Population of low-seniority isomeric states of ^{206}Hg by two-proton knockout reactions at relativistic energies

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(Received 5 October 2009; published 8 December 2009)

We consider calculations of direct two-proton removal reactions from a relativistic-energy ^{208}Pb beam incident on a ^9Be target, leading to the population of two-proton-hole isomeric configurations in ^{206}Hg . The calculated 5^- and 10^+ isomeric state population ratios of 18.8% and 4.7%, respectively, are in fair agreement with the values deduced experimentally ($21.9^{+1.2}_{-2.9}\%$ and $3.1^{+1.0}_{-1.2}\%$); however, the high density of final states in this case complicates the prediction of isomeric ratios for those states fed by unobserved prompt γ -ray cascades. We also consider and quantify the effects of selection of events, according to the longitudinal momenta of the reaction residues, on the measured isomeric ratios for high-spin final states and, hence, of the potential to exploit such sensitivity to gain further insight into the reaction mechanism.

DOI: [10.1103/PhysRevC.80.064608](https://doi.org/10.1103/PhysRevC.80.064608)

PACS number(s): 24.50.+g, 23.35.+g, 25.70.-z, 27.80.+w

I. INTRODUCTION

Intermediate-energy, direct, one- and two-nucleon removal reactions from near-stable and very exotic projectile beams, performed in inverse kinematics on light target nuclei, such as beryllium or carbon, have been studied and exploited as an effective probe of nuclear structure in nuclei with masses in the range $A \approx 8\text{--}70$ [1–8]. The measured nucleon removal cross sections have been analyzed quantitatively by combining an eikonal, adiabatic description of the reaction dynamics with either (a) semimicroscopic or shell-model one-nucleon overlap functions or (b) shell-model two-nucleon structure amplitudes.

Our interest here is the extension of such direct reaction models to make a first quantitative prediction of cross sections for simple excitations in heavy nuclei. Specifically, we consider population of low-seniority isomeric configurations. Our test case is the use of the $^{208}\text{Pb}(-2p)$ two-proton removal reaction, at a relativistic energy, to populate well-known, low-lying, two-proton-hole isomeric states in ^{206}Hg (see, e.g., Ref. [9]).

Rather little experimental information exists on direct two-nucleon removal reactions for nuclear systems with $A > 100$ incident on light nuclear targets. The available measurements are limited to final-state-inclusive cross sections [10–13]. The expected production cross sections from peripheral few-nucleon removal reactions have typically been estimated using fragmentation models (see, e.g., Refs. [10–12,14,15]) that involve the abrasion of some nucleons, followed by the ablation of others.

The population of isomeric states in exotic neutron-rich nuclei by projectile fragmentation has been advanced and exploited by the RISING Collaboration, in particular by exploiting stopped beams and γ -decay spectroscopy [16,17]. The γ decay of isomeric states (and their subsequent cascades) have provided level information on a wide range of different nuclei with $A \approx 100\text{--}200$ [18]. Thus, our ability to predict isomeric population ratios quantitatively, particularly in fast peripheral collisions that offer an efficient and highly selective route to specific exotic (N,Z) nucleon combinations, is of significant interest.

Here, we describe the removal of two protons from ^{208}Pb within the previously developed eikonal, adiabatic model. This has been applied and tested extensively throughout the *sd* shell [5] and, more recently, in the lower part of the *fp* shell [7]. Here, we extend the very preliminary estimates of the direct reaction yields of Ref. [19]. Specifically, calculations of the isomeric ratios (the ratio of the measured yield of a given isomeric state to the total two-proton removal yield) require an estimate of the summed two-proton removal cross sections to all bound final states of the heavy residue. We will assume here that the shell model adequately predicts the appropriate two-proton spectroscopic strength up to the first (neutron) threshold in ^{206}Hg . So, we calculate the cross sections to all predicted (shell-model) bound final states. This approach was used and worked well in the case of reactions along the carbon [20] and other light *sp**sd*-nuclear isotopic chains [21]. We also comment on the accuracy of simpler estimates, based on the use of coefficients of fractional parentage (cfp) and on assumed positions of the centroids of the two-proton strength distributions.

