An Accurate Closed-Form Approximation of the Ergodic Capacity over Log-Normal Fading Channels

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Abstract—The log-normal probability distribution is commonly used in wireless communications to model the shadowing and, more recently, the small-scale fading for indoor ultra-wide-band communications. In this paper, an accurate closed-form approximation of the ergodic capacity over log-normal fading channels is derived. This expression can be easily used to evaluate and compare the ergodic capacity of communication systems operating over log-normal fading channels.

I. INTRODUCTION

The log-normal distribution has long been used predominantly in communications to model the effect of shadowing, i.e., medium-scale fading, due to large obstructions [1]. Recently, the log-normal distribution has been applied to describe the small-scale fading of the IEEE 802.15.3a channel model [2], for indoor Ultra-Wide-Band (UWB) communications [3]. And more generally, the amplitude distribution of slowly varying communication channels, e.g., indoor environments, tends to be log-normal [4].

Channel capacity and probability of error are common metrics used to evaluate the performance of communication systems. The derivations of the channel capacity and the probability of error over typical fading channels, such as Rayleigh and Rician, have attracted considerable research interests over the past decade, [5]–[8]. The error probability of communication systems operating over log-normal fading channels was commonly expressed using a computable formula in [8] until an accurate closed-form approximation was derived in [9]. As for the capacity, it was first upper and lower bounded in [10], for log-normally distributed shadowing. These two bounds are closed-form expressions but unfortunately are loose for low Signal-to-Noise-Ratio (SNR) values. Recently in [4], an accurate computable formula, which is the truncation of an infinite alternating series and therefore is not a closed-form expression, was proposed to evaluate the ergodic capacity over a log-normal channel. This work was then extended in [11] and a closed-form approximation of the ergodic capacity was obtained. However, as it is evidently shown later in the paper, this expression is inaccurate for low SNR and high dB spread values of the log-normal distribution.

In this paper, we derive a closed-form approximation of the ergodic capacity over log-normal fading channels, relying on the system model proposed in Section II. It differs from the

well-know integral formulation by less than 0.01 nats/s/Hz for any SNR and dB spread values lying in the range of interest for wireless communications. We first derive in Section III two very tight closed-form approximations of the integral formulation of the ergodic capacity for low and high dB spread values, respectively. Our results show that the closed-form expression derived for low dB spread is an absolute lower bound on the ergodic capacity, i.e., it is expressed independently of the dB spread. We then design a parametric function based on hyperbolic functions that best fits the difference between the exact ergodic capacity and its absolute lower bound. In Section IV, we obtain an accurate closed-form approximation of the ergodic capacity, which is a function of the SNR of the system and the dB spread of the log-normal fading channel, by minimizing this difference. Our closed-form expression is simpler than the computable formula proposed in [4] and far more accurate than the expression derived in [11], for evaluating the channel capacities of communication systems over log-normal fading channels. We also use our expression to interpret the behaviour of the capacity at high SNRs and compute the ergodic capacities of Single-Input-Single-Output (SISO) and Multiple-Input-Single-Output (MISO) UWB systems operating over the IEEE 802.15.3a channel in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL FOR LOG-NORMAL FADING CHANNEL

In wireless communications, medium-scale fading is usually modelled as a multiplicative and slowly time varying random process. The received signal $r(t)$ can be expressed as [1]

$$r(t) = \sqrt{\epsilon} s(t) h(t) + n(t), \quad (1)$$

where ϵ is the average transmitted signal energy, $s(t)$ is the transmitted signal, $h(t)$ is the random process that characterizes the medium-scale fading, and $n(t)$ is an additive white gaussian noise process with a double-sided variance of $N_0/2$. The medium-scale fading is considered as log-normally distributed for any t value. In a slow-varying environment, $h(t)$ can be considered constant over multiple symbol periods such that $h(t) = \beta$ is a log-normally distributed Random Variable (RV). The multipath attenuation factors of the channel model released by the IEEE for indoor communications in

[2], namely the IEEE 802.15.3a channel model, exhibit a log-normal distribution. In this case, the received signal $r(t)$ can be expressed as in (1) with $h(t) = \sum_l G_l \delta(t - \tau_l)$, as shown in [12] and [13], where $\delta(t)$ denotes the Dirac delta function, $G_l = \beta_l e^{j\theta_l}$ is the l -th multipath attenuation, τ_l is its respective time delay factor, β_l is a log-normal RV, and $\theta_l \in \{0, \pi\}$ [2].

