Joint Power and Subcarrier Allocation Optimization for Multigroup Multicast Systems with Rate Splitting

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Abstract—In this article, we investigate a resource allocation problem for multicarrier multiuser MISO (multiple-input-single-output) downlink systems, where multiple co-channel multicast groups are served simultaneously. We consider a rate-splitting transmission scheme to address the inevitable inter-group interference under an overloaded multigroup multicast scenario, where the insufficient number of transmit antennas prevents the conventional schemes from neutralizing the interference. We first formulate an optimization problem for maximizing the minimum multicast group rate among all groups on all available subcarriers. This problem involves a joint power and subcarrier allocation optimization, and is non-convex. We apply an iterative scheme based on successive convex approximation (SCA) to find the locally optimal solution. Simulation results demonstrate the performance gain of the proposed scheme compared to the state-of-the-art transmission schemes.

I. INTRODUCTION

3GPP Release 15 has specified New Radio (NR) with changes and improvements from unicast perspective. However, unicast has been shown its drawback to support emerging use cases, where a possible population of millions of concurrent users consume the same bandwidth-heavy content [1]. To fulfil requirements of such scenarios, research community and industry have regained significant attention to point-to-multipoint (PTM) transmission technologies [2], [3], e.g., physical layer multicast [4], [5], due to its efficiency of using a fixed amount of resources to deliver common content to a group of users. PTM has been identified as an essential candidate in the development of 5G (5th Generation) wireless communications towards vertical use cases [6]–[8], e.g., multicasting public warning message and object-based broadcasting (OBB) [1]. In state-of-the-art multicast systems, different multicasting services are separated by Time Division Multiplexing (TDM) to avoid inter-group interference (IGI) [1]. The spectral efficiency of these TDM-based systems can be significantly improved by multiplexing multiple groups onto the same time and frequency resources and applying proper precoding schemes to mitigate the IGI [9], [10]. The classic precoding methods such as zero-forcing [11] require a sufficient number of transmitter antennas (offering a sufficient number of spatial degree of freedom) to eliminate the IGI. Nevertheless, such requirements could be difficult to fulfil in practice where the number of groups/users is relatively large, and can be even more challenging for multicast technologies in the current LTE multicast/broadcast standardization with a restricted number of transmitter antennas [1].

To address such a problem, a Rate-Splitting (RS) based precoding strategy has been shown to provide a non-zero degree of freedom for systems with insufficient transmit antennas, or namely, overloaded systems [11]. With the RS strategy, the original message is split into degraded and designated parts. The concatenated degraded part is superposed onto the

designated part, then the superposed message is transmitted to all users after both parts being precoded. In multigroup multicast systems, the degraded part is intended to be decoded by all groups while the designated part is decoded by the individual group. Our prior work [12] studied the RS-based power allocation algorithm to enable a non-orthogonal radio resource multiplexing for multiple multicast services in overloaded scenarios, but with a pre-determined radio resources (e.g., subcarriers in multicarrier systems) to services mapping policy. In this work, we aim to design an adaptive mapping scheme to better allocate multicast services to available subcarriers, in order to further improve the efficiency of utilizing radio resources based on the algorithm in [12]. To this end, we consider a non-orthogonal group allocation where it allocates a smaller number of multicast groups than the total number on each subcarrier. Note that the proposed scheme can be degraded into two cases, i.e., orthogonal allocation (i.e., one group per subcarrier) and over-aggressive multiplexing (e.g., all groups on every subcarrier), and it shall outperform these cases, in the sense that the proposed scheme strikes a balance between improving the radio resource efficiency and reducing the IGI. The contributions of this paper include:

- We study the performance of RS-based precoding in an overloaded multicarrier multigroup multiple-input-single-output (MISO) system, and formulate a max-min fairness optimization problem to achieve a balanced multicast rate.
- This precoding design jointly optimizes the power and subcarrier allocation for all user groups in the available radio resources. The formulated optimization problem is non-convex.
- We find a locally optimal solution of the formulated non-convex optimization problem, by using the fast converging successive convex approximation method.
- Simulation results verify the multicast group rate enhancement and comparable convergence, by using the proposed RS-based scheme with the joint precoding and subcarrier allocation optimization process.