The recent theoretical developments in the calculation of residue momentum distributions following two-nucleon knockout reactions [22,23] lead us to consider the isomeric ratio as a function of residue momentum. The primary sensitivity of the shape of the residue momentum distribution is to the total angular momentum I of the nucleon pair, with, quite generally, larger angular momentum couplings giving wider residue momentum distributions. This allows final-state spin assignments to be made, but it also means that the isomeric ratio for high-spin states should be rather sensitive to the residue momentum. Furthermore, an accurate isomeric ratio will only be obtained from experiment if the full residue momentum distribution is transmitted through the spectrometer.

In Sec. II we will discuss briefly the inputs specific to the knockout reaction model on ^{208}Pb . Our approach to the projectile (^{208}Pb) and reaction residue (^{206}Hg) structure inputs is essentially the same as that used for lighter mass systems. The two-nucleon amplitudes (TNA) are taken from the shell

model. The complication here is the high level density of (^{206}Hg) final states and the large number of these states that carry a significant spectroscopic strength. This is discussed in Sec. III. Results for two-proton removal from ^{208}Pb are presented and discussed in Sec. IV. We also consider what additional information could be obtained if one were able to select reaction events with respect to the longitudinal momentum of the reaction residues and, associated with this sensitivity, the implications of experimental cuts on the full momentum distribution for the measured isomeric yields. Our results are summarized in Sec. V.

II. KNOCKOUT REACTION MODEL

We study the two-proton knockout cross sections from a 1 GeV/u ^{208}Pb beam incident on a ^9Be target, relevant to recent experiments carried out at GSI (Helmholtzzentrum für Schwerionenforschung GmbH), Darmstadt, Germany. The adiabatic, eikonal, and spectator-core approximations are used. The associated formalism for these fast, two-like-nucleon removal events is discussed in detail in Ref. [5]. This is the approach adopted here. The underlying approximations used are expected to be particularly accurate at the 1 GeV/u beam energy of interest here. The calculations that follow include two-proton removal by both the stripping (inelastic breakup) and the diffraction dissociation (elastic breakup) mechanisms, as are discussed in detail in Ref. [5]. The individual contributions from each mechanism are not shown, though these were presented in a preliminary discussion [19].

Considerations of the dominance of the direct two-proton removal mechanism in this ^{208}Pb case are addressed in Fig. 1. The figure shows the relevant one- and two-nucleon thresholds of the ^{208}Pb , ^{207}Tl , and ^{206}Hg systems. In particular, the possible indirect population of bound states in ^{206}Hg by the two-step mechanism, involving one-proton removal to excited (continuum) configurations in ^{207}Tl followed by proton evaporation (the thin blue arrows labeled $-1p$ in Fig. 1), will involve the excitation of intermediate states with excitation

FIG. 1. (Color online) The one- and two-nucleon thresholds for the ^{208}Pb , ^{207}Tl , and ^{206}Hg systems. The direct (thick red arrow labeled $-2p$) and indirect (thin blue arrows labeled $-1p$) two-proton removal paths to the 10^+ ^{206}Hg final state are indicated schematically.

energy in excess of 7.38 MeV [>11.1 MeV in the case of population of the $^{206}\text{Hg}(10^+)$ state]. Although the proton and neutron thresholds from ^{207}Tl are quite similar, there is also a considerable Coulomb barrier to proton emission and so the dominant decay mechanism from low-lying continuum states is assumed to be neutron emission. Furthermore, the majority of single-proton strength is expected to be exhausted below the ^{207}Tl proton separation threshold; the proton-hole state of the most deeply bound active orbital $\pi[0g_{7/2}]$ has an excitation energy of 3.48 MeV. The expectation, therefore, is that the direct one-step mechanism (the thick red arrow labeled $-2p$ in Fig. 1) will be the completely dominant path to bound states in the residue.

Fundamentally, sudden two-nucleon removal reactions are highly constrained geometrically by the requirement that two valence nucleons undergo violent and absorptive interactions with the light target, but the remaining heavy residue of nucleons does not. At relativistic energy the highly absorptive nature of the elastic S matrices of the removed protons and the residue with the target results in the very strong, near-surface localization of the reaction mechanism. The essential dynamical reaction requirements are therefore these elastic S matrices of the removed protons and the residue with the target as a function of their collision impact parameters. In constructing these, theoretical considerations stressed in Ref. [6] were (a) the importance of using realistic sizes for the target (^9Be) and residue (^{206}Hg) nuclei; these dictate the ranges of the residue- and proton-target absorptive interactions and so locate the position and impact parameters of the surface, and (b) the use of geometries (for the potentials that bind the protons in ^{208}Pb) that generate proton single-particle orbitals with root-mean-squared (rms) radii that are consistent with the size used for the residue in (a).

