20\text{ }\mu\text{m}$ radius sphere this gives a maximum frequency of 304.34 MHz. Figures 2.5 to 2.7 show some typical results for the solid/air/solid systems listed above.

At the lower limit of the shell thickness used (0.1nm) for the metal/air/glass systems, the backscattered spectra obtained are quite different from those for the same systems with the shell excluded (fig.2.6). This implies that the introduction of even such a thin air layer has a very significant effect on the scattering i.e. the boundary conditions at the outer radius of the shell are such as to give effects which are very similar to those of a free surface.

As the air shell thickness is increased up to a maximum value of 10nm, the spectra in all cases (including the Polyethylene /air/glass system, fig.2.7) approach those for the relevant solid/air systems. An example of this is the Cu/air system shown in figure.2.8.

In effect the shell scatterer is acting as a spherical air filled void in a metallic solid matrix. The only case where the

Figure 2.5: Backscatter form function for a glass sphere surrounded by a thin air shell in a copper matrix. The three shell thicknesses displayed are 0.1nm, 1nm and 10nm.

Figure 2.6: The calculated backscattering from four solid elastic spheres in solid elastic matrices.

Fig. 2.6

Figure 2.7: The effect of thin air layers surrounding a glass sphere in a polyethylene matrix, shell thicknesses displayed are 0.1nm, 1nm and 10nm.

Figure 2.8: The backscatter form function for an air-filled spherical void in a copper matrix.

Fig. 2.8

system with a 0.1nm shell shows any transitional behaviour from a solid/solid to a solid/air system is the Polyethylene/air/glass system (fig. 2.7). This is presumably due to the reflection coefficient at the outer radius of the shell being lower than it is for the metal matrix systems, although in both cases the reflection coefficient is almost 1. Further support for this can be seen from the fact that for a 1nm shell thickness, the spectrum is practically the same as that for a gaseous spherical inclusion. Primarily then it is the boundary conditions at the outer radius that have the most (if not the only) effect on the backscattering in these particular systems.

2.2.1.2 Solid/air/liquid.

The following specific systems were studied for this section of the work, the physical parameters are given in table 1 and table 2 (see appendix 2):

Polyethylene/air/Pentane,	Polyethylene/air/Diiodomethane,
Al/air/Pentane,	Al/air/Diiodomethane.

Pentane and Diiodomethane were chosen for their contrasting properties, notably their densities and viscosities. The solids selected represent a low density lossy medium (Polyethylene) and a (relatively) lossless metal. These materials were felt to represent a reasonably large range of properties.

Al/air/liquid:

For an Aluminium matrix, the backscattering was largely the same as that calculated for a 20 μm radius air sphere when an air shell of 1nm thickness or greater is included in the system (fig. 2.9).

Figure 2.9: The backscatter form functions for a thin air shell surrounding a pentane sphere in an aluminium matrix. Shell thicknesses displayed are 0.1nm and 10nm.

When the air gap was reduced to 0.1nm the resulting sharp resonant structure calculated (fig. 2.9) still shows an underlying similarity to that of an air filled void (fig. 2.8).

Changing the material for the core liquid had little effect on the backscattered spectra when a thin air shell was present:- the difference between the properties of Diiodomethane and Pentane have a much smaller effect on the scattering than the great difference between Aluminium and air at the outer radius of the shell. Only when the shell thickness is of the order of 0.1nm or less does the effect of the core material become apparent, with sharp resonant structures appearing in the spectra. However it should be noted that for either a Pentane or Diiodomethane inclusion in an Aluminium matrix the backscattered spectra exhibit marked differences above $ka = 3$ compared to the systems with a 0.1nm shell included, (compare fig.2.10 with the curve for a 0.1nm shell in fig.2.9).

Polyethylene/air/liquid;

The results obtained when using 'Polyethylene as the matrix material are quite different to those calculated for an Aluminium matrix [c.f. fig.2.9 with fig.2.11]. For the Polyethylene/air/Diiodomethane system with a 0.1nm thick air layer surrounding the fluid inclusion, the backscattering spectrum shows a marked similarity to the same system with no shell present (fig.2.12). There are two main groups of resonant structure, these being centered around $1.0 \leq ka \leq 2.5$ and $3.5 \leq ka \leq 5.0$. The effect of this thin air shell appears to be a sharpening of the peaks with an increase in their magnitudes. In addition there is a shift of

Figure 2.10: The backscatter form function for a pentane sphere in an aluminium matrix.

Figure 2.11: Effect of a thin air shell on the scattering from a diiodomethane sphere in a polyethylene matrix. Shell thicknesses used are 0.1nm and 10nm.

Fig. 2.11

Polyethylene/Air/Diodomethane

these peaks to higher values of ka . With the air shell set to a thickness of 10nm the backscattered spectrum becomes essentially that for an air filled void in a Polyethylene matrix, revealing a surprisingly rapid change from the behaviour of a solid/liquid system to a solid/air one as the shell thickness is changed. With a 1nm thick shell the spectrum shows behaviour consistent with both extreme cases: at the lower ka values the structure is similar to that for the solid/liquid system and at the higher values it approaches that for a solid/air system.

Calculations performed for the Polyethylene/air/Pentane system show the same general trends as described above i.e. the spectrum changes rapidly from the behaviour of a Polyethylene/Pentane system to a Polyethylene/air system as the shell thickness is increased from 0.1nm to 10nm.

2.2.1.3 Liquid/air/solid.

The same materials were used here as in the previous section (i.e. Pentane, Al etc), the only difference being that the matrix materials were liquids and the core materials, solids. In all the four particular systems studied (Pentane/air/Al, Pentane/air/Polyethylene, Diiodomethane/air/Al etc) It was found that when the air shell was set to 0.1nm, the backscattering spectra show the same structure as those for the same system with the shell not included (fig. 2.13). There were some differences, these being small changes in the magnitudes and positions of features within the structure.

In contrast, with the air layer set to 10nm, the backscattering

Figure 2.12: Comparison of the scattering from a diiodomethane sphere in a polyethylene matrix to that from a diiodomethane sphere surrounded by a 0.1nm shell of air.

Figure 2.13: Backscatter form function for an aluminium sphere surrounded by a thin air shell in a diiodomethane matrix. Shell thicknesses used are 0.1nm, 1nm and 10nm.

Polyethylene/Air/Diodomethane

Fig. 2.13

Diiodomethane/Air/Al

spectra all showed the same form for the relevant liquid/air system, i.e an air bubble of $20\mu\text{m}$ radius. Above $ka \approx 1.5$, the curves are flat and have a value approximately equal to one (fig.2.13). For the backscattered spectrum for a bubble in a liquid there is a large monopole resonance usually around (or less) than $ka=1$, and the spectrum is basically flat with a value approximately equal to one.

In the systems with a boundary layer included there is a sharp double resonance in the region of $ka \approx 0.5$ dropping rapidly towards zero below this (the lowest value of ka used in the calculations was 0.05). The size, width and structure of this feature are dependent on the properties of the matrix and core medium as well as on the thickness of the shell. When the shell was set to 1nm it was found that in all the cases examined the positions of the large initial resonances shifted to higher values of ka .

2.1.2.4 Summary of the results of the gross shell model;

a) The outer radius boundary conditions have in general the most significant effect on the structure of the backscattering spectra. The inner boundary only has a significant effect when the boundary layer is extremely thin (the order of 0.1nm or less), or in the cases where the materials for the matrix and shell are both fluids i.e. Pentane/air/Al. In these cases it is the boundary conditions at the inner radius of the shell which have the most significant effect on the backscattering when the thickness is 0.1nm.

b) In general, above a thickness of 10nm the air shell looks like an air sphere with the effect of the core material being

negligible.

2.2.2 Subtle Shell Model.

The second part of the study was concerned with systems where the properties of the shell material were similar to those of the matrix and inclusion. In particular fluid shells with fluid cores and matrix were examined. This was done primarily to provide a greater insight to the effect of surface layers on the scattering from inclusions, but these results may have direct applications in other contexts. In the food industry a great deal of work has been carried out on the properties of emulsions and aerated foodstuffs [Fairley et al 1991, Robbins et al 1991]. Chemicals (surfactants) are often added to ensure that large globules do not form in the emulsions and a well dispersed system of droplets is formed, e.g. in salad dressing, which is an emulsion of fat droplets suspended in water. In optically opaque emulsions ultrasound offers one of the few ways of monitoring processes within an emulsifying process. As the surfactant coats the droplets it is of interest to determine whether or not the shell is detectable using ultrasound.

Three systems were examined in detail; Water/Anilene/Water Water/Morpholine/Water and Hexanol/Morpholine/Hexanol. The physical and thermal properties of the components are listed in table 2 in appendix 2. The second and third combinations of liquids were studied to compare the effects of:-

i) similar acoustic impedances but different thermal properties; Water/Morpholine/Water; and

ii) different acoustic impedances but similar thermal properties Hexanol/Morpholine/Hexanol.

(Anilene was originally studied since it has similar properties to water).

With the low levels of backscattering expected it was thought that thermal properties might well have a significant effect on the backscattering. As before, the backscattering form function was calculated over the range $0 \leq ka \leq 6$, and the shell thickness was varied from 0.1nm to a maximum of 30 μ m . The radius of the inclusion was kept constant at a value of 20 μ m.

For all three systems, with the shell thickness less than 1 μ m, the backscattering spectra all show the same structure. As expected, the magnitude of the features are very small: for Water/Anilene/Water with the shell set to 10nm the scattering values calculated are of the order of 10^{-6} . Figure 2.14 shows the calculated backscattering from a water filled Morpholine shell in water. The curves in descending order of magnitude are for a 10 μ m, 1 μ m, 0.1 μ m and a 10nm thick shell respectively. Below 10 μ m the structure in each case is basically the same, with only the magnitude of the scattering changing with the shell size. Only when the shell is set to 10 μ m (of the same order of size as the inclusion) does the shape of the backscattering spectrum significantly change.

It was noted that for shells whose thickness was greater than 1 μ m, the positions of the features in the spectra were shifted to higher values of ka as the shell thickness was increased. This was

Figure 2.14: Backscatter form function for a sphere of water surrounded by a shell of morpholine in a water matrix. Shell thicknesses range from 1nm to 10 μ m increasing by an order of magnitude for each curve. The curve for a shell of 1nm is not observable on the scale used.

Figure 2.15: Height of the second peak against shell thickness for a water sphere surrounded by an anilene shell in water (the lower line) and also for a hexanol sphere with a morpholene shell in hexanol.

Fig. 2.14

water/morpholine/water
shell thickness from $1e-10m$ to $1e-5m$

Fig. 2.15

height of 2nd peak against shell thickness
for water/anilene/water (lower curve)
and hexanol/morpholine/hexanol (upper)

very pronounced when the shell was set to $10\mu\text{m}$ (fig. 2.14). Below $0.1\mu\text{m}$ this shift seems to be negligible (if indeed it exists at all). As had been stated above, the curves obtained for shells of $0.1\mu\text{m}$ thickness or less appear to be very similar, the only differences being the magnitude of the spectra. If these were in fact the same curves, then it should be possible to rescale them using a scaling factor related to the thickness of the shell (this being the only parameter that was changing). Using the data calculated for the Hexanol/Morpholine/Hexanol system a scaling factor was defined as the ratio of the size of the second maxima for a given pair of curves.

This rescaling procedure was then carried out for the curves calculated for Hexanol/Morpholine/Hexanol, i.e rescaling the curve for the $1\mu\text{m}$ shell and then overlaying this on the curve for the $10\mu\text{m}$ shell. Good agreement was seen until the re-scaled curve for the $0.1\mu\text{m}$ Morpholine layer was super-imposed on that for the $1\mu\text{m}$ layer. Although the magnitudes of the features of the two curves were close there were differences in the positions of the structures in the two curves. The upper limit of the shell thickness before the structure of the spectrum starts to show this shift is around $0.1\mu\text{m}$. A graph of the magnitude of the second resonance (this being a defined and convenient point) was plotted as a function of shell thickness for both the Water/Anilene/Water and Hexanol/Morpholine/Hexanol systems on a log-log scale. A straight line was obtained (fig. 2.15).

For larger values of shell thickness $> 0.1\mu\text{m}$ a scaling factor was also defined for the k_a values. This was taken as the ratio of

the positions of the second minimum (this being an arbitrary but very well defined point). The calculated backscattering curve for the Hexanol/Morpholine/Hexanol combination with the shell set to $1\mu\text{m}$ was multiplied by the two scaling factors described above. This curve was then overlaid on that for the same system with the shell set to $10\mu\text{m}$ (fig. 2.16 ,continuous line is for the scaled $1\mu\text{m}$ shell, the dashed line is the $10\mu\text{m}$ shell). Reasonable agreement is seen for the two sets of data up to about $ka=4.5$. Beyond this, both the magnitudes and the positions of the features no longer coincide. As a generalisation, the shell affects the size and, to a lesser extent, the positions of characteristics of the backscattered spectra, but it does not appear to alter the basic form of the backscattering. This generalisation is only valid up to a certain limit, which in this particular case discussed here is around a shell thickness of $10\mu\text{m}$.

In figure 2.17 the position of the second minimum as a function of shell thickness for the Hexanol/Morpholine/Hexanol combination is shown. The main feature is that the curve approaches an upper limit of ka as a function of shell thickness. In this example the limiting value was found to be approximately 4.41 with the shell set to $30\mu\text{m}$. When the backscattered spectrum for a $50\mu\text{m}$ Morpholine sphere suspended in Hexanol is compared with a Hexanol/Morpholine/Hexanol system with the shell set to $30\mu\text{m}$ and inner inclusion $20\mu\text{m}$, it is apparent that the two curves are similar. The main difference is that the magnitude of the second resonance differs, the other features are in reasonable agreement. It is felt that the upper limiting value of ka of the second

Figure 2.16: The calculated backscattering from a hexanol sphere with a morpholine shell in hexanol overlaying the scattering with a $1\mu\text{m}$ shell on that with a $10\mu\text{m}$ shell.

Figure 2.17: Position of the second minimum for a hexanol sphere with a morpholine shell in hexanol as a function of shell thickness.

Fig. 2.16

1 μ m shell overlaid on curve
for 10 μ m shell (dashed curve)

Fig. 2.17

position of 2nd minimum
as a function of shell
thickness for
hexanol/morpholine/hexanol

minimum as a function of shell thickness identifies the region where the shell system appears to the acoustic wave to be effectively a sphere of the shell material without any inclusion.

The last part of the study was concerned with checking that the backscattering depended on the relative size of shell and inclusion and not on the absolute size of the shell. Backscattering curves were calculated with the internal radius (radius of inclusion) set to $200\mu\text{m}$ (to be compared with the scattering from a $20\mu\text{m}$ sphere) and the layer thickness set to $0.01\mu\text{m}$ (c.f. 1nm) and $100\mu\text{m}$ (c.f. $10\mu\text{m}$). When the relevant sets of data were compared with those for the $20\mu\text{m}$ inclusion based systems no obvious differences were found. This suggests that the backscattering is determined by the ratio of the shell thickness to sphere radius.

2.2.2.1 Summary of the results of the subtle shell model;

a) The shell becomes significant in its effect when the thickness is of the same order of magnitude as the inclusion.

b) The backscattering as a function of ka , for various shell thicknesses, gives basically the same curve with different scaling factors on the axes. These scaling factors are related primarily to the shell thickness.

c) There is a maximum limiting factor of ka for the position of the second minimum as a function of the shell thickness. For a $20\mu\text{m}$ sphere of Hexanol this value was found to be 4.41 and is reached with the shell thickness equal to $30\mu\text{m}$.

d) With the shell set to $30\mu\text{m}$ the Hexanol/Morpholine/Hexanol system described above shows many of the characteristics of the

backscattered spectrum for a 50 μ m radius Morpholine sphere suspended in Hexanol. This suggests that a change-over point has been reached where the system behaves much like a single spherical inclusion rather than a shell system.

e) The scattering curve depends on the relative size of the shell and internal sphere, not just on the absolute shell thickness.

2.3 Conclusions of the preliminary study.

The effect of a thin air layer on the ultrasonic backscattering from a spherical inclusion is very significant. In fact the scattering approaches that for a spherical void. In addition there is a sharp resonance feature in the low ka region, the magnitude and position of which appears to be dependent on the shell thickness. Since the method used to calculate the scattering from spheres is essentially the same as that for cylindrical inclusions, it is reasonable to expect these effects to be observed in the backscattering spectra for cylindrical inclusions with thin air shells. In view of these positive results it was decided to proceed and develop a model to calculate the ultrasonic backscattering from a cylindrical inclusion in a homogeneous isotropic matrix.

CHAPTER 3. CALCULATION OF LONGITUDINAL WAVE SCATTERING FROM A SOLID CYLINDRICAL INCLUSION IN A SOLID MATRIX (PROGRAM DEVELOPMENT AND TESTING).

3.0 Introduction.

The preliminary study had modelled a debonded scatterer as being surrounded by a thin air filled crack, the faces of this crack were assumed to be non-contacting. The results of this study had been particularly interesting, showing that with the debond modelled in such a manner the scatterer behaved almost as if it was an air-filled void. There was also a sharp resonance present in the low ka region of the backscatter form function. It was now necessary to develop a scattering model for a cylindrical scatterer to test whether the effects seen for the spherical shell model were present for cylinders.

The theoretical problem of ultrasonic backscattering from a cylindrical or spherical obstacle has in the past been studied by many authors, especially with regard to solid bodies and shells immersed in a fluid [Veksler 1989, Gaunaurd and Werby 1991, Faran Jr 1951]. Experimental verification of the theoretical predictions for such systems has also been widely addressed [Humphrey et al 1990, Neubauer et al 1974]. The corresponding problem for a solid scatterer embedded in an elastic matrix has, in contrast, received far less attention with a few notable exceptions.

Ying and Truell [1955] first examined the problem of scattering from solid spheres in an elastic isotropic solid theoretically, while Flax et al [1981] presented calculations using

a resonant scattering theory analysis for lucite spheres in an aluminium^{matrix}, and also for iron inclusions^{in this matrix}. In fact almost all the work published on scattering from spherical solid inclusions in solid matrices is of a theoretical nature. Johnson and Truell [1965] show numerically that the scattering of elastic waves in an elastic medium diverges from Rayleigh scattering for $ka > 0.5$. Iwashimizu [1972] calculated the scattering from a rigid movable sphere whilst McBride and Kraft [1972] studied transverse wave scattering from a sphere. Lain and Kanawal [1978] investigated both longitudinal and shear wave scattering from spherical inclusions and cavities using a Green's function approach. Olsson et al [1990] have modelled the effect of thin interface layers (typically a tenth of the scatterer's radius) on the scattering from a spherical inclusion, concluding that the parameters of the shell have a significant effect on the scattering. The scattering of longitudinal and shear waves from a solid cylindrical scatterer has been investigated by White [1958]. However his work only presents the scattering as a function of angle around the axis of the cylinder at a few fixed values of ka , all the experimental work he presents concerns only cylindrical holes.

3.1 Analysis of a longitudinal wave scattered by a cylindrical obstacle.

The equation of motion for an isotropic elastic medium is given by [Ying and Truell, 1955];

$$(\lambda + \mu)\nabla\nabla \cdot \mathbf{s} + \mu\nabla^2 \mathbf{s} = \rho \frac{\partial^2 \mathbf{s}}{\partial t^2} . \quad (3.1)$$

Here λ and μ are the Lamé constants and ρ is the density of the

solid medium, s is the displacement. Introducing scalar and vector potentials Φ and Ψ , the displacement can then be expressed as [Achenbach, 1973];

$$s = \nabla\Phi = \nabla \times \Psi, \quad (3.2)$$

where the time dependence, $e^{-i\omega t}$ is suppressed. It can be shown that

$$(\lambda+2\mu)\nabla^2\Phi = \rho \frac{\partial^2\Phi}{\partial t^2}, \quad (3.3)$$

$$\mu\nabla^2\Psi = \rho \frac{\partial^2\Psi}{\partial t^2}, \quad (3.4)$$

where Φ represents the compressional wave potential, and Ψ the shear wave potential.

For a plane longitudinal wave incident on an inclusion, two waves will be scattered (a compressional wave and a shear wave) and two waves will be refracted into the inclusion (a compressional wave and a shear wave), see fig. 3.1. No thermal waves have been included since their effect on backscattering can be shown to be insignificant for systems described in the preliminary study. The incident, refracted and scattered waves, as described above, can be represented by potentials (Φ for compressional and Ψ for shear waves) which obey the equations;

$$(\nabla^2 + k_c^2)\Phi = 0 \quad \text{:compressional wave} \quad (3.5)$$

$$(\nabla^2 + k_s^2)\Psi = 0 \quad \text{:shear wave} \quad (3.6)$$

These potentials expressed in cylindrical coordinates are given by the following equations.