II. SYSTEM MODEL

We consider a multicast downlink system which comprises a single base station (BS) with N_t transmit antennas and K users each with a single-antenna receiver, denoted by a set $\mathbb{K} \triangleq \{1, \dots, K\}$. $N_t < K$ always holds. Users are grouped into M multicast groups, and the set of all groups is denoted by $\mathbb{M}$, based on users' interests of the specific multicast content. $\mathbb{G}_l$ denotes the set of users that belong to the l -th group. We assume an equal number of users per group and that one user cannot belong to two groups. The total bandwidth is equally partitioned into N_{sc} orthogonal subcarriers, denoted by a set

$\mathbb{N}_{sc} \triangleq \{1, \dots, N_{sc}\}$, therefore each subcarrier has a bandwidth equals B Hz. On each subcarrier $i \in \mathbb{N}_{sc}$, m out of the total M groups ($1 < m \leq M$) are selected to be multiplexed. We denote the set of these m groups and the set of users that are allocated on the i -th subcarrier as $\mathbb{M}^i \subseteq \mathbb{M}$ and $\mathbb{K}^i \subseteq \mathbb{K}$, respectively. Note that the optimization of the value m is not in the scope of this paper, and we will evaluate the system performance in simulations with all possible values of m for a given M . The received signal at user $r \in \mathbb{K}^i$ is given by:

$$y_r^i = (\mathbf{h}_r^i)^H \mathbf{x}^i + n_r^i, \quad (1)$$

where $\mathbf{h}_r^i \in \mathbb{C}^{N_t}$ is the channel vector between the transmitter and the r -th user on the i -th subcarrier, and $\mathbf{x}^i \in \mathbb{C}^{N_t}$ is the signal vector transmitted on the i -th subcarrier, while $n_r^i \sim \mathcal{CN}(0, \sigma_n^2)$ denotes the complex additive white Gaussian noise. Channel state information can be obtained at the BS (i.e., CSIT) through uplink training in Time Division Duplex (TDD) or quantized feedback in Frequency Division Duplex (FDD) systems. We assume that the BS has access to perfect CSIT for all users in this work, while a further investigation with imperfect CSIT can be carried out by taking into account similar techniques as in [13].

Let t be the index for the allocated m groups on i -th subcarrier, and $g(t)$ be the function to map the index to a specific group in M , thus group messages $W_{g(t)}^i, \forall t \in \mathbb{M}^i$ are delivered to corresponding users in $\mathbb{G}_{g(t)}, \forall t \in \mathbb{M}^i$. Different approaches can be applied for such a group message delivery: (a) Single-stream (SS) transmission approach, where all the group messages are packed into one single data stream, and broadcasted to all groups. This approach is equivalent to the state-of-the-art multicast/broadcast transmission scheme such as TDM; (b) Non-orthogonal multiple access (NOMA) approach, where the group messages are superposed into data streams which are decoded and cancelled in a successive manner, while users are forced to fully decoded data streams for some other groups. Nevertheless, the SS approach suffers from the inefficient use of available resources, while the enforced ordering and fully decoding of other users/groups' streams hinder the NOMA approach achieving all spatial multiplexing gains. We consider the superiority of RS over NOMA [11], [14] and propose to use a RS-based approach in this work. With this approach, the degraded and designated parts of each message are generated as: $W_{g(t)}^i \rightarrow \{W_{g(t)0}^i, W_{g(t)1}^i\}$. Same applied to other groups on the i -th subcarrier. Hence, the super message consists of $\{W_{g(1)0}^i, \dots, W_{g(t)0}^i, \dots, W_{g(m)0}^i\}$ and is encoded as s_c^i . The designated part of the group message is independently encoded, e.g., $W_{g(t)1}^i \rightarrow s_{g(t)}^i$. Therefore, the transmitted signal on the i -th subcarrier is:

$$\mathbf{x}^i = \mathbf{p}_c^i s_c^i + \sum_{t=1}^m \mathbf{p}_{g(t)}^i s_{g(t)}^i, \quad (2)$$

where $\mathbf{p}_c^i \in \mathbb{C}^{N_t}$ and $\mathbf{p}_{g(t)}^i \in \mathbb{C}^{N_t}$ represent the precoding matrix on the i -th subcarrier for the degraded part (or the super message intended for all groups) and the designated parts for the $g(t)$ -th group messages, respectively. Defining $\mathbf{s}^i \triangleq [s_c^i, s_{g(1)}^i, \dots, s_{g(t)}^i, \dots, s_{g(m)}^i]$ and further assuming that $\mathbb{E}\{(\mathbf{s}^i)^H (\mathbf{s}^i)\} = \mathbf{I}$, the power constraint follows $\sum_{i=1}^{N_{sc}} \mathbb{E}\{(\mathbf{x}^i)^H (\mathbf{x}^i)\} \leq \sum_{i=1}^{N_{sc}} P_i$, where P_i is the power