Here, these S matrices are calculated based on the residue and target point nucleon densities using the optical limit of Glauber's multiple scattering theory [24,25]. A zero-range nucleon-nucleon (NN) effective interaction was used with strength determined, in the usual way [26], from the free nn and np total cross sections. The real-to-imaginary ratios of the forward scattering NN amplitudes were taken to be zero because the NN amplitude is essentially absorptive for the energy in question. The neutron and proton radial densities in the ^{206}Hg residue were calculated using spherical Skyrme (SkX interaction) Hartree-Fock (HF) calculations [27]. These point-nucleon densities had rms radii of 5.59 fm (neutrons) and 5.42 fm (protons). The density of the ^9Be target was assumed to be of (spherical) Gaussian form with rms matter radius of 2.36 fm. These default ^9Be geometry parameters have been used extensively and calculations have only a very small sensitivity to their choice, provided the rms radius is maintained.

The proton single-particle wave functions were calculated in a Woods-Saxon (central plus spin orbit) and Coulomb potential field. The SkX HF calculations for ^{208}Pb give $\pi 2s_{1/2}$, $\pi 1d_{3/2}$, $\pi 0h_{11/2}$, $\pi 1d_{5/2}$, and $\pi 0g_{7/2}$ orbitals with rms radii $r_{\text{HF}} = 5.246, 5.434, 6.111, 5.399$, and 5.649 fm, respectively. The HF proton separation energies were $S_{\text{HF}} = 7.69, 8.48, 9.16, 10.03$, and 12.15 MeV. Having used the HF calculations to constrain the Woods-Saxon geometry, and after taking

account of the small center-of-mass (c.m.) correction [6], the single particle wave functions were calculated for the empirical effective proton separation energies $S_p^{\text{eff}} = 8.00, 8.36, 9.36, 9.70, \text{ and } 11.48$ MeV (taken from the low-lying states in ^{207}Tl). The resulting single-proton orbitals have rms radii $R_{\text{sp}} = 5.256, 5.451, 6.152, 5.422, \text{ and } 5.675$ fm. Following the results of Ref. [6], the calculated nucleon removal cross sections are insensitive to details of the particular potential parameters used to bind the protons, provided that each proton single-particle wave function has the stated separation energy and R_{sp} .

The small production cross sections for exotic nuclei in peripheral nucleon knockout necessitates the use of thick (typically g/cm^2) production targets. As the target thickness increases, the probability of multiple interactions also increases, such that the final reaction residue may be populated by multiple-step rather than direct two-proton removal reactions. To be confident of the isomeric ratios calculated on the basis of the direct population mechanism we estimate the magnitude of such contributions. We estimate that for the present case, based on the calculated one- and two-nucleon removal cross sections and the target thickness, the contributions from two, sequential, one-proton removal events is $\sim 8\%$ of the total. These are therefore neglected in the following. However, in events where more than two nucleons are removed, the number of multistep routes for population of a given residue will increase. Any study of the direct mechanism contribution in multinucleon knockout would thus necessitate the use of thinner targets.

III. STRUCTURE INPUT

Structure input was taken from the shell model. For the even-even, spin-zero ^{208}Pb projectile, the structure detail enters through the (shell-model) two-proton overlap that is sampled at and near the nuclear surface in grazing collisions with the light target. For a ^{206}Hg final state with spin I , the two-particle overlap of spin I and projection μ (in the incident beam direction) can be written [4] as

$$\Psi_{I\mu}(1, 2) = \sum_{j_1 j_2} (-1)^{I-\mu} \frac{C(j_1 j_2 I)}{\sqrt{2I+1}} [\phi_{j_1}(1) \otimes \phi_{j_2}(2)]_{I\mu}, \quad (1)$$

with $C(j_1 j_2 I)$ the two-nucleon amplitudes for each active two-nucleon configuration that contributes to the state of spin I . Here, $[\phi_{j_1}(1) \otimes \phi_{j_2}(2)]_{I\mu}$ represents the antisymmetrized,

angular momentum coupled product of the two struck protons' single-particle wave functions.

