Curved bistable composite slit tubes with positive Gaussian curvature

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An investigation into the bistability of positively curved laminated composite slit tubes is presented, establishing a natural extension in this area which has previously been focused on straight tubes. Curved slit tubes are modeled as the surface segments of a torus. The design space is explored through a parametric study to investigate the effect on the second stable state, representing a small coil. This includes the effects of longitudinal curvature, cross-section subtending angles, non-uniform transverse curvature and spatially varying laminate properties. The second equilibrium state is determined through strain energy minimization using the Rayleigh-Ritz method. To verify the model, samples were manufactured from glass-fiber braid and polypropylene-resin. This investigation finds: (i) the initial curvature along the length of the tube has little effect on coil radius, however, the coil has a distinct barrel shape; (ii) highly enclosed and (iii) highly curved cross-sections result in higher edge strains of the second equilibrium, enabling identification of practical bistable tubes, and conversely; (iv) the greater the initial curvature along the length of the tube, the lower the second equilibrium strain.

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Nomenclature

R	= major circle radius of the torus
r	= minor circle radius of the torus
K_G	= Gaussian curvature
$(\dots)_0$	= initial, undeformed tubes and their properties are assigned subscript 0
κ_x	= azimuthal principal curvature (in the x direction)
κ_y	= cross-sectional principal curvature (in the y direction)
$\mathbf{T}$	= slit tube cross-section
s	= cross-sectional arc-length parameter
w_0	= tube width (in the y direction)
β_x, β_y	= azimuthal and cross-sectional subtended angle
n	= number of trial function terms
u_b, u_s	= bending and stretching strain energy per unit area
U_b, U_s	= bending and stretching strain energy per unit length
$\mathbf{D}, \mathbf{A}, \mathbf{B}$	= bending, extensional and coupling stiffness matrices
$\Delta\kappa_x, \Delta\kappa_y, \epsilon_x$	= change in longitudinal and transverse curvature, and stretching
l	= tube length (in the x direction)
M_x, M_y, M_{xy}	= longitudinal, transverse and torsional moments

I. Introduction

Bistable composite slit tubes are open-section tubular structures that can be rolled up and extended, analogous to tape measures, but are stable in both configurations without the need of a support structure. These lightweight, stiff, compact, quick and easy to use materials lend themselves to deployable structure applications.

Bistability in neatly coiling slit tubes may be engineered by the arrangement of fibers in each ply with respect to the deployment direction through an antisymmetric lay-up scheme. Theoretically, bistability may be achieved using any material of sufficiently high Poisson ratio and stiffness [1]. Also known as bistable reeled composites (BRCs), their invention and initial investigation [2] has led

to many valuable commercial and potential applications in the sectors of security, civil engineering, energy [3], mining, consumer, defense and aerospace [4].

Multiple deployment schemes for various types of multistable composite laminates have been devised. Small or low force actuation of neutrally stable tape springs [5, 6] improve control, reduce mass and are suitable for applications where it is desirable to eliminate deployment shock. Similarly, one-way actuation [7, 8] is another method for achieving large shape changes with little energy input. Another simple low cost approach installs tape spring hinges into closed section booms by cutting diametrically opposite slots [9–11] to enable self-deployable booms for small spacecraft.

The latest advances in aerospace have been focused on tailoring morphing and snap-through behavior for use in aircraft control surfaces [12–15]. These enable simple, lightweight and passive control methods for generating lift augmentation whereby geometry change may be induced at a specific temperature or aerodynamic pressure e.g. low velocity during aircraft takeoff and high velocity during aircraft climb or cruise.

Various approaches, modeling and analyses of shells [16] and BRCs over the years include finite element analysis [17] to predict and characterize the rolled up configuration, and beam models [18–20] to predict second equilibrium configuration and its stability. Progressing to a shell model [21] enabled the study of edge effects and an inextensional bending model [22] utilized the property of Gaussian curvature [23] for developable surfaces, which remains zero everywhere, to predict BRC coiled radius by parameterizing the deformation of a curved plate superimposed upon the surface of a cylinder. Investigating the bistability of curved, *non-developable* composite slit tubes establishes a natural extension from past work, which has been focused on straight, *developable* BRCs [24] and curved isotropic tape springs [25]. Furthermore, exploration of the design space is required in order to study the effects of multiple design parameters. This work builds on the recently developed ‘toroidal BRC model’ [26] by conducting a detailed study of curved bistable slit tubes that have: (i) initial, positive longitudinal curvature; (ii) cross-sections subtending more and less than 180°; (iii) non-uniformly curved cross-section shape; (iv) variable laminate properties; and (v) been manufactured for experimental verification. Curved tubes, either positively or negatively curved are defined by their azimuthal, or longitudinal curvature, and slit location.