Fig. 3.1

Figure 3.1: Geometry for the scattering problem as applied to a cylinder at normal incidence. The axis of the cylinder is into the page.

The incident compressional wave is given by:

$$\Phi_i = \sum_n \epsilon_n(i)^n J_n(k_{c1}r) \cos(n\theta) \quad (3.7)$$

where $\epsilon_n = 1$ for $n=0$ and $\epsilon_n = 2$ for $n \geq 1$. J_n is the cylindrical Bessel function of order n . The argument of the Bessel function is the product of the compressional wave number in the matrix - medium 1 - (hence the subscript $c1$) and r , the radius.

The potentials for the two reflected waves are:

$$\Phi_{c1} = \sum_n A_n H_n^{(1)}(k_{c1}r) \cos(n\theta) , \quad (3.8)$$

$$\Psi_{s1} = \sum_n B_n H_n^{(1)}(k_{s1}r) \cos(n\theta) . \quad (3.9)$$

Ψ_{s1} is the reflected shear wave potential, $H_n^{(1)}$ is a cylindrical Hankel function of the first kind (representing an outgoing wave since the time dependence is $e^{-i\omega t}$, if it was $e^{i\omega t}$ then Hankel functions of the second kind would be needed to represent outgoing waves.) with an argument which is the product of the shear wave number in medium 1 (k_{s1}) and the radius. A_n and B_n are undetermined constants which may be found by applying the boundary conditions at the surface of the inclusion and then solving the resulting equations in the undetermined constants.

The two refracted wave potentials are given by;

$$\Phi_{c2} = \sum_n C_n J_n(k_{c2}r) \cos(n\theta) , \quad (3.10)$$

$$\Psi_{s2} = \sum D_n J_n(k_{s2}r) \cos(n\theta) . \quad (3.11)$$

k_{c2} and k_{s2} are the compressional and shear wave numbers respectively in the inclusion (medium 2). C_n and D_n are the undetermined constants, and θ is the angle at which the scattering is to be calculated ($\theta = 0^\circ$ gives forward scattering and $\theta = \pi$ gives backscattering).

For the purposes of this work the following assumptions were made;

- i) the inclusion is an infinitely long cylinder.
- and
- ii) the incident wave is normally incident to the axis of the cylindrical inclusion.

The first assumption means that end effects for a finite cylinder are ignored. This is a reasonable approximation provided the length to breadth ratio is large [see Stanton, 1990]. The second assumption means that no waves are transmitted along the axis of the cylinder, and the computational problem has cylindrical symmetry. For small angles of incidence this appears to be an acceptable approximation [Stanton, 1990]^(a,b), and the materials which are ultimately of interest (fibre reinforced composites) are all in general likely to have their fibres oriented normal to the incident beam.

At the boundary between the media i.e $r=a$ (the radius of the inclusion) the following boundary conditions apply for a well-bonded inclusion;

$$\text{i) continuity of radial velocity} \quad v_{r1} = v_{r2} , \quad (3.12)$$

$$\text{ii) continuity of tangential velocity} \quad v_{\theta 1} = v_{\theta 2} , \quad (3.13)$$

$$\text{iii) continuity of radial stress} \quad \tau_{r1} = \tau_{r2} , \quad (3.14)$$

$$\text{and iv) continuity of tangential stress} \quad \tau_{\theta 1} = \tau_{\theta 2} . \quad (3.15)$$

It should be noted that some authors use continuity of displacement [Ying and Truell 1955, White 1958] instead of continuity of velocity [Linn and Raptis 1983, Habeger 1982] in the boundary conditions. In general where fluids are to be included in the analysis then continuity of velocity is used, however the only difference is a multiplicative term, $i\omega$, which cancels in the subsequent analysis. Irrespective of which convention is used the backscattering is unchanged.

The expressions for the velocity and stress components are given by:

$$v_r = -i\omega \left[\frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial \Psi}{\partial \theta} \right] , \quad (3.16)$$

$$v_\theta = -i\omega \left[\frac{1}{r} \frac{\partial \Phi}{\partial \theta} - \frac{\partial \Psi}{\partial r} \right] , \quad (3.17)$$

$$\tau_r = \lambda \nabla^2 \Phi + 2\mu \left[\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \Psi}{\partial \theta} \right) \right] , \quad (3.18)$$

$$\tau_\theta = \mu \left[2 \left(\frac{1}{r} \frac{\partial^2 \Phi}{\partial \theta \partial r} - \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} \right) + \left(\frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} - r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \Psi}{\partial r} \right) \right) \right] , \quad (3.19)$$

[see Linn and Raptis, 1983].

λ and μ are the Lamé constants which can be determined in terms of the longitudinal and shear wave speeds ;

$$\mu = \rho c_s^2, \quad (3.20)$$

$$\lambda + 2\mu = \rho c_c^2, \quad (3.21)$$

ρ being the density of the medium.

The compressional and shear wave numbers are defined respectively as [Anson and Chivers, 1990];

$$k_c = \frac{\omega}{c_c} + i\alpha_c \quad (3.22)$$

$$k_s = \frac{\omega}{c_s} + i\alpha_s \quad (3.23)$$

where c_c and c_s are the compressional and shear wave speeds. α_c and α_s are the compressional and shear attenuation constants given by;

$$\alpha = Af^m \quad (3.24)$$

where A is a constant, in general $m \approx 1$ for solids and 2 for liquids, although there are exceptions such as castor oil. If either or both media are viscous fluids, then k_s is defined as:

$$k_s = (1+i) \left[\frac{\omega\rho}{2\eta} \right]^{1/2}, \quad (3.25)$$

where η is the shear viscosity [Linn and Raptis, 1983].

Applying the boundary conditions given above at $r = a$ we obtain a system of four equations with four unknown coefficients, A_n to D_n . These can be written in matrix form ;

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad (3.26)$$

The expressions for x_{ij} and y_i are given in appendix 1. It is assumed that the solution of the equations is separate for each order of n .

At the time of the development of the cylindrical scattering model, the experimental work of the research group was restricted to the propagation and detection of only longitudinal waves. Consequently only the coefficient A_n needed to be evaluated (this represents the backscattered compressional wave). A_n is related to the scattered compressional wave by [Hickling, 1958]

$$P_s = P_o \sqrt{\frac{a}{2R}} f(ka) e^{i(kr - \omega t)} \quad (3.27)$$

where P_s is the scattered pressure

P_0 is the incident pressure

a is the radius of scatterer

R is the distance from the centre of scatterer

$f(ka)$ is the form function where

$$f(ka) = \sum_{n=0}^{\infty} \frac{2}{\sqrt{i\pi ka}} A_n \cos(n\theta) \varepsilon_n \quad (3.29)$$

In all calculations of scattering it is the form function as a function of acoustic radius (ka) that is determined. Since ka is dimensionless, the results for lossless media can be scaled for different frequency ranges or sizes of scatterers. However if losses are included in the model (i.e. α_c and α_s do not equal zero) then the results will not scale since the attenuation depends directly on the frequency.

3.2 Testing the model.

Much work has been previously done on the scattering from cylinders [e.g. Faran 1951, Brill and Gaunard 1983, Veksler 1989]. The majority of the published work has been on cylinders in a fluid medium (usually water) since the theory can be tested experimentally and water is effectively lossless [Faran 1951]. However little work appears to have been carried out with regards to scattering from cylindrical objects in a solid matrix, one exception is R.M. White's work [1958] although the results (computational and experimental) are confined to the angular scattering from a cylindrical hole in a block of Aluminium. Recently a paper has been published on the scattering from a SiC

(Silicon Carbide) fibre embedded in a Titanium matrix [Addison and Sinclair, 1991]. To this date the author has found no other work on the backscattered spectrum for a solid cylinder in a solid matrix. For this reason it was necessary to test the model by comparison with other worker's results for solid cylinders in viscous (and non-viscous) fluids.

Brill and Gaunard [1983] have published the backscattered form function for an Aluminium cylinder in water over the ka range

$$0 \leq ka \leq 40 .$$

The model developed by the present author was tested first against the results in this publication. Two other systems of solid cylinder in viscous fluids were investigated; crown glass in water and a crown glass cylinder in Mercury. Dr N.D.Veksler from the Institute of Cybernetics in Tallinn was kind enough to supply the results of his calculations for the systems listed above for comparison. The parameters used in the calculations are given in tables 1 and 2 (see appendix 2).

The results of these three calculations are shown in figures 3.2 to 3.4. The parameters for the materials listed in tables 1 and 2 were taken from Anson and Chivers [1990] and from Veksler [Private Communication]. Veksler's results are shown in figures 3.5 and 3.6.

For both systems there is good agreement between this author's model and the results produced by Veksler's calculations. There is a good correlation between the form and position of features between figures 3.3 and 3.5 and also for figures 3.4 and 3.6. Some of the sharper resonances in Veksler's results do not appear in the

Figure 3.2: Backscatter form function for an aluminium cylinder in water at normal incidence.

Figure 3.3: Backscattering from a glass cylinder in water.

Fig. 3.2

Al cylinder in Water

Fig. 3.3

Glass in Water

Figure 3.4: Backscattering from a glass cylinder in mercury.

Figure 3.5: Backscattering from a glass cylinder in water as calculated by Veksler.

Fig. 3.5

Figure 3.6: Backscattering from a glass cylinder in mercury as calculated by Veksler.

Figure 3.7: Backscattering from an aluminium cylinder in water with only the first nine partial waves included in the analysis as calculated by Gaunard.

Fig. 3.7

results of the model presented here. This can be explained by the larger (or more coarse) step size (0.1 ka) being used in the present calculations compared to the one (0.01 ka) that Veksler used. This was done to increase the speed of the computation. An example of this phenomenon can be seen in figure 3.4 in comparison with Figure 3.6, at $ka \approx 18$ in figure 3.6 there is a sharp resonance of a magnitude of 0.52, in figure 3.4 there is only a hint of this feature. Occasionally the step values used in the different models agree favourably, for example the sharp peak at $ka \approx 10$ is seen in figures 3.4 and 3.6.

Problems arose when the results of the model were compared with the calculation given in Brill and Gaunaud's work [1983], see fig. 3.7. Up to about $ka = 15$ there is close agreement (see figures 3.2 and 3.7), above that the position of some features coincide (e.g. at $ka = 18$), but the magnitudes differ significantly. There are three major differences between the two calculations:

i) Gaunaud and Brill assume that the surrounding water is inviscid. Consequently there is no backscattered viscous wave.

ii) The parameter values used for the two media are significantly different from those used here. Values quoted by Brill and Gaunaud are:

$$\text{Al: } c_c = 6380 \text{ m/s, } c_s = 3100 \text{ m/s, } \rho = 2790 \text{ kg/m}^3$$

$$\text{Water: } c_c = 1476 \text{ m/s, } \rho = 1000 \text{ kg/m}^3.$$

These values do not agree with those in Kaye and Laby [1973].

iii) In Brill and Gaunaud's work only the first nine orders are summed in the form function shown (fig. 3.7). In this

author's model the form function is summed over enough orders of n until the condition

$$\frac{|f_n|}{\left| \sum_n f_n \right|} < 10^{-8}$$

is fulfilled, the next five orders are also included in the sum in order to increase the accuracy.

This third point is quite important since the higher the ka value the larger the number of orders of n are needed to obtain a convergent series.

A modified version of the author's program was run assuming the water was inviscid and using the parameters quoted by Brill and Gaunaud. The results of this computation are presented in figure 3.8. Comparison of figures 3.7 and 3.8 still show quite large discrepancies above ka 20, although there is some agreement from ka 27 to 30. In view of these discrepancies the model was further modified so that only the first nine orders were summed ($n=0$ to 8) for the form function. The result is given in figure 3.9. As can be seen this agrees with the plot published by Brill and Gaunaud [fig. 3.7], in addition this also highlights the importance in the calculation of taking a large enough number of orders ^(typically 60) to obtain an accurate result.

In view of the success of the model in being able to reproduce the results of different authors, albeit for solid cylinders in fluids the present author felt confident to expand the model to include scattering from shells.

Figure 3.8: Backscatter form function for an aluminium cylinder in water.

Figure 3.9: Backscattering from an aluminium cylinder in water with only the first nine partial waves included in the calculation.

Al in Water

Fig. 3.9

Al in Water

first 9 orders

3.3 The Shell Model.

The use of a thin air shell separating the fibre and matrix as a possible model of a debonded fibre has been mentioned in the previous chapter. In order to model this, the previous program for calculating the scattering from an elastic fibre in elastic matrix had to be extended to include a shell around the fibre. (see figure 3.10). The refracted wave potentials inside the shell are given by;

$$\Phi_{c2} = \sum_n^n [E_n J_n(k_{c2}r) + F_n H_n^{(1)}(k_{c2}r)] \cos(n\theta) , \quad (3.28)$$

$$\Psi_{s2} = \sum_n^n [G_n J_n(k_{s2}r) + H_n H_n^{(1)}(k_{s2}r)] \cos(n\theta) . \quad (3.29)$$

The terms with the cylindrical Bessel functions represent in-going waves, whilst those with the Hankel functions of the first kind represent outgoing waves. It should be noted that H_n in equation 3.29 is a constant not to be confused with $H_n^{(1)}$ which represents the Hankel function.

Applying the boundary conditions given in equations 3.12 to 3.15 at $r=a$ and $r=b$ (the outer and inner radii of the shell) leads to a system of eight equations in eight unknowns, in matrix form as shown overleaf. The elements x_{ij} and y_i are given in Appendix 1 where the elements with i and $j \leq 4$ are the same as those for equation 3.26.

Fig. 3.10

Matrix Material

Figure 3.10: Geometry of the scattering problem for a cylindrical shell at normal incidence, the axis of the cylinder is into the page.

$$\begin{bmatrix}
 x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} \\
 x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} \\
 x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} \\
 x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} \\
 x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} \\
 x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} \\
 x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} \\
 x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88}
 \end{bmatrix} \cdot \begin{bmatrix}
 A_n \\
 B_n \\
 C_n \\
 D_n \\
 E_n \\
 F_n \\
 G_n \\
 H_n
 \end{bmatrix} = \begin{bmatrix}
 Y_1 \\
 Y_2 \\
 Y_3 \\
 Y_4 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix} \quad (3.30)$$

As before this is only solved to obtain the value for A_n , since we are only currently interested in the backscattered compressional wave. A library routine was used for the solution of the above equations. The form function is then calculated using A_n as in equation (3.27).

3.4 Testing The Shell Model.

A similar approach to section 3.2 was used to gain an appreciation of the validity of the predictions made using the shell model. The shell and core material were both set to the parameters of Aluminium (see table 1 appendix 2), the matrix material was given the properties of Polyethylene (see table 1). Polyethylene was chosen since it is a lossy material and if there were any tendencies for the new code to become unstable then the inclusion of losses would tend to highlight the presence of instabilities. To compare the results of the shell model the cylindrical model was used, this was now felt to be an accurate model in view of the previous testing. Both programs were run over

the range $0 \leq ka \leq 30$ with a step size of $0.1 ka$. The results of both calculations are shown in figure 3.11.

It is immediately apparent that there appears to be no significant difference between the two curves. To gain a better appreciation of the scale of the discrepancies between the two sets of data, the two sets were subtracted from each other, the result is displayed in figure 3.12. The differences do not go beyond 10^{-6} , and in general are of the order of 10^{-7} .

These differences are little more than rounding errors and may be considered to be insignificant. Consequently it was felt that the shell model could be used confidently as the basis for calculating the ultrasonic backscattering from debonded fibres surrounded by a thin air gap and for further modelling of the effect of fibre debonding on ultrasonic backscattering. This investigation is described in the following chapter.

Figure 3.11: Backscatter form function from an aluminium cylinder at normal incidence embedded in a polyethylene matrix calculated using the shell model (with inclusion and shell materials set to aluminium) and the two phase model. The latter is not distinguishable on the plot.

Figure 3.12: Difference of the two calculations presented in fig. 3.11.

Al in Polyethylene Matrix

Fig. 3.12

CHAPTER 4. MATHEMATICAL MODELLING OF THE EFFECT OF DEBONDS ON ULTRASONIC BACKSCATTERING.

4.1 Introduction.

The results from the preliminary study (chapter 2) on spherical inclusions indicated that a thin air shell would have a very significant effect on the structure of the backscattered ultrasonic longitudinal wave as a function of acoustic radius (ka). The general effect of including a thin air shell between the matrix and inclusion was to cause the backscattering to appear basically the same as would be expected from a void. In addition to this there was a sharp resonance in the low ka region (below $ka = 0.5$), the position of which depended upon the thickness of the shell.

In the previous chapter the scattering theory used to calculate the ultrasonic backscattering from a cylindrical inclusion at normal incidence was discussed, as well as the expansion of the program to include a shell surrounding the inclusion material. This chapter is concerned with the different scattering models used to calculate the potential effect of a debonded scatterer on the backscattering. The basis of all the approaches described below is the scattering model described in the previous chapter. The three models of debonding discussed in this chapter are;

- i) the thin air shell model, where the debond is modelled as a thin crack, with non-contacting faces, surrounding the fibre.
- ii) The slip model which assumes the fibre and matrix remain in contact but allows tangential slip of the fibre which is

governed by an effective shear modulus, K ,

and iii) The visco-elastic shell model where the debond is assumed to act like a thin layer of visco-elastic material separating the fibre and matrix.

4.2 The thin air shell model.

The assumption used for this model is that the debond behaves as a thin crack surrounding the inclusion. In addition it is assumed that the faces of the crack are non-contacting. A similar approach has been used recently by Yang and Norris [1991] for scattering of shear waves from a partially debonded fibre. In their paper they assumed that the crack was discontinuous but the faces were non-contacting. To model a continuous crack, the cylindrical shell model described in the previous chapter was used with the properties of the shell set to those of air. The matrix material was assumed to have the properties of perspex and the inclusion (the fibre) those of glass (see tables 1 and 2 in appendix 2).

4.2.1 Since the commonly used fibre-reinforced composites generally have polymeric matrices (e.g. glass or carbon-fibre reinforced composites) it was felt that the wave speeds and density of perspex would provide a good approximation to those for a general epoxy resin. Measurements were later made of the longitudinal and shear wave speeds and density on samples of araldite epoxy resin used for the experimental work (see chapter 6). The values obtained from time of flight measurements are given below in table 4.1:

Table 4.1	c_l	c_s	ρ
epoxy resin	2640 m s^{-1}	1339 m s^{-1}	1129 kg m^{-3}
perspex	2690 m s^{-1}	1340 m s^{-1}	1200 kg m^{-3}
Sayers and Smith	2640 m s^{-1}	1200 m s^{-1}	1202 kg m^{-3}

The measured values for the epoxy resin are within 5% tolerance. Also shown are the figures quoted by Sayers and Smith [1983] from the analysis they carried out of the measurements made by Kinra et al (1982) of the velocity and attenuation in an epoxy matrix containing lead inclusions. As can be seen from table 4.1 perspex gives a close approximation for the wave speeds in an epoxy resin although the density is a little high compared with the measured density. Comparing the values for perspex with those for epoxy published by Sayers and Smith [1983], it can be seen that the density values are in close agreement and it is the shear wave velocities that are in some disagreement. The effect of using the values for perspex as the matrix material is compared with the effect of using the measured values of the epoxy for the backscattering from a copper cylinder in a polymeric matrix (see fig. 4.1). Overall the effect is relatively minor, with only a slight shift to higher ka with the epoxy resin matrix. The shape and size of the resonant structure is unchanged. All calculations presented in this chapter were made assuming the matrix material had the properties listed for perspex in table 4.1.

Figure 4.2 displays the calculated backscatter form function

Figure 4.1: Comparison of the backscattering from a copper cylinder with either a perspex matrix (dotted line) or an epoxy resin matrix (continuous line).

Figure 4.2: Comparison of the backscattering from a well-bonded glass fibre (dotted line) and that with a 10nm air shell surrounding the fibre (continuous line).

Copper in Polymeric Matrix

Fig. 4.1

Glass Cylinder in Perspex Matrix

Fig. 4.2

for both a perfectly bonded glass cylindrical scatterer embedded in a perspex matrix (the dotted line), and the scattering calculated using the thin air shell model (the continuous line). The radius of the scatterer was set to $10\mu\text{m}$ and the shell thickness was 10nm ($0.01\mu\text{m}$). It is immediately apparent that the two curves are completely different. The bonded scatterer shows a typical resonant structure with several strong resonances, whilst the scattering from the air shell model oscillates around and converges towards a value of one. This is exactly what was seen for the spherical shell model in the preliminary study (see section 2.1). Also visible is the sharp resonance in the low ka region (approximately $ka = 0.2$), as was identified in the preliminary study. The overall general structure of the scattering calculated using the shell model is that for a cylindrical air filled void, with the addition of the resonance at low ka .