The ergodic capacity of the log-normal fading channel can be expressed as follows [10]

$$C_e = E[\ln(1 + \gamma\alpha)] = 2A \int_0^{+\infty} \ln(1 + \gamma\alpha) p(\alpha) d\alpha, \quad (2)$$

where $\gamma = \epsilon/N_0$ is the average SNR, $A = \frac{1}{2}$ if the capacity is expressed in (nats/s/Hz) or $A = \frac{1}{2\ln(2)}$ if the capacity is expressed in (bits/s/Hz), and $p(\alpha)$ is the probability density function (pdf) of a log-normally distributed RV α , given by

$$p(\alpha) = \frac{1}{\alpha\sqrt{2\pi}\sigma_\alpha} \exp\left(-\frac{1}{2}\left(\frac{\ln(\alpha) - m_\alpha}{\sigma_\alpha}\right)^2\right), \quad (3)$$

with m_α and σ_α being the mean and standard deviation of α , respectively. The RV $\alpha = \beta^2$ for medium-scale fading, and $\alpha \approx \sum_l \beta_l^2$ for the indoor UWB channel, as shown in [12] and [13]. Using the results in [14] and [15], $\sum_l \beta_l^2$ can be considered as a log-normally distributed RV since both β and β^2 are log-normally distributed [13]. Thus, the ergodic capacity formula in (2) can be applied to the medium-scale fading and to the indoor UWB channel. In wireless communications, parameters (m_y, σ_y) are introduced to characterize a log-normal distribution, where $m_y = Bm_\alpha$, $\sigma_y = B\sigma_\alpha$ and $B = 10/\ln(10)$ [14]. The parameter σ_y , known as the dB spread, is between 6 and 12 dB for medium-scale fading [14] and between 2 and 5 dB for the indoor UWB channel [2]. Without loss of generality, its range of values can be aggregated as $\sigma_y \in [2, 12]$ for most communication systems operating over log-normal fading channels. Using the change of variables $x = \left(\frac{\ln(\alpha) - m_\alpha}{\sqrt{2}\sigma_\alpha}\right)$ in (3), C_e in (2) is rewritten as

$$C_e(k, m) = A \int_{-\infty}^{+\infty} \ln\left(1 + e^{\left(\frac{x}{k} + m\right)}\right) \frac{2}{\sqrt{\pi}} e^{-x^2} dx, \quad (4)$$

where $m = m_\alpha + \ln(\gamma) = \frac{m_y + \gamma(\text{dB})}{B}$, $k = \frac{1}{\sqrt{2}\sigma_\alpha} = \frac{B}{\sqrt{2}\sigma_y} \geq 0$. Then, using the hyperbolic function $\cosh(u) = (e^u + e^{-u})/2$, (4) can be re-expressed as

$$C_e(k, m) = A \int_{-\infty}^{+\infty} \left[\left(\ln(2) + \frac{m}{2} \right) + \ln\left(\cosh\left(\frac{x + km}{2k}\right)\right) + \frac{x}{2k} \right] \frac{2}{\sqrt{\pi}} e^{-x^2} dx. \quad (5)$$

Moreover, knowing that $\int_{-\infty}^{+\infty} x e^{-x^2} dx = 0$ and $\int_0^{+\infty} e^{-x^2} dx = \sqrt{\pi}/2$, (5) can be rewritten as

$$C_e(k, m) = A \left[2\ln(2) + m + \int_{-\infty}^{+\infty} f(x) dx \right], \quad (6)$$

where $f(x) = \ln\left(\cosh\left(\frac{x + km}{2k}\right)\right) \frac{2}{\sqrt{\pi}} e^{-x^2}$.

