Influence of the prediction model complexity on the performance of model predictive anti-jerk control for on-board electric powertrains

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Abstract. Anti-jerk controllers compensate for the torsional oscillations of automotive drivetrains, caused by swift variations of the traction torque. In the literature model predictive control (MPC) technology has been applied to anti-jerk control problems, by using a variety of prediction models. However, an analysis of the influence of the prediction model complexity on anti-jerk control performance is still missing. To cover the gap, this study proposes six anti-jerk MPC formulations, which are based on different prediction models and are fine-tuned through a unified optimization routine. Their performance is assessed over multiple tip-in and tip-out maneuvers by means of an objective indicator. Results show that: i) low number of prediction steps and short discretization time provide the best performance in the considered nominal tip-in test; ii) the consideration of the drivetrain backlash in the prediction model is beneficial in all test cases; iii) the inclusion of tire slip formulations makes the system more robust with respect to vehicle speed variations and enhances the vehicle behavior in tip-out tests; however, it deteriorates performance in the other scenarios; and iv) the inclusion of a simplified tire relaxation formulation does not bring any particular benefit.

Keywords: Model predictive control, anti-jerk control, electric vehicle, on-board powertrain.

1 Introduction

Swift torque demand variations imposed by the driver or automated driving system excite the half-shaft dynamics and cause torsional drivetrain oscillations in vehicles with on-board powertrains [1] [2]. These torsional oscillations are transmitted to the vehicle body through the tire-road interaction, and thus provoke longitudinal acceleration oscillations and passenger discomfort. Anti-jerk controllers modify the driver torque demand, T_{ref} , to attenuate this behavior.

Model predictive anti-jerk controllers have been proposed in the literature, with prediction models ranging from two-inertia linear models [3] [4] to more complex formulations including backlash [5], tire relaxation [6] [7], and nonlinear tire characteristics

[7]. The prediction model should effectively capture the drivetrain dynamics. At the same time, a model with low number of states and parameters facilitates controller development and real-time implementation. However, to the best of our knowledge, the literature lacks an analysis of the effect of the prediction model on anti-jerk control system performance. To cover the gap, this paper adopts a unified tuning procedure and an objective performance indicator to assess the influence of different prediction models, which are embedded in anti-jerk MPC implementations for electric vehicles with multiple on-board motors.

The remainder is organized as follows: Section 2 describes the plant model for control system assessment; Sections 3 and 4 deal with the controller prediction models and MPC formulations; Section 5 discusses the results; and Section 6 summarizes the main conclusions.

Table 1. Main vehicle parameters.

Parameter	Value	Parameter	Value
M_v vehicle mass	2140 kg	$a_{x,1}$ relaxation model coeff.	0.251 m
J_w wheel inertia	$2.72 \text{ kg} \cdot \text{m}^2$	$a_{x,2}$ relaxation model coeff.	0.104 m
J_m motor inertia	$0.1 \text{ kg} \cdot \text{m}^2$	$a_{x,3}$ relaxation model coeff.	110
K_d drivetrain stiffness	$22600 \text{ N} \cdot \text{m}/\text{rad}$	$a_{x,4}$ relaxation model coeff.	-26
C_d drivetrain damping coeff.	$90 \text{ N} \cdot \text{m} \cdot \text{s}/\text{rad}$	a relaxation model coeff.	$4.04\text{e-}4 \text{ m} \cdot \text{N}^{-1}$
R_w wheel radius	0.36 m	b relaxation model coeff.	-0.018 m
i gear ratio	9.73	$F_{z,0}$ relaxation model coeff.	5200 N
θ_{bl} drivetrain play measured 2 deg at the wheel			