In the preliminary study it had been seen that the position of the resonance for the thin air shell model shifted to higher ka as the thickness of the shell was reduced. This effect is also apparent in the cylindrical shell model and is displayed in fig. 4.3. The calculation was carried out for shell thicknesses ranging from $0.01\mu\text{m}$ to $0.0025\mu\text{m}$ for a scatterer of radius $10\mu\text{m}$. The ka range is $0 < ka < 3$. These limits of the computation were chosen primarily since for higher ka with thin shells (e.g. shell thickness $\ll$ inclusion radius), the solution of the equations used to calculate the scattering coefficients, A_n , becomes rather unstable because the equations themselves are rather ill-conditioned. The effect of the instabilities can be seen for

Figure 4.3: The low ka resonance in the backscatter form function for a glass cylinder with a thin air shell in a perspex matrix.

Figure 4.4: The first four partial waves for a 10nm air shell surrounding a glass cylinder in a perspex matrix.

Perspex/Air/Glass

Fig. 4.3

Glass Cylinder in Prespex

Fig. 4.4

the results for a shell of $0.0025\mu\text{m}$ thickness above $ka = 0.25$. A library routine was used to solve the equations for A_n and consequently access to this source code was not possible. The problem of stabilising the solution of these type of scattering equations has been discussed at length by Anson and Chivers [1990]. The origin of the instabilities in the cylindrical shell model lies in the terms for waves in the shell becoming vanishingly small as the shell thickness is reduced. Despite these problems it is quite clear that the position of the low ka resonance does indeed move to higher ka as the shell thickness is reduced.

It had been discussed in Chapter 2 that there appeared to be an indication that the sharp low ka resonance was due to a translational motion of the glass inclusion within the cavity. This was based on the fact that the resonance occurs in the $n=1$ partial wave (see section 2.1). It was expected that this would also be the case for the cylindrical shell model. In figure 4.4 the first four partial waves are shown for a shell 10nm thick. As can be seen the low ka resonance occurs in the $n=1$ partial wave as was seen for the spherical shell model.

To summarise the results of the thin air shell model, if the debond is modelled as a continuous crack with non-contacting faces, then the calculations show that the scattering is essentially that for a cylindrical air-filled void. In addition there is a sharp resonance in the low ka region the position of which depends on the thickness of the shell and appears to be due to motion of the inclusion within the cavity.

4.3 The slip model.

For the thin air shell model it was assumed that the surfaces of the crack surrounding the inclusion are non-contacting. The effect of this air shell was to mask any of the resonances of the inclusion, causing the system to act basically as a void. It was not clear whether this was a physically realistic model of how a debonded fibre would behave. It seems likely that there would still be regions of contact across a debond, allowing some transmission of the incident wave. This would suggest that a debond could be modelled as an inclusion still in contact with the matrix material, but with the possibility of tangential motion of the matrix not being completely transmitted to the inclusion. Such a system was proposed by Mal and Bose [1974].

In their work on the elastic moduli of a suspension of imperfectly bonded spheres, it was assumed that the shear stresses at the interface would be proportional to the amount of slip. To model this effect they assumed that the inclusion and matrix are separated by a thin layer of visco-elastic material that prevents voids at the interface, but does allow relative tangential displacement. From these assumptions the boundary conditions become (u is the displacement):

$$\left. \begin{aligned} u_{r1} &= u_{r2} \\ \tau_{r1} &= \tau_{r2} \\ \tau_{\theta 1} &= K(u_{\theta 1} - u_{\theta 2}) \end{aligned} \right\} \quad (4.1)$$

K is termed the effective shear modulus, and is in general a frequency - dependent complex number. For an interfacial layer of

thickness δ then the shear modulus , S , of that layer would be:

$$S = K\delta . \quad (4.2)$$

There are two special cases of K , the first is when $|K| \rightarrow \infty$, then there is no slip at the interface and the boundary conditions for perfect bonding are reached, with continuity of radial and tangential stresses and displacements. At the other extreme, when $K = 0$, then there is perfect slip at the interface. For the interface being lubricated by a viscous fluid then $\text{Re}(K) = 0$ and $\text{Im}(K) = -\omega\eta/\delta$ where η is the coefficient of viscosity of the fluid. For the calculations presented in this thesis, K (the effective shear modulus) is assumed to be real since it was not clear what form the frequency dependence of the imaginary part should take. The effect of this is to reduce the amount of tangential coupling between the matrix and inclusion, but to assume that the boundary is lossless.

The work presented by Mal and Bose [1974] was limited to the low ka region, where the wavelength is much larger than the scattering inclusion. Their results indicate that in this region, the inclusion of slip causes attenuation of both longitudinal and shear waves.

The boundary conditions given in equations 4.1 were now applied to the cylindrical scattering model described in chapter 3. The backscatter form function was calculated using this modification of the program for a glass cylinder embedded in a perspex matrix with different values for the effective shear modulus, K , to determine what effect it would have on the scattering, the results of these calculations are displayed in

figures 4.5 and 4.6.

The effective shear modulus, K , used to define the amount of tangential slip, only begins to have a significant effect on the backscattering when it is of the order of 10^{15}N m^{-3} (fig 4.5) which is approximately three orders of magnitude greater than the shear modulus for perspex. Below $ka = 3$ the general effect as K is reduced is for the backscattering to be attenuated; this is in keeping the results published by Mal and Bose [1974]. Above $ka = 3$ there are noticeable changes in the structure of the scattering. For $K = 10^{15} \text{N m}^{-3}$ (the dotted line in fig. 4.5) the backscattering follows closely that for a well-bonded fibre over most of the ka range. There are a few deviations though: at $ka = 5$ the single resonance becomes a double resonance, and the sharp minimum becomes increasingly pronounced as K is reduced (see the dashed line for $K = 10^{14} \text{N m}^{-3}$). Above $ka = 8$ sharp resonances are seen but the original shape of the scattering for a bonded scatterer is still discernible. With the effective modulus set to 10^{14}N m^{-3} , the backscattering above $ka = 3$ is significantly different from that for a bonded scatterer. The most obvious change is the sharp resonance in place of the minimum just above $ka = 3$. The sharp resonances above $ka = 8$ are slightly more pronounced, but it is the appearance of the sharp resonance which is the most significant change as the effective shear modulus is decreased.

In figure 4.6 the scattering from a perfectly bonded cylinder is compared with the scattering from a cylinder with perfect slip ($K = 0$). The region below $ka = 3$ shows the attenuation of the broad resonance which has been identified in the previous graph

Figure 4.5: The comparison of the scattering from a perfectly bonded fibre (continuous line) with the scattering from a cylinder with tangential slip. The dashed line is calculated with the effective shear modulus, K , set to 10^{15} N m^{-3} , the dashed line has K set to 10^{14} N m^{-3} .

Figure 4.6: Comparison of the scattering from a perfectly bonded cylinder with the scattering from one with perfect slip, $K=0$.

Fig. 4.5

Fig. 4.6

(figure 4.5) The rest of the backscatter form function for the perfect slip case (dotted line) is basically the same as that seen for $K = 10^{14} \text{N m}^{-3}$ (see figure 4.5).

In general the effective shear modulus has significant effect on the backscattering in the range $10^{15} > K > 10^{14} \text{N m}^{-3}$. Above this range there is little difference between the scattering from a perfectly bonded and a "slipping" scatterer. Below 10^{14}N m^{-3} , the scattering is the same as that for a scatterer with perfect slip ($K = 0$). Below $ka = 3$ the effect of slip is to attenuate the backscattering. Above $ka = 3$ the scattering is basically that for a solid cylindrical scatterer with a number of sharp additional resonances present .

4.4 Visco-elastic shell model.

Mal and Bose [1974], as mentioned above, assumed that the matrix and inclusion were separated by a thin layer of visco-elastic material for a system of imperfectly bonded scatterers. Since a shell model had already been used to describe the scattering from a thin air shell surrounding a scatterer (see section 4.2) it was decided to investigate the effect of setting the shell properties to those of a viscoelastic solid. This would have the advantage over the slip model of including losses in the shell.

Although there appears to be very little in the literature on scattering from imperfectly bonded elastic inclusions some work has been published for poorly bonded flat interfaces [Rokhlin and Wang 1991, Baik and Thompson 1985, Angel and Achenbach 1985]. Rokhlin

and Wang have published work on two imperfectly bonded surfaces separated by a thin visco-elastic layer to model the effect of an epoxy bonding layer on the ultrasonic scattering. They compare several approximations with the exact case where the layer is modelled as a visco-elastic layer showing that the approximations only hold for limited ranges of frequency and layer thickness. Both Angel and Achenbach [1985] and Baik and Thompson [1985] model an imperfect interface as an array of thin cracks. Angel and Achenbach [1985] report that the thin strings of material between the cracks are able to transmit a considerable amount of energy at certain frequencies. Baik and Thompson [1985] model the imperfect interface by a thin layer (thickness $\ll$ incident wavelength) with an effective stiffness and mass comparing the results with those obtained by Angel and Achenbach [1985]. They found good agreement with Angel and Achenbach's result for the region where layer thickness $\ll$ incident wavelength. Since the cylindrical shell model could be easily modified to include visco-elastic effects in the shell material it was decided to investigate the effect of a thin visco-elastic layer on the backscattering from a cylindrical inclusion. This would also be in keeping with the assumption made by Mal and Bose [1974] that an imperfectly bonded inclusion can be assumed to be separated from the matrix by a thin layer of visco-elastic material.

The Kelvin-Voigt model was used to describe the visco-elastic behaviour of the shell, this follows work by Anson and Chivers [1990] and Gaunard and Kalmins [1982]. In Gaunard and Kalmins' work the Lamé constants are assumed to be complex with the real

part being the elastic quantity, and the imaginary part the viscous quantity. Following the work by Anson and Chivers, defining the problem and solving the Helmholtz equations in terms of the wave numbers, changes can be simply made by re-defining the appropriate wave number. This can be seen in chapter 3 for the inclusion of viscous fluid shells (equation 3.25). For the case of a visco-elastic material, the complex longitudinal and shear wave numbers are given by:

$$k_c^2 = \frac{\omega^2}{c_c^2 \left[1 - \frac{i\omega(\lambda_v + 2\mu_v)}{\rho c_c^2} \right]}, \quad (4.3)$$

and

$$k_s^2 = \frac{\omega^2}{c_s^2 \left[1 - \frac{i\omega\mu_v}{\rho c_s^2} \right]}, \quad (4.4)$$

where λ_v and μ_v are the viscous components of the Lamé constants. The parameters for the shell material were chosen to be those of RTV 615 silicone rubber, ^{and are close to those of the materials under consideration} (since they are known). They are given below in table.4.2.

Table.4.2 Properties of RTV 615.

c_c	c_s	ρ	μ_v	$\omega\lambda_v$
1083 m s ⁻¹	65.7 m s ⁻¹	1020 kg m ⁻³	0.149 Nsm ⁻²	6.3410 ⁶ Nm ⁻²

These values were taken from Madsen et al [1983], Folds [1984] and Gupta [1979].

With the shell set to a visco-elastic solid with the parameters of RTV 615 silicone rubber, the backscattering was calculated for a $10\mu\text{m}$ radius glass cylinder in a perspex matrix. The shell thickness was varied from $0.01\mu\text{m}$ to $1\mu\text{m}$. The ka range covered was $0 < ka < 4$, $ka=4$ being equivalent to 171.25 MHz. This was felt to give a realistic maximum frequency for NDE work (obviously techniques such as acoustic microscopy use much higher frequencies but are usually restricted to the laboratory). The step size used in the calculation was $\Delta ka=0.025$ in order to show up any sharp resonant structures which may be present. Figure 4.7 shows the results of the computations.

With a shell of thickness just $0.01\mu\text{m}$ there is little obvious difference between the bonded fibre (no shell included) and the curve for such a thin layer. This scattering curve is quite different to that obtained with thin air shell model of a debonded fibre [fig.4.2]. With increasing shell thickness, the level of backscattering drops over the range $0 < ka < 2$, possibly due to the very lossy nature of the visco-elastic layer. It is quite noticeable that for the 1 micron shell the form of the backscattering is markedly different from that of the bonded fibre, while the $0.1\mu\text{m}$ and $0.01\mu\text{m}$ layers still show the same (or similar) structure as the bonded system. This general attenuation of the region below $ka = 3$ is very similar to that seen for the slip model as the effective shear modulus was decreased, although there is no indication here of the sharp resonance seen around $ka = 3$ for $K < 10^{14} \text{N m}^{-3}$.

Figure 4.7: Backscatter form function from a glass fibre surrounded by a thin layer of visco-elastic material. Shell thicknesses used are $0.01\mu\text{m}$, $0.1\mu\text{m}$ and $1\mu\text{m}$.

Although the results of the visco-elastic shell model look promising, the major problem is deciding what the parameters of such a shell should be to model best an imperfectly or debonded scatterer. As can be seen from table 4.2, five different parameters are needed to characterise the behaviour of the shell material. In the above calculations silicone rubber was used to model the effects only because it was a material whose properties were known.

The slip model is attractive since there only one parameter is needed to describe the effect of debonding (K , the effective shear modulus), and further work to include losses (by allowing K to become complex) needs to be done. The major question is whether a debond is best modelled as a thin crack, in which case the scattering would be expected to be that from a void, or if a model allowing the matrix and fibre to remain in contact is better, the debond having an attenuative effect on the scattering. The only way to determine this is by an experimental investigation. For practical reasons this was done on a scale model of a debonded fibre, and it is this which is the subject of the remainder of the thesis.

CHAPTER 5

EXPERIMENTAL PROCEDURES.

5.1 In order to investigate the predictions made using the theoretical models it became necessary to determine the best manner in which to carry out detailed experimental studies. The problem was how to make backscattering measurements over a wide range of frequencies on a solid cylindrical inclusion embedded in a solid, elastic matrix. This had to be done with and without damage at the interface of the fibre (i.e. the cylindrical inclusion) and the matrix, and accurately enough to be able potentially to discriminate between a fibre with damage and an undamaged fibre.. Three approaches were looked at as the possible basis of an experimental technique, these were:

- i) Tone burst methods
- ii) Time Delay Spectrometry
- iii) Short Pulse Techniques

5.2 Tone Burst Methods.

Figure 5.1 shows the arrangement of the equipment for making tone burst measurements. The pulse generator is used to gate the signal from the signal generator thus creating a short pulse of several cycles (typically of the order of 20 cycles) at a preset centre frequency. This means that the transmitted frequency is quasi-continuous, oscillating at the frequency set on the signal generator. The repetition rate of the pulse generator is chosen to be such that the time between tone bursts is greater than the time needed for reverberations in the water tank to die down to an

Figure 5.1: Diagram of the system used for making tone burst measurements.

Figure 5.2: Angular scattering measurements from an aluminium cylinder in water at $ka=23$. $\pm$ indicates the measurement uncertainties (see p.84).

Fig. 5.1 Tone Burst System

Fig. 5.2

insignificant amplitude. The "synch" output of the signal generator is used as an external trigger for the oscilloscope.

The great advantage of this method is that a good signal to noise ratio can be achieved, since the majority of the transmitted energy is centred about a narrow frequency band. However to make measurements over a wide frequency band with a very fine step size, (since resonance features in backscattering form functions can be very sharp, see fig. 3.4), can be very time consuming unless an automated system is used to increment the frequency of the transmitted signal sufficiently precisely.

At a single frequency, angular scattering measurements can be made on cylinders. This type of measurement was used on an aluminium cylinder immersed in water (see fig. 5.2). In order to calculate the form function, $F_c(ka)$, from the measured voltage, V_c , at the receiving hydrophone the relation [Hickling, 1958]

$$F_c(ka, \theta) = \frac{V_c(\theta)}{V_o} \left[\frac{2R}{a} \right]^{1/2} \quad (5.1)$$

was used where $F_c(ka, \theta)$ was the value of the form function at a given value of ka and θ , and V_o is the voltage for the incident pressure at the centre of the scatterer. This was measured by removing the cylinder and placing the receiving hydrophone at the position where the centre of the cylinder had been. R was the radial distance from the centre of the scatterer to the hydrophone. The hydrophone was rotated about the axis of the cylinder on an arm fixed to a point on the axis, hence it was straightforward to measure the radial distance, R . The radius of the scattering cylinder was a .

The agreement between theory and the experimental measurements of the backscatter form function as a function of angle for the solid aluminum cylinder was not particularly good but of the correct order of magnitude. There is a sizable discrepancy in the forward scattering direction ($\theta = 0^\circ$), this is due to the the problems of making forward scattering measurements.

In general the measured values of the scattering follow the calculated curve as displayed in figure 5.2. However it should be born in mind that the uncertainties in the measurements, especially in the precision with which the angle was measured, are very large. Typically the error in the angle was of the order of a degree due to the play in the shaft to which the hydrophone was attached, this in turn could lead to variations in the received signal associated with a given scattering angle of about 50% since the scattering varies very rapidly with angle.

In taking the voltage measurements it was necessary to try and take the measurement at the same point in the tone burst, since the envelope shape of the tone burst was seen to change as the position of the receiver was changed. This was most noticeable at the forward and rear parts of the tone burst where transient effects in the transducer are most prevalent, for this reason the centre of the tone burst (well away from the transient effects) was the region where the measurements were taken. The variation in the envelope was seen to be quite small over about 10 cycles that covered this region, any errors due to this were felt to be insignificant to those introduced due to variation in the hydrophone's angular position.

The angular scattering measurements were made primarily to gain some appreciation as to whether or not the theoretical model described in chapter 2 was physically reasonable and a suitable basis for the investigations into the effect of debonding on ultrasonic backscattering. From the results shown in Fig. 5.2 it can be seen that the theory ^{is in keeping with} the angular scattering.

Although the tone burst method was used for the angular scattering measurements it was felt, for the reasons discussed above, that it was not a viable method to use for making broad-band backscattering measurements, primarily because it is a narrow band technique.

5.3 Time Delay Spectrometry

The technique of time delay spectrometry (TDS) was developed by Heyser in 1967 in order to make acoustic measurements on transducers. Since then the technique has been developed for use in ultrasonic systems [Heyser and Le Croisette 1974, Pedersen et al 1988]. It is basically a swept frequency measurement system that allows measurements to be made under reverberant conditions. Figure 5.3 shows a block diagram of a typical arrangement to make TDS measurements.

The majority of TDS work has been involved with transmission measurements, and this is the technique that will be outlined here. The oscillator which drives the transmitting transducer is repeatedly swept across the frequency range of interest. In addition to exciting the transducer the oscillator also drives a narrow-band tracking filter which is swept at the same rate as the

Figure 5.3: Diagram of a time delay spectrometry system.

Figure 5.4: Diagram of the TDS system used by Gammell to make backscattering measurements.

Fig. 5.3 TDS System.

Fig. 5.4 Gammell's Single Transducer TDS System.

transmitted signal. The output of the receiver passes through the tracking filter and on to whatever display device is in use (CRO, x-y plotter etc.).

Since the transmitter and receiver are separated by some distance, x , there is a time delay between the signal at a particular frequency being transmitted and being received, which depends upon the transit time between the transmitter and the receiver,

$$\frac{x}{c} = t \quad (5.2)$$

where c is the phase velocity in the medium separating the receiver and transmitter, and t is the transit time. This means that the tracking filter will be set to a higher frequency than the signal currently arriving at the receiver. Consequently the centre frequency of the tracking filter needs to be offset. The offset is given by;

$$f_t - f_r = \frac{x}{c} \cdot \frac{df}{dt} \quad (5.3)$$

where f_t is the frequency of the transmitter, f_r is the central frequency of the receiving filter, and $\frac{df}{dt}$ is the linear frequency sweep rate. The frequency offset, rather than being a problem, is in fact of particular value since by adjusting the frequency offset the system can be "tuned" to a particular propagation path, and will discriminate against all other signals incident on the receiver at the same time. Typically it is the direct path that is of main interest and any reverberations will fall outside the bandwidth of the narrow-band tracking filter on the receiver.

In addition to being able to make frequency response

measurements under reverberant conditions the use of the narrow-band filter on the receiver allows very high signal to noise ratios to be achieved. By reducing the width of the filter the signal to noise ratio can be increased. It is this feature of TDS technique that allows measurements to be made over a very large frequency range. However, as the tracking filter width is reduced the time needed to make a swept frequency measurement is increased, this is the limiting factor to making TDS measurements.

As was stated above the majority of work using TDS has been involved in making transmission measurements. Since it is the backscattering that is of interest here this means that a single transducer would have to be used.