II. The positively curved BRC model and manufacturing method

The model presented predicts the second equilibrium state for curved composite slit tubes (Fig. 1) and its bistability. The second equilibrium state is determined via the Rayleigh-Ritz method i.e. bending and stretching strain energy minimization whilst stability is verified by the local Hessian matrix being positive-definite.

Fig. 1 A positively curved slit tube (A); a negatively curved slit tube (B); the coiled up tube in A (C) and; the coiled up tube in B (D)

A. The geometrical considerations in this model

Curved slit tubes are modeled as surface segments of a torus. The torus is an example of a surface of revolution, which can be generated by sweeping a circular cross-section through an angle about an axis of revolution to produce a 3D solid or shell that may be expressed using two radii, R and r for the major and minor circles, respectively (Fig. 2).

The torus is also an example of a non-developable surface given its non-zero Gaussian curvature ($K_G \neq 0$). The Gaussian is the product of both principal curvatures ($K_G = \kappa_1 \kappa_2$, where subscript 1 and 2 represent the principal curvature directions) at any point on a surface. This curvature varies around the cross-section of a torus. As illustrated in Fig. 2 the Gaussian is: a negative maxima at point M ; zero at both N points and; a positive maxima at point P .

Non-developable surfaces require cutting or stretching in order to be flattened whereas developable surfaces do not. For example, a surface segment of a baseball, which is spherical and non-developable due to its positive Gaussian curvature, would need to be stretched and/or torn in order to be completely transferred to a flat surface.

Now consider the point P located at $X_0(s=0)$ on the torus with azimuthal and cross-sectional principal curvatures $\kappa_{x_0} = \frac{1}{R+r}$ and $\kappa_{y_0} = \frac{1}{r}$, respectively. It is appropriate to use the azimuthal and cross-section lines to define local longitudinal and transverse directions, respectively with curvilinear co-ordinate axes, x - y and use a global X - Y - Z Cartesian co-ordinate system. Subscript 0 signifies the initial, unstressed slit tube before any deformation has occurred. The transverse curvature (κ_{y_0}) is always taken to be positive, in the direction to the center of the minor circle. The sign of the longitudinal curvature (κ_{x_0}) depends upon location on the tube. Both principal curvatures are positive between points P and N but $\kappa_{x_0} = 0$ at N . The longitudinal curvature flips sign to become negative between points N and M , characterizing the saddle-shape region of the surface (negative Gaussian curvature, $K_G < 0$).

This is a symmetric problem allowing a single function, X_0 of a single independent variable, s to be used. The initial slit tube cross-section, $\mathbf{T}_0(s)$ is modeled as [26],

$$\mathbf{T}_0(s) = \begin{bmatrix} X_0(s) \\ 0 \\ Z_0(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{\kappa_{x_0}} + \frac{1}{\kappa_{y_0}} [\cos(s \kappa_{y_0}) - 1] \\ 0 \\ \int_0^{\pm s} \sqrt{1 - \left(\frac{dX_0(s)}{ds}\right)^2} ds \end{bmatrix} \quad (1)$$

where the cross-sectional arc-length, $s \in [-\frac{w_0}{2}, \frac{w_0}{2}]$ defines line segments along the width, $w_0 = \beta_{y_0} r$.

