Mixing effects on K -forbidden transition rates from the 6^+ isomers in the $N = 104$ isotones

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Abstract.

The problem of the very different $E2$ decay rates from the two-quasineutron $K^\pi = 6^+$ isomers in the $N = 104$ isotones ^{172}Er , ^{174}Yb , ^{176}Hf , and ^{178}W is investigated using the triaxial projected shell model with inclusion of multi-quasiparticle configurations. It is demonstrated that the highly K -forbidden transition from the 6^+ isomer to the ground-state band is sensitive to mixing with the 6^+ state of the γ -vibrational band. Thus the inter-band transitions, and lifetimes, depend on the relative position of the γ -band and the isomeric state in each isotone.

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1. Introduction

As the lowest collective vibrational excitation mode in nuclei, study of the γ -vibrational band, its mixing with the ground-state band, and its influence on the mixing with other nearby states have been key topics in discussions of nuclear structure [1, 2, 3, 4]. For example, evidence was shown [5] for strong coupling of quasiparticle (qp) excitations with the γ vibration in rare-earth transitional nuclei, and this was used to explain

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the systematic behavior of the low-lying $K^\pi = 3^+$ 2-qp band. With a quantitative reproduction of the experimental data, that study showed clearly, as expected, that the band mixing depended sensitively on the proximity of the members of the γ vibrational band to those of the 3^+ 2-qp band.

K -isomeric states [6] in deformed nuclei are generally understood to arise from approximate conservation of the K quantum number, the projection of the total angular momentum on the nuclear symmetry axis, and the fact that changes in the projection are constrained by the multipolarity of the electromagnetic decays. Since such transitions are (in principle) forbidden, the isomer lifetimes depend strongly on the degree of violation of the K quantum number, and hence on small K -admixture.

Experimentally, many examples of 2-qp and 3-qp K -isomers are known (see for example Ref. [7]), with lifetimes that vary significantly even for states with similar decay energies and configurations in adjacent nuclei. One extreme example is that of the $K^\pi = 6^+$ 2-qp isomer in the $N = 104$ isotones ^{172}Er [8], ^{174}Yb [9], ^{176}Hf [10, 11], and ^{178}W [12], arising from the $\{5/2^- [512] \otimes 7/2^- [514]\}$ 2-quasineutron configuration, with an additional 2-quasiproton admixture in the ^{176}Hf case. Although the initial-state configurations are closely related and these nuclei are similar in terms of deformation etc., the transition rates of decays from the 6^+ isomers differ by several orders of magnitude. The strength of the $E2$ transition to the 4^+ member of the ground-state band, $B(E2, 6_i^+ \rightarrow 4_g^+)$, is very small in the case of ^{174}Yb (about 10^{-11} W.u.), but it increases to $\sim 10^{-7}$ W.u. in the neighboring isotone ^{176}Hf , and again to 10^{-4} W.u. in the next isotone ^{178}W . For ^{172}Er , the $\Delta I = 0$ (mixed M1/E2) transition from the isomer to the ground-state band has been observed, but not the $\Delta I = 2$ (E2) transition. The $B(E2)$ value of the transition to the γ -band is measured to be $\sim 10^{-4}$ W.u., corresponding to a reduced hindrance factor $f_\nu = 69$ [8].

As implied above, in principle, γ -ray transitions between states where the difference in K exceeds the transition multipolarity, λ , are forbidden, but such transitions do proceed, with very low probabilities that scale approximately with the magnitude of the nominal forbiddenness (given by $\nu = \Delta K - \lambda$). At some level, the fact that the decays proceed is related to the breaking of the K quantum number in the initial or final states, or both. The physical origin of this violation is thought to be due to axial asymmetry in the nuclear potential, so that K is not well-defined at the outset or is modified through shape changes, or to other dynamical effects such as the K -mixing induced by the large Coriolis forces in nuclei. Accordingly, the problem has been studied through γ -tunneling calculations [13] and with the cranking approach [14, 15]. These studies have successfully explained some aspects of the isomeric transitions but the methods do not treat the two K -mixing mechanisms on the same footing. The authors of these works also suggested that angular-momentum-conservation is needed for a proper treatment.