This problem of making single transducer swept frequency ultrasonic backscattering measurements has been addressed [Gammell, 1979] for use on human tissues. Gammell indicates that the electronic signal at the transducer will consist of a very strong electrical excitation in combination with the much weaker received signal. In his measurements the difference frequency was extracted, and the amplitude of this signal was plotted against the time from the beginning of the sweep, which is proportional to the transmitted frequency. The method Gammell used (see Fig. 5.4) for the detection of the difference frequency was a simple envelope detector consisting of a full-wave rectifier and two stages of R-C filtering. In addition he used a power splitter in order to reduce the strength of the signal applied to the detector. Gammell indicates that this system was far from perfect in operation, but was adequate for the frequency range of interest to him (2 MHz to

9 MHz).

Although TDS appears to be a very attractive solution to the problem of making accurate backscattering frequency measurements it does require some specialized equipment, namely a narrow band tracking filter which can be swept linearly in time. Some Hewlett Packard spectrum analysers incorporate a tracking generator with frequency offset but the problems of cost and long term availability meant that this method could not be implemented for the present work.

For the reasons discussed above a pulse-echo technique was chosen as the method to be pursued with which to carry out the main experimental scattering investigations.

5.4 Short Pulse Technique.

Figure 5.5 shows the arrangement of the apparatus that was used to make the broad-band backscattering measurements. A single transducer is used as both the transmitter and the receiver. A high voltage, short duration pulse is used to excite the transducer. This causes the transducer to ring for a few cycles (dependent on the Q factor of the transducer). This short duration pulse, when examined in the frequency domain, is composed of the frequency response of the transducer and also the spectral characteristics of the source used to drive the transducer.

The transducer used for the initial measurements was a commercially available Panametrics broad-band transducer with a nominal centre frequency of 7.5 MHz. This was the only broad-band

Fig. 5.5 Experimental Arrangement

Figure 5.5: Diagram of the experimental arrangement used to make broadband backscattering measurements.

transducer with a high sensitivity that was available within the group at that time. Immersion transducers were chosen since it was felt that the problems associated with reproducibility of coupling a contact transducer to the epoxy surface of the test sample with a consistent pressure would be quite large whereas immersion transducers are not subject to this type of problem. In addition the acoustic impedance mismatch between water and epoxy is not particularly large ($1.485 \text{ kgm}^{-2}\text{s}^{-1}$ for water, $3.168 \text{ kgm}^{-2}\text{s}^{-1}$ for epoxy) and hence the ultrasound is reasonably well coupled into the sample.

The pulser-receiver used to excite the transducer was made by Ultrasonic Sciences Ltd and was controlled via an interface bus with a personal computer. The pulser-receiver was capable of producing 200V pulses with a bandwidth of 2 to 50MHz (at the -3dB points). Attenuation of the received signal could be selected from 0 to 63dB in 1dB steps.

The output signal from the pulser-receiver was captured on a digital storage oscilloscope, and the signal from the scatterer, easily identified by the time of the returned signal, gated out. To improve the signal to noise ratio multiple averaging was used, this was one of the functions incorporated in the oscilloscope. It was found that the maximum averaging of 256 acquisitions per average produced the best improvement (as would be expected), but this tended to make the measuring time quite long: 15 minutes per measurement was typical.

Once the signal had been captured and digitised it was then transferred to the personal computer via an IEEE 488 interface bus

for signal processing.

5.4.1 Signal processing

The transferred signal consisted of a 1024 data point file with each point being given an ASCII value relative to the bottom of the screen on the oscilloscope (this being 0). This meant that the whole r.f. signal is digitised as positive numbers and not relative to the electrical ground. Consequently the first process was to convert the data file normalising this to the peak voltage of the signal, which was measured directly from the oscilloscope screen. Ground was determined by taking an average of the first ten data points. This was then subtracted from the file. The resulting data file was now ready for Fourier transformation and deconvolution of the transducer response.

It is assumed that the signal pathway (pulser electronics, transducer, water bath and through the target) is linear and time invariant, and consequently the signal detected and displayed is given by;

$$y(t) = x(t)*g(t), \quad (5.1)$$

where $x(t)$ is the signal pathway (transducer response, measurement electronics etc.), $g(t)$ is the response of the scattering object, and $*$ represents a convolution in time. By taking Fourier transforms the following relation is obtained;

$$Y(\omega) = X(\omega) \cdot G(\omega), \quad (5.2)$$

where $Y(\omega)$, $X(\omega)$ and $G(\omega)$ are the Fourier transforms of $y(t)$, $x(t)$ and $g(t)$ respectively, with ω as the angular frequency. From 5.2 it can be seen that by dividing $Y(\omega)$ by $X(\omega)$ the frequency response of the scatterer is obtained.

In order to do this, it is necessary to be able to determine the frequency response of the signal pathway. This was done by measuring the front face reflection from a large flat reflector at normal incidence to the incident beam, the reflector was approximately 3 cm from the transducer face. Since the reflector is flat and the ultrasonic beam impinges at normal incidence, the frequency response of it is basically equal to 1 (i.e. $G(\omega) = 1$). Consequently the Fourier transform of this signal is the frequency response of the signal pathway. It has been shown [Chivers and Duck, 1978] that it is important that the dimensions of the reflector must be larger than the dimensions of the beam. For this work two different reflectors were used, the first was a block of perspex with polished faces and the other was a block of glass. In both cases the surface roughness was much less than the wavelength of the interrogating ultrasonic beam (for 10 MHz in water this is 0.15 mm).

The above outlines the analysis used in the experimental investigations of model composites. However before the effect of debonding was examined, the experimental procedure had to be tested to see if it was a viable method of making measurements of backscattering form functions for cylinders. To do this, measurements were made on cylindrical scatterers immersed in water.

5.4.2 Solid scatterers in water.

The problem of calculating the backscattered form function for a cylindrical scatterer immersed in water has been addressed by

several authors [Humphrey et al 1990, Neubauer et al 1974]. Consequently it was known that theory and experiment agreed well for this type of problem and would be a good test of the present pulse-echo system as a quantitative tool for future investigations of solid scatterers embedded in solid matrices.

Brass and copper wire were used as the different cylindrical scatterers, the elastic parameters are:

	c_e (m s ⁻¹)	c_s (m s ⁻¹)	ρ (kg m ⁻³)
Copper	4759	2325	8933
Brass	4372	2100	8500

[Anson and Chivers, 1990]

The radius of the brass wire was 0.397 mm and that for the copper wire, 0.450mm. For the bandwidth of the transducer (3 MHz to 10 MHz) this leads to a ka range of $5.0 < ka < 15.0$ for the brass scatterer, and $8.0 < ka < 18.0$ for the copper one.

The window used to gate out the signal from the scatterer was 20 μ s, the delay time base being set to 2 μ s/div. It was important to ensure that no portion of the reverberations within the water tank were accidentally captured along with the desired signal. This was achieved by moving the position of the wire closer to the transducer so that the returned signal from the wire was time resolved from tank reverberations. In addition, if the wire was too close to the transducer then the relatively large main r.f. pulse (when compared to the specular reflection from the wire, typically 10:1) can overload the amplifiers on the oscilloscope, when the gain is increased to increase the resolution of features in the backscattered signal. This leads to distortion of leading part of

the backscattered signal.

In order to ensure that the wire was at normal incidence to the ultrasonic beam the transducer's position was varied until the specular reflection from the cylinder was a maximum.

The gated signal was digitised into 1024 data points, this leads to a step size of 20 ns, which equates to a sampling frequency of 50 MHz. From Nyquist sampling theory this means that the maximum bandwidth possible is 25 MHz (i.e. the maximum sampled frequency is the sampling frequency divided by 2). After the digitised signal had been Fast Fourier Transformed (FFT) the resulting file was composed of frequencies from 0 to 25 MHz (the first 512 points) and from -25 MHz to -48 kHz (the second 512 data points, i.e. from 513 to 1024). Obviously only the positive frequencies are of interest.

Since the FFT program produces a file of complex numbers the actual data file produced is the amplitude of the frequency power spectrum which is given by:

$$Z(\omega) = \sqrt{X(\omega)^2 + Y(\omega)^2} \quad (5.3)$$

where $X(\omega)$ and $Y(\omega)$ are the real and imaginary parts of the FFT, $Z(\omega)$ is the amplitude of the power spectrum at the frequency ω . The step size in the frequency domain was 48.828 kHz.

After the FFT and deconvolution of the signal pathway the resulting frequency response of the cylindrical scatterer had to be multiplied by a factor to compensate for the cylindrical spreading of the backscattered signal. This was given by [Humphrey et al, 1990]:

$$\text{factor} = \sqrt{\frac{2r}{a}} \quad (5.4)$$

5.4.3 Results of backscattering measurements on immersed cylinders.

The measured backscattering from the brass rod in water is shown in Fig. 5.6. In comparison the theoretical prediction is given (the continuous line), the water was assumed to be viscous and attenuation was included in the calculations. Parameters for water are given in Chapter 2. The ka range is from $ka = 5.0$ to $ka = 15.0$, the radius of the scatterer being 0.397 mm.

The structure of the backscattered form function is basically six large, equally spaced resonances (at ka 6.48, 8.0, 9.49, 11.06, 12.54 and 13.99), with further features, such as the minima at $ka=8.43$ and 10.91 superimposed. These resonances can be explained in terms of surface wave modes circumnavigating the cylindrical scatterer and then interfering with the specularly reflected wave [Flax et al, 1981]. Different families of waves resonantly excited on the cylinder surface correspond to the regularly spaced features seen in the backscattered form function (i.e. the six main resonances seen have a spacing of approximately $\Delta ka = 0.15$ suggesting that this is a particular family of waves). In order to analyse the form functions in detail, to identify all the modes of vibration excited on the cylinder it is necessary to use Resonant Scattering Theory [Flax et al, 1981].

The basic assumption behind this theory is that the scattered wave is composed of two parts, the first being a background component which for a solid cylinder in a fluid medium is the

Figure 5.6: Comparison of the theoretical backscatter form function from a brass cylinder in water (continuous line) and the experimental measurement (crosses joined by dashed line).

Figure 5.7: Comparison of theory (continuous line) and experiment for the scattering from a copper cylinder in water.

Fig. 5.6

Fig. 5.7

response of a rigid cylinder, and the second is the resonance component. By subtracting the rigid background component from the scattering and decomposing the result in terms of the partial waves, it is possible to identify particular resonances and characterise families of waves. A detailed explanation of Resonant Scattering Theory is given in Flax et al [1981]. To analyse properly all the resonance components is very time consuming and, for the purposes of this work, unnecessary (the final intention being to detect changes in the backscattering response from a fibre due to debonding). Consequently backscattering responses (measured and calculated) will only be described in general terms rather than by a detailed identification of the resonances.

The agreement between theory and experiment in fig. 5.6 is reasonable with regards to detecting the positions of the main resonances successfully. (The theoretical curve was calculated using the model described in sections 3.1 and 3.2 with a step size of $\Delta ka=0.05$.) However there are some discrepancies between theory and experiment around $ka = 11$ where it is debatable whether the two sharp minima either side of the large resonance have been detected. The minima at $ka = 8.5$ is clearly identified but the measured value sits far too high.

A problem with making the measurements for the scatterers immersed in water was that the backscattered signal in the time domain contained components (due to circumferential waves) which arrived at the receiver outside the acquisition window. The full effects of this are not totally appreciated as yet, but may well account for some of the discrepancies seen between theory and

experiment. In addition there is the problem of transducer near field effects which have been ignored (typically the last axial maximum for the transducers used would be 28 cm from the transducer face, measurements are taken in the region of 1 cm from the transducer.)

Figure 5.7 shows the measured and calculated backscattering for the copper wire immersed in water. The structure of the backscattered form function for the copper shares many features in common with the brass cylinder, which is not surprising since they have very similar wave speeds and densities. The radius of the copper wire was 0.45 mm which was slightly larger than that for the brass wire, which explains why the ka range is from 8.0 to 18.

The first four large resonances are very similar in structure to those seen from $ka = 8.0$ in fig. 5.6, as is the sharp minimum at $ka = 8.4$. However the resonances in fig. 5.7 occur at slightly higher values of ka , for example the $ka = 8.0$ resonance in fig. 5.6 appears at $ka = 8.8$ in fig. 5.7. This shift is directly attributable to the increase in the wave speeds from brass to copper, the most significant parameter here being the shear wave speed. If the shear wave speed for the brass is increased to 2300 m/s with the longitudinal wave speed and density kept the same the effect on the backscattered form function (according to auxiliary calculations performed) is to shift the main resonances to higher ka with some minor changes to the structure of the scattering. The shift corresponds to that seen between figures 5.6 and 5.7, which is thus expected since the shear wave speed for copper is 2325 m/s. The spacing between the main resonances in

Fig. 5.7 is approximately $\Delta ka = 1.6$ which is slightly larger than that seen for the brass wire, this is consistent with the slightly larger circumference of the copper wire.

The agreement between the theoretical curve and experiment was somewhat better than seen for the previous measurement, the positions of the resonances can be clearly determined and are in accord with those predicted by theory. In addition the size of features is clearly in better agreement than was previously the case. The sharp minimum at $ka = 12$ has not been captured, as was the situation for the resonance at $ka = 11$ for Fig. 5.6.

The larger radius wire helped to improve the signal to noise ratio by providing a significantly larger scattering target, this in turn helped to reduce the effect that noise had on the signal processing. It is fairly certain that poor signal to noise ratio is a limiting factor in accurately measuring the backscattered form function. This can be seen at the extremes of the ka range where the transducer sensitivity is relatively low, i.e. beyond the -3 dB points.

5.5 Summary.

The reasons for deciding to use a short duration pulse-echo method as the basis of the experimental method have been discussed above. It has been shown that in principle (from the measurements made on scatters submersed in water) it is possible quantitatively to investigate the backscattering form function. The major problem with this approach is obtaining a reasonable signal to noise level which will allow features in the backscattering response to be

identified. With this problem in mind it was decided to investigate the backscattering from solid cylindrical scatterers embedded in a solid matrix, and to see if it was possible to detect the effects of debonding on the backscattering.

CHAPTER 6. ULTRASONIC LONGITUDINAL BACKSCATTERING MEASUREMENTS ON SOLID CYLINDRICAL SCATTERERS IN A SOLID MATRIX

6.1 Introduction

In chapter 4 several models were proposed to describe the effect of fibre debonding on ultrasonic backscattering. These included the thin air shell model, where a thin layer of air was assumed to separate the fibre and matrix mimicking a crack, the introduction of a thin viscoelastic layer between fibre and matrix, and finally the slip model which allows tangential slip between the fibre and matrix related to an effective shear modulus. As was seen these models gave varying predictions as to the effect on the backscattering (especially comparing the thin air shell with the other two). Consequently it was necessary to investigate experimentally the effects (if any) that could be detected using ultrasonic backscattering. The basic method was explained in detail in the previous chapter where it was shown in principle accurate, repeatable measurements can be made on cylindrical scatterers (albeit only immersed in water).

Although the problem of scattering from solid inclusions in solid matrices has been studied by previous authors [Ying and Truell 1955, White 1958, Olssen et al 1990] the majority of the work has only been concerned with theory, little work has been published on experimental investigations. Sayers and Smith [1983] have investigated velocity and attenuation measurements published by Kinra et al [1982] in an epoxy resin containing low volume

concentrations of lead inclusions (5% and 20%). These measurements were compared with the predictions using Ying and Truell's single scattering theory. In a recent paper Addison and Sinclair [1991] studied the backscattering response of a Silicon Carbide (SiC) fibre embedded in an Aluminium matrix. The experimental method that they used was not outlined in any detail but they present the measured backscattered response in the frequency range $0 < f < 100\text{MHz}$. For the SiC fibre in the samples they examined, this equated to a ka range of $0 < ka < 7.2$. The experimental data was compared with the theoretically expected scattering, as calculated using the work of both Faran [1951] and White [1958].

Their results showed a good agreement between theory and experiment for the ka range $2.5 < ka < 6.5$ on some sections of the fibres examined. In other regions of the fibre there was no correlation seen between theory and experiment. This Addison and Sinclair attribute to debonding of the fibre and matrix, although they gave no further analysis of the result.

The subsequent sections of this chapter will deal with the backscattering measurements on solid cylindrical scatterers embedded in an epoxy resin matrix and with the effects of debonding of the scatterers on the backscattering.

6.2 Experimental samples.

The samples used for the experimental investigations were composed of a single wire (the fibre) cast in a block of epoxy resin (the matrix). Several considerations had to be taken into account in making the samples, these included the dimensions of the

sample and the materials used in the construction.

The dimensions of the epoxy block are particularly important. The near field of the transducers used extended (at 7.5MHz) for 12.5cm. Since it was thus necessary to work in the near field, it was decided to work as near to the transducer as possible to optimise the signal to noise ratio. However the backscattered signal from the front face of the block and from the embedded fibre need to be resolved in time. Typically a 7.5MHz transducer can be expected to ring for a few cycles after being excited by the high voltage, short duration electrical pulse. Assuming that the transducer rings for 5 cycles, then the duration of the resulting signal is $0.67\mu\text{s}$. This was rounded up to $1\mu\text{s}$. For a longitudinal wave propagating in perspex the wave speed is 2690 ms^{-1} , and in $1\mu\text{s}$ the wave will travel 2.69mm. Consequently if the fibre is set 3mm back from the front face of the epoxy block then the signal from the fibre will be time-resolved from the front face reflection. Since the width of the window for the delay time base was $20\mu\text{s}$ ($2\mu\text{s cm}^{-1}$) it was necessary to have the distance from the front face to the back face of the epoxy block sufficiently great to be able to gate out the back face reflection from the block. In $20\mu\text{s}$ a longitudinal wave in perspex will travel 5.31cm, hence the thickness of the block was chosen as 3cm. In considering the dimensions of the front face of the block, the size of the incident beam was considered. The transducer element had a radius of 5mm so typically the width of the incident beam is going to be of the order of 1cm in diameter. Hence the front face dimensions were 3cm $\times$ 3cm, figure 6.1 shows the dimensions of the samples.

Fig. 6.1 Dimensions of sample.

Note: Retaining screws for holding wire captive in the tensioning mechanism have been omitted from the drawing.

Fig. 6.2 Mould for casting wire reinforced samples

resin. The mould had been constructed from perspex and obviously a hot curing epoxy could not be used. In addition, using a cold setting resin meant that the samples could be made in the laboratory without needing an oven. The resin was commercially available Araldite MY753 and the hardener HY951. Typically it was found that a mix of resin to hardener in the ratio of 11:1 gave the best results in terms of producing clear samples and little effects due to the exothermic reaction of hardener and resin which can cause small areas on unset resin.

The mould was designed and built in the departmental workshop. Figure 6.2 shows a diagram of the mould. The inner surfaces of the mould had to be rubbed with silicone grease to ensure the epoxy resin would not bond to them. This treatment was also applied to the joints of the mould to allow it come apart easily. The wire was tensioned so that it was parallel to the face of the block, this would help to ensure that the incident ultrasonic beam during measurements would be normal to the wire.

After the mould had been filled with resin/hardener mix it was then placed in a water bath, the purpose of this was to reduce the heat build-up in the resin by acting as a heat sink. If this is not done then the epoxy resin will set hard in some places, and not at all in others. While setting, the mould and water bath were placed under vacuum to force any air bubbles in the resin to be expelled. Typically the assembly would be left for 24 hours to ensure proper setting.

The choice of copper and steel wires as the fibres was due to availability of wire of suitable gauges. Since the ^{only} transducers

available at the time were the 7.5MHz immersion transducers (bandwidth: $3\text{MHz} < f < 10\text{MHz}$) the only parameter left with which to define the ka range of interest is the radius, a . From Chapter 3 it was shown that low ka ranges were likely to be of practical interest (for a $20\mu\text{m}$ radius fibre $ka = 5$ gives a frequency of 105MHz in an epoxy resin matrix). Steel and copper wire were readily available with a radius of 0.35mm . Using the 7.5MHz transducers this gave a ka range of $2 \leq ka \leq 8$ which was essentially the region of interest. Smaller gauge wire could have been used but since (as was discussed in the previous chapter) the signal to noise ratio is very important in making accurate measurements of the backscattering, a larger scatterer was desirable. Essentially the choice of scatterer was a trade off between the availability of material and the gauge of wire.

After the sample had been removed from the mould, the surfaces were polished (mainly the front face) in order to reduce surface roughness to a level less than that of the wavelength of the incident ultrasonic wave (0.2mm). In practice no difference was seen between the results from samples with the surface unpolished and those with the surface polished. The major advantage to a polished front face is that it is much easier to get a clear view of the wire.

6.3 Scattering from embedded cylindrical scatterers.

The basic method for making backscattering form function measurements has been discussed in Chapter 5. In order to use the same method for a solid scatterer embedded in a solid matrix a

further correction has to be made to compensate for transmission through the water/epoxy interface. This factor is given by:

$$T = \frac{4R_1R_2}{(R_1+R_2)^2} . \quad (6.1)$$

where R_1 and R_2 are the respective acoustic impedances for the water and matrix materials.