2 Plant model for control system assessment

The plant is a front-wheel-drive electric vehicle with two on-board motors, each one connected to the respective wheel through a single-speed transmission and a half-shaft. Table 1 shows the main vehicle parameters. Tire dynamics are modeled with the Pacejka magic formula, coupled with the relaxation formulation in [8], which has been modified to generate plausible results at low vehicle speed. The relaxation length, σ_k , is a function of the vertical tire load, F_z , and longitudinal slip ratio, k :

$$\sigma_k = (\sigma_{k,0} - X_1) \exp(-X_2|k|) + X_1 \quad (1)$$

$$X_1 = a_{x,1} + a_{x,2}[(F_z - F_{z,0})/F_{z,0}] \quad (2)$$

$$X_2 = a_{x,3} + a_{x,4}[(F_z - F_{z,0})/F_{z,0}] \quad (3)$$

$$\sigma_{k,0} = a F_z + b \quad (4)$$

where $\sigma_{k,0}$ is a function of F_z ; and $a_{x,1-4}$, a , b and $F_{z,0}$ are the relaxation model coefficients. The relaxed slip ratio, k' , is calculated as:

$$\dot{k}' = (v/\sigma_k)k - \text{sat}_\varepsilon^{inf}[(\dot{\sigma}_k + v)/\sigma_k]k' \quad (5)$$

where v is the vehicle speed. The saturation function ensures that the term $(\dot{\sigma}_k + v)/\sigma_k$ is larger than a small positive threshold, ε , and therefore prevents the system time constant from becoming negative.

3 Prediction models

Fig. 1 shows the considered prediction model layouts. Each of them is identified by the letter M, with a subscript indicating the number of states, and a superscript, BL, indicating whether backlash is considered.

Fig. 1. Prediction model layouts.

The assessed prediction models are:

- M_3 , adopted in [3] and [4], which has two mass moments of inertia: i) the motor inertia referred to the wheel, $J_{m,w}$; and ii) the equivalent inertia corresponding to half of the vehicle and wheels, J_{eq} . Its states are: i) the electric motor speed referred to the wheels, i.e., at the transmission output, $\dot{\theta}_{m,w}$; ii) the driven wheel speed, $\dot{\theta}_w$; and iii) the torsion angle of the drivetrain, $\Delta\theta = \theta_{m,w} - \theta_w$. M_3 is described by:

$$\ddot{\theta}_{m,w} = (T_{m,w} - T_{hs})/J_{m,w} \quad (6)$$

$$\ddot{\theta}_w = (T_{hs} - 0.5 T_{aero} - 0.5 T_{roll})/J_{eq} \quad (7)$$

$$T_{hs} = K_d \Delta\theta + C_d \Delta\dot{\theta} \quad (8)$$

$$T_{aero} = 0.5 \rho S_v C_x (\dot{\theta}_w R_w)^2 R_w \quad (9)$$

$$T_{roll} = [f_0 M_v g + f_2 M_v g (\dot{\theta}_w R_w)^2] R_w \quad (10)$$

where $T_{m,w}$ is the motor torque referred to the transmission output; T_{hs} is the half-shaft torque; T_{aero} and T_{roll} are the aerodynamic drag and rolling resistance torques; K_d and C_d are the torsional drivetrain stiffness and damping coefficient; ρ is air

density; S_v is the frontal area of the vehicle; C_x is the aerodynamic drag coefficient; R_w is the wheel radius; f_0 and f_2 are the tire rolling resistance coefficients; M_v is the vehicle mass; and g is the gravitational acceleration.

- M₄, which includes a linear tire model without relaxation. M₄ has four states, i.e., $\dot{\theta}_{m,w}$, $\dot{\theta}_w$, $\Delta\theta$ and v , and is described by (6), (8), (10) and (11)-(14):

$$\ddot{\theta}_w = (T_{hs} - F_t R_w - 0.25 T_{roll})/J_w \quad (11)$$

$$F_t = B_x k \quad (12)$$

$$\dot{v} = [2F_t - (T_{aero} + 0.5 T_{roll})/R_w]/M_v \quad (13)$$

$$T_{aero} = 0.5 \rho S_v C_x v^2 R_w \quad (14)$$

where F_t is the longitudinal tire force, and B_x is the longitudinal tire slip stiffness, which is considered constant.

- M₅, from [6], which includes tire relaxation, and is described by the same equations as M₄, except for (12), which is replaced by (15) and (16):

$$F_t = B_x k' \quad (15)$$

$$\dot{k}' = (k - k') v / \sigma_k \quad (16)$$

where the relaxed slip, k' , is the fifth state of the model, and σ_k is constant.