III. TIGHT APPROXIMATIONS OF THE INTEGRAL OF $f(x)$

The integral part of (6) is equivalent to

$$\int_{-\infty}^{+\infty} f(x) dx = \int_0^{+\infty} [f(-x) + f(x)] dx. \quad (7)$$

For any value of k , using (7) and the integration by parts defined as $\int_a^b w(x) \frac{dy(x)}{dx} dx = [w(x)y(x)]_a^b - \int_a^b \frac{dw(x)}{dx} y(x) dx$, with $w(x) = \ln\left(\cosh\left(\frac{\pm x + km}{2k}\right)\right)$, $\frac{dw(x)}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$ and hence $\frac{dw(x)}{dx} = \pm \frac{1}{2k} \tanh\left(\frac{\pm x + km}{2k}\right)$, $y(x) = \text{erf}(x)$, the ergodic capacity in (6) can be re-expressed as follows

$$\begin{aligned} C_e(k, m) &= A \left\{ (2\ln(2) + m) + \left[\text{erf}(x) \left\{ \ln\left(\cosh\left(\frac{-x + km}{2k}\right)\right) + \ln\left(\cosh\left(\frac{x + km}{2k}\right)\right) \right\} \right]_0^{+\infty} + \frac{1}{2k} \int_0^{+\infty} \text{erf}(x) \right. \\ &\quad \times \left. \left(\tanh\left(\frac{-x + km}{2k}\right) - \tanh\left(\frac{x + km}{2k}\right) \right) dx \right\} \\ &= Am + \frac{A}{2k} \int_0^{+\infty} g(x) dx, \end{aligned} \quad (8)$$

where $g(x) = \text{erf}(x) \left(\tanh\left(\frac{-x + km}{2k}\right) - \tanh\left(\frac{x + km}{2k}\right) \right) + 2$, and $\text{erf}(x)$ is the error function. We also know that $\tanh(ax)$ is a good approximation of $\text{erf}(x)$, for any $x \in [0, +\infty)$ and $a \in [1.1, 1.3]$. Using this approximation, we obtain $g(x) \approx \tilde{g}(x) = \tanh(ax) \left(\tanh\left(\frac{-x + km}{2k}\right) - \tanh\left(\frac{x + km}{2k}\right) \right) + 2$. Knowing that $\tanh(x) = 1 - 2/(\exp(2x) + 1) = -1 + 2/(\exp(-2x) + 1)$, $C_e(k, m)$ can be approximated by substituting $g(x)$ with $\tilde{g}(x)$ in (8), as follows

$$\begin{aligned} C_e(k, m) &\approx Am + \frac{A}{k} \int_0^{+\infty} \left[\left(1 - \frac{2}{e^{2ax} + 1} \right) \left(\frac{1}{e^{\frac{x}{k} - m} + 1} + \frac{1}{e^{\frac{x}{k} + m} + 1} - 1 \right) + 1 \right] dx. \end{aligned} \quad (9)$$

Applying the change of variable $u = e^x$ in (9) and after some simplifications, (9) can be re-expressed as follows

$$\begin{aligned} C_e(k, m) &\approx 2A \left[\left(1 + \frac{1}{2ak} \right) \ln(2) + \frac{m}{2} + \ln\left(\cosh\left(\frac{m}{2}\right)\right) \right] \\ &\quad - \frac{2A}{k} \int_1^{+\infty} \left(\frac{1}{p(u, m, k, a)} + \frac{1}{p(u, -m, k, a)} \right) du, \end{aligned} \quad (10)$$

where $p(u, m, k, a)$ is a polynomial given by

$$p(u, m, k, a) = u(e^m u^{2a + \frac{1}{k}} + u^{2a} + e^m u^{\frac{1}{k}} + 1). \quad (11)$$

The integral part in (10) can be solved by expanding the rational functions $1/p(u, \pm m, k, a)$ into partial fractions.

The expression in (10) is a generic and accurate closed-form approximation of the ergodic capacity as long as the roots of $p(u, m, k, a)$ in (11) can be obtained. Using this expression, we derive a simpler expression that is a tight approximation of $C_e(k, m)$ for $k \gg 1$, or equivalently $\sigma_y \ll 1$ dB. In the case of $k \gg 1$, the integral part in (10) becomes negligible, and (10) can simply be re-expressed as

$$C_{e,0}(m) = 2A \left[\ln(2) + \frac{m}{2} + \ln\left(\cosh\left(\frac{m}{2}\right)\right) \right]. \quad (12)$$

Fig. 1. Comparison of $C_e(k, m)$, $C_{e,0}(m)$, $C_{e,1}(k, m)$ vs. σ_y , for various m values.

For $k = 3$, $C_e(k, m)$ and $C_{e,0}(m)$ differ by less than 0.01 nats/s/Hz for any m value. This difference decreases as k goes to infinity, or reciprocally σ_y goes to zero, as shown in Figs. 1 and 2. Notice also that $C_{e,0}(m)$ is independent of k , and thus $C_{e,0}(m)$ is an absolute lower bound of the ergodic capacity over log-normal fading channels.