The purpose of the first measurements was to test the accuracy of the scattering models discussed in Chapter 3. These had already shown a good agreement with the measurements on cylindrical scatterers immersed in water (Chapter 5). Unfortunately initial attempts at making measurements met with failure. There were two main reasons for this. Firstly since the signal from the embedded wire was not particularly strong (approximately 0.25V p-p compared with 4V p-p for the front face reflection), the effect of noise on the signal was quite marked, masking most of the signals following the specular reflection from the wire's surface. The signal to noise ratio could be improved by increasing the amount of time averaging performed by the oscilloscope (This could be increased up to the maximum of 256 acquisitions of the waveform per average.)

Increasing the amount of averaging revealed a further problem. Owing to slight jitter in the signal, continued averaging had the effect of averaging the signal to zero, once again losing any useful information content. By sampling the signal at four times the previous sampling rate (200MHz as opposed to 50MHz) it became possible to time average the signal accurately. However if this signal is then Fourier transformed there is insufficient resolution in the part of the frequency domain of interest. Thus 200MHz

divided into 1024 points (all waveforms were digitised into 1024 points) leads to a frequency step size of $\Delta f = 195.31\text{kHz}$. Over the frequency range of interest ($3\text{MHz} < f < 10\text{MHz}$) this would mean only 35 points. This is really too coarse for accurate measurements. By only using every fourth point in the digitised time domain signal the sampling rate could be artificially reduced to 50MHz, thus increasing the number of data points in the relevant frequency range to 140. In this way accurate measurement of the time domain signal can be made without the disadvantage of little resolution in the frequency domain.

Using this technique it was possible to make reliable measurements of the backscattered form function for a solid cylindrical scatterer embedded in a solid matrix.

6.3.1 Copper wire embedded in an araldite matrix.

The results of the backscattering measurement for the copper embedded in araldite are shown in figure 6.3 as crosses; the theoretical prediction is the continuous line. Agreement between the theory and the experimental data is very good over much of the ka range, especially for $2.5 < ka < 7.5$. Over all of the range of ka displayed the positions of features such as the minima in the form function are clearly defined, and the relative sizes of the resonances are in keeping with those predicted by the model.

For $ka < 2.5$ and for $ka > 7.5$ the experimental points diverge quite noticeably from theory. This can be explained on the basis of errors occurring during deconvolution of the signal pathway contribution in the frequency ranges where the transducer

Figure 6.3: Comparison of the theoretical prediction (continuous line) and measured backscatter form function from a copper wire in an epoxy resin matrix (crosses).

Figure 6.4: Frequency response of the signal pathway.

Copper Wire in Polymeric Matrix

Fig. 6.3

Signal Pathway Response

Fig. 6.4

sensitivity is quite low. Figure 6.4 shows the response of the signal pathway (essentially the response of the transducer) as a function of acoustic radius indicating that at $ka = 2.5$ and $ka = 7.5$ the response is approximately $1/3$ of the maximum. In addition in fig. 6.3 there appears to be a systematic error which causes the experimental data to lie slightly above the theoretical curve, which maybe due to small errors in the terms used to compensate for transmission through the water-epoxy interface of the specimen, and for the cylindrical spreading of the backscattered signal from the wire.

6.3.2 Steel wire embedded in an araldite matrix.

The measured backscattering form function for a steel wire embedded in an araldite matrix is given in Figure 6.5 as a series of crosses (the data points) , the theoretical curve is a continuous line. As for the copper wire the overall agreement between theory and the experimental data is good. Since the gauge of the steel wire was slightly larger than the copper (0.37mm as opposed to 0.36mm for the copper) the ka range here is $3.0 < ka < 9.0$. The major divergence between theory and experiment is below $ka = 4.0$ where the experimental points lie a long way above the theoretical curve. This is almost certainly due to the problems with deconvolving the signal pathway response from the total signal in a region where the transducer is not particularly sensitive. Above $ka = 7.5$ there is a further divergence but the positions of the two resonances and the minima predicted by the theory are clearly detected in the experiment, this is once again a

Figure 6.5: Comparison of theoretical prediction (dashed line) and measured backscattering from a steel wire in an epoxy resin matrix (crosses joined by line).

deconvolution problem.

Overall Figs. 6.3 and 6.5 show a high degree of correlation between the theoretical predictions for the backscattering and the measured backscattering. Positions of features such as sharp resonances and the minima can be clearly defined. This is very important if, as expected, the shape of the resonance spectra are altered by debonding of fibre and matrix. The absolute size of the structure of the measured backscattered form functions is not so reliable being subject to errors in the deconvolution process. In addition there may well be errors occurring in the correction factors for transmission through the water-epoxy interface and the cylindrical spreading of the backscattered signal. These errors would be due to possible variations of the wavespeeds and densities for the matrix material and for water from the published values [Kaye and Laby 1973, Anson and Chivers 1990] as well as errors in measuring the radial distance from the scatterer to the transducer. In addition there are other uncertainties, for example the measurements are taken in the near field of the transducer, this may well introduce some factors into the observed scattering. Problems such as these have yet to be investigated, but in view of the generally good agreement between theory and experiment may well be relatively unimportant in a practical application.

6.3.3. Reproducibility of measurements.

Figures 6.6 and 6.7 show the backscattered form functions measured from different samples as an investigation of the reproducibility of the measurements. Fig. 6.6 is for samples

Figure 6.6: Four graphs showing the typical agreement between theory and experiment for the backscattering from a copper wire in an epoxy resin matrix.

Fig. 6.6

Copper Wire in Polymeric Matrix

Copper in Araldite

Cu in Araldite

Copper in Araldite

containing the copper wire. Here there is a very high degree of reproducibility between the measurements on the different samples. In all four cases the positions of minima are very clearly identified, this is a feature that has always been noticed in the measurements of the backscattering. The resonant structure from $ka = 3$ to $ka = 7$ is very well picked out in terms of its position. There is some variation in the absolute magnitudes of the resonances, for example the resonance at $ka = 4.5$ ranges in size from 1.25 to 1.6, the theoretical value is 1.5. Typically within the range $3 < ka < 7$ errors in the magnitudes of the structure of around 12% can be expected. Outside this region (i.e. in the low transducer sensitivity region) the errors are much larger. The actual structure of the resonances is reasonably well identified, in two of the graphs in fig. 6.6 the double resonance at $ka = 5.5$ is clearly seen with correct relative magnitudes of the two peaks. In the other two graphs, a double resonance is possibly identified but is not particularly clear. The problem in this case is whether the difference is due to experimental deviation or some difference between the four different samples (damage to the interface between the fibre and matrix, variation in the elastic properties of the materials, etc).

Figure 6.7 shows the backscattered form functions for four samples with the steel wire inclusion. The variation between the four measurements is rather more marked than for the copper wire samples (fig. 6.6). However for each case in the range $4.5 < ka < 7.5$ (the most sensitive region of the transducer) the two large resonances are clearly identified. Typically the peak at

Figure 6.7: Four graphs showing the typical agreement between theory and experiment for the backscattering from a steel wire in an epoxy resin matrix.

Steel Wire in Araldite Matrix

Steel Wire in Araldite Matrix

Steel in Araldite

Steel Wire in Araldite Matrix

$ka = 5.5$ shows a variation from 1.25 to 1.4 (theory predicts a value of 1.4). Thus the best accuracy that can be expected in the measurement of the magnitude of a resonance using this method will be about 10%. The parts of the scattering at the limits of the transducer sensitivity are subject to much larger errors in the magnitude of the structure. This suggests that the 10%-12% error seen at the centre of the transducer frequency band may well be due to errors such as the effect of noise masking some of the more subtle features in the time domain signal and introducing artifacts into the form functions.

To summarise, the position of the resonant structure in the measured backscatter form functions fits very well with the predictions made by the theory. The amplitudes of peaks in the backscatter form functions show a fairly good agreement with the theory but it is reasonable to expect at least a 10% uncertainty in the measured values.

6.4 Measurements on debonded wires.

Having shown that the backscatter form function for a solid cylindrical scatterer embedded in a solid matrix could be accurately measured, it was necessary to investigate the effect that debonding the inclusion would have on such measurements, and whether a particular feature or features could be identified. If this could be done then this effect could then be incorporated in a scattering model to predict the effect of debonds in engineering composites (i.e. carbon fibre reinforced plastics) on the backscattering, this could then possibly form the basis of a new

diagnostic technique.

The first problem to be overcome was how to ensure that the wire in the sample had indeed been debonded. This was carried out in a particularly crude but effective way by pulling the wire free from the block and then carefully re-inserting the wire in the hole. The re-insertion of the wire had to be done carefully since if the wire kinked, it became impossible to push it into the hole.

6.4.1 Debonded copper wire.

In figure 6.8 the measured backscatter form function from a sample with a debonded copper wire is shown as a series of crosses, the continuous line is the theoretical scattering for a well-bonded copper wire. There are a number of differences noticeable between the theory and experiment, namely the much smaller peak at $ka = 4$, the loss of the double resonance feature at $ka = 5.5$ and, most obviously, the difference in the positions of the minima at $ka = 7.5$ (theory) and $ka = 7.75$ (experiment). Yet these differences are relatively minor and are not very much larger than the variation between different measurements. If it was not for the fact that the wire was known to be debonded, then the experimental data could be claimed as a non-too-unreasonable fit to the scattering from a bonded wire! In order to detect any differences the measured backscattering before debonding and after must be compared.

In figure 6.9 the measured backscatter form functions before (continuous line) and after debonding (dashed line) are compared.

Figure 6.8: Comparison of the theoretical prediction of the scattering from a well bonded wire with the measured scattering from a debonded copper wire in an epoxy resin matrix.

Figure 6.9: Comparison of the measured backscattering from a copper wire embedded in an epoxy resin matrix before (continuous line) and after (dot-dashed line) debonding.

Copper Wire in Araldite Matrix

Fig. 6.8

Copper Wire in Araldite Matrix

Fig. 6.9

The most striking difference between the two curves is the shift to higher ka of the curve for the debonded wire. This is particularly clear for the minima at $ka = 3.75$ and $ka = 6.5$. In addition, for the bonded wire the double resonance at $ka = 5.5$ is present but not for the debonded wire. It would appear that the only sizable effect of debonding on the scattering is a small shift to higher ka for some of the features in the backscattering. The same experiment was carried out for a sample with a steel wire embedded.

6.4.2 Debonded steel wire.

Figure 6.10 shows the experimentally measured backscatter form function for the debonded steel wire (crosses joined by a continuous line) compared with the theoretical prediction for a well-bonded wire. In contrast to the results in Fig. 6.8 there is quite a marked difference between the scattering for a bonded wire and that for a debonded wire, these differences are significantly larger than the size of variation seen in the measurements on bonded samples. The minimum at $ka = 7.5$ is still picked out suggesting that the shift in ka seen for Fig. 6.9 is related to the scatterer being copper. Above $ka = 7$ the resonances appear to have been heavily attenuated. The signal in the time domain was rather smaller than observed for the bonded samples. Why the sample with the steel wire behaves so differently to the sample with the copper wire is not clear.

6.4.3 Discussion of initial debonding measurements.

The scattering from the debonded copper wire (fig. 6.8 and

Figure 6.10: Comparison of the theoretical prediction of the scattering from a perfectly bonded steel wire (dashed line) and the measured scattering from a debonded steel wire in an epoxy resin matrix.

6.9) indicate that the effect of debonding on the backscattering is rather small. It seems to be limited to a slight shift in ka of the positions of the resonances and minima, and an attenuation of the resonances. In contrast the steel sample (fig. 6.10) shows a large difference between the scattering for bonded and debonded samples, but no obvious shift in ka for any the features common to the bonded and debonded cases.

Since the frequency bandwidth of the incident ultrasonic wave has not been changed, the shift in ka suggests that the effective radius of the scatterer has increased. When the wire is pulled out there will be some damage caused in the matrix material surrounding the wire, in the form of micro-cracking for example. This may have the effect of increasing the effective radius of the scatterer. In addition to this, when the copper wire is being pulled out it will stretch. This has an acousto-elastic effect which may change the wave speeds in the copper wire.

The effect of changing the wave speeds in the copper is shown in figure 6.11. The longitudinal and shear wave speeds were increased by an arbitrary 2.5%. The dashed line is the scattering from the copper with the increased wave speed. There is an obvious shift in ka which is of the same order of magnitude as seen in fig. 6.9. Also the resonance at $ka = 6.5$ is about 10% larger as is seen in fig. 6.8. The reduced height of the resonance at $ka = 4.5$ (see fig. 6.8) is not predicted. This suggests very strongly that the ka shift is due to a change of the elastic parameters of the copper but the attenuation of the resonances is due to some other effect, possibly debonding. The steel wire being much stiffer does

Figure 6.11a: Effect of increasing the wave velocities of copper on the scattering from a copper cylinder in an epoxy resin matrix.

Figure 6.11b: Calculated effect of the acousto-elastic effect on the backscattering from a copper wire in an epoxy resin matrix.

Copper in Perspex Matrix

Fig. 6.11

(a)

Acousto-elastic Effect

(b)

not show this ka shift but does show the attenuation of the resonant structure.

6.4.4 Acousto-elastic effect in copper.

In all the previous analyses of ultrasonic propagation in solid materials it has been assumed that all the materials are linear and isotropic in their elastic properties (see Chapter 3). For an isotropic elastic material the longitudinal and shear wave speeds only depend upon the Lamé constants λ and μ . If an elastic material is subjected to an external stress then its properties will be anisotropic [Smith, 1963]. In this situation the wave speeds will also depend on the third order elastic constants l , m , and n (as defined by Murnaghan [1951] and also Hughes and Kelly [1953]). The longitudinal and shear wave speeds perpendicular to an applied tension are given by [Smith, 1963]:

$$\rho c_c^2 = \lambda + 2\mu - \frac{T}{3K} \left(\frac{2\lambda}{\mu} (\lambda + 2\mu + m) - 2l \right), \quad (6.2)$$

$$\rho c_s^2 = \mu - \frac{T}{3K} \left(2\lambda - m + \frac{n}{2} + \frac{\lambda}{\mu} \frac{n}{2} \right). \quad (6.3)$$

where ρ is the density, c_c and c_s are the longitudinal and shear wave speeds respectively, T is the applied tension and K is the bulk modulus. If the tension is zero then equations 6.2 and 6.3 reduce to the case for an isotropic elastic solid.

For copper the values for the 3rd order elastic constants are [Wasserbach, 1990]:

$$l = -261 \text{ GPa}, m = -523 \text{ GPa}, n = -775 \text{ GPa}.$$

K was calculated from [Smith, 1963]:

$$K = \frac{1}{3}(3\lambda + 2\mu). \quad (6.4)$$

The tension that was applied to the wire was estimated from the strain which was measured as $5.3 \times 10^{-3} \pm 30\%$, this gave a tension of $6.8 \times 10^8 \text{ N/m}^2 \pm 30\%$. Substituting the above values into equations 6.2 and 6.3 leads to values for the longitudinal and shear wave speeds of:

$$c_c = 4700 \text{ m/s}, \quad c_s = 2300 \text{ m/s}.$$

It should be born in mind that to make accurate and reproducible measurements of the third order elastic constants is rather difficult since to a certain degree they will depend upon the history of the material being examined. This can be seen in the variation in the values published by Wasserbäch, which can show differences of 100%. In general all that can be safely predicted is whether the relevant velocity will increase or decrease with the applied stress.

Figure 6.11b shows the calculated effect on the backscattering from a copper wire embedded in an araldite matrix of a stress applied along the axis of the wire. The dotted curve has been calculated assuming the stress estimated above ($6.8 \times 10^8 \text{ N/m}^2$). It can be seen that although there is a slight shift to higher ka it is much smaller than that observed in the backscattering measurement on the debonded copper wire (fig. 6.9). As the stress is increased the shift becomes far more significant: at an applied stress of $2.05 \times 10^9 \text{ N/m}^2$ the degree of shift is in keeping with that observed in the experiment. The calculated velocities at this level of stress are:

$$c_c = 4800 \text{ m/s} \quad c_s = 2400 \text{ m/s}.$$

Carrying out the same calculations for a steel wire assuming

the same applied stresses as for the copper wire shows negligible effect on the backscattering. When the stress is set to $2.05 \times 10^9 \text{ N/m}^2$, there is only an increase in the longitudinal wave speed of 15 m/s, and of 2 m/s for the shear wave speed. This is in agreement with the measurement on the debonded steel wire (fig. 6.10) where no shift to higher k_a was seen.

From these calculations it has been shown that the shift to higher k_a of features in the backscatter form function for the debonded copper wire are most probably due to the acousto-elastic effect in the material of the scatterer, and are not a general characteristic of debonding.

6.5 Measurements on partially debonded wires.

After the initial results of experiments on the debonded samples, further measurements were made on different samples to determine which features were common to the backscattering from debonded wires, and to discriminate those effects due to the changes in the material properties of the inclusions.

This time when the wires were debonded they were not removed from the block, they were only debonded for about 2/3 to 3/4 of the length of the wire. Debonding of the wire could be clearly seen with the naked eye as the stress on the wire was increased. Viewing the block through crossed polarised filters showed quite well the point where the wire changes from being bonded to debonded.

On these samples the backscattering from two separate regions of the debonded wire was measured. The reason for doing this was to provide results which would be consistent for the two areas of the

wire, and not subject to variations due to the casting of the sample. Both regions of the wire were well clear of the ends of the wire, to ensure there were no end effects (although there had been no indication that this might be a problem).

Figure 6.12 shows a diagram of a partially debonded sample and the two areas of the wire that were investigated using ultrasonic backscattering, region I is the upper part of the wire, region II is the lower part of the debonded wire.

6.5.1 Measurements on copper reinforced sample.

Immediately after debonding the wire region I and region II were inspected using ultrasonic backscattering. (Figures 6.13 and 6.14). Figure 6.13 (upper wire) shows little correlation between the scattering for a well-bonded wire and the backscattering from the debonded wire. There are no obvious indications of resonant structure at all, this would seem to suggest that it is possible to detect debonding using backscattering measurements. Figure 6.14 (lower wire) shows a similar situation with all the resonant structure above $ka = 3.5$ that would be expected for a well-bonded wire absent. In addition there are two clearly defined minima that were not present in fig. 6.13, these are at $ka = 3.75$ and $ka = 6.7$. From their positions they appear to be the two minima predicted for a perfectly bonded cylinder but shifted to slightly higher ka . This shift is just the acoustoelastic effect explained above (section 6.4.4). Below $ka = 3.75$ there are large fluctuations in the measured backscattering with the resonance predicted at $ka = 3$ possibly being identified. However since the time domain signal was

Fig. 6.12 Diagram of partially debonded sample.

Figure 6.13: Comparison of the backscattering from a perfectly bonded copper wire (dot-dashed line) in an araldite (epoxy resin) matrix and the measured scattering immediately after debonding (crosses joined by a line); region I.

Figure 6.14: Comparison of the scattering from a perfectly bonded copper wire (dot-dashed line) in an epoxy resin matrix and the measured scattering immediately after debonding (crosses joined by a line); region II.

Copper Wire in Araldite Matrix

Fig. 6.13

Copper Wire in Araldite Matrix

Fig. 6.14

smaller than that seen for a well-bonded wire, it is likely that this structure may be no more than artifacts introduced during the signal processing, especially since the the transducer's sensitivity in this region of ka is limited (fig. 6.4). In general these measurements appear to indicate a marked change in the backscattering due to debonding.

The measurements on region I and region II were repeated (approximately 30 minutes after the wire was debonded) to check that the effects described above were reproducible. Displayed in figures 6.15 and 6.16 are the results of these measurements compared, as before, with the backscattering for a perfectly bonded wire. Comparing fig. 6.15 with fig. 6.13 (both are measurements of the upper wire) it is obvious that there is a very large difference between them. Whereas in fig. 6.13 there is no correlation between the scattering from the debonded curve and that for a bonded wire, in fig. 6.15 there are definite similarities between the two curves. Below $ka = 5$ the two large resonances predicted at $ka = 3$ and $ka = 4$ for the bonded wire are present in the measured backscatter form function for the debonded wire, although the peak at $ka = 4$ is heavily attenuated. The minimum at $ka = 7.5$ is also clearly present (and is also seen in fig. 6.14). Apart from this one feature all the resonances above $ka = 5$ are missing from the measured backscattering. Figure 6.16 shows essentially the same scattering as fig. 6.14.

Although fig. 6.15 indicates significant differences between the scattering from bonded and debonded wires, it is very unsatisfactory that the same region of wire should give two

Figure 6.15: Comparison of backscattering from a perfectly bonded wire (dot-dashed line) and the measured scattering from a copper wire in an epoxy resin matrix (crosses joined by a line) approximately 30 minutes from debonding; region I.

Figure 6.16: Comparison of the backscattering from a perfectly bonded wire (dot-dashed line) and the measured scattering from a copper wire in an epoxy resin matrix (crosses joined by a line) approximately 30 minutes after debonding; region II.

