EXIT Chart Analysis of Space-Time Turbo Equalizer

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Abstract — In this paper, we study the performance of the turbo equalization schemes for systems with space-time block code (STBC) using the extrinsic information transfer (EXIT) chart, which is shown to be very useful for analyzing the convergence behavior of turbo equalizers, predicting the expected BER performance of the STBC coded systems, determining the SNR threshold for a target BER, as well as facilitating the proper choice of equalizers and channel codes for specific channel conditions.

I. INTRODUCTION

Most radio communication systems suffer from the inter-symbol interference (ISI) induced by frequency-selective fading channels. The received signal includes multiple delayed and attenuated versions of the transmitted waveform. By extending the idea of iterative decoding that was used to decode turbo codes [1], a method for joint equalization and decoding (also called turbo equalization) was proposed, e.g., in [2], where soft-input, soft-output equalizer and channel decoder are integrated in an iterative manner. Turbo equalization has been shown to provide an effective mean to combat the effect of ISI.

Convergence behavior of iterative decoding schemes has been investigated in several papers. The extrinsic information transfer characteristics based on mutual information to describe the flow of extrinsic information through constituent decoders was introduced in [3] for the analysis of iterative decoders for parallel concatenated codes. The system components are characterized by their extrinsic information transfer (EXIT) functions, which describe the output mutual information as a function of the input mutual information. The exchange of extrinsic information between the decoders is visualized as a decoding trajectory in the EXIT chart. Simulation results suggest that EXIT chart accurately predicts the convergence behavior of the iterative decoder for large interleaving depth, and mutual information has been shown to predict the convergence behavior better than a variety of other measures. With EXIT chart, no resource-intensive BER simulations of the iterative decoding scheme itself are required, only simulations of individual constituent decoders are needed to obtain the desired transfer characteristics. The same technique is extended to the analysis of turbo equalizer in [2,4,5] and turbo multiuser detectors in [6].

Space-time coding techniques employ multiple antennas at both transmitter and receiver. They combine antenna array signal processing and channel coding technique, and

provide significant capacity gains over traditional single antenna systems in wireless fading channels. Space-time codes mainly fall into two categories, i.e., space-time trellis codes (STTCs) [7] and space-time block codes (STBCs) [8,9]. The former achieves both diversity and coding gain, but is complex to design and to decode since maximum likelihood sequence detection is required. On the other hand, the latter achieves full diversity as the STTC although they do not provide any coding gain, but can be easily decoded with maximum likelihood decoding via linear signal processing at the receiver. For the purpose of this work, we consider the use of STBC, in particular, the two-antenna transmit diversity scheme [8] proposed by Alamouti. In [10], the STBC originally developed for flat fading channels is generalized for frequency selective channels, where the transmitted signals are coded on a block-to-block basis rather than a symbol-by-symbol basis. This is so-called time-reversal (TR) STBC. However, TR-STBC only converts a multiple-input, multiple-output (MIMO) channel into a single-input, single-output (SISO) channel, it does not, however, eliminate ISI to which conventional SISO equalization has to be applied. Another alternative to combat ISI in a STBC coded system is space-time equalization which integrates the space-time decoding and equalization. In [11], we took the integrated approach and developed a space-time turbo equalizer under the constraint of linear-processing at the receiver. It is shown to be superior to the TR-STBC solution. Some SISO turbo equalization schemes have been analyzed, e.g. in [2,4,5] using EXIT chart. However, the analysis of space-time turbo equalizers is not yet available in the literature, and it will be the focus of this study.

II. SYSTEM MODEL AND TURBO EQUALIZATION SCHEMES

Fig. 1 shows the baseband representation of the two branch transmit diversity scheme under study. The information sequence $\{b_n\}$ is convolutionally encoded into coded bits $\{u_n\}$, which are subsequently interleaved and each block of two coded and interleaved bits $u'_n[0], u'_n[1]$ is mapped into one of the four QPSK symbols $s_n = x_n^0 + jx_n^1$. The interleaver and deinterleaver are denoted as Π and Π^{-1} , in Fig. 1, 2, and 3. We use the space-time coding scheme proposed in [8]. The transmitted symbols are grouped into blocks of 2 symbols at each antenna. At a given time, two symbols are simultaneously transmitted from two antennas. At time instance t , the symbol transmitted from the first antenna is denoted as s_n^0 , and the symbol transmitted from the second antenna is denoted as s_n^1 . During the next symbol period $t + T$, symbol $-s_n^{1*}$ is transmitted from the first antenna, and s_n^{0*} is transmitted from the second antenna,

Figure 1: Diagram for the STBC coded system with 2TX-2RX antennas.

