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VALIDATION OF ELECTRO-OPTIC ENSORS FOR MEASUREMENT OF DC FIELDS IN THE PRESENCE OF SPACE CHARGE

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Abstract – The paper presents theoretical analysis and experimental verification of an electro-optic sensor for measurements DC electric fields in space charge environment. Effects of both unipolar and bipolar charge were investigated. It was shown that in both cases the sensor output is linearly dependent on the intensity of measured electric field and independent of the space charge density. The achieved measurement resolution is 100 V/m with a dynamic range of 80 dB and a temperature stability of 0.1 %/°C for temperatures near 20 °C.

Keywords: space charge, electric field measurement, Pockel's effect, linear electro-optic effect.

I. INTRODUCTION

AC electric fields are most frequently measured using dipole probes with diodes or thermocouple square-law sensing elements [1]. For an optimal measurement, the dimension of the dipole is required to be comparable to one tenth of the wavelength of the measured field. At extra low frequencies (ELF) the dimension of an optimal dipole probe would become unrealistic, and therefore dipole probes are normally used for measurement of electric fields at frequencies above 100 MHz. Short dipole antennas are used at frequencies down to 1 kHz, but this is at the cost of reduced measurement sensitivity.

DC electric fields, however, are measured mainly using potential probes or field meters utilising the principle of electric induction [4]. In order to avoid the spurious effects, drifts, offsets and charge effects the sensor output is usually modulated by mechanically rotating or vibrating the sensing plates forming the known *rotating vane field meter* or *vibrating fork field meter*, respectively. Typically, these sensors measure electric fields between 1 and 1000 kV/m and have a high spatial resolution. However, they tend to perturb the measured field by the presence of metallic parts and by the grounding effects. Typical measurement accuracy is not better than 1 dB.

The electro-optic effect has been successfully utilised for electric field measurement at both low [5] and high frequencies [2], but mainly at the frequencies above 20 Hz. The initial DC field optical probes [6][7] utilise mechanical rotation of the sensing crystal to avoid detrementing effect of the space charge deposited on the surface. With this design, however, the concomitant sensor complexity and inaccuracy in optical alignment becomes apparent. In contrast, we have shown [4] that the measurement of the DC electric fields in the present of the space charge can be achieved by the electro-optic sensor without the need for mechanical rotation. More importantly, the presence of the space charge can be beneficially utilised to improve the measurement sensitivity.

In this paper we present a detailed analysis of the electric field measurement utilising electro-optic effect in crystals, as well as side effects that influence the measurement resolution and accuracy. The external charge effects were analysed using Laplace equation. Two sources of the internal charge were considered: the crystal conductivity and photoconductive effect. Jones calculus has been used to assess the temperature effects. The results have been experimentally verified.

II. THEORETHICAL ANALYSIS

In contrast to interferometric optical schemes, the selected polarimetric optical scheme for the sensor shown in Figure 1 offers a better compromise between the sensor sensitivity and optical complexity. Light from the light source is polarised by a polariser into a linearly polarised light beam with its plane of polarisation along the fixed x_2 axis. The quarter-wave plate, aligned with its fast axis at an angle α ($= 45^\circ$), converts the linearly polarised light from the polariser into a circularly polarised light. Consequently, both x' and y' eigen-axes of the crystal are populated regarding of their azimuth β . In order to achieve the maximum sensor sensitivity, particularly for low intensity electric fields, the analysers must be aligned at an angle of $\zeta = 45^\circ$ with respect to the crystal's eigen-axes (y'). As such, the analyser resolves the light beam into one component, and therefore modulates the intensity of the output light beam I proportionally to the applied electric field E [4].

Figure 1 Schematic diagram of the electro-optic sensor

a) *Effects Of Internal Space Charge*

Under the effect of a DC or sufficiently low-frequency electric field E , the free charge carriers in the crystal of finite resistivity starts drifting along the electric field lines and tend to accumulate on crystal boundaries. The accumulated charge generates an electric field with the opposite direction cancelling out the induced electric field E_{m0} . The speed of this process depends on the density of free charge carriers, which is proportional to crystal conductivity. Consequently, the internal macroscopic field E_m changes in time as:

$$E_m(t) = E_{m0} \cdot e^{-\frac{t}{\tau}} \quad (1)$$

where $\tau (= \epsilon_r \epsilon_0 / G)$ is the charge relaxation time, G is specific conductivity of the crystal, and ϵ_0 and ϵ_r are permittivity of vacuum and relative permittivity of the crystal, respectively. In turn, the sensor output does not correspond to the measured external electric field, but to decaying internal field E_m . It is evident, therefore, that if the crystal is not to be physically rotated, its conductivity must be the lowest possible assuring a high charge relaxation time τ . As the conductivity generally depends on the lattice defects and the bandgap of the material, this implies that the crystal must be of the high quality, impurity free, and with the large bandgap. Figure 2 shows the time relaxation time for readily available crystals indicating that only Bismuth Germanate ($\text{Bi}_4\text{Ge}_3\text{O}_{12}$) and Lithium Niobate (LiNbO_3) crystals are practically suitable for DC and ELF electric field measurement enabling the theoretical time measurement of ~ 100 s and ~ 100000 s, respectively. Practical values are somewhat lower as they depend on the crystal purity.

Figure 2 Charge relaxation time constant τ of selected crystals

Another source of free internal charge carriers inside crystals is photoconductive effect, where the excitation energy necessary to generate free carriers is provided by a flux of photons with energies $h\nu$, where ν is the frequency of the light and h is the Plank constant. The wavelength of the light at which the photon energy is equal to the bandgap energy defines intrinsic absorption edge and is usually shown as a

critical wavelength λ_c . At wavelengths $\lambda < \lambda_c$ the laser radiation is strongly absorbed by the crystal, generating additional charge, hence increasing the conductivity of the crystal. Using available data on bandgaps of the crystals of Lithium Niobate and Bismuth Germanate, we found that their critical wavelengths are 350 nm and 394 nm, respectively [8]. The critical wavelength represents the limitations not only for DC field measurement, but also for AC field measurement due to a strong absorption of the light beam. We have also experimentally established that the defects in the crystals, such as impurities and dislocations, generally increase the critical wavelength λ_c and that this increase is proportional to the level of the defects.

b) Effects Of External Space Charge

In the presence of DC electric field, the environmentally charged particles, if present, will move along the field lines; the positively charged particles in the direction of the electric field intensity and the negatively charged particles in the opposite direction. Inevitably, some of the particles will hit the surface of the dielectric probe placed in the field and will stay attached to it due to its polarisation and low surface conductivity. This process will continue until the attractive forces acting upon the charge caused by the original field are equal to the repulsive force of the deposited charge. The deposited charge, however, will generate the electric field on its own, hence altering the internal electric field in the probe, as well as measurement sensitivity [8]. Considering that the space charge contribution to the electric field is negligible, the potential of the electric field around a charged dielectric probe can be calculated using Laplace equation as:

$$\Delta V = 0 \quad (2)$$

where Δ is the Laplace operator. To better understand the charging process and also to simplify the calculation, the probe is considered as an isotropic ball of the diameter a and relative permittivity ϵ_r , immersed in a homogeneous electric field. The orientation of the electric field E and the coordinate system are defined in Figure 3. Because of the geometrical symmetry of the probe, the polar coordinate system is more suitable because the E_ϕ component of the electric field, as well as derivative $\partial V / \partial \phi$, are equal to zero. Thus the original 3-dimensional problem is reduced to 2-dimensional and the 2-dimensional Laplace equation (2) is readily solved using the method of separation of variables, as

$$V = \sum_{n=0}^{\infty} A_n r^n P_n(\cos \theta) + \sum_{n=0}^{\infty} B_n r^{-(n+1)} P_n(\cos \theta) \quad (3)$$

where A_n and B_n are coefficients satisfying boundary conditions and $P_n(\cos \theta)$ are Legendre's polynomials of n -th order with argument $\cos \theta$.

Figure 3 polar coordinate system for the ball-shaped dielectric probe

The equation (3) is a general solution of the Laplace equation (2) and it is valid for both inside and outside the dielectric ball, but not on its surface where we assume a layer of deposited charge. The solution can be found by imposing the following boundary conditions:

1. The electric field is uniform in the absence of the sphere;
2. Potential V is continuous across the boundary;
3. The normal components D_{n1} , D_{n2} of the electrical displacement vector $\mathbf{D}$ are continuous and satisfy

$$D_{n1} - D_{n2} = \sigma \text{ for } r = a, \text{ where } \sigma \text{ is a surface charge density.}$$