In Ref. [16], Saitoh *et al.* examined K -forbidden transitions in the context of rotor models and found qualitatively that the enhancement in transition probabilities was correlated with γ -softness. They speculated that there would be levels that mediate transitions between high- and low- K states through two-state mixing.

The aim of the present study is, by applying the triaxial projected shell model approach, to elucidate the factors that control both the absolute magnitudes of the forbidden decays, and in the specific example of the 6^+ isomers in the $N = 104$ isotones, to explain quantitatively the rapid changes observed as a function of Z .

2. Model calculations

2.1. Projected shell model with axially deformed basis

The projected shell model (PSM) [17] treats the deformed states with angular momentum projection. In the ordinary version of the PSM, the rotational invariant Hamiltonian with the quadrupole plus pairing interaction,

$$\hat{H} = \hat{H}_0 - \frac{\chi}{2} \sum_{\mu} \hat{Q}_{\mu}^+ \hat{Q}_{\mu} - G_M \hat{P}^+ \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^+ \hat{P}_{\mu}, \quad (1)$$

is diagonalized in a basis composed of angular-momentum-projected qp states. The basis set $|\Phi\rangle$ is obtained by applying a BCS approach to a set of deformed Nilsson single-particle states with axial symmetry. If $|\Phi\rangle$ is taken as a set of 0-qp and 2-qp states:

$$|\Phi\rangle = \{|0\rangle, \alpha_{\nu}^+ \alpha_{\nu}^+ |0\rangle, \alpha_{\pi}^+ \alpha_{\pi}^+ |0\rangle\}, \quad (2)$$

then the total wavefunction can be written as

$$|\psi_M^{I,\sigma}\rangle = \sum_{\kappa, K \leq I} f_{\kappa}^{I,\sigma} \hat{P}_{M,K}^I |\Phi_{\kappa}\rangle = \sum_{\kappa} f_{\kappa}^{I,\sigma} \hat{P}_{M,K_{\kappa}}^I |\Phi_{\kappa}\rangle. \quad (3)$$

The index σ labels states with the same angular momentum I and κ labels the basis states. $\hat{P}_{M,K}^I$ is the angular-momentum-projection operator, and the coefficients $f_{\kappa}^{I,\sigma}$ are weights of the basis states that are determined by diagonalization. The deformed states are obtained with an axially symmetric potential, and each of the basis states in Eq. (2) has a well-defined K . In this case, a shell model diagonalization is equivalent to K -mixing. The amount of mixing can be read from the resulting wavefunction in Eq. (3). The amplitudes can then be used to calculate the isomeric transitions. This corresponds to a fully quantum-mechanical treatment of K -violation.

Calculations for the four $N = 104$ isotones ^{172}Er , ^{174}Yb , ^{176}Hf , and ^{178}W were carried out with this model. For ^{174}Yb , the energy levels for both the ground-state band and the isomer band, and the strength of the $E2$ transitions between them, were correctly reproduced. In particular, the calculation yielded a small, but non-zero, value for the strength of the inter-band $E2$ transitions as observed [9]. (Note that without mixing in the wavefunction, the transition from the $K = 6$ isomer to the $K = 0$ ground-state band would vanish.) Although experimentally the isotones ^{176}Hf and ^{178}W have more enhanced inter-band $E2$ transitions from the 6^+ isomer to the ground-state band [10, 11, 12], the PSM calculations predict essentially the same (small) strengths as those calculated and observed for ^{174}Yb . Table 1 summaries the results.

Table 1. Inter-band $B(E2)$ values (in W.u.) of transitions from the $K^\pi = 6^+$ isomer to the ground band 4^+ state in the $N = 104$ isotones. The calculation was performed with an axially deformed basis in the PSM.

	Exp.	Cal.
^{172}Er	— ^a	6.72×10^{-11}
^{174}Yb	$7.2 \times 10^{-11\text{b}}$	4.10×10^{-11}
^{176}Hf	$3.16 \times 10^{-7\text{c}}$	4.36×10^{-11}
^{178}W	$4.67 \times 10^{-4\text{d}}$	5.64×10^{-11}

^a Not observed, see Ref. [8].

^b Data taken from Ref.[9].

^c Data taken from Ref. [10, 11].