Copper Wire in Araldite Matrix

Fig. 6.15

Copper Wire in Araldite Matrix

Fig. 6.16

different results for what should be the same measurement, especially since the scattering from lower down the wire (region II) shows a reproducible result. Either the initial measurement was carried out incorrectly, or the conditions existing for the wire have changed.

After a period of consideration the backscattering from regions I and II was re-measured, this was approximately 2 days from when the wire was debonded. The results are presented in figures 6.17 and 6.18 (regions I and II respectively). These measurements were taken approximately two days after the wire was debonded.

Figure 6.17 indicates that the backscattering for $ka < 5$ is very similar to the backscattering that you would expect to see for a well-bonded wire. The peak at $ka = 4$ is somewhat larger than in fig. 6.15 (approximately 20% larger) but the width of this resonance is rather less than is expected for a well-bonded wire. Above $ka = 5$, only the minimum at $ka = 7.5$ is clearly identified, All the other structure appears to have little relation to that for a perfectly bonded wire.

Figure 6.18 is in stark contrast to the results seen for region II before (figs. 6.14 and 6.16). Instead of there being none of the resonances present for a bonded wire, the peaks at $ka = 3$ and $ka = 4$ are now present. The scattering now displays the features seen for region I with the scattering below $ka = 5$ being effectively that for a well-bonded wire but with obvious differences above $ka = 5$.

These results show (figs. 6.13 to 6.18) a behaviour in the

Figure 6.17: Comparison of the backscattering from a perfectly bonded wire (dot-dashed line) and the measured scattering from a copper wire in an epoxy resin matrix (crosses joined by a line) approximately 2 days after debonding; region I.

Figure 6.18: Comparison of the backscattering of a perfectly bonded wire (dot-dashed line) and the measured scattering from a copper wire in an epoxy resin matrix (crosses joined by a line) approximately 2 days after debonding; region II.

Copper Wire in Araldite Matrix

Fig. 6.17

Copper Wire in Araldite

Fig. 6.18

partially debonded samples that changes with time, the scattering from the debonded wire initially shows little in common with the scattering from a bonded wire. As time increases the scattering from the debonded wire "relaxes" back to the scattering for a wire in good contact with the matrix material. In addition, this relaxation occurs at different rates for different regions of the wire.

When the wire is being debonded it has to be stretched in order that the wire will ^{pull} away from the matrix material. As the debond propagates down the fibre (the debond always starts at surface of the block where the end of the wire protrudes and is gripped) so the already debonded wire is going to be subjected to a greater strain than the newly debonded material. This can explain why the upper part of the wire (region I) relaxes back faster than the lower part of the wire (region II). The wire in region I has been subjected to a greater strain and experiences a greater force trying to pull it back into the block.

In addition to the force that the wire will feel due to the strain that it was subjected to, the matrix material (epoxy resin) will also be slowly flowing back into improving contact with the wire. Thus initially after debonding there is poor contact between wire and matrix. After two days they are once again in good contact but the interface is still not perfect, hence the differences between the backscatter form function for a bonded wire and that for the debonded wire.

6.5.2 Measurements on the steel reinforced sample.

The same experiment as was carried out in the previous section 6.5.1 above was carried out on a steel wire cast in an araldite epoxy resin block. As before, the wire was debonded over approximately $2/3$ of its length, and two sets of measurements made on the debonded part of the wire. Figures 6.19 to 6.24 display the backscattering form functions for the upper and lower regions of the debonded steel wire over a period of one day.

Figures 6.19 and 6.20 display the measured backscattering from the upper and lower debonded parts of the steel wire (regions I and II respectively) immediately measured after debonding the wire. The experimental data is presented as crosses joined by a continuous line. The calculated scattering for a bonded wire is shown as a continuous line. These two figures are relatively similar, with the backscattering being significantly lower than that expected for a bonded wire. This was also noticed in the time domain where the amplitude of the signal was smaller than that for the bonded wire. Although the overall level of the backscatter form function is significantly lower (approximately 50% of the bonded curve) the structure is quite similar to that for a perfectly bonded steel wire. For example in figs. 6.19 and 6.20 the peaks at $ka = 4.25$, 5.5 and 7 are identified, as are as the minima at $ka = 3$ and 7.75 (this latter minimum was not identified in fig. 6.20).

After an hour the measurements were repeated to investigate whether the same relaxation type of process as had been seen for the copper would be observed for the steel. The results for these measurements are displayed in Figures 6.21 (region I) and 6.22 (region II).

Figure 6.19: Comparison of the scattering from a perfectly bonded steel wire (dot-dashed line) with the measured scattering from a steel wire in an epoxy resin matrix immediately after debonding (crosses joined by a line); region I.

Figure 6.20: Comparison of the scattering from a perfectly bonded wire (dot-dashed line) with the measured scattering from a steel wire in an epoxy resin matrix immediately after debonding (crosses joined by a line); region II.

Steel Wire in Araldite Matrix

Fig. 6.19

Steel in Araldite Matrix

Fig. 6.20

Figure 6.21: Comparison of the backscattering from a perfectly bonded steel wire in an epoxy resin matrix (dot-dashed line) with the measured scattering from a steel wire approximately one hour after debonding (crosses joined by a line); region I.

Figure 6.22: Comparison of the scattering from a perfectly bonded wire (dot-dashed line) with the measured backscattering from a steel wire embedded in an epoxy resin matrix approximately one hour after debonding (crosses joined by a line); region II.

In figure 6.21 there is a small amount of evidence of relaxation around the peak at $ka = 7$, with the peak being about double the previous height. It should be born in mind that this is near the limit of the transducer sensitivity and this apparent increase in the height of the resonance may be largely due to signal processing errors. The rest of the spectrum shows no signs of the relaxation that was seen for the copper. Figure 6.22 also shows no sign of the epoxy/steel system having relaxed back into better contact.

The epoxy/steel sample was left for a day and the experiment was repeated once more. The results are shown in figures 6.23 (upper wire) and 6.24 (lower wire). From fig. 6.23 it can be clearly seen that the backscattering is now effectively the same as that from a well-bonded steel wire embedded in an epoxy matrix (see fig. 6.5). There were problems with the deconvolution process (see section 6.3.1.) which leads to the large discrepancies between the calculated scattering and the measured for $ka < 4.5$ and $ka > 7.5$. There are no signs of any significant changes in the structure of the form function, possibly indicating that for this part of the wire the longitudinal backscattered wave is insensitive to the presence of the debond. Figure 6.24 also has the same overall features as fig. 6.23 in that it is essentially the scattering from a well-bonded steel wire in an araldite matrix, and it also displays the same discrepancies for $ka < 4.5$ and $ka > 7.5$. However this time there are small differences to the structure of the form function over the rest of the ka range.

Figure 6.23: Comparison of the scattering from a perfectly bonded steel wire in an epoxy resin matrix (dot-dashed line) with the measured backscattering from a steel wire approximately one day after debonding (crosses joined by a line); region I.

Figure 6.24: Comparison of the scattering from a perfectly bonded wire (dot-dashed line) with the measured backscattering from a steel wire in an epoxy resin matrix approximately one day after debonding (crosses joined by a line); region II.

Steel Wire in Araldite Matrix

Fig. 6.23

Steel Wire in Araldite Matrix

Fig. 6.24

In particular the resonance at $ka = 5.5$ is a little over 10% smaller, with a small double resonance incorporated into it. These changes are small and of the same order of magnitude as the uncertainty in the size of features in the measured form function (see section 6.3.3).

6.5.3 Summary of results on partially debonded wires.

The measurements that have been carried out on the epoxy/copper and epoxy/steel samples indicate that initially after debonding it is possible to detect debonding using ultrasonic longitudinal backscattering measurements. However there is a relaxation of these epoxy resin matrix samples which causes the backscattering after a period of time (of the order of 24 hours) to return to a state that largely reacts to an incident ultrasonic longitudinal wave as a well-bonded scatterer. Since, after the system has relaxed back, there are some small changes to the backscattering (especially the epoxy/copper sample) it appears that longitudinal backscattering is not totally insensitive to debonds. It is very likely that in other types of composites this relaxation will not occur and the debonding effects will remain very significant.

Addison and Sinclair [1991] have published backscattering measurements for Titanium reinforced with Silicon Carbide (SiC) fibres. Their results are in the ka range $0 < ka < 7$ (the frequency range was $0 < f < 100\text{MHz}$ since the fibre was $70\mu\text{m}$). They found on some sections of their samples that the measured backscattering showed no correlation with the theory for a perfectly bonded fibre (following the analysis by Faran [1951] and White [1958]). This

they had attributed to debonding of the fibre and matrix. Their work suggests that debonding can be detected in metal matrix composite materials, but the discrepancies between theory and experiment they report have not been analysed sufficiently to show they are due conclusively to debonds.

If a fibre reinforced composite structure is going to be continuously stressed cyclicly (i.e. fatigued) then it is likely that the sample would not have the opportunity to relax and allow the fibre and matrix material to achieve good contact. Consequently backscattering measurements may well provide a useful NDE tool for detecting debonds.

6.6 15MHz longitudinal backscattering measurements.

A pair of Panametrics 15MHz immersion transducers became available during the course of the study, and it was felt that it would be of interest to measure the backscatter form function for a cylindrical scatterer embedded in a solid matrix at higher ka , and to determine whether or not the effects of debonding were more apparent at higher ka ranges.

The same experimental arrangement as was used for the previous measurements at 7.5MHz (described in detail in Chapter 5) was used for the 15MHz measurements. The 15MHz transducers were significantly less sensitive than the 7.5MHz transducers, and this was expected to pose some problems in making accurate backscattering measurements. The response of the signal propagation pathway is displayed in figure 6.25 (measured using the perspex reflector). Compared with the response for the 7.5MHz transducers

Figure 6.25: The frequency response of the signal pathway using the 15MHz longitudinal immersion transducers.

Figure 6.26: Comparison of the theoretical prediction of the backscatter form function for a copper cylinder in an epoxy resin (dot-dashed line) with the measured backscatter form function (crosses joined by a line).

Signal Pathway Response

Fig. 6.25

Copper Wire in Araldite

Fig. 6.26

(fig. 6.4) it can be seen that the peak 15MHz response is 40% the peak value of the 7.5MHz response. The useful measurement bandwidth of the 15MHz system is $9.6\text{MHz} < f < 19.2\text{MHz}$. The ka range that equates to $\sqrt{\lambda}$ (where k is the wave number in araldite and a is the radius of the wire, 0.35mm) is $8 < ka < 16$.

A sample was cast with a copper wire embedded in it and a measurement of the backscatter form function made, the result of this measurement is shown in figure 6.26.

The experimental data in fig. 6.26 is displayed as crosses joined by a continuous line; the theoretical prediction for the backscatter is the dashed line. The agreement in view of the previous backscatter results is relatively poor (c.f. fig. 6.3). Some of the resonances measured coincide with the theory e.g. $ka = 9, 11.5, \text{ and } 13$. The minima are reasonably well defined. Although several of the large features, such as the minima at $ka = 8.75, 11, 12.5, 13.5 \text{ and } 14.5$ and the resonances mentioned above, appear in the measured backscattering it is particularly worrying that ^{a distinct feature} like the series of small resonances in the region $9 < ka < 11$ are significantly diminished. There is a small peak in the measured data at $ka = 9.5$ but it is very much smaller than might have been expected. Also features like the double resonance at $ka = 11$ are not present, whereas in the measurements at 7.5MHz these kind of features are clearly seen (fig. 6.3).

Although the agreement is not particularly good between theory and experiment, the majority of the main features in the structure are evident. The reasons for the discrepancies could be due to several causes. Firstly perhaps the assumptions made in the

scattering calculations are not completely valid for the higher ka range (e.g. normal incidence of the ultrasonic beam, plane wave assumption). It could also be possible that the scattering at higher ka is more sensitive to the conditions existing at the interface of the fibre and the matrix. In addition there is the problem of a less sensitive transducer being used for the measurements, which may well introduce errors into the measured data.

With the above discrepancies in mind, the wire was partially debonded as before and a series of backscatter measurements made primarily to check whether the relaxation of the epoxy/copper system would be apparent at higher ka as was expected. In addition, it was considered to be of interest to see if the debond would be detectable after the system had stopped relaxing at higher ka . As for the previous measurements, two regions of the debonded wire were observed for signs of a different relaxation rate between the upper and lower regions of the debonded wire.

Figure 6.27 show the measured backscatter form functions from the debonded copper wire immediately after debonding (the dashed line), one hour after being debonded (the dotted line), and one day after being debonded (the continuous line). There is a definite indication that the relaxation that was described in section 6.5 is occurring here and is being detected. It is interesting to note that there is very little change between the backscattering one hour and one day after the sample was debonded. The backscattering immediately after debonding does show definite differences in comparison with the other two curves, the main resonances at

Figure 6.27: The measured backscattering from a copper wire embedded in an epoxy resin matrix immediately after debonding (dashed line), one hour after (dotted line) and one day after debonding (continuous line). Measurements taken in region II.

Figure 6.28: Comparison of the measured backscattering from region I (continuous line) and region II (dashed line) after approximately one day after debonding.

Copper Wire in Araldite

Fig. 6.27

Copper Wire in Araldite

Fig. 6.28

$ka = 9.5$, 11 , and 12 are noticeably attenuated. The resonance at $ka = 13.5$ appears to be shifted and the peak at $ka = 14$ is missing altogether. Comparing the result for one day after debonding (fig. 6.27 continuous line) with the scattering observed for the bonded wire (fig. 6.26) the main four large resonances at $ka = 11$, 12 , 13 and 14 are all present. There is some sharply peaked structure over laying the whole spectrum, but this is most probably due to the effects of a poor signal to noise ratio that was characteristic of these measurements.

The backscattered signal from the upper region of the debonded wire was not ^{initially} of a sufficient amplitude to make accurate measurements, consequently it was not until a day after the wire had been debonded that the backscattering form function from the upper wire was successfully measured. In figure 6.28 the result for the upper wire is compared with that for the lower wire (a day after debonding). The upper wire is the continuous line, the lower wire is the dashed line.

The only sections of the two spectra which are coincident are the two large resonances at $ka = 12$ and 13 . The resonance at $ka = 14$ (for the lower wire) might be present in the backscatter from the upper wire but it is rather spread out and the centre frequency of the peak is shifted. Below $ka = 12$ there is no correlation between the results for the upper and lower wire after the wire has relaxed.

Since the theoretical prediction and experimental measured backscatter form function for the well-bonded copper wire in an araldite matrix were not in very good agreement, it is rather

difficult to determine whether the discrepancies between the scattering from the upper and lower parts of the wire are due to the effects of debonding. It may well be that the wire was not particularly well bonded initially. However the relaxation process that was observed at the lower ka region is present as expected. Assuming that the wire was well-bonded initially there are changes between the scattering from the bonded wire and from the debonded wire after the wire has apparently relaxed back into contact with the matrix. These changes are rather small and of the same order as the variations seen in the 7.5MHz measurements (of the order of 10%).

6.7 Effect of temperature change on the backscattering from debonded samples.

Since the wire inclusions had been debonded from the matrix materials it was suggested that temperature change might well affect the backscattering due to the differing expansion rates of the matrix and inclusion materials, leading to increased (or conversely reduced) frictional forces across the interface. The linear thermal expansion coefficients of copper, steel and epoxy resin are given below [Kaye and Laby, 1975]:

$$\text{Copper: } 20 \times 10^{-6} \text{ K}^{-1},$$

$$\text{Steel: } 11 \times 10^{-6} \text{ K}^{-1},$$

$$\text{Epoxy Resin: } 50 \times 10^{-6} \text{ K}^{-1}.$$

Typically the linear coefficient of thermal expansion for epoxy resin is two and a half times that for copper, so it may be reasonable to expect some temperature dependence on the backscatter

form function for a debonded copper wire in an araldite matrix. Firstly the effect of temperature on the scattering from bonded samples was examined to see if there was any sensitivity in the samples to environmental effects such as temperature.

The temperature range chosen was from 15°C to 20°C , this being the typical temperature range that the water tank used for the scattering measurements might suffer within the lab. It was only those temperature effects that may have been falsely assumed to be due to debonding that were of particular interest, and this is why this temperature range was chosen and not a wider one. Temperature control was effected by partially submerging the water tank in a water bath which was in turn placed on a hot plate. It was found that the water tank, having reached a particular temperature of interest, was thermally stable to within 0.25°C over the time needed to make a backscattering measurement (typically 15 minutes). Although a certain temperature could be maintained in this way, heating to a pre-determined temperature was far more difficult.

The output of the transducer was checked over the temperature range for any variation in sensitivity by examining the reflection from the front face of a flat glass block at normal incidence. No significant changes were noticed (p-p output changed by less than 2%).

The backscattering from a well-bonded copper wire is shown in figures 6.29 (at 15.75°C) and 6.30 (18.75°C). In both experiments the measured backscattering fits the theory for a well-bonded copper wire particularly well. The same experiment was carried out for a well-bonded steel wire, the results are displayed

Figure 6.29: Comparison of the theoretical prediction of the backscatter form function for a copper wire in an epoxy resin matrix (continuous line) with the measure scattering at 15.75°C (crosses joined by a line).

Figure 6.30: Comparison of the predicted scattering from a copper wire in an epoxy resin matrix (continuous line) with the measured scattering at 18.75°C (crosses joined by a line).

Cu in Araldite

Fig. 6.29

Cu in Araldite

Fig. 6.30

in figures 6.31 (at a temperature of 15.5° celsius) and 6.32 (18.5° celsius). The agreement between the experimental data for the steel wires and the theory is even better than for the copper wires. These results suggest very strongly that for the typical temperature range over which these (and all previous) measurements were taken there are no significant temperature related effects on the backscattering.

Three separate samples that had been previously used in the partial debonding experiments (section 6.5) were chosen to be studied. Two of the samples contained a copper wire the other a steel wire. Firstly the backscatter form function for each was measured, the copper reinforced samples are shown in figure 6.33 and the steel reinforced sample in figure 6.34, all three measurements were taken at 16.5° Celsius. The results in figure 6.33 came as a surprise, since these samples had both been seen to relax back to a state that acted mostly as a wire in good contact with the araldite (see fig. 6.17). The backscattering now being observed from both samples was that for a newly debonded wire (c.f. fig. 6.14). This was taken to suggest that the cycling of room temperature over the weeks since the wire had relaxed back into good contact with the araldite had once again destroyed this contact. However when the steel reinforced sample was examined (fig. 6.34) it was seen that it behaved as a wire in good contact with the matrix material. This was in contrast to the results for the copper wire.

When the initial partial debonding measurements were made on these blocks the weather was significantly warmer than when the

Figure 6.31: Comparison of the theoretical scattering from a steel wire embedded in an epoxy resin matrix (continuous line) and the measured scattering at 15.5 °C (crosses joined by a line).

Figure 6.32: The comparison of the predicted scattering from a steel wire embedded in an epoxy resin matrix (continuous line) and the measured scattering at 18.5 °C (crosses joined by a line).

Figure 6.33: The measured backscattering from previously debonded copper wires in epoxy resin matrices at 16.5°C (dashed and dotted lines) compared with the predicted scattering for a bonded wire (continuous line).

Figure 6.34: Comparison of the measured scattering from a previously debonded steel wire in an epoxy resin matrix at 16.5°C (crosses joined by a line) with the predicted scattering from a perfectly bonded wire (continuous line).

Cu in Araldite

Fig. 6.33

Steel in Araldite

Fig. 6.34

later measurements were made. Since the epoxy resin has a higher linear coefficient of expansion than the copper or steel, it would seem reasonable to expect the frictional forces between the wire and matrix to increase as the temperature was decreased since, the hole in which the wire sits would shrink faster than the wire. This would suggest that at cooler temperatures the debonded wires would behave more like a system in good contact.

A series of measurements of the backscattering from the debonded samples was carried out at different temperatures. The backscattering from the copper reinforced samples is displayed in figures 6.35 to 6.37. The results for the steel reinforced sample are displayed figure 6.38. From figs. 6.35 to 6.37 it can be seen that temperature change has negligible effect on the backscattering. Even when the temperature was increased to 25° Celsius (fig. 6.37) there was no change. The steel reinforced sample (fig. 6.38) did produce more variation with temperature than the copper reinforced samples, but the backscattering was essentially that for a steel wire in good contact with the araldite matrix. No effects that have been attributed to debonding were apparent with temperature change.

The conclusion that can be drawn from this is that the backscattering form function for both bonded and debonded scatterers is not affected by temperature changes in the region investigated. Why the copper reinforced samples were now acting as if they were freshly debonded, when previously the backscattering had indicated they were in good contact with the matrix material is not clear. However it is not due to a temperature effect. Possibly

Figure 6.35: Comparison of the measured scattering from previously debonded copper wires in epoxy resin (dotted and dashed lines) at 17.25°C with the theoretical prediction of the scattering from a perfectly bonded wire (continuous line).