Our shell-model calculations for ^{208}Pb and ^{206}Hg made use of the code OXBASH [28] with the $jj56pn$ model space for protons (i.e., the $2s_{1/2}, 1d_{3/2}, 0h_{11/2}, 1d_{5/2}, \text{ and } 0g_{7/2}$ orbitals) and the $khhe$ [29] interaction. Of the 56 final states obtained, 52 are predicted below the first (neutron) threshold in ^{206}Hg , the remaining four (neutron-unbound) states being predominantly $\pi[0g_{7/2}]^{-2}$ configurations. These four states were thus excluded from our calculations of the total $-2p$ reaction yields and the isomeric ratios.

The calculated total two-proton removal cross section, including both elastic and inelastic mechanisms as described in Ref. [5], to bound states in ^{206}Hg is 4.59 mb, and the direct populations of the $5^-, 7^-, 8^+, \text{ and } 10^+$ states, based on the shell-model calculations, were 0.22, 0.27, 0.16, and 0.22 mb. The shell-model TNA for these states are presented in Table I. An alternative, simpler cross-section estimate is based on the use of the two-proton cfp. These, $\sqrt{2I+1}$ for $2p$ removal from filled subshells and a final state of spin I [19,30,31], have unit strengths for the dominant two-hole configurations in Table I. This approximation is rather good in the case of the current doubly magic test-case calculation. Using these pure cfp amplitudes, we obtain total and partial cross sections of 4.55, 0.16, 0.25, 0.16, and 0.22 mb, respectively. Because, in our chosen model space, the yrast 8^+ and 10^+ states are pure $[0h_{11/2}]^{-2}$ configurations, these cross sections are unchanged. However, the 5^- and 7^- states are both populated more strongly when using the full shell-model wave functions, indicating the presence of constructive coherent contributions from the smaller components in the wave function, as identified in Table I. The good agreement between shell-model and cfp approaches for the total cross sections suggests that such estimates may be useful in simplifying calculations of the inclusive cross section, particularly in mass regions inaccessible to complete shell-model calculations.

In the 1 GeV/u $^{208}\text{Pb} + ^9\text{Be}$ stopped-beam experiment of Steer *et al.* [18], two ^{206}Hg isomers were observed: the 5^- state ($[0h_{11/2}]^{-1}[2s_{1/2}]^{-1}$) at 2.102 MeV [with half-life 2.09(2) μs] and the 10^+ state ($[0h_{11/2}]^{-2}$) at 3.723 MeV [with half-life 112(4) ns]. The 5^- isomer is known to be fed by (at least) the $7^-, 8^+, \text{ and } 10^+$ states, observed via isomer decay spectroscopy. It is also expected that other (unobserved) prompt γ -ray transitions will feed the 5^- isomer. The observed levels, transitions, and lifetimes are shown schematically in Fig. 2. The experiment did not monitor the primary beam

TABLE I. The shell-model two-nucleon spectroscopic amplitudes (scaled by $\sqrt{2I+1}$) for the required $^{208}\text{Pb}(0^+) \rightarrow ^{206}\text{Hg}(I^\pi)$ transitions, and the $2s_{1/2}, 1d_{3/2}, 0h_{11/2}, 1d_{5/2}, \text{ and } 0g_{7/2}$ proton model space and the $khhe$ interaction. Thus, the amplitudes $C(j_1 j_2 I)$ in Eq. (1) are $\sqrt{2I+1}$ times the entries shown in the table. For the corresponding cfp, the major components shown in the table are unity and all other components are zero.

I^π	$[0h_{11/2}]^{-1}[0g_{7/2}]^{-1}$	$[0h_{11/2}]^{-1}[1d_{5/2}]^{-1}$	$[0h_{11/2}]^{-1}[1d_{3/2}]^{-1}$	$[0h_{11/2}]^{-1}[2s_{1/2}]^{-1}$	$[0h_{11/2}]^{-2}$
10^+	—	—	—	—	1.0
8^+	—	—	—	—	1.0
7^-	0.03446	0.13024	0.99088	—	—
5^-	0.01431	0.13333	0.28217	0.94995	—