In the case of $k \ll 1$, or equivalently $\sigma_y \gg 1$ dB, the functions $f(-x)$ and $f(x)$ can be tightly approximated by $\tilde{f}(-x)$ and $\tilde{f}(x)$ =

$$\begin{cases} \left(\frac{x - km}{2k} - \ln(2) \right) \frac{2}{\sqrt{\pi}} e^{-x^2}, & x \in [k(2\ln(2) + m), +\infty), \\ \left(\frac{x + km}{2k} - \ln(2) \right) \frac{2}{\sqrt{\pi}} e^{-x^2}, & x \in [k(2\ln(2) - m), +\infty), \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

respectively. Consequently, an accurate closed-form approximation of the ergodic capacity can be obtained by substituting $f(-x)$ and $f(x)$ with $\tilde{f}(-x)$ and $\tilde{f}(x)$ in (7) as follows

$$\begin{aligned} C_e(k, m) &\simeq 2A \ln(2) + Am \\ &+ A \int_{k(2\ln(2)+m)}^{+\infty} \left(\frac{x - km}{2k} - \ln(2) \right) \frac{2}{\sqrt{\pi}} e^{-x^2} dx, \\ &+ A \int_{k(2\ln(2)-m)}^{+\infty} \left(\frac{x + km}{2k} - \ln(2) \right) \frac{2}{\sqrt{\pi}} e^{-x^2} dx. \end{aligned} \quad (14)$$

After some simplifications, it can be read as follows

$$\begin{aligned} C_{e,1}(k, m) &= Am - \frac{A}{2} \left\{ (m - 2\ln(2)) \operatorname{erf}[-k(m - 2\ln(2))] \right. \\ &\quad \left. - (m + 2\ln(2)) \operatorname{erf}[k(m + 2\ln(2))] - \frac{2}{k\sqrt{\pi}} \right. \\ &\quad \left. \times e^{-k^2(m^2 + (2\ln(2))^2)} \cosh(4\ln(2)mk^2) \right\}. \end{aligned} \quad (15)$$

For $k = 0.025$, $C_e(k, m)$ and $C_{e,1}(k, m)$ differ by less than 0.01 nats/s/Hz for any m value. The accuracy of (15) increases

Fig. 2. Comparison of $C_e(k, m)$, $C_{e,0}(m)$, $C_{e,1}(k, m)$ vs. m , for various σ_y values.

as k goes to zero, or reciprocally σ_y goes to infinity, as shown in Figs. 1 and 2.

The closed-form approximations derived in (12) and (15) are highly accurate for low and high σ_y values, respectively, as depicted in Figs. 1 and 2, but are less accurate for more practical values of dB spread. In order to obtain a closed-form expression for practical dB spread values, i.e., $\sigma_y \in [2, 12]$, or equivalently $k \in [0.256, 1.535]$, we propose a function $\phi(k, m) \approx \frac{A}{2k} \int_0^{+\infty} g(x) dx - 2A \ln(2 \cosh(\frac{m}{2}))$ that estimates tightly the difference between $C_e(k, m)$ and $C_{e,0}(m)$. An in-depth study of $\phi(k, m)$ for various (k, m) values reveals that this function decreases towards zero as k increases and decreases exponentially as m increases. Relying on this study and the approximation method in [9],

$$\phi(k, m) = \frac{A\eta_0}{2k} e^{-\eta_1 \ln(\cosh(\frac{m}{2\eta_2}))} \quad (16)$$

has been found to be an excellent choice of a parametric function, with $\eta_0, \eta_1, \eta_2 \in \mathbb{R}$.