- M₃^{BL}, M₄^{BL} and M₅^{BL} have the same sets of equations as M₃, M₄ and M₅, except for (8), which is replaced by (17):

$$T_{hs} = \frac{1}{2} K_d (\Delta\theta - \theta_{bl}/2) \{ \tanh[K_1 (\Delta\theta - K_2)] + 1 \} + \frac{1}{2} K_d (\Delta\theta + \theta_{bl}/2) \{ \tanh[-K_1 (\Delta\theta + K_2)] + 1 \} + C_d \Delta\dot{\theta} \quad (17)$$

where θ_{bl} is the nominal backlash, measured at the wheel, and K_1 and K_2 are constants.

The inputs variables – either measured or estimated – required by M₃ and M₃^{BL} are $\dot{\theta}_{m,w}$, $\dot{\theta}_w$ and $\Delta\theta$; M₄, M₅, M₄^{BL} and M₅^{BL} also require v . In the remainder, the controllers will be named after their prediction model, e.g., “controller C₃” will refer to the MPC implementation based on M₃.

4 Optimal controller design

The MPC cost function, V , is designed to reduce: i) the drivetrain torsion rate, $\Delta\dot{\theta}$, which is the root cause of vehicle jerk [9]; and ii) the anti-jerk torque correction, T_{corr} , which is the control action:

$$V(t_k) = \int_{t_k}^{t_k + N_p H} [q \Delta\dot{\theta}(t)^2 + r T_{corr}(t)^2] dt \quad (18)$$

where t_k is the current time step; N_p is the number of prediction steps; H is the discretization time; q and r are weights; and t is time. T_{corr} is defined as $T_{corr} = T_{ref} - T_{plant}$, where T_{ref} , which is considered constant along the prediction horizon, is the motor torque demand before the correction imposed by the anti-jerk controller, and T_{plant} is the corrected torque demand applied to the plant.

Constraints are incorporated in the optimal control problem formulation to account for the torque and power limitations of the electric machines. The software library IPOPT, based on a primal dual interior point algorithm with a filter line search method [10], has been used to solve the nonlinear programming problem.

The values of q and r have significant influence on the anti-jerk control performance. In this study they were tuned to minimize an objective performance index, J , along a tip-in maneuver simulated with the model in section 2, from an initial vehicle speed $v_0 = 10$ km/h, and with a final value of motor torque demand $T_{tip-in} = 50$ Nm on each front corner. This maneuver will be referred to as the “nominal case” in the remainder. q and r were optimized for $H = 1, 5$ and 10 ms, and $N_p = 2, 4, 6, 8$ and 10 , which corresponds to 15 control tunings for each prediction model, and a total of 90 anti-jerk controller configurations evaluated in the analysis. The controller sampling time, T_s , was imposed equal to H , while the number of control steps was set to 2. The *fmincon* function of Matlab with the interior point algorithm was used to minimize J .

J is defined as:

$$J = W_1 VDV_{a_x^*} + W_2 RMS_{a_x^*} + W_3 \Delta_{v_x} + W_4 \Delta_{t,a} + W_5 RMS_{T_{corr}} + W_6 RMS_{T_{corr}SS} \quad (19)$$

where W_{1-6} are constant weights, selected through a trial-and-error process. The terms in (19) are drivability performance indicators:

- The vibration dose value, $VDV_{a_x^*}$, and the root-mean square value of the longitudinal vehicle acceleration, $RMS_{a_x^*}$, which evaluate the comfort level:

$$VDV_{a_x^*} = \sqrt[4]{\int_{T_1}^{T_2} a_x^{*4} dt} \quad (20)$$

$$RMS_{a_x^*} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} a_x^{*2} dt} \quad (21)$$

where T_1 and T_2 define the beginning and the end of the relevant part of the tip-in or tip-out test ($T_2 - T_1 = 0.5$ s in this study); a_x^* is the high-frequency component of the longitudinal vehicle acceleration, filtered with a high-pass Butterworth filter with a cut-off frequency of 8 Hz.