allocated on the i -th subcarrier and $P_i \geq 0$. Therefore, the received signal at user $r \in \mathbb{K}^i$ in (1) can be rewritten as:

$$y_r^i = (\mathbf{h}_r^i)^H \mathbf{p}_c^i s_c^i + \sum_{t=1}^m (\mathbf{h}_r^i)^H \mathbf{p}_{g(t)}^i s_{g(t)}^i + n_r^i. \quad (3)$$

III. RESOURCE ALLOCATION PROBLEM FORMULATION

The signal to interference plus noise ratio (SINR) experienced by the r -th user at different stages of successive interference cancellation (SIC) operations can be given as: **Stage 1:** User decodes the degraded part, having a SINR as

$$\eta_r^{c,i} = \frac{|(\mathbf{h}_r^i)^H \mathbf{p}_c^i|^2}{\sum_{t=1}^m |(\mathbf{h}_r^i)^H \mathbf{p}_{g(t)}^i|^2 + \sigma_n^2}, \quad (4)$$

thus the achievable rate for the degraded part can be given as $R_r^{c,i} = B \log_2(1 + \eta_r^{c,i})$. All users on the i -th subcarrier should be able to decode the degraded part, and the maximum transmission rate of the degraded part is $R^{c,i} = \min_{r \in \mathbb{K}^i} R_r^{c,i}$.

Stage 2: User removes the degraded part and decodes the designated part individually. We assume in this work that the degraded part is always decoded before the designated part when performing SIC, as in [11]–[13]. Further investigations can be carried out for different ordering and receiver architecture. Defining $\mu(r)$ as the group (on the i -th subcarrier) that the r -th user belongs to, the SINR for the designated part is

$$\eta_r^{p,i} = \frac{|(\mathbf{h}_r^i)^H \mathbf{p}_{\mu(r)}^i|^2}{|\sum_{t=1, g(t) \neq \mu(r)}^m (\mathbf{h}_r^i)^H \mathbf{p}_{g(t)}^i|^2 + \sigma_n^2}, \quad (5)$$

thus the achievable rate for the designated part is given by $R_r^{p,i} = B \log_2(1 + \eta_r^{p,i})$. Due to the fact that the rate of multicast systems is decided by the lowest user's data rate, therefore the group rate for the $\mu(r)$ -th group on the i -th subcarrier is given by: $R_{\mu(r)}^{p,i} = \min_{r \in \mu(r)} R_r^{p,i}$. With RS, each group is only interested in a fraction of the degraded part. Therefore, we can define $C_{\mu(r)}^i = \frac{\mathbb{L}(W_{\mu(r)0}^i)}{\sum_{t=1}^m \mathbb{L}(W_{g(t)0}^i)} R^{c,i}$, where $C_{\mu(r)}^i$ is the rate contributed from the degraded part to the $\mu(r)$ -th group, and $\mathbb{L}(W_{\mu(r)0}^i)$ represents the length of $W_{\mu(r)0}^i$. Then, the overall group rate can be given by: $R_{\mu(r)}^i = C_{\mu(r)}^i + R_{\mu(r)}^{p,i}$. In this paper, we use Max-Min fairness (MMF) strategy which can maximize the minimum rate among all users. The MMF rate on i -th subcarrier can be formulated as:

$$T^i = \left\{ \sum_{j=1}^J a_j^i (\max_{\mathbf{p}^i, \mathbf{C}} \min_{\mu(r) \in \mathbb{M}^i} R_{\mu(r)}^i) \mid \mathbf{a}, m \right\}, \quad (6)$$