Figure 2: Diagram for the turbo equalization scheme.

where T is the symbol duration and $*$ denotes the conjugate transpose operation when applied to vectors and matrices, and simply the complex conjugate when applied to scalars.

The task of the receiver is to detect the transmitted information bits $\{b_n\}$ given the received observations $\{r_n, y_n\}$. To this end, we need first to detect the transmitted QPSK symbols $\{s_n^0, s_n^1\}$ which are corrupted with ISI and AWGN noise. An equalizer is required to remove the detrimental effect of ISI. The estimated symbols are then converted to coded bits, which are subsequently deinterleaved and decoded to obtain an estimate of the information sequence. Here, we focus on the space-time turbo equalization algorithm which combines space-time decoding, equalizer and channel decoder in an iterative fashion.

The space-time turbo equalizer structure is illustrated in Fig. 2. First, we use a training sequence to acquire a channel estimate $\hat{\mathbf{H}}$ using some channel estimation algorithm. In the meantime, a modified Alamouti algorithm is used to obtain the soft values of the transmitted symbols in the form of log-likelihood ratio (LLR) $\{\lambda(s_n) = \lambda(x_n^0) + j\lambda(x_n^1)\}$ where s_n denotes either s_n^0 or s_n^1 . The channel estimate $\hat{\mathbf{H}}$ and symbol estimates $\{\lambda(s_n)\}$ are passed to the equalizer, which computes $\{\tilde{s}_n\}$, the soft decision of s_n . The soft estimate of the symbol is then mapped to the LLR values of coded bits $\{\lambda(u'_n; O)\}$ by the symbol-to-bit converter (SBC), which are deinterleaved to yield $\{\lambda(u_n; I)\}$. Based on the soft inputs, a Log-MAP decoder computes the LLR for each information bit $\lambda(b_n; O)$ and each coded bit $\lambda(u_n; O)$. The former is used to make decisions on the transmitted information bit at the final iteration, and $\lambda(u_n; O)$ is interleaved and passed through a bit-to-symbol converter (BSC) to derive a soft symbol estimate $\lambda(s_n)$, which is used for equalization at the next iteration. We use the notations $\lambda(\cdot; I)$ and $\lambda(\cdot; O)$ to denote the input and output ports of a soft-input and soft-output device. Refer to [11] for detailed description of this space-time turbo equalization algorithm.

III. EXIT ANALYSIS OF TURBO EQUALIZATION

The idea of EXIT chart analysis is to predict the behavior of

an iterative algorithm by solely looking at the input/output relations of individual constituent decoders. The receiver components are modeled as devices mapping a sequence of observations and the a priori information L_i to a new sequence of extrinsic information L_o .

We denote $I_i \in [0, 1]$ (I_i^E for equalizer; I_i^D for decoder) and $I_o \in [0, 1]$ (I_o^E for equalizer; I_o^D for decoder) as the mutual information measures of the input and output LLRs, respectively. The priori information L_i is usually modeled as Gaussian random variable with PDF

$$f_{L_i}(l|x) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(l - x\sigma_i^2/2)^2}{2\sigma_i^2}\right), \quad (1)$$

i.e., $L_i \sim \mathcal{N}(\frac{1}{2}\sigma_i^2, \sigma_i^2)$. The mutual information I_i is essentially a function of the single parameter σ_i [3, 6]. For a range of values of σ_i , a histogram of the output LLRs $P_{L_o}(l|x)$ is generated based on extensive simulations. The PDF of the random variable L_o of output LLRs is then estimated for the equalizer and decoder from this histogram. Mutual information I_i and I_o can then be computed numerically as [4]

$$I_i = \frac{1}{2} \sum_{x \in \{\pm 1\}} \int_{-\infty}^{\infty} f_{L_i}(l|x) \log_2 \frac{2f_{L_i}(l|x)}{f_{L_i}(l+1) + f_{L_i}(l-1)} dl$$

$$I_o = \frac{1}{2} \sum_{x \in \{\pm 1\}} \sum_l \log_2 \frac{2P_{L_o}(l|x)}{P_{L_o}(l+1) + P_{L_o}(l-1)}. \quad (2)$$

Fig. 3 shows the mechanism of the EXIT chart analysis for the turbo equalization algorithms under investigation. The extrinsic information transfer (EXIT) chart traces the evolution of the mutual information I_i^E/I_o^E between input/output LLRs and x_n (stands for either x_n^0 or x_n^1) for equalization; and the mutual information I_i^D/I_o^D between input/output LLRs and u_n for decoding. The random variable x in Equation (1) stands for either x_n or u_n . The output LLRs of the equalizer I_o^E are forwarded to the decoder as input, i.e., $I_i^D = I_o^E$; the output LLRs of the decoder I_o^D are fed back to the equalizer, i.e., $I_i^E = I_o^D$, and so on. Note that the blocks SBC, BSC, Π , and Π^{-1} do not alter mutual information.