IV. AN ACCURATE CLOSED-FORM APPROXIMATION OF THE ERGODIC CAPACITY

The tightness of $\phi(k, m)$, or equivalently $\phi(\sigma_y, \gamma(\text{dB}))$, in (16) can be adjusted by using the parameters η_0 , η_1 and η_2 . Our method for finding a very tight approximation of the ergodic capacity in (6) by $C_{e,0}(\gamma(\text{dB})) + \phi(\sigma_y, \gamma(\text{dB}))$ is to minimize the Mean Squared Error (MSE) criterion given by

$$\begin{aligned} \frac{1}{2N+1} \sum_{\gamma(\text{dB})=-N}^N \left| C_e(\sigma_y, \gamma(\text{dB})) - \left(C_{e,0}(\gamma(\text{dB})) \right. \right. \\ \left. \left. + \frac{A\eta_0\sigma_y}{\sqrt{2B}} e^{-\eta_1 \ln[\cosh(\frac{\gamma(\text{dB})+m_y}{2B\eta_2})]} \right) \right|^2 \leq \varepsilon_0, \end{aligned} \quad (17)$$

where $\varepsilon_0 \ll 1$. Notice that m_y is a term that shifts the SNR value, i.e., it sets the origin of the SNR. Using this MSE criterion, we have obtained η_0 , η_1 , and η_2 , considering

$\sigma_y \in [0.01, 12]$ with an increment step of 0.1 dB for σ_y values, over a range of SNR $\gamma(\text{dB}) = [-N, N]$, $N = 100$, with $\varepsilon_0 = 5.10^{-6}$, and then interpolated the resulting values for η_0 , η_1 and η_2 . The results are as follows:

For $\sigma_y \in [0.01, 6]$:

$$\begin{cases} \eta_0 = -1.0658 \cdot 10^{-4} \sigma_y^3 - 0.0019047 \sigma_y^2 + 0.083954 \sigma_y \\ \quad - 0.0004047, \\ \eta_1 = -0.0160 \sigma_y^2 + 0.3180 \sigma_y + 1.6580, \\ \eta_2 = (0.045 \sigma_y + 0.385) \ln(10). \end{cases}$$

For $\sigma_y \in [6, 12]$:

$$\begin{cases} \eta_0 = 8.7552 \cdot 10^{-5} \sigma_y^3 - 0.0043629 \sigma_y^2 + 0.093625 \sigma_y \\ \quad - 0.011684, \\ \eta_1 = -0.0080 \sigma_y^2 + 0.2200 \sigma_y + 1.9580, \\ \eta_2 = (0.045 \sigma_y + 0.385) \ln(10). \end{cases}$$

Finally, adding (12) and (16) together and using the values of η_0 , η_1 and η_2 obtained above, we derive a highly tight closed-form approximation of the ergodic capacity as follows

$$C'_e = 2A \left\{ \ln \left(2 \cosh \left(\frac{\gamma(\text{dB}) + m_y}{2B} \right) \right) + \frac{1}{2B} \left[\gamma(\text{dB}) + m_y + \frac{\sigma_y \eta_0}{\sqrt{2}} \cosh^{-\eta_1} \left(\frac{\gamma(\text{dB}) + m_y}{2B \eta_2} \right) \right] \right\}, \quad (18)$$

which differs from C_e in (2) by less than $2 \cdot 10^{-4}$ nats/s/Hz for any $\sigma_y \in [0.01, 6]$ and by less than $8 \cdot 10^{-3}$ nats/s/Hz for any $\sigma_y \in [6, 12]$, and for any $\gamma(\text{dB})$ value. In Fig 3, we compare our closed-form approximation C'_e in (18) with C_e in (2) and the closed-form approximation $C''_e = \left(1 + \exp \left(\frac{\gamma(\text{dB}) + m_y}{B} + \frac{\sigma_y^2}{2B^2} \right) \right) \times \left[1 + 2 \exp \left(\frac{\gamma(\text{dB}) + m_y}{B} + \frac{\sigma_y^2}{2B^2} \right) + \exp \left(2 \frac{\gamma(\text{dB}) + m_y}{B} + 2 \frac{\sigma_y^2}{B^2} \right) \right]^{-\frac{1}{2}}$ proposed in [11] as functions of $\gamma(\text{dB})$, for various dB spread values, and $m_y = -\sigma_y^2/(2B)$. The graph in Fig. 3 shows a highly tight fitness between C_e and C'_e for any dB spread value and, hence, indicates graphically the accuracy of our proposed closed-form approximation in (18). It also depicts that C''_e is only accurate for low values of σ_y and that C'_e is not as accurate as our proposed expression C'_e . Notice that the accuracy of C'_e can be improved for lower ε_0 values by an appropriate choice of the parameters η_0 , η_1 and η_2 in (17).