- Δ_{v_x} , i.e., the absolute value of the difference between the vehicle speeds without and with anti-jerk controller at the end of the tip-in. Δ_{v_x} measures the degradation of the longitudinal acceleration performance caused by the controller.
- $\Delta_{t,a}$, i.e., the pure time delay between T_1 , the time at which the torque demand variation is triggered, and the time at which a reference vehicle acceleration, a_{ref} , is achieved. a_{ref} is defined as:

$$a_{ref} = 0.9 (2T_{tip-in} i \eta) / (M_v R_w) \quad (22)$$

where η is the drivetrain efficiency. $\Delta_{t,a}$ evaluates the vehicle responsiveness to the motor torque requests.

- $RMS_{T_{corr}}$, i.e., the root mean square value of T_{corr} , which measures the control effort:

$$RMS_{T_{corr}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} (2T_{corr})^2 dt} \quad (23)$$

where the factor 2 in (23) is introduced because of the symmetry of the front electric powertrains and tire-road friction conditions in the simulated scenarios.

- $RMS_{T_{corrSS}}$, i.e., the root mean square value of T_{corr} , calculated from 0.5 s to 0.6 s, to evaluate the steady-state control effort.

5 Results

5.1 Nominal case

Fig. 2 reports the values of J for the nominal case. Each bar chart refers to a single controller, optimized for different values of H and N_p . The cases that show persistent control action oscillations are marked by a grey spot, instead of a white bar, and their J value is omitted.

The controllers, especially C_4 , C_5 , C_4^{BL} and C_5^{BL} , tend to be unstable for high values of H . This is related to the fact that the inclusion of the fast wheel dynamics in the prediction models makes the system of differential equations stiff [7]. All controller formulations achieve their best performance for $H = 1$ ms; nonetheless, a longer step size facilitates the controller implementation on a real vehicle. With respect to the influence of the number of prediction steps, the trend qualitatively confirms the results in [7]. Interestingly, the best performance is achieved by the controllers with low N_p values, i.e., for $N_p = 2$, which poses questions on the real benefit of the MPC prediction.

The introduction of the relaxation length in C_5 does not bring any performance benefit in terms of J values, with respect to the C_4 controllers. However, for $H = 1$ ms the tire relaxation formulation in the prediction model allows stable controller operation for a higher number of prediction steps.

In Fig. 2 all formulations including backlash achieve lower J values than the corresponding implementations without backlash; this is especially evident for C_3^{BL} with $H = 10$ ms (J reduced up to 13%) and C_4^{BL} (J reduced up to 22%). In this respect, for $N_p = 2$ and $H = 10$ ms, Fig. 3 compares the torque and acceleration profiles for: i) the passive vehicle; ii) the vehicle with C_3 ($J = 9.53$); and iii) the vehicle with C_3^{BL} ($J = 8.83$). The time histories in Fig. 3 highlight the vehicle responsiveness benefits of the backlash formulation, which does not affect comfort. A second example of the benefit of introducing backlash in the prediction model formulation is in Fig. 4, showing

the smaller overshoot of $\dot{v}$ achieved by C_4^{BL} ($J = 7.53$) with respect to C_4 ($J = 8.93$), for $N_p = 2$ and $H = 1$ ms.

Fig. 2. J results for the nominal tip-in test for different controller formulations, number of prediction steps, N_p , and time step size, H .

Fig. 3. Comparison of C_3 and C_3^{BL} along the nominal tip-in test, for $N_p = 2$ and $H = 10$ ms.

Table 2 reports the values of J and its individual performance indicator contributions for $N_p = 2$ and $H = 1$ ms, which provide the best performance for all prediction models. Surprisingly, C_3 and C_3^{BL} show the best performance in terms of J . In particular, they achieve low values for the comfort indicators, $VDV_{a_x}^*$ and $RMS_{a_x}^*$, and for the steady-state torque reduction, $RMS_{T_{corr-ss}}$. However, they are characterized by reduced responsiveness, i.e., higher values of Δv_x and Δt_a , and increased control effort, i.e., higher values of $RMS_{T_{corr}}$, compared to the controllers including wheel dynamics.

Fig. 4. Comparison of C_4 and C_4^{BL} , for $N_p = 2$ and $H = 1$ ms.

Table 2. Performance indicators for the nominal tip-in test.