Longitudinal curvature is defined such that the longitudinal radius of curvature, $d(s)$ is a straight line perpendicular to the point on the cross-section intersecting the axis of revolution [16]. Conversely, the initial transverse curvature is uniform at every point and describes the arc of a circle. It follows that the initial (subscript 0) and deformed (without subscript) principal curvatures are:

$$\kappa_x(s) = \frac{\sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2}}{X(s)}, \quad \kappa_y = \pm \frac{\left|\frac{d^2X(s)}{ds^2}\right|}{\sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2}} \quad (2)$$

Parameterizing deformation of the cross-section is attempted using both polynomial and cosine series trial functions. For example, the effect of the first two ($n = 2$) terms of a cosine series, c_1 and

Fig. 2 A torus may be expressed using a major (R) and minor (r) radius. The surface curvature may be calculated using $X(s)$ and $d(s)$. The toroidal surface is described using curvilinear axes x and y for the azimuthal (longitudinal) and cross-sectional (transverse) directions

c_2 is illustrated in Fig. 3. Using a cosine trial function, the deformed slit tube cross-section, $\mathbf{T}(s)$ is modeled as:

$$\mathbf{T}_{\cosine}(s) = \begin{bmatrix} X(s) \\ 0 \\ Z(s) \end{bmatrix} = \begin{bmatrix} X_0(s) - c_1 + \sum_{i=2}^n c_i \cos\left(\frac{(2i-3)\pi s}{w_0}\right) \\ 0 \\ \int_0^{\pm s} \sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2} ds \end{bmatrix} \quad (3)$$

If appropriately implemented, c_1 represents coiling radius of the tube whilst c_2 represents uniform flattening of the cross-section. Adding more terms, $n > 2$ incorporates higher frequency, lower magnitude functions to better match complex coil shapes. Likewise, using a polynomial trial function, the deformed tube is modeled as,

$$\mathbf{T}_{polynomial}(s) = \begin{bmatrix} X(s) \\ 0 \\ Z(s) \end{bmatrix} = \begin{bmatrix} X_0(s) + \sum_{i=1}^n c_i \bar{s}^{2i-2} \\ 0 \\ \int_0^{\pm s} \sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2} ds \end{bmatrix} \quad (4)$$

where $\bar{s} = \frac{2s}{w_0}$ is the normalized arc-length. Equations 3 & 4 use cosine and polynomial trial functions for $X(s)$, respectively. They are developed from previous models [26] and necessary in order to describe non-uniformly curved cross-sections, which are non-circular, by using even/symmetrical trial functions.

Fig. 3 Deformation of the cross-section due to c_1 and c_2 , coefficients of the cosine trial function

The strain energy model used is derived from Iqbal's work [2] and takes into account: (i) non-zero initial longitudinal curvature; (ii) non-uniform transverse curvature and; (iii) longitudinal stretching. The bending and stretching strain energies per unit area [27] are defined as:

$$u_b(s) = \frac{1}{2} \begin{bmatrix} \Delta\kappa_x(s) & \Delta\kappa_y(s) & \Delta\kappa_{xy}(s) \end{bmatrix} \begin{bmatrix} \mathbf{D}(s) \end{bmatrix} \begin{bmatrix} \Delta\kappa_x(s) \\ \Delta\kappa_y(s) \\ \Delta\kappa_{xy}(s) \end{bmatrix} \quad (5)$$

$$u_s(s) = \frac{1}{2} \begin{bmatrix} \epsilon_x(s) & \epsilon_y & \epsilon_{xy}(s) \end{bmatrix} \begin{bmatrix} \mathbf{A}(s) \end{bmatrix} \begin{bmatrix} \epsilon_x(s) \\ \epsilon_y \\ \epsilon_{xy}(s) \end{bmatrix} \quad (6)$$

Change in the longitudinal and transverse curvature, and longitudinal stretching at each point on the surface are,

$$\Delta\kappa_x(s) = \frac{\sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2}}{X(s)} - \frac{\sqrt{1 - \left(\frac{dX_0(s)}{ds}\right)^2}}{X_0(s)} \quad (7)$$

$$\Delta\kappa_y = \frac{\frac{d^2X(s)}{ds^2}}{\sqrt{1 - \left(\frac{dX(s)}{ds}\right)^2}} - \frac{1}{r} \quad (8)$$

$$\epsilon_x(s) = \frac{l(s) - l_0(s)}{l_0(s)} \quad (9)$$

where l_0 and l are the initial and deformed lengths of imaginary and parallel longitudinal ‘fibers’:

$$l_0(s) = \beta_{x_0} X_0(s) \quad (10)$$

$$l(s) = \beta_x X(s) \quad (11)$$

The bending energy per unit length of tube is determined by integrating Eq. 5 across the slit tube surface:

$$U_b = \int \int_A u_b(s) \, dA = 2 \int_0^{\frac{w_0}{2}} u_b(s) \cdot l_0(s) \, ds \quad (12)$$

Similarly, the stretching energy per unit length is:

$$U_s = \int \int_A u_s(s) \, dA = 2 \int_0^{\frac{w_0}{2}} u_s(s) \cdot l_0(s) \, ds \quad (13)$$

The cross-section width, w_0 is assumed constant because transverse stretching is considered negligible ($\epsilon_y = 0$), given $l_0 \gg w_0$.