Figure 3: Model for EXIT analysis of turbo equalization algorithm.

IV. RESULTS OF EXIT ANALYSIS

Results of EXIT analysis are presented in this section to assess the performance of the space-time turbo equalization schemes. We employ a rate 1/3 Maximum Free Distance convolutional code with constraint length $L_c = 5$ and generator polynomials (25, 33, 37) in octal form. During each Monte-Carlo run, the block size is set to 12000 information bits followed by 4 tails bits to terminate the trellis, which corresponds to $12004 \times 3 = 36012$ coded bits or 18006 QPSK symbols. The coded bits are interleaved by a random interleaver. QPSK symbols are transmitted over the MIMO ISI channels. Alamouti's STBC with two transmit and two receive antennas is employed. For simplicity, each channel is assumed to have 3 taps. However, the extension of analysis to generic ISI channels is straightforward. The setting of channel coefficients is given in Table 1. The channel coefficients are normalized such that $\sum |h_{i,j}^l|^2 = 1.0$, where the summation is carried out over all possible values of $i, j \in \{0, 1\}$ and $l \in \{0, 1, 2\}$.

We analyze the low complexity turbo equalizers for space-time block coded systems introduced in [11]. They are based on the Alamouti algorithm, which has been modified in such way that both spatial and temporal diversities are exploited and the effect of ISI is mitigated. The basic idea is to apply soft interference cancellation technique to cancel the contribution of the ISI before applying the Alamouti detection scheme. In addition to the direct implementation of the Alamouti detection on the ISI canceled signals, we can also apply the Alamouti detection on the two strongest or all the three taps and combine desired signals from different taps in order to take advantage of multipath propagation and obtain diversity gain. We call these turbo equalization schemes as 1-path, 2-path and 3-path scheme, respectively.

Different turbo equalizers are compared in Fig. 4. The BER performance of each scheme is plotted as a function of E_b/N_0 , where E_b refers to the received bit energy. As shown in Fig. 4, off all the discussed strategies, the 3-path combining scheme achieves the best results; whereas the 1-path scheme has the worst performance. The 2-path scheme lies somewhere in between. This is also verified by the extrinsic information transfer characteristics at 3.5 dB for different turbo equalizers shown in Fig. 5. The 3-path equalizer provides the best mutual information, and the 1-path is the worst among the 3 schemes. Obviously, in order to fully achieve the temporal diversity from the multipath propagation, we need to combine the signals from all the paths. As indicated by the transfer curves, the output LLR becomes more reliable (the value of I_o^E/I_o^D increases) as the

Figure 4: Comparison of different turbo equalization after convergence.

Figure 5: Extrinsic information transfer characteristics of different turbo equalizers and Log-MAP decoder.

input LLR becomes more reliable in the equalizer/decoder (bigger a priori input leads to bigger extrinsic output).

Fig. 4 shows the results at the 4th stage since it is observed that only 4 stages are needed for the turbo equalizers to converge. This is verified by the BER plots shown in Fig. 6 for the 3-path turbo equalizer. We also noticed that the higher SNR, the more gain is obtained by the iterative equalization and decoding process.

The EXIT chart can be used to obtain an estimate of the BER performance. Fig. 7 plots the BER as a function of I_i^D for the chosen rate 1/3 convolutional code. The curve is obtained with simulations. It gives an indication of the minimum required mutual information at the input of the channel decoder in order to achieve a target BER. For example, if the target BER is $10^{-2}/10^{-3}/10^{-4}$, then the mutual information I_i^D has to be no less than 0.47/0.56/0.65. By observing the intersection point at which the equalizer

Table 1: The setting of 2Tx-2Rx channel coefficients.