^d Data taken from Ref. [12].

One has to conclude that although the PSM with axial symmetry in the basis includes a K -mixing mechanism, effectively introducing some axial asymmetry, the mixing is too weak to reproduce the observed strengths in the higher- Z isotones. Conversely, one might also conclude that the agreement for ^{174}Yb was fortuitous.

Approaching the problem from a different perspective, a γ -tunneling model [13] was introduced in an attempt to describe the (relatively) enhanced inter-band $E2$ values from various isomers. In this model, the γ degree of freedom is taken into account explicitly, thus breaking the axial symmetry of the mean-field. The spontaneous symmetry breaking helps to produce electromagnetic transitions that are otherwise impossible because of the (nominal) K -selection rule. In this way, the authors in [13] were able to describe the relatively large inter-band $E2$ s in ^{176}Hf and ^{178}W but they could not reproduce the very small $6_i^+ \rightarrow 4_g^+$ $E2$ strength observed in ^{174}Yb . This is essentially the complementary situation to that arrived at with the PSM.

Both approaches, the configuration mixing implemented in the PSM [17] and the γ -tunneling model of Narimatsu *et al.* [13], introduce mechanisms that break the axial symmetry, although the degree of symmetry breaking is different. If the physical process is a perturbation in K -space, then it is described better by the PSM based on the axially symmetric mean field. If it is not a perturbation, axial asymmetry in the mean field should be introduced, as in the γ -tunneling model. The two methods may be viewed as simplifications of the many-body problem that each emphasize a different aspect. A unified microscopic description is clearly desirable.

2.2. Triaxial projected shell model and the γ degree of freedom

To efficiently incorporate the K -breaking mechanism into the projection theory, one must give up the axial symmetry of the mean-field, employ a triaxially deformed basis, and carry out three-dimensional angular momentum projection on it [18]. Such triaxially deformed single-particle states can be obtained by solving a Nilsson Hamiltonian of the

Table 2. The deformation parameters ϵ_2 and ϵ' in Eq. (4), used in the TPSM calculations.

	ϵ_2	ϵ'
^{172}Er	0.330	0.124
^{174}Yb	0.307	0.091
^{176}Hf	0.287	0.102
^{178}W	0.263	0.106

form

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3}\hbar\omega \left[\epsilon_2 \hat{Q}_0 + \epsilon' \frac{\hat{Q}_{+2} + \hat{Q}_{-2}}{\sqrt{2}} \right]. \quad (4)$$

Recently, the triaxial projected shell model (TPSM) that includes various multi-qp configurations became available [5, 19, 20, 21]. In the following, we take advantage of this development. As we shall discuss, within this framework the proposed K -mixing components - triaxial deformation in the mean-field and the Coriolis coupling through rotation - are taken into account properly. We note that the initial shape of the nucleus is fixed by deformation parameters in the TPSM basis (see Table 2). Shape changes with rotation are not explicitly considered in the deformed basis, but for different spin states, some dynamical changes may be introduced through mixing of configurations.

The deformation parameters used in the calculation are given in Table 2. The triaxial deformation ϵ' is treated as a parameter in the model, and is adjusted to reproduce the energy of the observed 2^+ state of the γ band in each nucleus. As discussed in detail in Ref. [22], a non-zero ϵ' is needed to shift the γ band down to the right position even for nuclei that are thought to be axially symmetric. The ground-band energies of these nuclei are not affected by the value of ϵ' [22]. Use of a fixed triaxiality may correspond to a γ -rigid picture, as discussed in the Davydov-Filippov model [23]. We note that the calculated deformations of Möller *et al.* [24] suggest similar values for the four isotones, with $\epsilon_2 = 0.267$ for ^{172}Er , $\epsilon_2 = 0.258$ for ^{174}Yb , 0.250 for ^{176}Hf , and 0.242 for ^{178}W . However, the deformations deduced from experimental B(E2) values by Raman *et al.* [25] suggest larger values for ^{174}Yb and ^{176}Hf , similar to the values in Table 2. As our model Hamiltonian that describes deformed nuclei is constructed through a self-consistent relation with deformation (see Ref. [26]), we require that the chosen input deformation reproduces the B(E2) values of the ground band (see results in Table 3). The B(E2) values of the ground-state band in ^{172}Er are not known. We choose a deformation parameter similar to the listed deformation of lighter Er isotopes [25].