A constrained, nonlinear, multivariable MatLab optimizer is used to calculate local equilibrium configurations. Equilibria are determined by minimizing the total strain energy with respect to all trial function coefficients, ϵ_x and c :

$$\left[\frac{\partial}{\partial \epsilon_x}, \frac{\partial}{\partial c_i}, \frac{\partial}{\partial c_{i+1}}, \dots, \frac{\partial}{\partial c_{n-1}}, \frac{\partial}{\partial c_n} \right] U_t = 0 \quad (14)$$

where $U_t = U_b + U_s$ is the total strain energy per unit length. Given the tube is modeled as a free body, an equality constraint is applied to ensure the resultant transverse moment at the edge of the equilibrium configuration ($s = \frac{w_0}{2}$), which presumably resembles a coil shape, is zero:

$$M_y \left(\frac{w_0}{2} \right) = 0 \quad (15)$$

The moments are calculated as,

$$M_x(s) = B_{11}(s) \epsilon_x(s) + D_{11}(s) \Delta \kappa_x(s) + D_{12}(s) \Delta \kappa_y(s) \quad (16)$$

$$M_y(s) = B_{21}(s) \epsilon_x(s) + D_{21}(s) \Delta \kappa_x(s) + D_{22}(s) \Delta \kappa_y(s) \quad (17)$$

$$M_{xy}(s) = B_{31}(s) \epsilon_x(s) + D_{31}(s) \Delta\kappa_x(s) + D_{32}(s) \Delta\kappa_y(s) \quad (18)$$

because $\epsilon_y(s)$, $\epsilon_{xy}(s)$ and $\Delta\kappa_{xy}(s)$ are taken to be zero. Shearing is neglected given $A_{13}(s)$ and $A_{23}(s)$ are zero. In addition, $B(s)$, $D_{31}(s)$ and $D_{32}(s)$ are all equal to zero given an antisymmetric layup with an even number of plies, so no torsion or twisting develops, $M_{xy} = 0$.

B. Manufacturing and the mechanical considerations in this model

The mechanics are derived from classical lamination theory [28, 29] to calculate the elastic properties of the laminate. Spatially varying properties are modeled along the slit tube cross-section. This is appropriate given the manufacturing process for positively curved tubes decreases both the fiber volume fraction ratio and laminate thickness along the centerline. The opposite occurs in negatively curved tubes.

Consider imaginary ‘longitudinal strips’ along the centerline (Fig. 4). These stripped areas encounter the greatest Gaussian curvature change upon introducing the second principal (longitudinal) curvature as the laminate was originally flat. Conversely, the top and bottom of the toroidal composite (corresponding to points N , in Fig. 2) encounter no change in Gaussian - and remain zero during this phase of pre-processing - when the developable surface transitions into a non-developable one.

Changes in the fiber volume fraction and thickness are assigned a range of values, shown in Table 1 to explore the design space. This range is considered to be the typical variability achievable using hand manufacture. Additionally, given $\Delta K_G > 0$ about the centerline, which translates into modification of the fiber angles, it may be reasonable to include this for the spatially varying laminate properties already considered. However, possible effects from this are neglected in this model but discussed later. The **ABD**-matrix along the centerline of a positively curved tube (point

Fig. 4 Illustration of introducing two principal curvatures into a flat laminated composite and the effects of this on the thickness and fiber angles about the centerline (red highlighted region)