where $\mathbf{p}^i \triangleq [\mathbf{p}_c^i, \mathbf{p}_{g(1)}^i, \dots, \mathbf{p}_{g(m)}^i]$, and $\mathbf{C} \triangleq [C_1^i, \dots, C_m^i]$ includes the corresponding degraded rate of each group. We denote the indicator vector for the i -th subcarrier as a column vector $\mathbf{a}_j^i$ with dimension of $[J \times 1]$, and $J = \binom{M}{m}$ as the number of possible group combinations, and we have $\mathbf{J} \triangleq [1, \dots, J]$. Individual entry of $\mathbf{a}_j^i$ is denoted as $a_j^i, j \in \{1, \dots, J\}$, where $a_j^i = 1$ means that the j -th group allocation combination is applied on the i -th subcarrier. $\mathbf{a} \triangleq [\mathbf{a}_1^1, \dots, \mathbf{a}_J^{N_{sc}}]$ denotes the collection of indicator vectors on all subcarrier. Given (6), we can formulate an optimization

problem for the RS-based joint subcarrier allocation and transmit precoder design problem, as following:

$$\max_{\mathbf{a}} \sum_{i=1}^{N_{sc}} T^i, \quad (7a)$$

$$\text{s.t. } a_j^i \in \{0, 1\}, \forall j, i \quad (7b)$$

$$\sum_{j=1}^J a_j^i \leq 1, \forall i \quad (7c)$$

$$\sum_{j=1}^J \sum_{i=1}^{N_{sc}} a_j^i (\|\mathbf{p}_c^i\|^2 + \sum_{t=1}^m \|\mathbf{p}_{g(t)}^i\|^2) \leq P \quad (7d)$$

$$\sum_{t=1}^m C_{g(t)}^i \geq R^{c,i}, C_{g(t)}^i \geq 0. \quad (7e)$$

The conditions (7b) and (7c) restrict the group allocation on each subcarrier, and (7d) takes account of the transmission power restriction, and (7e) guarantees the decodability of the degraded part. It can be found that the optimization problem in (7) is non-convex. In Section IV, this non-convex problem is reformulated, and a locally optimal solution is achieved by using a Successive Convex Approximation (SCA) method.

IV. OPTIMIZATION

Due to the uncertainty of the message splitting ratio i.e. $\frac{\mathbb{L}(W_{\mu(r)0}^i)}{\sum_{t=1}^m \mathbb{L}(W_{g(t)0}^i)}$, the objective function in (7) can not be directly reformulated. Our solution is to first formulate an equivalent total achievable rate of the r -th user, given as

$$R_r^{c,i} + R_{\mu(r)}^{p,i} = \sum_{j=1}^J B \log_2 \left(1 + \frac{a_j^i |(\mathbf{h}_r^i)^H \mathbf{p}_c^i|^2}{\sum_{t=1}^m a_j^i (|(\mathbf{h}_r^i)^H \mathbf{p}_{g(t)}^i|^2) + \sigma_n^2} \right) + \sum_{j=1}^J B \log_2 \left(1 + \frac{a_j^i |(\mathbf{h}_r^i)^H \mathbf{p}_{\mu(r)}^i|^2}{\sum_{t=1, g(t) \neq \mu(r)}^m a_j^i (|(\mathbf{h}_r^i)^H \mathbf{p}_{g(t)}^i|^2) + \sigma_n^2} \right). \quad (8)$$

where $R_r^{c,i}$ in (8) contains the common symbol/stream from other group(s) (e.g., $g(t) \neq \mu(r)$). We then introduce new auxiliary variable sets to separate the useful part, and reformulate (7) as follows:

$$\max_{\mathbf{p}^i, \mathbf{a}, \mathbf{C}, \mathbf{z}} \sum_{i=1}^{N_{sc}} z^i, \quad (9a)$$

$$\text{s.t. } (7b) \sim (7e)$$

$$C_{\mu(r)}^i + R_{\mu(r)}^{p,i} \geq z^i, \forall r \in \mathbb{K}^i \quad (9b)$$

$$R_r^{c,i} \geq R^{c,i}, \forall r \in \mathbb{K}^i \quad (9c)$$

where $\mathbf{z} \triangleq [z^1, \dots, z^{N_{sc}}]$, with z^i denoting the MMF rate on the i -th subcarrier. We define $\mathbf{H}_r^i = \mathbf{h}_r^i (\mathbf{h}_r^i)^H$, and consider semidefinite programming (SDP) with $\tilde{\mathbf{P}}_{j,c}^i = a_j^i \mathbf{p}_c^i (\mathbf{p}_c^i)^H$ and $\tilde{\mathbf{P}}_{j,g(t)}^i = a_j^i \mathbf{p}_{g(t)}^i (\mathbf{p}_{g(t)}^i)^H$. Hence, we have (9b) and (9c) equivalent to (using Tr to denote the matrix trace):