The strength of the quadrupole-quadrupole interaction χ in Eq. (1) is determined through a self-consistent relation with the input quadrupole deformation ϵ_2 [17, 26]. The strength of the monopole pairing G_M in (1) is

$$G_M = \left(21.24 \mp 13.86 \times \frac{N - Z}{A} \right) / A \quad (\text{MeV}) \quad (5)$$

Table 3. Ground-band $B(E2)$ values (in W.u.) of in-band transitions in the $N = 104$ isotones. The calculation is performed with a triaxially deformed basis in the TPSM. The experimental data are taken from Refs. [27] (for ^{174}Yb), [28] (for ^{176}Hf), and [29] (for ^{178}W).

	^{172}Er	^{174}Yb	^{176}Hf	^{178}W
$B(E2, 2_g^+ \rightarrow 0_g^+)(\text{Exp.})$	—	201(7)	183(7)	153(2)
$B(E2, 2_g^+ \rightarrow 0_g^+)(\text{Cal.})$	204	204	169	136
$B(E2, 4_g^+ \rightarrow 2_g^+)(\text{Exp.})$	—	280(9)	—	—
$B(E2, 4_g^+ \rightarrow 2_g^+)(\text{Cal.})$	254	271	223	178
$B(E2, 6_g^+ \rightarrow 4_g^+)(\text{Exp.})$	—	370(50)	—	—
$B(E2, 6_g^+ \rightarrow 4_g^+)(\text{Cal.})$	245	278	229	182

with a $(-)$ sign for neutrons and a $(+)$ sign for protons. The strength of the quadrupole pairing is assumed to be $G_Q = 0.16G_M$ for all three nuclei. These are the standard parameters of the PSM [17].

3. Results and discussion

3.1. Comparison with experimental data

The calculated level schemes for the $N = 104$ isotones are compared with experimental data in Figs. 1 – 4. It can be seen that the energy levels of the ground-state band, the γ band, and the $K^\pi = 6^+$ band based on the isomer are all described well, including the second 6^+ band in ^{176}Hf . In Table 3, the agreement of the calculated ground-band $B(E2)$ values with available data is also good, indicating that the chosen deformation parameters (Table 2) are appropriate. In all $B(E2)$ calculations here and for the following isomer transitions, standard effective charges of $0.5e$ for neutrons and $1.5e$ for protons are used. In the calculation, the main components of the ground-state band and the γ band are obtained from projection of the triaxially deformed 0-qp state onto $K = 0$ and $K = 2$, respectively. The $K^\pi = 6^+$ band is calculated to have a major component of projection onto $K = 6$ from the lowest triaxially-deformed 2-qp state. It is clear that these configurations all originate from the same triaxially deformed potential, and therefore have a close intrinsic relationship. (K in this case is solely a label and by no means a good quantum number.)

The calculated inter-band $B(E2)$ values for the 6^+ isomer decays are given in Table 4, and compared with known experimental data. It is seen, for example, that starting from ^{174}Yb , the increase of about two orders of magnitude in the $B(E2)$ values for the $6_i^+ \rightarrow 4_g^+$ transitions in successive even isotones is approximately reproduced by the calculations. This is a considerable improvement on the calculations described earlier.

In order to isolate the reason for the large difference in transition rates in the isotones, we show in Fig. 5 the energy separation between the 6^+ isomer and the $I = 6$

Figure 1. Calculated level scheme of ^{172}Er , compared with the experimental data taken from Ref. [8].

Figure 2. Calculated level scheme of ^{174}Yb , compared with the experimental data taken from Refs. [30, 31, 9].

Figure 3. Calculated level scheme of ^{176}Hf , compared with the experimental data taken from Refs. [32, 10].

Figure 4. Calculated level scheme of ^{178}W , compared with the experimental data taken from Refs. [33, 34].

Table 4. $B(E2)$ values (in W.u.) of transitions from the $K^\pi = 6^+$ isomer in the $N = 104$ isotones. The calculation is performed with a triaxially deformed basis in the TPSM. The experimental data are taken from Refs. [8] (for ^{172}Er), [9] (for ^{174}Yb), [10, 11] (for ^{176}Hf), and [12] (for ^{178}W).