FIG. 2. Experimentally observed excited states in ^{206}Hg and their dominant configurations. The isomeric 5^- and 10^+ states are shown with decay half-lives taken from Ref. [18]. The decay half-lives given by Ref. [9] are $2.15(21) \mu\text{s}$ and $92(8) \text{ ns}$, respectively. The unobserved yrast 4^+ and 6^+ states have (shell-model) energies of 2.498 and 3.590 MeV, respectively.

intensity and thus, although isomer production ratios could be determined, absolute isomer production cross sections could not be deduced. Further high-spin excited states above the 10^+ isomer have been observed in heavy-ion $^{208}\text{Pb} + ^{238}\text{U}$ collisions [9], but these involve core excitations across the neutron shell gap and are thus not expected to be populated by the present, highly selective, direct reaction mechanism.

The 10^+ isomer can be described as a pure $\pi[0h_{11/2}]^{-2}$ configuration and is thus expected to be only directly populated and not fed by γ cascades, partially because of its high spin and partially because of the absence of $E1$ transitions for our assumed set of active nucleon orbitals. Thus, the 10^+ isomeric ratio should offer the best first benchmark of the knockout mechanism in these heavy nuclei. As was previously stated, this is not the case for the 5^- state, which is expected to be fed by (at least) the higher lying 7^- , 8^+ , and 10^+ states. In principle, the full γ -decay scheme for ^{206}Hg could be calculated from the shell-model levels to obtain an estimate of the feeding of the 5^- isomer. However, the model space used does not allow $E1$ transitions. These were believed to be important (through core excitations) in the analogous transitions in ^{130}Sn [32], in particular, the $8^+ \rightarrow 7^-$ transition. Undue reliance on the theoretical shell-model decay scheme is thus difficult to justify.

IV. CALCULATIONS OF ISOMERIC RATIOS

The experimental [18] and calculated isomeric production ratios are collected in Table II. The deduced values use the isomer half-lives of the same reference, also shown in Fig. 2.

TABLE II. Theoretical and experimentally deduced [18] isomeric ratios for the 5^- and 10^+ isomers in ^{206}Hg . The errors on the 10^+ state ratio include ambiguities associated with the experimentally unobserved electromagnetic decay intensity of the 364-keV $7^- \rightarrow 5^-$ transition. Basing the isomeric ratio only on the observed $10^+ \rightarrow 7^-$ and $8^+ \rightarrow 7^-$ transitions gives value (a). Additionally, including information on the 364-keV transition gives (b) [18]. For the 5^- state we also show the theoretical estimate of Ref. [14], based on the yields from the abrasion-ablation code ABRABLA [15].

	State I^π	Isomeric ratio (%)
Experiment	5^-	$21.9^{+1.2}_{-2.9}$
	$(10^+)^a$	$3.1^{+1.0}_{-1.2}$
	$(10^+)^b$	$2.2^{+0.7}_{-0.8}$
Theory	5^- (direct only)	4.8
	5^- (includes 7^- , 8^+ , and 10^+)	18.8
	10^+	4.7
	5^-	32.5 [14]

For the 5^- state we show calculations of the isomeric ratio based on both (a) the direct 5^- state population (4.8%) only and (b) the value calculated assuming the observed 5^- state yield is also fed by the direct strength calculated through the 7^- , 8^+ , and 10^+ states (18.8%). These theoretical calculations were carried out at an assumed reaction energy of 1 GeV/u. The slow energy variation of the underlying NN cross sections (and of the derived S matrices) near 1 GeV/u, and the calculation of cross-section ratios in the present work, will result in very minimal changes if one recalculates the theory at the midtarget energy. Because the theoretically calculated quantities in the following relate to a thin-target limit, we adopt the 1 GeV/u calculations throughout.

Although the calculated 10^+ and 5^- (including 7^- , 8^+ , and 10^+ feeding) isomeric ratios are in good agreement with the experimental values, there are certain caveats: (a) We have ignored two-step reactions within the thick target, which we estimated may contribute $\sim 8\%$ of all events. (b) The final states that such two-step processes will populate is unclear. We might expect the fractional population of high-spin states to decrease, though the effect is probably small. (c) Although the 5^- isomeric ratio is in reasonable agreement, we have included only the direct population and the strength from observed γ -ray cascades that are apparent from the decay of the 10^+ state. In reality, there could be other (unobserved) prompt γ -ray feeding of the 5^- state. The reasonable agreement when neglecting these contributions may indicate that such feeding is small. We conclude that the available experimental data are consistent with a picture in which the low-seniority two-proton-hole isomers are populated predominantly by a direct reaction mechanism.