$$C_{\mu(r)}^i + \sum_{j=1}^J B \log_2 \left(\frac{\sum_{t=1}^m \text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,g(t)}^i) + \sigma_n^2}{\sum_{t=1, g(t) \neq \mu(r)}^m \text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,g(t)}^i) + \sigma_n^2} \right) \geq z^i, \quad (10a)$$

$$\sum_{j=1}^J B \log_2 \left(\frac{\text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,c}^i) + \sum_{t=1}^m \text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,g(t)}^i) + \sigma_n^2}{\sum_{t=1}^m \text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,g(t)}^i) + \sigma_n^2} \right) \geq R^{c,i}, \quad (10b)$$

with additional rank constraints $\text{rank}(\tilde{\mathbf{P}}_{j,c}^i) = 1$ and $\text{rank}(\tilde{\mathbf{P}}_{j,g(t)}^i) = 1$. We remove these rank constraints from the optimization problem due to their non-convexity, but will discuss the conditions for SDP relaxation at the end of this section. Both equations in (10) fall in the form of $\log_2(X) \geq Y$ thus can be reformed, taking (10b) as an example, as follows:

$$\begin{aligned} R^{c,i} &= \max_{\mathbf{P}^i, \mathbf{C}, \tilde{\mathbf{P}}_{j,c}^i} \min_{r \in \mathbb{K}^i} F(\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i) - G(\mathbf{P}^i) \\ &= \max_{\mathbf{P}^i, \mathbf{C}, \tilde{\mathbf{P}}_{j,c}^i} \min_{r \in \mathbb{K}^i} \sum_{j=1}^J B \log_2 (\text{Tr}(\mathbf{H}_r^i \tilde{\mathbf{P}}_{j,c}^i) + \text{Tr}(\tilde{\mathbf{H}}_r^i \mathbf{P}^i) \\ &\quad + \sigma_n^2) - \sum_{j=1}^J B \log_2 (\text{Tr}(\tilde{\mathbf{H}}_r^i \mathbf{P}^i) + \sigma_n^2), \end{aligned} \quad (11)$$

where $\mathbf{P}^i \triangleq [\tilde{\mathbf{P}}_{j,g(1)}^i, \dots, \tilde{\mathbf{P}}_{j,g(m)}^i]^T \in \mathbb{C}^{(mN_t) \times N_t}$. $\tilde{\mathbf{H}}_r^i \in \mathbb{C}^{N_t \times (mN_t)}$ is the Kronecker product of an all-one matrix and channel matrix $\mathbf{H}_r^i$, i.e., $\tilde{\mathbf{H}}_r^i = \mathbf{1}_{1,m} \otimes \mathbf{H}_r^i$, where $\mathbf{1}_{1,m}$ denotes the all-one matrix and has the dimension of $1 \times m$. Both $F(\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i)$ and $G(\mathbf{P}^i)$ are summation of $\log_2$ functions, thus (11) satisfies that

$$-R^{c,i} = \min_{\mathbf{P}^i, \mathbf{C}, \tilde{\mathbf{P}}_{j,c}^i} \max_{r \in \mathbb{K}^i} -F(\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i) - (-G(\mathbf{P}^i)). \quad (12)$$

Note that $-F(\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i)$ and $-G(\mathbf{P}^i)$ are convex functions and the problem in (12) can be solved by the difference of convex function programming. In this work, we apply the successive convex approximation to find a locally optimal solution for (12). Since $-G(\mathbf{P}^i)$ is a differentiable convex function w.r.t $\mathbf{P}^i$, a feasible solution, denoted by $\mathbf{P}^{i,(k)}$, can be initialized to form the following inequality:

$$-G(\mathbf{P}^i) \geq -G(\mathbf{P}^{i,(k)}) - \text{Tr}(\nabla_{\mathbf{P}^i} G(\mathbf{P}^{i,(k)})^T (\mathbf{P}^i - \mathbf{P}^{i,(k)})), \quad (13)$$

where ∇ denotes the partial derivative. Therefore, we can obtain an upper bound for $-R^{c,i}$, as given by:

$$\begin{aligned} -R_r^{c,i} &\leq -F(\mathbf{P}^i, \tilde{\mathbf{P}}_{j,c}^i) - (-G(\mathbf{P}^{i,(k)}) \\ &\quad - \text{Tr}(\nabla_{\mathbf{P}^i} G(\mathbf{P}^{i,(k)})^T (\mathbf{P}^i - \mathbf{P}^{i,(k)}))), \end{aligned} \quad (14)$$

where

$$\nabla_{\mathbf{P}^i} G(\mathbf{P}^{i,(k)})^T (\mathbf{P}^i - \mathbf{P}^{i,(k)}) = \sum_{j=1}^J B \frac{\tilde{\mathbf{H}}_r^i (\mathbf{P}^i - \mathbf{P}^{i,(k)})}{(\text{Tr}(\tilde{\mathbf{H}}_r^i \mathbf{P}^i) + \sigma_n^2) \ln 2}. \quad (15)$$

The same SCA-based reformulation of (10a) can be applied, which yields:

$$\begin{aligned} -C_{\mu(r)}^i + [-G(\mathbf{P}^i) - M(\tilde{\mathbf{P}}^i)] &\leq -C_{\mu(r)}^i + \\ &\quad [-G(\mathbf{P}^i) - M(\tilde{\mathbf{P}}^{i,(k)}) - \text{Tr}(\nabla_{\tilde{\mathbf{P}}^i} M(\tilde{\mathbf{P}}^{i,(k)})^T (\tilde{\mathbf{P}}^i - \tilde{\mathbf{P}}^{i,(k)})]), \end{aligned} \quad (16)$$

where $\tilde{\mathbf{P}}^i \triangleq \mathbf{P}^i \setminus \tilde{\mathbf{P}}_{j,\mu(r)}^i \in \mathbb{C}^{(N_t \times (m-1)) \times N_t}$,

$$M(\tilde{\mathbf{P}}^i) = - \sum_{j=1}^J B \log_2 (\text{Tr}(\tilde{\mathbf{H}}_r^i \tilde{\mathbf{P}}^i) + \sigma_n^2), \quad (17)$$

$$\nabla_{\tilde{\mathbf{P}}^i} M(\tilde{\mathbf{P}}^{i,(k)})^T (\tilde{\mathbf{P}}^i - \tilde{\mathbf{P}}^{i,(k)}) = \sum_{j=1}^J B \frac{\tilde{\mathbf{H}}_r^i (\tilde{\mathbf{P}}^i - \tilde{\mathbf{P}}^{i,(k)})}{(\text{Tr}(\tilde{\mathbf{H}}_r^i \tilde{\mathbf{P}}^i) + \sigma_n^2) \ln 2}, \quad (18)$$

and $\bar{\mathbf{H}}_r^i = \mathbf{1}_{1,(m-1)} \otimes \mathbf{H}_r^i$. Therefore, we obtain an upper bound of (9), as given by

$$\min_{\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i, \mathbf{C}, \mathbf{z}} \sum_{i=1}^{N_{sc}} -z^i, \quad (19a)$$

s.t. (7b) ~ (7e),

$$-C_{\mu(r)}^i + [-G(\mathbf{P}^i) - M(\tilde{\mathbf{P}}^{i,(k)}) - \text{Tr}(\nabla_{\mathbf{P}^i} M(\tilde{\mathbf{P}}^{i,(k)})^T (\tilde{\mathbf{P}}^i - \tilde{\mathbf{P}}^{i,(k)}))] \leq -z^i \quad (19b)$$

$$-F(\mathbf{P}^i, \tilde{\mathbf{P}}_{j,c}^i) - (-G(\mathbf{P}^{i,(k)}) - \text{Tr}(\nabla_{\mathbf{P}} G(\mathbf{P}^{i,(k)})^T (\mathbf{P}^i - \mathbf{P}^{i,(k)}))) \leq -R^{c,i}. \quad (19c)$$