	^{172}Er	^{174}Yb	^{176}Hf	^{178}W
$B(E2, 6_i^+ \rightarrow 4_g^+)(\text{Exp.})$	$< 4.4 \times 10^{-8}$	7.2×10^{-11}	3.16×10^{-7}	4.67×10^{-4}
$B(E2, 6_i^+ \rightarrow 4_g^+)(\text{Cal.})$	2.66×10^{-5}	7.52×10^{-9}	3.94×10^{-7}	3.74×10^{-5}
$B(E2, 6_i^+ \rightarrow 6_g^+)(\text{Exp.})$	—	8.5×10^{-9}	2.82×10^{-6}	—
$B(E2, 6_i^+ \rightarrow 6_g^+)(\text{Cal.})$	1.19×10^{-4}	4.50×10^{-9}	1.91×10^{-7}	7.78×10^{-6}
$B(E2, 6_i^+ \rightarrow 4_\gamma^+)(\text{Exp.})$	2.07×10^{-4}			9.01×10^{-2}
$B(E2, 6_i^+ \rightarrow 4_\gamma^+)(\text{Cal.})$	3.39×10^{-4}			4.84×10^{-3}

Figure 5. (Color online) (a) Calculated energy separation ΔE between the $K^\pi = 6^+$ isomer and the $I^\pi = 6^+$ member of the γ -band. (The actual values of the theoretical ΔE are 54 keV for ^{172}Er , 732 keV for ^{174}Yb , 428 keV for ^{176}Hf , and 94 keV for ^{178}W .) Note that for ^{172}Er , ^{174}Yb , and ^{178}W , the 6_γ^+ state is not known experimentally. (b) $B(E2, 6_i^+ \rightarrow 4_g^+)$ values (including the measured limit for ^{172}Er [8]) for the $N = 104$ isotones.

member of the γ -band (ΔE) for each isotone, together with the $B(E2, 6_i^+ \rightarrow 4_g^+)$ values. A clear anti-correlation is evident in Fig. 5 for both experiment and calculation: an isotone with smaller ΔE has a larger $B(E2, 6_i^+ \rightarrow 4_g^+)$ value, and vice versa.

The obvious implication is that the strengths are related to the proximity of the states in the γ -vibrational band to the 6^+ isomer. Even with a small interaction, if the energy separation is relatively low as in ^{172}Er and ^{178}W , the 6^+ isomer could mix with the 6^+ state of the γ band, and then the $(6_i^+ \rightarrow 4_g^+)$ transition could be enhanced progressively through the isomer- γ and the γ - g mixing. Thus the existence of the γ band in between plays a key role.

To verify this, we compare the results of two calculations for isomer transitions in the three isotones. In Table 5, we repeat the calculated $B(E2, 6_i^+ \rightarrow 4_g^+)$ values

Table 5. Calculated $B(E2, 6_i^+ \rightarrow 4_g^+)$ values (in W.u.) with and without the γ -configuration.

	with γ -configuration	without γ -configuration
^{172}Er	2.66×10^{-5}	2.92×10^{-8}
^{174}Yb	7.52×10^{-9}	4.22×10^{-9}
^{176}Hf	3.94×10^{-7}	1.77×10^{-9}
^{178}W	3.74×10^{-5}	3.65×10^{-11}

Table 6. Comparison of the calculated $B(E2, 6_i^+ \rightarrow 4_g^+)$ values (in W.u.) from this work (TPSM) with those from Refs. [13, 35, 36] (γ -tunneling model).