As will be pointed out later, a stopped-beam experiment necessarily sacrifices potentially valuable diagnostic information as is contained in the momentum distributions of the reaction residues. Access to fast-beam data, where the momenta of the recoiling reaction residue (denoted by κ_c in

the projectile rest frame) and selected in-flight decay γ rays can be identified, would offer additional information and insight.

A. Momentum distributions and alignment

In-flight γ -ray angular distribution measurements following one-nucleon knockout reactions, in combination with differential data on the heavy reaction residue longitudinal momentum distribution, predict significant alignment of the spins of the knockout reaction residues with respect to the beam direction (see, e.g., Refs. [3,33]). As was also reported recently [22,23], theoretical calculations of two-nucleon removal also predict (a) residue momentum distributions with shapes and widths characteristic of the final-state spin value I , with larger I giving increasingly broad residue momentum distributions, and (b) significant spin alignment of the reaction residues. The immediate consequence here is that the isomeric ratio for the high-spin states populated should be significantly greater in the tails of the residue momentum distribution. The alignment is particularly strong if one is able to select knockout events near the peak of the residue momentum distribution (see, e.g., Table I of Ref. [23]). In the present case, and for population of the 10^+ isomer, more than 50% of the calculated cross section populates magnetic substates with $|\mu| = 8, 9$, and 10 . Sensitivity of the isomeric ratio to the residue momentum has previously been observed in the (peripheral) fragmentation reaction $^{46}\text{Ti}(-nnp)$, populating the $J_f^\pi = 19/2^-$ isomer in ^{43}Sc [34]. It was observed that the isomeric ratio increased and the alignment decreased away from the central residue momentum, consistent with present theoretical expectations of two-nucleon knockout.

It follows that a more complete and quantitative understanding of the reaction yields, and of the applicability of the direct reaction mechanism description, could be gained by reference to the momentum distribution of the reaction residues in coincidence with selected in-flight decay γ rays.

It is important to note that in our calculated isomeric ratios it is assumed that the spectrometer setting in the experiment accepts the complete longitudinal momentum distribution of the heavy reaction residues. In practice, as the ^{206}Hg residues are so close in mass and charge to the ^{208}Pb beam, slits are placed in the spectrometer to best eliminate the unreacted beam and protect detectors further downstream. This could remove those reaction residues at the extremes of the longitudinal momentum distribution and affect disproportionately those final states with the broadest residue momentum distributions. Depending therefore upon (a) the severity of any such cut, (b) the direct-reaction-induced momentum distribution width, (c) the primary beam momentum resolution, and (d) the extent of additional experimental broadening (principally the differential energy loss in the reaction target), the measured isomeric ratio may be affected.

Because the direct-reaction-induced residue momentum distribution for a high-spin 10^+ state is predicted to be very broad, it is interesting to consider the calculated isomeric ratio as a function of the residue momentum. Naturally we would expect the differential isomeric ratio to be significantly enhanced for large κ_c , the residue momentum in the projectile

FIG. 3. Calculated 10^+ (dashed) and 5^- (including 7^- , 8^+ , and 10^+ feeding; dotted) isomeric ratios, as a function of heavy residue momentum κ_c in the projectile rest frame, obtained using the (dominant) stripping reaction mechanism. The top panel shows the actual residue momentum distributions. The 10^+ and 5^- state distributions have been scaled by the factors shown to facilitate comparisons of their shapes with that of the inclusive cross section to all bound final states (solid). The middle and bottom panels show the isomeric ratios, also as a function of the ^{206}Hg residue momentum. The experimentally measured, momentum-inclusive values and their errors are shown by the horizontal solid line and shaded areas. In both cases the calculated isomeric ratios are strongly enhanced toward the large $|\kappa_c|$ regions of the distributions.