Figure 6.36: Comparison of the scattering from previously debonded copper wires (dotted line for block A at 19°C , dot-dashed line for block B at 20.5°C) with the theoretical prediction for a perfectly bonded copper wire (continuous line).

Figure 6.37: Comparison of the measured backscattering from previously debonded copper wires at 25°C (dotted and dot-dashed lines) with the theoretical prediction of the scattering from a perfectly bonded copper wire in an epoxy resin matrix.

Figure 6.38: Comparison of the measured backscattering from a previously debonded steel wire at 15.5°C (dotted line) and at 18.5°C (dot-dashed line) with the theoretical prediction for a perfectly bonded steel wire in an epoxy resin matrix.

Copper in Araldite

Fig. 6.37

Steel in Araldite

Fig. 6.38

the resin has undergone some chemical change, the result of which has led to this result, but this is pure conjecture at the present time.

6.8 Summary.

In this chapter the effect of debonding in scale models of fibre reinforced composites on longitudinal ultrasonic backscattering has been studied. It has been shown that for a well-bonded scatterer, the measured backscatter form function can accurately be measured with a good degree of reproducibility. This has meant that differences between the theoretically predicted scattering and the measured scattering can be assumed to be due to changes to either the properties of the matrix and inclusion (i.e the acousto-elastic effect) or to changes at the interface of the matrix and inclusion.

It has been possible to detect the effect of debonding on the backscattering of longitudinal waves. However it was observed that the systems relax with time and behave, when a longitudinal wave is incident upon them, essentially as bonded scatterers embedded in a solid matrix. Consequently longitudinal ultrasonic waves will not only be sensitive to debonds early in their formation, ^{but also, sometimes, in their later history (see chapter 7)}. This is not particularly useful for a diagnostic tool.

In view of the partial success of using longitudinal incidence to detect debonding between a single cylindrical scatterer and a solid elastic matrix it was decided to attempt to use incident shear waves. It seemed intuitively obvious that a shear wave would be more sensitive to a debond even when the scatterer and matrix

were in good contact. The theoretical and experimental investigations using shear waves are discussed in the next chapter.

CHAPTER 7. SHEAR WAVE SCATTERING.

7.1 Introduction.

In the previous chapter the effect of debonding of the reinforcing wire from the matrix material was investigated using ultrasonic longitudinal wave backscattering measurements. It was seen that when the wire had been freshly debonded it was possible to detect the debond rather easily. In general the effect was to attenuate all the resonances that would be present in the backscatter form function for a well-bonded scatterer in an elastic matrix. However a further effect was seen which tended to negate what at first sight seemed to be a successful method of detecting a debonded fibre. This was a relaxation of the sample under investigation so that the inclusion began to act once more as a well-bonded wire. There were some minor effects present in the backscattering but these were not by themselves enough to determine confidently that the wire was debonded.

All this had suggested that longitudinal waves were not particularly sensitive to debonds if the fibre and matrix material were in a reasonably intimate contact. It was suggested that shear waves may well provide a better sensitivity to such a defect. Intuition suggests that if the displacement of the incident wave is parallel to the axis of the fibre (as opposed to normal in the longitudinal case) then a debonded region of the fibre may well have a strong attenuative effect on the backscattered wave due to friction of fibre and matrix. In addition, transmission across the interface is likely to be considerably reduced. All these factors

are likely to have a strong effect on the backscatter form function.

7.2 Theoretical modelling of shear wave scattering.

For all the calculations and experimental measurements of shear wave scattering it is assumed that the polarization of the incident wave is along the axis of the scattering cylinder (SH waves, see figure 7.1). In addition to the incident wave, Ψ_{inc} , there are two other waves, the reflected wave, Ψ_{s1} , and the refracted wave Ψ_{s2} .

As for the longitudinal wave scattering (see Chapter 3) the shear waves can be represented by wave potentials [Pao and Mow, 1973], these are for the incident, reflected and refracted waves respectively:

$$\Psi_{inc} = \sum_{n=0}^{\infty} \epsilon_n i^n J_n(k_{s1}r) \cos(n\theta) , \quad (7.1)$$

$$\Psi_{s1} = \sum_{n=0}^{\infty} E_n H_n^{(1)}(k_{s1}r) \cos(n\theta) , \quad (7.2)$$

$$\Psi_{s2} = \sum_{n=0}^{\infty} F_n J_n(k_{s2}r) \cos(n\theta) . \quad (7.3)$$

E_n and F_n are the undetermined coefficients for the reflected and refracted SH wave potentials respectively. k_{s1} and k_{s2} are the shear wavenumbers in the matrix and inclusion materials respectively. They are defined in Chapter 3 as are J_n and $H_n^{(1)}$ (the cylindrical Bessel function and cylindrical Hankel function of the first kind). ϵ_n is equals to 1 for $n = 0$, and 2 for $n \geq 2$. θ is the scattering angle as defined in fig. 7.1.

For a perfectly bonded fibre the boundary conditions are:

Fig. 7.1 Geometry of SH wave scattering.

Figure 7.1: Geometry of the SH wave scattering from a cylinder. Displacement of incident wave is parallel to the axis of the cylinder.

$$\text{continuity of tangential stress} \quad \sigma_{z1} = \sigma_{z2} , \quad (7.4)$$

$$\text{and continuity of tangential displacement } u_{z1} = u_{z2} . \quad (7.5)$$

The tangential stress is given by [Pao and Mow, 1973]:

$$\sigma = \mu \frac{1}{r} \frac{\partial \Psi}{\partial r} . \quad (7.6)$$

The tangential displacements are given by;

$$u = u_0 \Psi , \quad (7.7)$$

where u_0 is the incident displacement. Applying the boundary conditions leads to a simple 2×2 matrix in the unknowns E_n and F_n . Since only the undetermined constant for the backscattered wave is required (E_n), the solution is relatively simple and given below:

$$E_n = \frac{(\rho_2/\rho_1)k_{s1}^2 a^2 J_n'(k_{s2}a) \cdot k_{s1} a J_n(k_{s1}a) - k_{s2} a J_n(k_{s2}a) \cdot k_{s1}^2 a^2 J_n'(k_{s1}a)}{(\rho_2/\rho_1)k_{s1}^2 a^2 J_n'(k_{s2}a) \cdot k_{s1} a H_n(k_{s1}a) - k_{s1}^2 a^2 H_n'(k_{s1}a) \cdot k_{s2} a J_n(k_{s2}a)} \quad (7.8)$$

H_n in the above is a Hankel function of the first kind, and ' denotes the derivative with respect to the argument of the function. Having been calculated, E_n is then substituted in the expression for the form function, $f_\infty(\theta, ka)$, as defined in Chapter 3 (section 3.27).

7.2.1 Effect of slip on SH wave backscattering.

Following the same method as was defined in Chapter 4 slip was allowed at the interface of the matrix and inclusion by replacing the boundary condition in equation 7.5 by:

$$\sigma_{z1} = K(u_{z1} - u_{z2}) , \quad (7.9)$$

where K is the effective shear modulus (see Chapter 4). Introducing the relationship above, (equation 7.9) results in the solution of E_n

becoming:

$$E_n = \frac{-Kk_{s2}^2 a J_n(k_{s2}a) \cdot k_{s1}^2 a^2 J_n'(k_{s1}a) + L_n}{-k_{s1}^2 a^2 H_n'(k_{s1}a) \cdot Kk_{s2} a J_n(k_{s2}a) + M_n}, \quad (7.10)$$

where

$$L_n = (\rho_2/\rho_1) k_{s1}^2 a^2 J_n'(k_{s2}a) \cdot [Kk_{s1} a J_n(k_{s1}a) - k_{s1}^2 a^2 J_n'(k_{s1}a)], \quad (7.11)$$

$$M_n = (\rho_2/\rho_1) k_{s1}^2 a^2 J_n'(k_{s2}a) \cdot [Kk_{s1} a H_n(k_{s1}a) - k_{s1}^2 a^2 H_n'(k_{s1}a)]. \quad (7.12)$$

As before it can be seen that as the effective shear modulus tends to infinity, equation 7.10 tends to continuity of tangential displacement. At the other extreme ($K \rightarrow 0$) then the situation of perfect slip is reached and equation 7.10 reduces to:

$$E_n = \frac{J_n'(k_{s1}a)}{H_n(k_{s1}a)}. \quad (7.13)$$

This is the solution for a cylindrical cavity with no refracted shear wave propagating into the inclusion [Pao and Mow, 1973]. The effect of changing the amount of slip in the interface is displayed in figures 7.2 and 7.3 for a glass cylinder embedded in a perspex matrix.

In figure 7.2 the scattering from a perfectly bonded cylinder is shown as a continuous line for a ka range $0 < ka < 10$. The scattering consists primarily of three large peaks at $ka = 1, 5.5$, and 8.5 , this last peak is really a double peak. The effective modulus, K , is reduced and the results are displayed in comparison with the perfectly bonded scatterer in fig. 7.2. Significant effects on the backscattering are seen for $K \leq 10$. For an order of magnitude above this value of K the effects are insignificant.

Figure 7.2: The calculated backscatter form function for SH waves incident on a glass cylinder in a perspex matrix calculated at different values of the effective shear modulus, K .

Figure 7.3: The calculated backscattering of SH waves from a glass cylinder in a perspex matrix indicating the sharp low ka resonance.

Glass in Perspex, SH incidence

Fig. 7.2

Glass in Perspex, SH incidence

Fig. 7.3

Below $ka = 3$, as K is reduced, the backscattering is increasingly attenuated, although this attenuation is only of the order of 5% for $2.5 \leq K \leq 10$. At $K = 1$ there is an indication of a resonance appearing below $ka = 1$. The minima at $ka = 4.25$, 7.25 and 9 all exhibit a significant change with decreasing K . The general trend is for the width of the minimum to decrease with decreasing magnitude of the effective shear modulus. In addition to these two effects, there is a general increase in the overall level of backscattering above $ka = 3$. When $K = 1$ the majority of the backscatter form function maintains a value of approximately one, this is approaching the scattering that would be exhibited by a cavity.

If K is reduced below a value of one, then the scattering above $ka = 3$ is effectively the same as from a cylindrical cavity. Below $ka = 3$ there are still significant changes with decreasing K , these are highlighted in Figure 7.3. Above $ka = 1$, as the effective shear modulus, K , is decreased, the scattering shows a smooth oscillation about a value of one. This is essentially the same form as seen for a longitudinal wave scattered from a cavity, the oscillations in the form function being due to circumferential waves interfering either constructively or destructively with the specularly reflected wave [Flax et al, 1981].

Below $ka = 1$ as K is reduced a sharp resonance becomes apparent which shifts to lower ka as the effective shear modulus is reduced. This low ka resonance is similar in its position to a resonance reported by Yang and Norris [1991] for SH wave scattering from a partially debonded fibre. In their calculations they

modelled the debond as a discontinuous crack with non-contacting faces. They explained the presence of this resonance, which occurs when the crack has almost totally surrounded the fibre, as being due to an oscillation of the fibre which is only joined to the matrix by a thin neck of material. Such a low ka resonance has also been seen in Chapter 2 for the spherical shell model, and in Chapter 4 for the cylindrical thin air shell model. For the spherical shell model it was shown that the resonance occurred for the $n=1$ partial wave (see fig. 2.3) which is associated with bodily motion of the scatterer. This is also the case for the cylindrical shell model (see section 4.2 and fig. 4.4).

The scattered waves for a particular value of n (see equation 7.5) are called partial waves. Figure 7.4 displays the first three partial waves scattered from a SH wave incident on a glass cylinder in a perspex matrix with the effective shear modulus set to 0.01. The sharp low ka resonance occurs only in the $n=0$ partial wave. For a longitudinal incident wave this partial wave is associated with a breathing motion [Anson, private communication] of the scatterer. For an incident shear wave this motion is clearly not possible since it would mean that there had been mode conversion from an SH wave to a longitudinal wave. As yet it is unclear what mode of vibration this resonance is, although it has been suggested [Anson, private communication] that it may be an axial motion of the cylinder which is not possible for a better bonded scatterer.

Figures 7.5 and 7.6 show the results of calculations of the effect of the effective shear modulus for a copper cylinder in

Figure 7.4: The first three partial waves in the backscattering of SH waves from a glass cylinder in a perspex matrix with the effective shear modulus set to $K=0.01 \text{ N m}^{-3}$.

Figure 7.5: The calculated backscattering of SH waves from a copper cylinder in a perspex matrix with different values of the effective shear modulus, K .

Glass in Perspex

Fig. 7.4

Cu in Perspex, SH incidence

Fig. 7.5

Figure 7.6: The calculated SH wave backscattering from a copper cylinder in a perspex matrix at different values of K , indicating the effect on the low ka resonance.

Figure 7.7: Graph of the position of the low ka resonance as a function of the effective shear modulus, K .

Cu in Perspex, SH incidence

Fig.7.6

Glass in Perspex

Fig. 7.7

perspex. From fig. 7.5 it is evident that as for the glass in perspex there is the general trend for the scattering to tend to a value of one as K is reduced. The attenuation of the region below $ka = 3$ is far more significant. The minima all show the same reduction of width as the modulus is reduced.

Figure 7.6 displays the low ka resonance for values of the effective shear modulus, $K \leq 1$. Here the resonance shifts to lower ka as the effective shear modulus is reduced. As a highly simplified and rather empirical explanation, it can be imagined that the scatterer acts like a simple mass spring system with K the spring constant (its actual units are those of a modulus $\times$ metres). Consequently as the effective shear modulus is reduced, the resonant frequency will decrease. Figure 7.7 shows the position of the resonance as a function of K showing quite clearly the shift in resonance to lower ka as K decreases.

7.3 Experimental measurements of the backscatter form function for SH waves.

For all the previous measurements longitudinal waves have been used. The type of transducers used were immersion transducers since the difference in the acoustic impedances for water and epoxy resin is not particularly large. In addition it is possible to characterise the propagation signal pathway (see chapter 5). Since shear waves do not propagate in a fluid, it is necessary to use a contact shear wave transducer. This introduces several problems that will be discussed below.

For the SH wave measurements a commercially available

Panametrics shear wave transducer was used. The first problem was how to measure the frequency response of the transducer with the attendant electronics. In the longitudinal wave measurements the front face reflection from a large, flat reflector at normal incidence was used. The target for the shear wave transducer was the back face of a perspex block. The resulting response is characteristic of the transducer, measurement electronics and the particular perspex block that the wave propagates through. Since the elastic properties of perspex and araldite epoxy resin are relatively close (see Chapter 4), it seems reasonable to use a perspex block as the target to characterise the signal propagation pathway. The measured spectrum of the signal propagation pathway is displayed in figure 7.8.

Since the shear wave contact transducer is, by its very nature, in contact with the face of the epoxy block this means it is only some 3 mm away from the wire inclusion. Consequently the backscattered signal from the wire is not completely time resolved from the transducer-front face signal. The effect of this is to 'pull down' the leading edge of the signal from the wire. This is displayed in figure 7.9 this can be compared with the typical time domain signal from a copper wire using the longitudinal scattered wave. Comparing figs. 7.9 and 7.10 it is clearly apparent how distorted the SH signal is. This problem was solved by placing the transducer on the face of the block away from the wire inclusion, the signal from this area was recorded and subtracted from the signal backscattered from the wire. The result should be just the signal from the wire inclusion. Once this procedure had been

Signal Pathway Response

Fig. 7.8

Figure 7.8: The frequency response of the signal pathway using the shear wave transducers.

Figure 7.9: Typical backscattered time domain signal from a copper wire in an epoxy resin matrix using SH wave incidence.

Figure 7.10: Typical backscattered time domain signal from a copper wire in an epoxy resin matrix using longitudinal incident wave.

Time Domain Signal

Fig. 7.9

SH wave from copper

Time Domain Signal

Fig. 7.10

longitudinal wave from copper

carried out, the backscatter form function can be determined using exactly the same method as described in Chapter 5.

SH wave scattering measurements were carried out on a sample containing a copper wire, and a second sample containing a steel wire. The results are displayed in figures 7.11 and 7.12.

In figure 7.11 the measured backscatter form function from the copper wire in an araldite epoxy resin matrix is compared with the expected scattering calculated using the theory above in section 7.2. The ka range for the measurements was $2.5 < ka < 8.5$, this is equivalent to a frequency range of $1.5\text{MHz} < f < 5.2\text{MHz}$. In general the agreement is particularly good especially since the amplitude of the backscattered signal in the time domain is considerably less than that for the comparable signal measured for the scattered longitudinal wave (c.f. figs 7.9 and 7.10). The main discrepancy is around $ka = 6.5$ where the measured resonances are approximately 50% larger than those predicted by theory. This looks very similar to the deconvolution errors that were seen in for the longitudinal scattered wave measurements (see Chapter 6). However these errors were only present at the extremes of the ka range, while the present difference is firmly in the most sensitive region of the transducer response (see fig. 7.8). This error may be due to the method by which the response of the propagation pathway was measured. Since perspex was used as the target, it may be that the loading on the transducer face is different to that when the transducer is in contact with the front face of the sample. This may cause the pathway response to be significantly different to that indicated in fig. 7.8. However since the overall shape of the

Figure 7.11: Comparison of the measured backscatter form function (crosses joined by a line) for SH waves incident on a copper wire in an epoxy resin matrix with the theoretical prediction (continuous line).

Figure 7.12: Comparison of the measured backscatter form function (crosses joined by a line) for SH waves incident on a steel wire in an epoxy resin matrix with the theoretical prediction (continuous line).

Cu in Araldite Matrix

Fig. 7.11

Steel in Araldite

Fig. 7.12

features in the form function agrees particularly well with theory, the experimental arrangement was considered adequate to investigate the potential of SH waves for detecting debonds.

Figure 7.12 displays the measured scattering from the steel wire embedded in an araldite epoxy matrix. In contrast to fig. 7.11 the agreement between theory and experiment is far less obvious. The only features that can be identified with any degree of confidence are the minima at $ka = 4.25$ and 7.25 . In view of the reasonable success of the previous measurement this was rather disappointing. The measurement was repeated several times but the result was the same as displayed in fig. 7.12.

It had been noted that the scattering showed similarities to that for longitudinal waves from a freshly debonded wire (see Chapter 6) with only the minima clearly identified. It is difficult to determine if all the resonant structure has been attenuated, since the SH wave scattering does not exhibit strong readily identifiable resonances. However the overall level of the backscattering from the steel reinforced sample was significantly less than that for the copper reinforced sample (approximately 50%).

In view of these anomalous results, it was decided to make longitudinal wave scattering measurements to determine if there were any obvious defects in the steel reinforced sample (i.e. debonding). The results of these measurements are displayed in figures 7.13 and 7.14.

The agreement between theory and experiment for the copper reinforced sample (fig. 7.13) is, as expected, very good, with only

Figure 7.13: Comparison of the measured backscatter form function (crosses joined by a line) for longitudinal waves incident on a copper wire in an epoxy resin matrix with the theoretical prediction (continuous line).

Figure 7.14: Comparison of the measured longitudinal backscattering (crosses joined by a line) from a steel wire in an epoxy resin matrix with the theoretical prediction (continuous line).

Cu in Araldite

Fig. 7.13

Steel in Araldite

Fig. 7.14

minor discrepancies below $ka = 3$. The backscattering from the steel reinforced sample (fig. 7.14) was expected (on the strength of the SH wave measurement) to have fairly poor agreement with the theory as it was thought that the wire might be poorly bonded to the resin. As can be seen from figure 7.14, this is not the case. The agreement between theory and experiment for this sample was probably the best that had been achieved during this work, even at the extremes of the ka range there is good agreement. From this measurement it would seem reasonable to assume, (as far as longitudinal backscattering is concerned), that the wire is well bonded or in very good contact with the matrix.

This suggested that either the steel wire possesses some quantities that made it particularly difficult to detect using SH wave scattering (the theory suggests otherwise), the experiment is at fault (the results for the copper reinforced sample do not support this), or the SH waves may be more sensitive to poorly bonded scatterers than longitudinal waves. To determine the effect of debonding on SH wave scattering, measurements were made on the previously debonded specimens. Two copper reinforced specimens and one steel reinforced specimen from the previous measurements described in Chapter 6 were available for the SH wave measurements.

The results for the SH wave measurements for debonded copper wires are displayed in figures 7.15 and 7.16. For both of these the measured backscattering is completely different from the calculated scattering, being much lower than expected and with only minor indications of the minima. The same overall type of scattering is seen in fig. 7.17 from the debonded steel wire. This effect, with

Cu in Araldite Matrix

Fig. 7.15

Cu in Araldite Matrix

Fig. 7.16

all the structure in the form function being attenuated, appears to be a common feature of the measured backscattering from a debonded wire. The measurements made using longitudinal waves on freshly debonded wires (see figs. 6.13 and 6.14) are very similar to those displayed in figs. 7.14 to 7.16.