P , $X_0(s = 0)$, recall Fig. 2 and 0cm in Fig. 6) using the properties in Table 1 is modeled as,

$$\begin{bmatrix} \mathbf{A}_P & \mathbf{B}_P \\ \mathbf{B}_P & \mathbf{D}_P \end{bmatrix} = \begin{bmatrix} 1.96 \times 10^7 & 1.86 \times 10^7 & 0 & 0 & 0 & 0 \\ 1.86 \times 10^7 & 1.96 \times 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.86 \times 10^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.77 & 0.73 & 0 \\ 0 & 0 & 0 & 0.73 & 0.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.73 \end{bmatrix} \quad (19)$$

and along the top and bottom of the tube (points N in Fig. 2 and 2.51cm in Fig. 6),

$$\begin{bmatrix} \mathbf{A}_N & \mathbf{B}_N \\ \mathbf{B}_N & \mathbf{D}_N \end{bmatrix} = \begin{bmatrix} 2.56 \times 10^7 & 2.44 \times 10^7 & 0 & 0 & 0 & 0 \\ 2.44 \times 10^7 & 2.56 \times 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.44 \times 10^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.11 & 1.06 & 0 \\ 0 & 0 & 0 & 1.06 & 1.11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.06 \end{bmatrix} \quad (20)$$

and on the innermost centerline (point M in Fig. 2),

$$\begin{bmatrix} \mathbf{A}_M & \mathbf{B}_M \\ \mathbf{B}_M & \mathbf{D}_M \end{bmatrix} = \begin{bmatrix} 3.23 \times 10^7 & 3.06 \times 10^7 & 0 & 0 & 0 & 0 \\ 3.06 \times 10^7 & 3.23 \times 10^7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.07 \times 10^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.54 & 1.46 & 0 \\ 0 & 0 & 0 & 1.46 & 1.54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.47 \end{bmatrix} \quad (21)$$

The materials used were dry glass-fiber braid and polypropylene (PP) resin. A new manufacturing technique is used to intentionally produce curved, composite slit tubes [30] (Fig. 5) that mitigate undesirable wrinkling - a problem that does not arise when producing straight slit tubes. Wrinkling arises when flat fabric or prepreg plies are forced to conform to a non-flat surface, in this process a flexible tube former is used. This stage of the process signifies the initially developable wet layup's transition into a non-developable sheet of material. Short samples were produced with dry, glass-fiber braid and polypropylene (PP) resin in the following manner:

1. Lay up $[\pm 45^\circ]$ glass-fiber/PP-resin film (Fig. 5A)
2. Form glass/PP laminated composite onto flexible former and wrap tightly with polyester shrink-wrap tape (Fig. 5B) - this step introduces the first principal (transverse) curvature
3. Tighten kevlar-string/turnbuckle (Fig. 5C) - this step introduces the second principal (longitudinal) curvature
4. Process in oven for 20-25 minutes at 194°C

5. Remove shrink-wrap tape, remove the processed sample from the former and clean up the edges of any loose glass-fibers and excess PP-resin (Figs. 5D & 5E)

Fig. 5 The manufacturing process for short ($\approx 50\text{cm}$), curved, laminated composite slit tubes: a glass/PP layup (A); a laminate wrapped onto flexible former (B); two examples of the laminate former being curved using kevlar-string/turnbuckle (C); a positively curved sample (D) and; a negatively curved sample (E)

III. Results

The following results are for glass/PP laminated composites with the mechanical properties modeled in Table 1. Two MatLab optimizers are used, FMINUNC (unconstrained, quasi-Newton algorithm) and FMINCON (equality constrained, interior-point algorithm) to minimize the total strain energy with respect to a polynomial trial function comprising thirteen terms ($n = 13$, selected following convergence studies) that parameterize deformation of the slit tube. An appropriate initial guess for the optimizer is determined using MULTISEARCH.

The results presented in Figs. 6, 8 & 9 are plotted for half the tube cross-section given symmetry about the X-Y plane. Figures 7–11 and Table 2 are results for positively curved tubes.

IV. Discussion

Verification of the model is presented with two examples in Fig. 6. The initial principal curvatures for both a positively and negatively curved slit tube are modeled correctly, exhibiting lon-

Table 1 Glass-fiber/PP-resin laminated composite mechanical properties. A range of fiber volume fraction and ply thickness are used for modeling spatially varying laminate properties

Property	Value/Range	Units
Fiber modulus	240×10^9	Nm^{-2}
Fiber shear modulus	95×10^9	Nm^{-2}
Fiber Poisson's ratio	0.22	-
Matrix modulus	1.33×10^9	Nm^{-2}
Matrix shear modulus	4×10^8	Nm^{-2}
Matrix Poisson's ratio	0.35	-
Fiber volume fraction	0.4 to 0.6	-
Ply thickness	$0.19 \text{ to } 0.21 \times 10^{-3}$	m
Layup	$[\pm 45, \pm 45]$	Degrees