	Tx0-Rx0	Tx1-Rx0	Tx0-Rx1	Tx1-Rx1
tap 1	$-0.1390 + 0.5060i$	$-0.2456 + 0.0051i$	$0.0669 + 0.0936i$	$0.0308 + 0.3451i$
tap 2	$0.2387 - 0.1335i$	$-0.0977 + 0.2637i$	$-0.1271 - 0.1605i$	$0.3558 + 0.0936i$
tap 3	$-0.0151 + 0.0280i$	$0.0746 - 0.0617i$	$0.3157 + 0.1003i$	$0.1672 - 0.2274i$

Figure 6: Performance of 4 stages turbo equalization with 3-path combining.

and the decoder transfer functions cross, the BERs after convergence can be predicted. It is also shown in Fig. 5 that the 2-path equalizer and decoder transfer curves intersect at a I_i^D value slightly higher 0.47; the 3-path equalizer and decoder transfer curves intersect at a I_i^D value around 0.56. Therefore, it can be predicted that the 3-path equalizer would achieve $BER \approx 10^{-3}$ at 3.5 dB after convergence, which is validated by Fig. 6. We notice that the BER estimates are quite close to the simulation results even for finite block length.

The convergence behavior of the 3-path equalizer is investigated at $E_b/N_0 = 3.5$ dB in Fig. 8 which shows the trace of the evolution of the mutual information. Noted that in this case, $I_i^E > 0$ at the beginning of the iterative process. This is due to the presence of the initial Alamouti detection stage (see the lower left corner in Fig. 2). Therefore, the a priori information is not zero when the turbo equalization starts. In Fig. 8, the iterative equalization and decoding process is depicted by a staircase trace between the transfer curves of the equalizer and decoder. The expected trace is shown in this figure. The real trace slightly deviates the predicted one due to the finite interleaver length. However, the discrepancy is small. The trace shows that only 3 stages of equalization/decoding are needed for the system to converge (reach the maximum I_i^D) at $E_b/N_0 = 3.5$ dB. (Note that the third stage equalization and decoding at the end of the trace is hardly visible as the diagram gets too cramped). This concurs with the BER results shown in Fig. 6.

In addition to being able to determine the number of iterations needed for a system to reach convergence as shown above, the EXIT chart can also be used to determine the

 Figure 7: BER vs. I_i^D for the log-MAP decoder. $I_i^D = 0.47 \rightarrow BER = 10^{-2}$, $I_i^D = 0.56 \rightarrow BER = 10^{-3}$, $I_i^D = 0.65 \rightarrow BER = 10^{-4}$.

 Figure 8: Exit chart analysis for 3-path turbo equalization at $E_b/N_0 = 3.5$ dB. In the plot, $\rightarrow$ represents the decoding process and $\uparrow$ represents the equalization process.

Figure 9: Extrinsic information transfer characteristics of the 3-path turbo equalizer at different SNRs.

SNR threshold for a target BER. Fig. 9 shows the transfer curves of the 3-path equalizer at different SNR levels. As mentioned earlier, the intersection point where the equalizer and decoder transfer curves meet determines the BER performance upon convergence. We see that if E_b/N_0 goes to 3.5 dB, there will be a tunnel between the transfer curve of the equalizer and that of the decoder leading to $I_o^D \geq 0.56$ at the intersection point. However, once E_b/N_0 is lower than this threshold, the system cannot find a trajectory leading to $I_o^D \geq 0.56$, and the target BER cannot be achieved. This coincides with the results shown in Fig. 6.

The EXIT chart also helps us choose a suitable channel code for the system. Fig. 10 shows the transfer curves of different convolutional codes. In fact, the transfer curves of all the rate 1/3 code with generator polynomials (5, 7, 7), (13, 15, 17), (25, 33, 37) intersect with the transfer curve of the 3-path equalizer almost at the same point. We therefore would expect a similar performance upon convergence regardless which one of them is used. Using the code with constraint length $L_c = 5$ is kind of overkill. It is also clear to see that the two rate 1/2 codes yield worse performance than the rate 1/3 codes if used in conjunction with the 3-path equalizer.

V. CONCLUSIONS

In this paper, we apply the EXIT chart to the analysis of the space-time turbo equalization schemes. In addition to its ability to accurately predict the convergence behavior of the system under study without running intensive simulations, we have also shown that EXIT chart facilitates the choice of equalizer and channel code and helps us determine the SNR threshold for a target BER and the number of iterations needed to reach convergence. It is therefore a very useful tool for designing an adaptive system which optimizes coding, equalization schemes, as well as other system parameters accordingly as channel condition varies.

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Figure 10: Transfer characteristics of different convolutional codes.

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