rest frame, over that for $\kappa_c \approx 0$. Similarly, because the higher spin states are observed to feed the 5^- isomer, we would again expect the isomeric ratio to be large at the extremes of the residue momentum. These calculated cross sections and isomeric ratios, based on the dominant stripping (inelastic) removal mechanism, are shown in Fig. 3 as a function of κ_c . It is important to note that in these theoretical calculations, without experimental broadening, an infinitely thin target has been assumed. The figure shows, however, that these differential effects might be observed by gating on the inclusive momentum distribution of a thin-target experiment, without the need to extract final-state-exclusive residue momentum distributions. The residue momentum distributions resulting from events where one nucleon is removed elastically and the other inelastically (previously called the diffractive-stripping events) are expected to be identical to the pure-stripping distributions used here because of the very similar projectile-surface localization of the cross section (see Ref. [23]). For the

5^- state yield it would be particularly interesting to investigate the isomeric ratio near the peak of the momentum distribution. Here, we would expect to obtain a value that is significantly smaller than the present (momentum-inclusive) experimental value. If this were not the case, after experimental broadening has been taken into account, it would indicate significant additional (and unobserved) feeding of the 5^- state from higher lying states of low spin.

Precisely how experimental cuts imposed on the laboratory frame residue momentum distributions would affect the (now partially momentum-integrated) isomeric ratios is not immediately apparent from Fig. 3. Although the isomeric ratio is much larger in the extremes of the momentum distribution, the smaller cross sections in these regions reduce their contribution to the total isomeric ratio. Thus, to understand the implications for experiments, we also need to take account of the experimental (predominantly target thickness) broadening of these theoretical, projectile rest frame results.

The ^{206}Hg laboratory frame momentum distribution after passage through the target (but excluding the reaction mechanism induced effects of Fig. 3) was calculated using LISE++ [35] by assuming a monoenergetic primary beam. This distribution reflects the differential energy loss in the reaction target because of the uncertainty in the reaction vertex and gives an approximately square distribution. Its width is ≈ 1.5 GeV/c for the current thick-target (2.525 g/cm 2) experiment [18]. The two-proton removal reaction-induced distribution has a width that depends on the final state; the 10^+ distribution has a laboratory frame full width at half-maximum (FWHM) of ≈ 1.2 GeV/c and the total (final-state-inclusive) distribution ≈ 0.75 GeV/c. We also calculate the expectations for a target one-third of the current thickness. The theoretical laboratory frame distributions are the convolution of the ^{206}Hg distributions with the theoretical rest-frame distributions shown in Fig. 3, after the latter have received the appropriate Lorentz boost (γ -factor stretch) to the laboratory frame.

Thus, in Fig. 4 we now show the calculated 10^+ and 5^- (including 7^- , 8^+ , and 10^+ feeding) isomeric ratios as a function of K_{max} , where these have been calculated by assuming that only those residues with laboratory momenta K_A in the symmetric interval $[-K_{\text{max}}, K_{\text{max}}]$ relative to the peak value of the momentum distribution are transmitted through the spectrometer. The dotted lines show the results for an infinitely thin target, the solid lines for the (2.525 g/cm 2) thick target [18], and the dashed lines for a target one-third of this thickness (0.842 g/cm 2). In addition to the reaction target itself, the latter two cases also include broadening from other material in the beam line before the S1 slits, most notably the 0.221 g/cm 2 Nb stripper. We note that, even in the presence of the experimental broadening, there is significant residual sensitivity and information to be gained from momentum-selective experiments (i.e., by gating on the differential isomeric yield distributions).

Finally, we consider the implications of the physical slit arrangement in the current experiment [18]. The slits were placed after the first dipole magnet following the reaction target, positioned to exclude the transmitted ^{208}Pb beam, at which point the transverse position of the residue is

FIG. 4. Calculated 10^+ and 5^- (including 7^- , 8^+ , and 10^+ feeding) isomeric state ratios as a function of the momentum acceptance, K_{max} , of the laboratory frame residue momenta, that is, the isomeric ratio for residue momenta integrated on the interval $[-K_{\text{max}}, K_{\text{max}}]$ relative to the peak value of the momentum distribution. The dotted lines show the results for an infinitely thin target, the solid lines for a (2.525 g/cm 2) thick target, and the dashed lines for a target one-third of this thickness (0.842 g/cm 2).

correlated with its longitudinal momentum. The resulting theoretical ^{206}Hg laboratory frame momentum distributions are shown in Fig. 5. The 10^+ (dashed line) and 5^- (dotted line) distributions have been scaled by factors of 20 and 5, respectively, to facilitate comparison with the total two-proton removal distribution (solid line). We find that the considerable (thick target) momentum broadening results in very similar postreaction momentum distributions for the two isomers and the inclusive two-proton removal distributions, reducing the potential sensitivity to differential cuts in the distributions.