Fig. 6 The initial principal curvatures (κ_{x0} and κ_{y0}) and spatially varying laminate properties (200 data points) for initially positive and negative curved tubes are plotted. The torus modeled is: $R = 80\text{cm}$, $r = 1.6\text{cm}$, $\beta_{x0} = \frac{1}{R}$ and $\beta_{y0} = \pi$ with the mechanical properties in Table 1. $D_{12} = D_{21}$ and $D_{22} = D_{11}$ given an antisymmetric laminate layup

Fig. 7 The tube and laminate from Fig. 6 is modeled (left) and its second equilibria (right) for four cases: unconstrained and constrained (Eq. 15) strain energy minimization and; constant and spatially variable laminate properties

itudinal curvature $\kappa_{x_0} = \frac{1}{R+r} \approx 1.2\text{m}^{-1}$ and $\kappa_{x_0} = -\frac{1}{R-r} = -1.3\text{m}^{-1}$, respectively along the centerline. The longitudinal curvature approaches zero at the edges and the transverse curvature is constant along the circular cross-section indicated by horizontal line at $\kappa_{y_0} = 62.5\text{m}^{-1}$. Two modifications for each tube are modeled by linearly varying the fiber volume fraction ratio and ply thickness to explore the design space, shown in Table 1. The centerline (0cm) is modeled with 0.4 fiber volume fraction and $0.19 \times 10^{-3}\text{m}$ thickness. These values change to 0.5 and $0.20 \times 10^{-3}\text{m}$ at the edges (points N in Fig. 2) for a tube subtending 180° , respectively. The volume fraction and thickness would be 0.6 and $0.21 \times 10^{-3}\text{m}$ at point M in the same figure. 200 data points were

Fig. 8 The in-plane moments plotted along the cross-section for each of the four cases modeled.

The legend from Fig. 7 is used

Fig. 9 The principal curvature changes and strain plotted along the cross-section for each of the four cases. The legend from Figs. 7 & 8 is used

Fig. 10 Convergence study using $n \in [4, 10]$ polynomial and cosine trial functions of optimized coils for: $R = 10^4$ cm, $r = 1.6$ cm, $\beta_{x_0} = \frac{1}{R}$ and $\beta_{y_0} = \pi$

Table 2 A comparison between Iqbal’s straight tube model [2], the curved tube model developed and experiment (Samples 1-3)

Sample	β_{y_0} ($^\circ$)	R (cm)	r (cm)	Coil Radius _{Iqbal} (cm)	Coil Radius _{Curved} (cm)	Experiment (cm)
Sample 1	210	∞	1.75	1.65	1.88	3.00
Sample 2	210	25.10	1.70	-	1.88	2.70
Sample 3	210	16.25	1.70	-	1.91	2.90
Straight 1	300	∞	1.8	1.64	1.91	-
Straight 2	180	∞	1.8	1.70	1.92	-
Straight 3	90	∞	1.8	1.71	1.96	-
Straight 4	300	∞	3.6	2.98	3.82	-
Straight 5	180	∞	3.6	3.35	3.83	-
Straight 6	90	∞	3.6	3.40	3.87	-

taken along the cross-section as they provided a good balance between accuracy and computational expense. The effect of these on the second equilibrium coil for a positively curved tube are presented in Fig. 7.

Considering constant or spatially varying laminate properties was found to yield negligible difference for the coil radii predicted i.e. fractions of a centimeter, although results showed 20% less residual strain energy for variable property laminates in both constrained and unconstrained cases. In principle, this is expected to occur given that positively curved tubes have lower stiffness properties due to lower fiber volume fraction ratios and ply thicknesses that effectively scale down the total strain energy. Furthermore, the peak strain during coiling will be lower. This provides a valuable criterion for identifying practical bistable tubes i.e. those that will not fracture or fail during coiling and deployment and can be investigated by following the ‘path’ from the initial state to the second equilibrium.

Although a constrained approach is physically accurate due to the tubes being modeled with free-free edges so that Eq. 15 is satisfied, the largest difference in coil radius predicted is 0.12mm.