However, the non-convex binary constraint (7b) still exists. To address this non-convexity, first, we relax the binary variable $a_j^i = \{0, 1\}$ to be a continuous value in the interval of $[0, 1]$, with an additional constraint:

$$\sum_{j=1}^J a_j^i - (a_j^i)^2 \leq 0, \quad \forall i \in \mathbb{N}_{sc}, \quad (20)$$

where, since $a_j^i > (a_j^i)^2$ always holds when $a_j^i \in (0, 1)$, the feasible solutions are $a_j^i = \{0, 1\}$. Next, we introduce an additional penalty factor γ [15] to further reformulate the objective function as $\min_{\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i, \mathbf{C}, \mathbf{z}} \sum_{i=1}^{N_{sc}} -z^i + \gamma(\sum_{j=1}^J a_j^i - (a_j^i)^2)$. Given the negative valued $-z^i$, in order to obtain the minimum value of the objective function, a_j^i should be as close as possible to $(a_j^i)^2$. Otherwise, their difference will penalize the objective function. The increase of the penalty factor γ will amplify the penalization. It is obvious that $a_j^i - (a_j^i)^2$ can also be reformed based on the difference of convex functions programming. Hence, the final optimization problem is reformulated as:

$$\min_{\tilde{\mathbf{P}}_{j,c}^i, \mathbf{P}^i, \mathbf{C}, \mathbf{z}} \sum_{i=1}^{N_{sc}} -z^i + \gamma(\sum_{m=1}^M \sum_{t=1}^M A^i(a)), \quad (21a)$$

s.t. (7c) ~ (7e), (14), (16)

$$\tilde{\mathbf{P}}_{j,c}^i \geq 0, \tilde{\mathbf{P}}_{j,g(t)}^i \geq 0, \quad \forall i, j, t \quad (21b)$$

$$\text{Tr}(\tilde{\mathbf{P}}_{j,g(t)}^i) \leq a_j^i P, \quad \forall i, j, t \quad (21c)$$

$$\text{Tr}(\tilde{\mathbf{P}}_{j,c}^i) \leq \text{Tr}(\|\mathbf{p}_c^i\|^2) \quad \forall i, j \quad (21d)$$

$$\text{Tr}(\tilde{\mathbf{P}}_{j,g(t)}^i) \leq \text{Tr}(\|\mathbf{p}_{g(t)}^i\|^2), \quad \forall i, j, t \quad (21e)$$

$$\text{Tr}(\tilde{\mathbf{P}}_{j,c}^i) \geq \text{Tr}(\|\mathbf{p}_c^i\|^2) - (1 - a_j^i)P, \quad \forall i, j \quad (21f)$$

$$\text{Tr}(\tilde{\mathbf{P}}_{j,g(t)}^i) \geq \text{Tr}(\|\mathbf{p}_{g(t)}^i\|^2) - (1 - a_j^i)P, \quad \forall i, j, t \quad (21g)$$

where $A^i(a) \triangleq a_j^i - [((a_j^i)^{(k)})^2 + 2(a_j^i)^{(k)}(a_j^i - (a_j^i)^{(k)})]$, $P = (\sum_{i=1}^{N_{sc}} P_i)$. An iterative algorithm is used to solve problem (21) until the objective function converge to a locally optimal point. To this end, the objective function (21a) along with all the constraints, are presented in convex form and can be efficiently solved using MATLAB based CVX toolbox with a polynomial time computational complexity [16].

After solving the optimization problem (21), following observations can be achieved:

- Due to the SDP relaxation in (10) and removal of the rank one constraints, the matrices $\tilde{\mathbf{P}}_{j,c}^i$ and $\tilde{\mathbf{P}}_{j,g(t)}^i$ being optimized in (21) are likely not to be rank one in general. In order to restore the corresponding precoding vectors, i.e., $\mathbf{p}_c^i$ and $\mathbf{p}_{g(t)}^i$, if the matrices have rank one after optimization,

Fig. 1: Achievable group MMF rate vs SNR

then their principal component is the optimal precoding vector; if not, randomization process can be used to recover the precoding vectors, such that a set of candidate weight vectors for $\mathbf{p}_c^i$ and $\mathbf{p}_{g(t)}^i$ can be generated, and from which set the best precoder vector combination can be selected by substituting the candidate vectors back to (21). One commonly used method for generating candidate vectors is using eigenvalue decomposition. Note that the computational cost of the above recovering process is negligible compared to that of solving the original problem.