Comparing fig. 7.17 with fig. 7.12 it is apparent that there are noticeable differences. In fig. 7.12 the minima at $ka = 4.25$ and 7.25 are clearly identified, which they are not in fig. 7.17 suggesting that full debonding of the steel wire is clearly detectable. However the relatively poor agreement between experiment and theory in fig. 7.12 of the "bonded" steel wire, in view of the excellent agreement seen in fig. 7.11 for the bonded copper wire, suggests that the sample used for the measurement in fig. 7.12 is not completely debonded. The discrepancy between theory and experiment may be highlighting a weak bond, this is further highlighted by the excellent agreement between theory and experiment seen for the longitudinal wave scattering (fig. 7.14). However, without measuring the scattering from several steel reinforced samples it is not possible to determine exactly how the measured scattering from a bonded wire compares with theory, although the results for the copper reinforced samples suggest the theory is correct.

It should be emphasised that there has only been an initial study of the problem of SH wave scattering. A far more detailed study is necessary. The initial results suggest that SH waves are more sensitive to debonding of the scatterer from the matrix and

Figure 7.17: Comparison of the measured SH wave scattering from a debonded steel wire in an epoxy resin matrix (crosses joined by a line) with the theoretical prediction for a perfectly bonded wire (continuous line).

this is some indication that weak bonds may be detectable.

The theoretical modelling of the SH wave scattering from a debonded fibre predicts that there will be a loss of the resonant structure in the backscatter form function as the effective shear modulus is reduced. This feature appears to be present in the measurements on the debonded samples. The theory also predicts that the level of the backscattering should average around a value of one for the form function as K becomes small. The experimental measurements show that debonded samples have a much lower form function than that for a bonded scatterer, indicating that there is some loss mechanism which has not been included in the model. This feature must be the focus of future theoretical work on this problem.

CHAPTER 8. CONCLUSIONS.

8.0 Introduction.

This thesis has been concerned with the problem of the effect of fibre debonding on ultrasonic backscattering. As described in chapter 1, the requirement to be able to detect such damage in a fibre-reinforced composite is particularly important, since not only does the strength of a laminate depend upon the properties of the fibre-matrix interface, but fibre debonding is the earliest sign of fatigue damage in a composite laminate. The approach presented in this thesis for investigating the interaction of ultrasound with a fibre-matrix debond has been to use a combination of mathematical modelling and experimental investigations to provide new insights to the processes occurring. The eventual aim of this work has been to use the improved understanding of the effects of debonds on ultrasonic backscattering to provide either a new technique for testing composites for this defect or to improve upon existing methods of non-destructive evaluation.

8.1 General conclusions.

The initial investigations of debonding were carried out using a computer program to calculate the ultrasonic backscattering from a spherical inclusion in a solid matrix (chapter 2). As an initial approximation the debond was modelled as a thin (thickness $\ll$ inclusion radius), air-filled crack with non-contacting faces surrounding the glass inclusion, the matrix was given the properties of perspex. The results of the

calculations indicate that the thin air shell model of a debond acts essentially the same as an air-filled void irrespective of the thickness of the shell. In addition there was a sharp resonance present in the low ka region of the backscattering form function. The position was shown to be dependent on the thickness of the shell; with decreasing shell thickness the resonance shifted to higher ka . From analysis of the partial wave scattering, it was seen that the resonance occurred in the $n=1$ partial wave indicating that it was due to motion of the inclusion within the void.

A new program (chapter 3) was developed to calculate the ultrasonic scattering from a cylindrical inclusion embedded in an elastic, isotropic matrix. This was further extended to include a shell surrounding the inclusion. It was this program that was used as the basis of three models of debonding. The first was the thin air shell model as described above. For the second and third models it was assumed that after debonding the inclusion would remain in contact with the matrix but would be allowed some tangential slip. This was the basis of the slip model where the amount of slip was governed by an effective shear modulus relating the tangential stress incident on the inclusion to the jump in the tangential displacements across the interface. In the third model the debond was modelled by a thin layer of visco-elastic material.

The thin air shell model showed the same effects as had been seen for the spherical inclusion; the debond acted as an air-filled void with a sharp resonance in the low ka region. As before this resonance occurred in the $n=1$ partial wave. The results for the slip model indicate that below $ka = 3$ as the effective shear modulus, K ,

was reduced, the backscattering was attenuated. Above $ka = 3$ the backscattering was similar to that for a perfectly bonded fibre ($K = \infty$) but with additional sharp resonances. For the visco-elastic shell model it was seen that with increasing layer thickness the attenuation of the backscattering increased.

The experimental work was of particular importance in order to indicate which of the above models would provide the best description of how a debond affects ultrasonic backscattering. In addition, the results of the experimental work would indicate the direction in which future modelling should be extended.

Broadband backscattering measurements were made (chapter 6) on scale models of a single fibre embedded in an epoxy resin matrix. Copper and steel wires were used as the reinforcing fibres. It was shown that excellent agreement between theory and experiment could be achieved. Consequently any significant differences between theory and experiment for measurements made on samples which had been debonded could be attributed to the act of debonding the fibre. The ka range of the measurements was $2 < ka < 8$. For a $10\mu\text{m}$ radius fibre this would equate to a frequency range of $84 \text{ MHz} < f < 336 \text{ MHz}$. This is well within the frequency ranges of current scanning acoustic microscopy techniques.

Initial measurements on debonded samples were made on specimens that had had the wire removed, then reinserted into the epoxy resin matrix. The results indicated that the backscattering from these samples was not significantly different to that from a well bonded wire. For those specimens that had a copper reinforcing wire a shift of the backscatter form function to higher ka was

seen. This was shown to be directly attributable to the acousto-elastic effect.

Subsequent debonded samples were only debonded over about 2/3 of the length of the reinforcing wire. These were termed the partially debonded samples. Initially after debonding the ultrasonic longitudinal backscattering was seen to be distinctly different to that from a well-bonded sample. All the peaks in the backscatter form function were seen to be highly attenuated producing a relatively flat function. Some features such as sharp minima were still present in the backscattering.

After some time from when the wire was debonded (typically one hour) the partially debonded system was seen to relax back to a system that acted effectively as a well-bonded system under interrogation by an incident longitudinal wave. This indicates that the epoxy resin and wire inclusion have relaxed back into intimate contact. Consequently it is only possible to detect fibre-matrix debonding with longitudinal waves immediately after debonding.

In view of the only partially successful attempts to detect fibre-matrix debonds using longitudinal wave backscattering, it was decided to investigate the possibility of using shear wave incidence. It was felt that a wave with its displacement parallel to the axis of the debonded wire (a SH wave) would potentially be sensitive to the debonded region.

The SH wave backscattering (chapter 7) was calculated allowing slip at the interface of the matrix and fibre defined by the effective shear modulus, K . It was shown that as $K \rightarrow 0$ the backscattering tends to that for a cylindrical cavity with a sharp

resonance in the low ka region. From a partial wave analysis it was shown that this resonance occurs in the $n=0$ partial wave indicating that it is a different mode of vibration to that seen for the thin air shell models used to calculate longitudinal wave backscattering.

Experimental measurements were made on both copper-reinforced and steel-reinforced samples. The agreement between theory and experiment for a well-bonded copper-reinforced specimen was excellent, however for the steel-reinforced specimen the agreement was poor with only the minima in the backscatter form function being clearly identified. It was suspected that the steel reinforcing sample may have been poorly bonded to the resin matrix. Longitudinal wave backscattering measurements on this sample however showed the best agreement with theory yet seen.

A series of SH wave backscattering measurements were carried out on previously debonded samples. The results indicated that the effect of debonds on SH waves was completely to attenuate all the peaks in the form function, in addition there were no minima identified in these measurements. These measurements show that the SH waves were sensitive to the presence of the debonds even when the longitudinal waves were not able to detect them.

To summarise briefly the effect of fibre-matrix debonds on ultrasonic backscattering; for both longitudinal and SH wave incidence, a fibre-matrix debond will cause the resonant structure in the backscatter form function to be heavily attenuated. In the case of longitudinal wave backscattering, if the matrix and fibre

materials are in intimate contact then the system will appear to the longitudinal wave as a well-bonded system. This is not the case for the SH wave which will remain sensitive to the presence of the debond.

8.2 Future work and recommendations.

From the experimental work it can be seen that the first model of debonding that was discussed (the thin air shell model) is not an accurate description of the effect of debonding on ultrasonic backscattering. Of the slip model and the visco-elastic shell model, the visco-elastic shell model is too complicated. Firstly the layer requires seven separate parameters to characterise it and it is not clear how to best choose these in order to model a debond realistically.

This leaves the slip model that only uses one parameter, the effective shear modulus K , to characterise the amount of bonding between fibre and matrix. For all the calculations presented in this thesis K has been assumed to be real. In order to incorporate losses at the interface K must be allowed to become complex, in addition it should be investigated how K depends upon frequency.

The results of successfully modelling this single scattering problem can then be introduced into a multiple scattering model. This would be used to calculate the effect of debonded scatterers on the velocity and attenuation in a fibre-reinforced lamina. This has obvious practical applications since from simple velocity and attenuation measurements it may well be possible to estimate quantitatively the amount of debonded reinforcing in the composite

structure under test. This could then be used to estimate the life time of a component and then decide if a costly replacement is needed.

Initially it was intended that longitudinal waves would be used in any future instrument or technique to detect debonding. Since water could be used as a couplant, a longitudinal wave transducer could then be scanned over a surface to detect differences in the ultrasonic backscattering form function of the reinforcing fibres. However, it has been shown that longitudinal waves are only sensitive to debonds under certain circumstances, consequently any further experimental work should be concentrated on using shear wave incidence. The second mode of shear wave vibration, the SV mode, with the polarisation of the displacement of the incident wave perpendicular to the axis of the fibre, has not been examined in this thesis. Although making experimental measurements with SV waves is no more difficult than for the SH waves, the problem of calculating the scattering is far more complex due to mode conversion of the SV waves into longitudinal waves. In any investigation of SV wave scattering this problem would have to be overcome.

The work presented in this thesis has only been concerned with the micromechanics of the effect of debonding on ultrasonic backscattering. For this reason all the scattering measurements have been on single fibre-reinforced samples. Future experimental work should be carried out to determine, in conjunction with theoretical modelling, the effects of multiple scattering on the received backscatter signal. Of particular interest is how the

ordered array like that of a fibre-reinforced laminar might change the ^{observed} backscattering.

To conclude, it has been shown in principle that it is possible to detect the presence of fibre-matrix debonds using ultrasonic backscattering. This form of damage has the effect of attenuating the resonant structure in the backscatter form function for the reinforcing fibre. Shear wave incidence has been shown to be more sensitive to the presence of debonds than longitudinal wave incidence and it is this mode of vibration that should be investigated for future research. In the long term, further research should be conducted to use these results to perform true quantitative non-destructive evaluation of composites.

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APPENDIX 1. ELEMENTS OF SCATTERING EQUATIONS.

$$x11 = -(2n^2 - k_{s1}^2 a^2) H_n(k_{c1} a) + k_{c1} a H'_n(k_{c1} a)$$

$$x12 = -2k_{s1} a n H'_n(k_{s1} a) + 2n H_n(k_{s1} a)$$

$$x13 = \frac{\rho_1}{\rho_2} \left[\left(2 \left(c_{2s}/c_{1s} \right)^2 n^2 - k_{s1}^2 a^2 \right) J_n(k_{c2} a) - 2 \left(c_{2s}/c_{1s} \right)^2 k_{c2} a J'_n(k_{c2} a) \right]$$

$$x14 = \frac{\rho_1}{\rho_2} \left(c_{2s}/c_{1s} \right)^2 \left[2k_{s2} a n J'_n(k_{s2} a) - 2n J_n(k_{s2} a) \right]$$

$$x15 = \frac{\rho_2}{\rho_1} \left[\left(2 \left(c_{2s}/c_{1s} \right)^2 m^2 - k_{s1}^2 a^2 \right) H_n(k_{c2} a) - 2 \left(c_{2s}/c_{1s} \right)^2 k_{c2} a H'_n(k_{c2} a) \right]$$

$$x16 = 2m \left(c_{2s}/c_{1s} \right)^2 \frac{\rho_2}{\rho_1} \left[k_{s2} a H'_n(k_{s2} a) - H_n(k_{s2} a) \right]$$

$$x17 = 0$$

$$x18 = 0$$

$$x21 = 2nk_{c1} a H'_n(k_{c1} a) - 2n H_n(k_{c1} a)$$

$$x22 = (2n^2 - k_{s1}^2 a^2) H_n(k_{s1} a) - 2k_{s1} a H'_n(k_{s1} a)$$

$$x23 = \frac{\rho_1}{\rho_2} \left(k_{1s}/k_{2s} \right)^2 \left[2n J_n(k_{c2} a) - 2nk_{c2} a J'_n(k_{c2} a) \right]$$

$$x24 = \frac{\rho_1}{\rho_2} \left(k_{1s}/k_{2s} \right)^2 \left[(k_{s2}^2 a^2 - 2n^2) J_n(k_{s2} a) + 2k_{s2} a J'_n(k_{s2} a) \right]$$

$$x25 = -2m \left(c_{2s}/c_{1s} \right)^2 \frac{\rho_2}{\rho_1} \left[k_{c2} a H'_n(k_{c2} a) - H_n(k_{c2} a) \right]$$

$$x26 = \left(c_{2s}/c_{1s} \right)^2 \frac{\rho_2}{\rho_1} \left[(2m^2 - k_{s2}^2 a^2) H_n(k_{s2} a) - 2k_{s2} a H'_n(k_{s2} a) \right]$$

$$x27 = 0$$

$$x28 = 0$$

$$x31 = -k_{c1} a H_n'(k_{c1} a)$$

$$x32 = -n H_n(k_{s1} a)$$

$$x33 = k_{c2} a J_n'(k_{c2} a)$$

$$x34 = n J_n(k_{s2} a)$$

$$x35 = k_{c2} a H_n(k_{c2} a)$$

$$x36 = -m H_n(k_{s2} a)$$

$$x37 = 0$$

$$x38 = 0$$

$$x41 = n H_n(k_{c1} a)$$

$$x42 = k_{s1} a H_n'(k_{s1} a)$$

$$x43 = -n J_n(k_{c2} a)$$

$$x44 = -k_{s2} a J_n'(k_{s2} a)$$

$$x45 = n H_n(k_{c2} a)$$

$$x46 = -k_{s2} a H_n'(k_{s2} a)$$

$$x47 = 0$$

$$x48 = 0$$

$$x51 = 0$$

$$x52 = 0$$

$$x53 = -(2n^2 - k_{s2}^2 b^2) J_n(k_{c2} a) + 2k_{c2} b J_n'(k_{c2} a)$$

$$x54 = 2nk_{s2} b J_n'(k_{s2} a) - 2n J_n(k_{s2} a)$$

$$x55 = -(2n^2 - k_{s2}^2 b^2) H_n(k_{s2} b) + 2k_{c2} b H_n'(k_{c2} b)$$

$$x56 = 2nk_{s2} b H_n'(k_{s2} b) - 2n H_n(k_{s2} b)$$

$$x57 = -\frac{\rho_3}{\rho_2} \left[(2 \left(c_{3s}/c_{2s} \right)^2 n^2 - k_{s2}^2 b^2) J_n(k_{c3}b) - 2 \left(c_{3s}/c_{2s} \right)^2 k_{c3} b J'_n(k_{c3}b) \right]$$

$$x58 = 2n \left(c_{3s}/c_{2s} \right)^2 \frac{\rho_3}{\rho_2} \left[k_{s3} b J'_n(k_{s3}b) - J_n(k_{s3}b) \right]$$

$$x61 = 0$$

$$x62 = 0$$

$$x63 = -2n(k_{c2} b J'_n(k_{c2}a) - J_n(k_{c2}a))$$

$$x64 = (2n^2 - k_{s2}^2 b^2) J_n(k_{s2}a) - 2k_{s2} b J'_n(k_{s2}a)$$

$$x65 = -2n(k_{c2} b H'_n(k_{c2}b) - H_n(k_{c2}b))$$

$$x66 = (2n^2 - k_{s2}^2 b^2) H_n(k_{s2}b) - 2k_{s2} a H'_n(k_{s2}b)$$

$$x67 = -2n \left(c_{s3}/c_{s2} \right)^2 \frac{\rho_3}{\rho_2} \left[k_{c3} b J'_n(k_{c3}b) \right]$$

$$x68 = \left(c_{s3}/c_{s2} \right)^2 \frac{\rho_3}{\rho_2} \left[(2n^2 - k_{s3}^2 b^2) J_n(k_{s3}b) - 2k_{s3} b J'_n(k_{s3}b) \right]$$

$$x71 = 0$$

$$x72 = 0$$

$$x73 = k_{c2} b J'_n(k_{c2}a)$$

$$x74 = -n J_n(k_{s2}a)$$

$$x75 = k_{c2} b H'_n(k_{c2}b)$$

$$x76 = -n H_n(k_{s2}b)$$

$$x77 = k_{c3} b J'_n(k_{c3}b)$$

$$x78 = -n J_n(k_{s3}b)$$

$$x81 = 0$$

$$x_{82}=0$$

$$x_{83}= nJ_n(k_{c2}a)$$

$$x_{84}= -k_{s2}bJ_n'(k_{s2}a)$$

$$x_{85}= nH_n(k_{c2}b)$$

$$x_{86}= -k_{s2}bH_n'(k_{s2}b)$$

$$x_{87}= nJ_n(k_{c3}b)$$

$$x_{88}= -k_{s3}bJ_n'(k_{s3}b)$$

$$y_1= \epsilon_n i^n \left[(2n^2 - k_{s1}^2 a^2) J_n(k_{c1}a) - 2k_{c1}a J_n'(k_{c1}a) \right]$$

$$y_2= \epsilon_n i^n \left[2n J_n(k_{c1}a) - 2nk_{c1}a J_n'(k_{c1}a) \right]$$

$$y_3= \epsilon_n i^n k_{c1}a J_n'(k_{c1}a)$$

$$y_4= -\epsilon_n i^n n J_n(k_{c1}a)$$

$$y_5=0$$

$$y_6=0$$

$$y_7=0$$

$$y_8=0$$

where ' denotes the derivative with respect to the argument.

APPENDIX 2.

TABLE 1: PROPERTIES OF SOLIDS.

TABLE 2: PROPERTIES OF LIQUIDS.

Fluid	ρ Density (kg/m^3)	C_l Wave Speed (m/s)	α Attenuation (np/m)	η Viscosity ($\text{Ns/m}^2 \times 10^3$)	C_p Specific Heat (J/kgK)	k Thermal Conductivity (W/m K)	β Thermal Expansivity ($\text{K}^{-1} \times 10^4$)
Air	1.17	344	1600×10^{-14}	0.02	1000	0.024	36.6
Pentane	630	1031	6.3×10^{-14}	0.24	2300	0.11	15.5
Diiodomethane	3310	962	25.4×10^{-14}	2.95	490	0.09	9.1
Water	1000	1485	2.5×10^{-14}	1.00	4182	0.59	2.1
Anilene	1030	1656	11.0×10^{-14}	4.39	2150	0.17	8.5
Morpholine	1000	1431	9.6×10^{-14}	2.23	2000	0.2	9.7
Hexanol	820	1331	12.0×10^{-14}	5.4	2280	0.15	8.4
Mercury ^a	13600	1450		1.55			

Reference: Anson and Chivers [1990]

^aReference: Kaye and Laby [1973]

Table 2. Properties of fluids.

Solid	C_l Longitudinal Wave Speed (m/s)	C_s Shear Wave Speed (m/s)	ρ Density (kg/m ³)	α_l Longitudinal Attenuation (np/m)	α_s Shear Attenuation (np/m)	C_p Specific Heat Capacity (J/kgK)	k Thermal Conductivity (W/m K)	β Thermal Expansivity (K ⁻¹ x10 ⁻⁴)
Copper	4759	2325	8933			384	400	0.5
Aluminium	6374	3111	2670			904	238	0.69
Magnesium	5823	3163	1738			983	155	0.75
Glass	5100	3200	2650	$0.012 \times 10^{-14} f^2$	$0.017 \times 10^{-14} f^2$	803	3.51	0.34
Polyethylene	2430	950	960	$20.5 \times 10^{-6} f$	$146 \times 10^{-6} f$	2000	0.5	4.5
Perspex	2690	1340	1200	$8.2 \times 10^{-6} f$	$25 \times 10^{-6} f$	1500	0.2	1.5
Steel ^a	5960	3235	7800					

Reference: Anson and Chivers [1990]

^aReference: Kaye and Laby [1973]

f is the frequency in Hz.

Table 1. Properties of solids.