The shape of the curved coil cross-section is an important design consideration. Results show the sagitta of the coil is 0.36mm, almost half the laminate thickness (0.8mm). Therefore implementation

Fig. 11 Coil radius parametric study of the parameters R , r and β_{x0} for a $[\pm 45, \pm 45]$ layup with constant laminate properties. Shallowly and highly curved tubes ($\frac{r}{R} < 0.05$ and $\frac{r}{R} > 0.05$) are plotted as black and red lines respectively

of a spheroid/barrel-shaped deployment mechanism spindle may be required, particularly for tubes with sagittas greater than the laminate thickness. For these types of tube, the residual curvature of the coil will dominate the deployment mechanism design, as shown in past work for deployable helical tubes [31]. Similarly for straight tubes, another primary concern is scalability, for which the laminate thickness dominates and causes the ‘50 pence’ effect [32], particularly in long tubes with small coil radii. Practically, this effect hinders tight coils and can induce blossoming or self-deployment.

The second equilibrium moments highlight further constraint effects. A build up of the longitudinal and transverse moments is observed in the last 10% of the cross-section at the tube edges, as shown in Fig. 8. This known and hereby verified edge effect [17, 21] (also referred to as the aforementioned ‘50 pence’ effect) prevents the transverse curvature from decreasing during roll up,

resulting in the edges sagging inwards once coiled up (Figs. 9 & 10). Furthermore, no twisting moments are induced indicating a neat and tight coil, which is to be expected for an initially untwisted tube with antisymmetric layup - this has been observed in samples manufactured e.g. Fig. 1.

Complete flattening of 80% of the cross-section is indicated by $\Delta\kappa_y \approx -62.5\text{m}^{-1}$ in the plot (recall that $\kappa_{y0} = 62.5\text{m}^{-1}$ in Fig. 6). Edge effects are observed consistently over all number of polynomial trial functions used to describe the cross-section (Fig. 10) and match Galletly's shell model [19, 21]. The model exhibits good convergence for relatively straight tubes with at least five polynomials ($n \geq 5$) with highly curved tubes requiring at least seven terms.

The curved model closely matches Iqbal's established beam model [2] and the discrepancy with experiment is most likely due to mismatch between the laminate properties (Samples 1-3, Table 2) given that spatially varying laminate properties were found to be negligible. The results show minimal effect between the initial longitudinal curvature and coil radius, however, investigations highlight the coil's sagitta is proportional to the longitudinal curvature. Consequences of this have been discussed.

For straight tubes ($R \rightarrow \infty$) both models exhibit similar values and proportionality of coil radii over the range of cross-section subtending angles (Straight 1-3 & 4-6) as is also the case for shallowly curved tubes ($\frac{r}{R} < 0.05$) as observed in Fig. 11 (black line plots). This arises due to $l_0 \gg w_0$ and $\kappa_{y0} \gg \kappa_{x0}$, so that the initial cross-section curvature is the most dominant parameter in the strain energy and influencer of the second equilibrium.

A wider range of diverging coil radii are predicted for highly curved and enclosed tubes ($\frac{r}{R} > 0.05$, red line plots in Fig. 11). This may be due to the growing contribution of the currently neglected transverse stretching (ϵ_y) in producing strain energy as the tube more closely resembles a square plate ($w_0 \rightarrow l_0$) demonstrating a limitation of the current approach. For example, two highly curved tubes of $R = 25\text{cm}$, $r = 11.25\text{cm}$ ($\frac{r}{R} = 0.45$) and $\beta_{y0} = [90^\circ, 240^\circ]$ are predicted to have coil radii of 10.8cm and 15.9cm, respectively. This is in conflict with theory, the results presented here (Table 2) and previous work [17], which has demonstrated an inverse relationship between coil radius and subtending angle.

V. Conclusion

Investigating the bistability of curved composite slit tubes with positive Gaussian curvature has been conducted to characterize these novel materials, which enable many high-value structural applications. The work presented focuses on positively curved tubes with the relationship between how the initial principal curvatures and cross-section subtending angle (R , r and β_{y_0}) affect the second equilibrium radius and shape - a compact barrel-shaped coil. Further model development is required to reliably simulate highly and negatively curved tubes which undergo high strain when transitioning to the second state. This could be rectified by implementing transverse stretching within the model.

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