	TPSM	γ -tunneling model
^{172}Er	2.66×10^{-5}	$2.8 \times 10^{-10\text{a}}$
^{174}Yb	7.52×10^{-9}	1.3×10^{-8}
^{176}Hf	3.94×10^{-7}	9.2×10^{-7}
^{178}W	3.74×10^{-5}	2.7×10^{-5}

^a This was calculated by assuming the quadrupole moment $Q_0 = 7.57$ eb taken from the ^{170}Er experimental value, see Ref. [35].

from Table 4 calculated with the full set of configurations. In particular, the 4_g^+ state wavefunction is exactly the one that produces the ground band B(E2) values given in Table 3. Next, we calculate the $B(E2, 6_i^+ \rightarrow 4_g^+)$ with the γ -band configuration removed from the model space. This is done by excluding the projected $K = 2$ state of the 0-qp vacuum. Without the γ -configuration, the value of $B(E2, 6_i^+ \rightarrow 4_g^+)$ decreases by six orders of magnitude (see Table 5) for ^{178}W . In contrast, if we remove all other ‘irrelevant’ configurations, leaving only the three ‘relevant’ configurations, 2-qp $K = 6$, 0-qp $K = 2$, and 0-qp $K = 0$, in the model space, the order of magnitude of the calculated $B(E2, 6_i^+ \rightarrow 4_g^+)$ values remains unchanged. Therefore, the accidental proximity of the $I = 6$ state of the γ band to the 6^+ isomer could explain the observed large $E2$ inter-transition rates in ^{178}W . Similar results can be found in ^{172}Er , where the removal of the γ -configuration leads to a decrease of the inter-band B(E2) value by three orders of magnitude.

In ^{176}Hf and ^{174}Yb , the 6^+ states are much further apart. For these two isotones, the calculated $B(E2, 6_i^+ \rightarrow 4_g^+)$ values without the γ -configuration are also listed in Table 5. For ^{176}Hf , the absence of the γ -configuration also leads to a decrease in $B(E2, 6_i^+ \rightarrow 4_g^+)$ but the effect is smaller than in the case of ^{178}W . Almost no effect is seen in ^{174}Yb .

3.2. Comparison with γ -tunneling calculations

Previously, γ -tunneling calculations were performed to study K -isomer decays in this mass region [13, 35, 36]. These calculations reproduced correctly the K -forbidden $B(E2)$ trend along the isotonic chain. It is interesting to directly compare the results in Refs. [13, 35, 36] with those of the TPSM presented in Table 4. Table 6 summarizes the comparison. As can be seen, for the three heavier isotones, the two models are in good agreement for the isomer decay with differences of less than a factor of three in the $B(E2, 6_i \rightarrow 4_g)$ values. This is rather surprising given the fact that the two models are very different. Nevertheless, a clear difference between the calculated $B(E2, 6_i \rightarrow 4_g)$ values from the two models is seen for ^{172}Er . In the γ -tunneling model, the mixing probability between the isomer and members of the ground-state band is calculated through the transmission coefficient, $T = [1 + \exp(2W/\hbar)]^{-1}$ controlling tunneling between the high- K ($\gamma = -120^\circ$) and the low- K ($\gamma \approx 0^\circ$) minima of the potential energy surface, given by Nilsson-Strutinsky calculations. Here the quantity W is the so-called WKB action, calculated semiclassically. For the $N = 104$ isotones, the values of W reduce considerably as Z changes from 68 to 74, consistent with the potential energy surface becoming softer in the γ -direction, i.e., the barrier height between the two minima becomes lower [13, 35, 36]. This trend is well known and is seen also through the energy of γ -vibration, $E(2_\gamma^+)$, which decreases with increasing Z for the isotones under discussion, except for ^{172}Er . As we have already pointed out, the coupling through the γ -band configuration is crucial for the present TPSM calculations, where the lowering of the γ -band energy makes the isomer-decay probabilities increase rapidly. We conclude that the γ degree of freedom plays a similarly important role in both models although the treatment by the TPSM is fully microscopic and that in the γ -tunneling model is semiclassical.

3.3. Effective mixing matrix elements

It is useful to put the discussion above into the context of two-state mixing involving isomers and the implied interaction strength [37]. Such (very small) interactions are known empirically (see, for example Ref. [37]) and a theoretical evaluation in the framework of the PSM has recently been carried out for the case of ^{171}Tm [38]. On this general basis, we proceed below to extract the effective interaction matrix element V contained implicitly in the present model.