Here, the surface charge density acquires the same axial symmetry as the electric field and can be, therefore, expressed as a function of only one variable θ . Since the Legendre's polynomial form a complete set, the surface charge density σ can be expressed in the form of Legendre's expansion as

$$\sigma = \sum_{n=0}^{\infty} \sigma_n P_n(\cos \theta) \quad (4)$$

where σ_n are the coefficients of the Legendre's expansion. The coefficients A_n and B_n are then found using (3) and (4) and the boundary conditions. The potential outside the probe V_{out} and the potential inside the probe V_{in} are finally found to be

$$\begin{aligned} V_{out} = & \frac{\sigma_0 a^2}{r} - E_0 r \cos \theta + \frac{[\sigma_1 + E_0(\epsilon_r - 1)]a^3}{r^2(\epsilon_r + 2)} \cos \theta + \dots \\ & + \frac{\sigma_n a^{n+2}}{(\epsilon_r + 1)n + 1} \frac{1}{r^{n+1}} P_n(\cos \theta) + \dots \quad n = 2, 3, 4, \dots \end{aligned} \quad (5)$$

$$V_{in} = \sigma_0 a + \frac{\sigma_1 - 3E_0}{2 + \epsilon_r} r \cos \theta + \dots$$

$$+ \frac{\sigma_n a^{1-n}}{(\epsilon_r + 1)n+1} r^n P_n(\cos \theta) + \dots \quad n = 2, 3, 4, \dots \quad (6)$$

where E_0 is the measured electric field intensity. Since the potential is defined as a negative gradient of the electric field intensity $\mathbf{E} = -\nabla V$, the components of the electric field intensity in polar coordinates inside and outside the electric field probe are easily obtained:

$$E_{rout} = \frac{\sigma_0 a^2}{r^2} + E_0 \cos \theta + 2 \frac{[\sigma_1 + E_0(\epsilon_r - 1)]a^3}{r^3(\epsilon_r + 2)} \cos \theta +$$

$$+ (1+n) \frac{\sigma_n a^{n+2}}{(\epsilon_r + 1)n+1} \frac{1}{r^{n+2}} P_n(\cos \theta) + \dots \quad n = 2, 3, 4, \dots \quad (7)$$

$$E_{\theta out} = -E_0 \sin \theta - 2 \frac{[\sigma_1 + E_0(\epsilon_r - 1)]a^3}{\epsilon_r + 2} \frac{1}{r^3} \frac{\partial P_1}{\partial \theta} -$$

$$- \frac{\sigma_n a^{n+2}}{(\epsilon_r + 1)n+1} \frac{1}{r^{n+2}} \frac{\partial P_n(\cos \theta)}{\partial \theta} - \dots \quad n = 2, 3, 4, \dots \quad (8)$$

$$E_{rin} = -\frac{\sigma_1 - 3E_0}{2 + \epsilon_r} \cos \theta - \frac{\sigma_2}{a[2(1 + \epsilon_r) + 1]} r P_2(\cos \theta) -$$

$$- \frac{\sigma_n a^{1-n}}{(\epsilon_r + 1)n+1} r^{n-1} P_n(\cos \theta) - \dots \quad n = 3, 4, 5, \dots \quad (9)$$

$$E_{\theta in} = -\frac{\sigma_1 - 3E_0}{2 + \epsilon_r} \sin \theta - \frac{\sigma_2}{a[2(1 + \epsilon_r) + 1]} r \frac{\partial P_2(\cos \theta)}{\partial \theta} -$$

$$- \frac{\sigma_n a^{1-n}}{(\epsilon_r + 1)n+1} r^{n-1} \frac{\partial P_n(\cos \theta)}{\partial \theta} - \dots \quad n = 3, 4, 5, \dots \quad (10)$$

In a bipolar environment both positive and negative charged particles drift in opposite direction along the field lines. Assuming that there is no charge migration on the surface of the probe and that the initial densities of positive and negative ions are the same, the accumulating charge on the surface of the probe will perturb the field and the force acting on the ions in the vicinity of the surface will change. Due to the axial symmetry, the electric field force in the vicinity of the surface $\mathbf{F}$ will have only two components: $\mathbf{F}_r$ and $\mathbf{F}_\theta$. The force $\mathbf{F}_r$ is pushing the ions away from the surface when positive (orientation away from the surface) or pulling them towards the surface when negative, whereas $\mathbf{F}_\theta$ tries to move them along the surface. The charging process will cease once the radial force $\mathbf{F}_r$ acting on the ions in the vicinity of the surface is equal to zero ($\mathbf{F}_r = 0$). Since the probe is symmetric and charge particles of both polarities are present, the equilibrium is expected to be reached when this condition $\mathbf{F}_r = 0$ applies on the whole surface of the probe. As in the case of surface charge density (4), the radial component of electric field intensity on the surface of