- We also obtain the optimal subcarrier allocation indicator and the optimal MMF rate on each subcarrier, collectively denoted as $\mathbf{a}_{\text{opt}}$ and $\mathbf{z}_{\text{opt}} \triangleq [z_{\text{opt}}^1, \dots, z_{\text{opt}}^{N_{sc}}]^T$, respectively. We map $\mathbf{a}_{\text{opt}}$ to a matrix $\mathbf{G} \in \mathbb{R}^{M \times N_{sc}}$ to indicate the group selection on all subcarriers, such that some entries of $\mathbf{G}$ equal to 1 if the corresponding groups are selected on the considered subcarrier while the rest of entries equals to 0. Given that an optimum MMF design achieves balanced group rates, all co-located groups should have the same rate. Hence, we can calculate the group MMF rate as $\mathbf{r}_g^{MMF} = \mathbf{G}\mathbf{z}_{\text{opt}}$ where $\mathbf{r}_g \in \mathbb{R}^{M \times 1}$, then the group MMF rate for the multicarrier multigroup multicast system can be achieved as $R_g^{MMF} = \min\{\mathbf{r}_g^{MMF}\}$.

V. SIMULATION RESULTS

The performance of the proposed RS-based multicarrier multigroup multicast with the optimized power and subcarrier allocation policy is simulated and compared with several baseline models. Four baseline models are considered: **1.** RS-based multicarrier multigroup multicast system without subcarrier allocation (SA) [12]. **2.** Single Stream transmission without SA [17], as described in Section II. This model represents the TDM transmission. **3.** No-RS system but with SA, where m out of M groups are served at the same time on each subcarrier. **4.** No-RS system without SA [4], but solved via SCA method. The same set of channel realizations is used in all simulation models to guarantee a fair comparison.

Fig. 2: Convergence comparison with/without SA

A. MMF Rate Comparison

Fig.1 compares the average achievable MMF group rate with different transmission schemes in a multicarrier environment. Simulation settings are $\{N_t, M, K, G, N_{sc}, m\} = \{4, 3, 9, 3, 3, 2\}$. As shown in the Fig1(a), baseline model 4 results in a saturating performance (1.8bps/Hz @30dB SNR) due to the lack of degree of freedom. The state-of-the-art multicast transmission (i.e., TDM) and all the RS schemes do not have this issue. In the low SNR regime, TDM is outperformed by No-RS scheme (2dB difference to achieve 1bps/Hz) as the former does not benefit from multiplexing gain. The gain of applying RS over No-RS (baseline 2 and 4) is pronounced, more than 5dB difference to achieve 3bps/Hz group MMF rate. Besides, the proposed RS with SA strikes a good balance between IGI increasing (multiplexing more groups on the same subcarrier) and spectrum efficiency reduction (orthogonal group-subcarrier mapping), and results in a better group MMF rate compared to baseline model 1. The gain is more obvious at high SNRs, e.g., the proposed RS with SA requires 2dB less SNR to achieve 3.72bps/Hz, which is the maximum MMF rate that baseline model 1 can reach @30dB SNR. However, in baseline model 3, the SA process helps No-RS working in a non-saturated situation, since there are fewer groups being multiplexed on each subcarrier. Corresponding results are shown in the Fig1(b), where the proposed RS with SA still outperforms baseline model 3 in the whole testing SNR region, with a constant 1dB less SNR requirement to achieve the same rate.

B. Convergence of the Proposed Algorithm

In Fig.2, we compare the convergence of the proposed SCA based RS algorithms with and without (i.e., baseline model 1) SA under different transmit SNRs. The system configuration is same as Section.V.A. Fig.2 shows that the SA process does not significantly increase the required number of iterations to make the algorithm converge. Both cases (with and without SA) require more iterations to converge along with an increase of the transmit SNR. More specifically, at 15dB SNR, both cases converge approximately with 7 iterations. While with 30dB SNR, the case with SA needs averagely 5 more iterations to

achieve convergence (11 and 16 iterations required for system without and with SA, respectively).

VI. CONCLUSION

We have studied the Rate-Splitting assisted power and sub-carrier allocation problem in the considered multicarrier multi-group multicast downlink system. We formulated the non-convex optimization problem and derived the approximated solution by using the SCA approach. In particular, we showed that with the optimized power and subcarrier allocation policy, the proposed scheme outperforms four baseline models in terms of the achievable rate. The proposed scheme provides benefits of the guaranteed fairness among all multicast groups due to the Max-Min fairness optimization policy, and the fast convergence thanks to the use of the SCA approach.

One step ahead can be introducing the optimization of m , i.e., to form a joint group and subcarrier allocation optimization for the considered system.

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