The two relevant levels (the $K^\pi = 6^+$ isomer and the $I = 6$ member of the γ -band) can be treated as a two-level system. Using the two-level-mixing approximation applied in Ref. [37], the wavefunction of the 6^+ isomer can be written as

$$|\psi\rangle = \alpha |6_i^+\rangle + \beta |6_\gamma^+\rangle, \quad (6)$$

where $|6_i^+\rangle$ and $|6_\gamma^+\rangle$ are the pure configurations in (2), obtained respectively from projection of the lowest 2-qp state onto $I = 6$ and $K = 6$, as well as the 0-qp state onto

Table 7. The extracted mixing matrix element V between the $K^\pi = 6^+$ isomer and the $I^\pi = 6^+$ member of the γ - band.

	^{172}Er	^{174}Yb	^{176}Hf	^{178}W
$V(\text{eV})$	170	231	506	482

$I = 6$ and $K = 2$. The amplitude β is defined [37] through the relative $E2$ strengths

$$\beta^2 = \frac{B(E2, 6_i^+ \rightarrow 4_\gamma^+)}{B(E2, 6_\gamma^+ \rightarrow 4_\gamma^+)}, \quad (7)$$

and $\alpha^2 + \beta^2 = 1$. The mixing matrix element is given by

$$V = \beta\alpha\Delta E \quad (8)$$

where ΔE is the energy separation between the two levels.

The calculated values of V are shown in Table 7, in which all quantities used to express V in Eq. (8) are theoretical. From the table, one can see that although the values differ somewhat, they are all of the same order of magnitude. Therefore, it can be concluded that the observed different orders of the 6^+ isomer transition in these $N = 104$ isotones originate from mixing matrix elements of similar magnitudes, being several hundred eV in each case. The factor governing the differences in observed rates in the isotones is the difference in separation between the $K^\pi = 6^+$ isomer and the $I = 6$ member of the γ band in each case. The values of V found in the present work lie between the known examples of very small values (a few eV in Refs. [37, 39]) and the tens of keV found in Ref. [40] for high-spin states. As discussed in Ref. [37] these interactions are approximately correlated with ΔK , with $\Delta K = 4$ values deduced from specific state mixing of a few keV being expected.

A further question relates to the dependence of the mixing matrix element V on rotation. Although not the focus of the present work, it is appropriate to evaluate the behavior, at least theoretically. As often discussed in the literature, the two leading mechanisms in nuclei that lead to K -violation are triaxiality and the rotation-induced Coriolis force. Earlier we discussed the role played by triaxiality of the nuclear potential. Now we see if the rotation would have any effect on the discussed mixing between the isomer band and the γ band. The results of a calculation of V for different angular momenta is shown in Fig. 6. In all four isotones V is predicted to increase with rotation. For ^{178}W , for example, the interaction climbs from about 600 eV at $I = 6$ to about 7200 eV at $I = 12$. This suggests that the mixing between the states of the 6^+ isomer band and those of the γ band could be significantly enhanced by rotation, although decays out of the band are unlikely to compete with in-band decays. Rotation in the projection theory is microscopically described by the angular-momentum-projection operator, which has an effect similar to that of the Coriolis force in semi-classical models.

Figure 6. (Color online) Calculated mixing matrix element V between members of the $K^\pi = 6^+$ isomer band and the corresponding member of the γ -band as a function of angular momentum.

4. Summary and conclusions

In summary, K -forbidden $E2$ transitions from the $K^\pi = 6^+$ isomers in the four $N = 104$ isotones, ^{172}Er , ^{174}Yb , ^{176}Hf and ^{178}W , have been studied using the triaxial projected shell model. Mixing of the γ -vibrational configuration with the 6^+ isomer is shown to be important in accounting for the inter-band $E2$ transitions, whose $B(E2)$ values are reasonably well reproduced. The large variation of the $B(E2, 6_i^+ \rightarrow 4_g^+)$ values across the $N = 104$ isotones originates from mixing matrix elements of the same order of magnitude ($V \sim 10^2$ eV), but with different energy separations of the two levels. This is an example where the three basic modes of rotational, vibrational, and quasiparticle excitation coexist and interact in the low-energy nuclear region.

A more sensitive measurement for ^{172}Er , for which only a limit exists at present, is needed to define properly the isotonic dependence of the $E2$ transition strengths from the 6^+ isomer to the ground-state band and possibly differentiate between the alternative model approaches.

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