Controller	Performance indicators						
$N_p = 2$	$VDV_{a_x^+}$	$RMS_{a_x^+}$	Δ_{v_x}	Δ_{t_a}	$RMS_{T_{corr}}$	$RMS_{T_{corr,SS}}$	J
$H = 1 \text{ ms}$	(m/s ^{1.75})	(m/s ²)	(km/h)	(s)	(Nm)	(Nm)	(-)
C ₃	0.102	0.063	0.129	0.263	38.86	0.05	7.51
C ₃ ^{BL}	0.119	0.073	0.098	0.214	32.38	0.26	7.38
C ₄	0.232	0.142	0.033	0.093	11.95	0.88	8.93
C ₄ ^{BL}	0.186	0.108	0.049	0.103	17.33	0.39	7.53
C ₅	0.227	0.138	0.036	0.094	12.73	0.90	8.87
C ₅ ^{BL}	0.214	0.128	0.038	0.093	13.30	1.13	8.72

5.2 Sensitivity analysis

The controllers are assessed along multiple maneuvers to evaluate their robustness with respect to v_0 , T_{tip-in} and $T_{tip-out}$ (the torque demand value before the tip-out application). All formulations are tested for $N_p = 2$ and $H = 1$ ms.

Fig. 5 (a) shows the results for $v_0 = 5, 10, 15$ and 20 km/h. For $v_0 = 5$ km/h all controllers achieve higher J values than in the nominal case (i.e., for $v_0 = 10$ km/h), because low vehicle speeds induce larger drivetrain oscillations. For higher v_0 the controllers including consideration of tire slip achieve lower J values than in the nominal case, while C_3 and C_3^{BL} experience a marginal increase of J . This behavior highlights the robustness of C_4 , C_5 , C_4^{BL} and C_5^{BL} with respect to vehicle speed.

The results for different T_{tip-in} (25, 50 and 75 Nm) are in Fig. 5 (b). J increases as a function of T_{tip-in} for all controllers. Δ_{t_a} , and therefore J , cannot be computed for controller C_3 and $T_{tip-in} = 75$ Nm. In fact, in this case the vehicle acceleration does not reach a_{ref} , because of a steady-state offset of the torque demand. This behavior is particularly noticeable for C_3 , but is common to all controllers without backlash formulation, for T_{tip-in} different from the nominal value of 50 Nm. Future research could evaluate the effect of an integral action, see [11], to reduce the steady-state offset.

In the tip-out maneuvers (Fig. 5 (c)) the trends are monotone, i.e., J decreases with $T_{tip-out}$ and the performance improves with model complexity. The best result is provided by C_5^{BL} ; however, all formulations behave significantly better than the passive vehicle.

Fig. 5. J results for different values of v_0 (a), T_{tip-in} (b) and $T_{tip-out}$ (c) .

6 Conclusions

This simulation study objectively compared six model predictive anti-jerk control formulations, to assess the influence of the prediction model complexity on anti-jerk control performance in several tip-in and tip-out tests, for a case study front-wheel-drive electric vehicle with on-board motors.

The results show that:

- As also the most advanced prediction model formulation is characterized by significant mismatch in comparison with the plant, all controllers performed better with a short prediction horizon ($N_p = 2$), confirming the results from the literature for similar anti-jerk control configurations.
- The controller sampling and discretization time, H , has a significant impact on the results. As a consequence, this parameter must be carefully selected to provide the appropriate trade-off between performance and control hardware requirements for real vehicle implementation.
- The inclusion of backlash in the prediction model always reduces the defined performance index, J . This is especially evident for the controllers based on the simple two-inertia model formulation and those with a steady-state tire model. In these cases J was reduced up to 13% and 22% with respect to the corresponding configurations without drivetrain backlash.

- The controllers including tire slip behavior showed good robustness with respect to the variation of initial vehicle speed, and superior performance during tip-out tests. On the other hand, in nominal conditions the inclusion of the tire slip model was not beneficial, as it made the system stiff and provoked persistent control action oscillations for high values of H .
- The introduction of relaxation length in the prediction model does not bring any substantial performance variation. On a minor note, with $H = 1$ ms the tire relaxation model allowed stable operation of the controllers for a higher number of prediction steps.

Based on the presented results, the recommended prediction model formulation is a two-inertia model with backlash, i.e., M_3^{BL} , as it provides the best results during tip-in tests, also outside nominal conditions.

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