V. APPLICATIONS OF C'_e

Our closed-form expression in (18) can be used to calculate effectively and accurately the ergodic capacity of communication systems over log-normal fading channels. For instance, we can easily compute the ergodic capacity of UWB systems operating over the IEEE 802.15.3a channel model, using the method proposed in [16] to combine the effect of N_f log-normal multipath attenuation factors into an equivalent log-normal RV.

Fig. 3. Comparison of C_e , C'_e and C''_e vs. average SNR, for various σ_y values.

In multiple-input-multiple-output UWB system, assuming perfect channel estimation and accurate timing synchronization, the instantaneous SNR related to Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC) can be expressed as $\hat{\gamma} = \gamma \alpha$ with α defined as follows [16]

$$\alpha = \begin{cases} \left(\sum_{\rho=0}^{N_t-1} \sum_{n=0}^{N_t-1} \sum_{f=0}^{N_t-1} |\beta_{\rho,n,f}| \right)^2 / (N_t N_t^2 N_t) , & \text{for EGC,} \\ \sum_{\rho=0}^{N_t-1} \sum_{n=0}^{N_t-1} \sum_{f=0}^{N_t-1} \beta_{\rho,n,f}^2 / N_t , & \text{for MRC,} \end{cases} \quad (19)$$

where N_r is the number of receive antennae, N_t is the number of transmit antennae, and N_f is the number of multipath over each receive-transmit antenna pair.

According to [14] and [15], the RV α , which is a sum of log-normal RVs, can be well-approximated as a log-normal RV. The mean m_α and standard deviation σ_α of the equivalent log-normal RV α have been obtained for different numbers of transmit antennae and using either EGC or MRC, in Tables 4.2, 4.3, 5.1 and 5.2 of [16]. Inserting these values of m_α and σ_α in (19), we plot in Figs. 4 and 5 the ergodic capacities of SISO and MISO-UWB systems, respectively, over various channel scenarios, i.e., CM1-4, of [2], applying either EGC or MRC, for $N_f = 20$ and $N_r = 1$. Fig. 4 depicts that MRC provides always an extra 0.2 nats/s/Hz capacity improvement compared to EGC, and that the capacity over the Line-Of-Sight (LOS) channel scenario CM1 is 1.2 nats/s/Hz higher than the capacity over the worst non-LOS channel scenario CM4. Fig. 5 shows that adding extra transmit antennae allows a small capacity improvement of around 0.2 nats/s/Hz at the SNR of 30 dB. These results are consistent with the results obtained in [17] for Rayleigh fading channel.

Our expressions in (12) and (15) can also be used to interpret the behaviour of the ergodic capacity $C_e(k, m)$ at high SNRs. Assuming in (12) that $m \gg 1$, or equivalently

Fig. 4. Ergodic capacity of SISO-UWB systems for different channel scenarios and various combining methods.

$\gamma(\text{dB}) + m_y \gg 1$, we have $C_{e,0}(m) \simeq 2Am$. Similarly, assuming in (15) that $m \gg 1$, we have $C_{e,1}(k, m) \simeq Am - \frac{Am}{2}(\text{erf}(-km) - \text{erf}(km)) - \frac{A}{k\sqrt{\pi}}e^{-k^2(m(m-4\ln(2)))}$, which can be simplified as $C_{e,1}(k, m) \simeq 2Am$ for $m \gg 1/k$. Therefore, the ergodic capacity of a log-normal fading channel can be simplified at high SNRs as follows

$$C_e \simeq \frac{2A}{B} (\gamma(\text{dB}) + m_y). \quad (20)$$

Thus, the capacity is a linear function of the SNR at high SNRs, which is consistent with the results obtained in [18] as well as in Figs. 4 and 5.

VI. CONCLUSION

An accurate closed-form approximation of the ergodic capacity for wireless communication systems operating over log-normal fading channels has been derived. Our proposed closed-form approximation is the sum of an absolute lower bound and a parametric function. Its accuracy has been obtained by finding values of η_0 , η_1 and η_2 that minimize the MSE criterion, and it has been shown experimentally for practical SNR and dB spread values. Its application in evaluating the ergodic capacities of SISO and MISO-UWB systems over the IEEE 802.15.3a channel model has been presented. Finally, our results have been used to interpret the behaviour of the ergodic capacity of log-normal fading channels at high SNRs.

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Fig. 5. Ergodic capacity of MISO-UWB systems for different channel scenarios and various numbers of transmit antennae.

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