In the current experiment the slits transmit all residues with S1 x positions between 0 and +10 mm, where $x = 0$ corresponds to a residue trajectory with a kinetic energy of 831.16 MeV/u. The peak of the residue momentum distribution has an energy of 830.31 MeV/u. The residue momentum distributions are not centered within the slits and the imposed cut is asymmetric about the peak momentum, being more severe on the high-momentum side. Based on these physical slit positions, the imposed momentum cuts are shown (Fig. 5, shaded areas). After integration over the transmitted region, the isomeric ratios are marginally affected; both the 5^- and 10^+ ratios are reduced by 0.2%. The weak effect in the present case is attributed to the position of the slits,

FIG. 5. Theoretical cross sections as a function of laboratory frame residue momentum K_A , following convolution of the Lorentz-boosted projectile rest frame and the target-broadened ^{206}Hg distributions. The 10^+ (dashed line, $\times 20$), 5^- (dotted line, $\times 5$), and the inclusive (solid line) distributions are shown. The vertical lines and shaded areas show the momentum cuts imposed by the spectrometer slits.

which essentially cut one-half of the distribution and do not preferentially cut the tails to a significant degree. Our estimate is that the slits exclude 38% of all two-proton removal events.

Clearly the relatively large differential energy losses in this thick-target experiment result in a loss of sensitivity to the direct-reaction-induced intrinsic momentum distributions associated with the different isomeric final states. Experiments using thinner targets would permit one to explore the sensitivities discussed and quantified in this section. To first approximation the differential energy loss broadening is proportional to the target thickness, and we have shown that a target three times thinner would lead to significant differential effects, but at the cost of decreased reaction yield.

V. SUMMARY

We have shown that the relative population of isomeric states in ^{206}Hg via direct two-proton removal on a ^9Be target is reasonably well described within an adiabatic, eikonal model methodology previously applied only to lighter nuclei. The primary theoretical complication in applications to heavy projectiles is the large number of residue final states and

ignorance of the feeding of isomeric states in the absence of measurements of prompt γ cascades. Knockout reactions with more exotic heavy nuclei, despite having lower particle separation thresholds, will be further complicated by the expectation of an increased density of low-lying, more collective states.

We have shown that the isomeric ratio should be rather sensitive to the residue longitudinal momentum for two-nucleon knockout reactions, particularly when the isomeric state has high spin relative to the other states populated. We conclude that thin-target experiments would permit a more careful consideration of the longitudinal momentum transmissions and such sensitivity. Such effects are obscured by the extent of the momentum broadening in the thick-target data set discussed here.

Further, simpler test cases of the knockout mechanism for heavy projectiles would be provided by single-neutron and proton removal from ^{208}Pb . Single-proton knockout will populate five (simple) proton-hole states in ^{207}Tl , in particular the $\pi[0h_{11/2}]^{-1}$ isomer with a half-life of 1.3 s. In addition to the isomeric ratios, the actual cross sections are needed to quantify deduced spectroscopic factors against, for example, $(e, e'p)$ analyses that are also available for this stable target [36]. Similarly, one-neutron removal from ^{208}Pb would populate the $T_{1/2} = 0.8$ s $\nu[0i_{13/2}]^{-1}$ isomeric state, providing an excellent large orbital angular momentum test case.

The removal of two protons from ^{136}Xe will, in principle, populate an analogous set of states to those considered here, but based on two particles in the $\pi 0g_{7/2}-2s_{1/2}$ orbitals as opposed to two holes. Given that there are then only four protons in this space, rather than the 32 in ^{208}Pb , we would expect a significantly reduced inclusive two-proton removal cross section, but having a far simpler distribution of strength between the final states. Cross sections for ^{136}Xe fragmentation on a Be target, also at 1 GeV/u, have recently been measured [13] and we propose that the most peripheral, few-nucleon removal reactions in this case warrant such a study within the direct knockout model.

ACKNOWLEDGMENTS

We gratefully acknowledge the contributions of members of the wider RISING Collaboration to the data discussed here. This work was supported by the United Kingdom Science and Technology Facilities Council (STFC) through Research Grant Nos. EP/D003628 and ST/F012012.

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