the dielectric probe can be written in the form of Legendre's expansion and the equilibrium condition can be written as:

$$E_r = \sum_{n=0}^{\infty} \varepsilon_n P_n(\cos \theta) = 0 \quad (11)$$

where ε_n is the n-th coefficient of the Legendre's expansion and $P_n(\cos \theta)$ is the n-th Legendre's polynomial with argument $\cos \theta$. The coefficients σ_n in equations (7) – (10) can be found using the orthogonal properties of Legendre's polynomials [9];

$$\int_{-1}^1 P_n(\cos \theta) P_m(\cos \theta) d(\cos \theta) = \frac{2\delta_{mn}}{2n+1}, \quad m, n = 0, 1, \dots \quad (12)$$

where δ_{mn} is the Kronecker delta function. Substituting $r = a$ (the surface of the probe), one gets the expansion (11). Finally, by integrating over the whole surface $(0, \pi)$, the σ_n coefficients are found to be:

$$\sigma_n = 0 \text{ for } n \neq 1, \quad \sigma_1 = -\frac{3}{2} E_0 \varepsilon_r \varepsilon_0 \quad (13)$$

The equilibrium surface charge density in a bipolar environment can be finally written as

$$\sigma(\theta) = -\frac{3}{2} E_0 \varepsilon_r \varepsilon_0 \cos \theta \quad (14)$$

The corresponding internal field in the probe is then obtained by substituting (14) into (9) and (10), and transforming it back into Cartesian coordinates we get:

$$\mathbf{E}_{in} = \frac{3}{2} \mathbf{E}_0 \quad (15)$$

The equation (15) shows that the electric field inside the dielectric ball-shaped probe is uniform with the same orientation as the original field $\mathbf{E}_0$. It is also independent on the permittivity of the probe and in intensity directly proportional to the external field. Moreover, it is 1.5 times higher, so, at least theoretically, the electric field probe measuring in bipolar environment has higher sensitivity.

In the unipolar environment with charge particles of only one polarity, the charge deposition is limited to only one side (hemisphere). Consequently, the internal electric field is not expected to be uniform any longer and can be written as:

$$E_m(\theta, r) = C(\theta, r) E_0 \quad (16)$$

where C is a constant for a given position (θ, r) and E_0 is measured or external field. The analytical solution even for a ball-shaped probe appears to be far too complex, and, as explained in Section III, we used numerical modelling to determine the coefficient C .

c) Temperature Stability Of The Sensor

The changes in the output intensity of a polarimetric sensor (Figure 1) caused by the changes in environmental temperature can be, as suggested in ref. [10], characterised by a temperature coefficient α_T defined as

$$\alpha_T = \left| \frac{1}{I_0} \frac{\partial I}{\partial T} \right| \quad (17)$$

where I_0 is the light intensity at the photodetector without electric field, I is the light intensity at the photodetector with electric field, and T is the temperature. Using the Jones calculus and following the procedure in ref. [10], it is not difficult to show that the intensity at the photodetector I with electric field applied on the sensing crystal is related to the electrically induced phase retardation in the electro-optic crystal Γ and natural phase retardation in the quarter wave-plate γ ($= \pi/2$) as

$$I = \cos^2 \left(\frac{\gamma + \Gamma}{2} \right) \quad (18)$$

It is worth mentioning here that induced phase retardation in the crystal is directly proportional to the applied electric field and depends on the crystal symmetry. For cubic crystals, for example, the phase retardation Γ is

$$\Gamma = \frac{2\pi}{\lambda} n^3 r_{41} l E \quad (19)$$

where n is the refractive index of the crystal, r_{41} is the only electro-optic coefficient which characterises the electro-optic behaviour of the cubic crystal, l is the length of the crystal traversed by the light, λ is the wavelength of the light and E is electric field. Similarly, for anisotropic crystals of point symmetry 3m (Lithium Niobate) and the light beam propagating along the optical axis, the phase retardation Γ is given by

$$\Gamma = \frac{2\pi}{\lambda} n_o^3 r_{22} l E \quad (20)$$

where n_o is ordinary refractive index and r_{22} is the electro-optic coefficient.

Applying the equation (17) on (18), the temperature coefficient α_T is found to be

$$\alpha_T = \left| \sin(\gamma + \Gamma) \left(\frac{\partial \gamma}{\partial T} + \frac{\partial \Gamma}{\partial T} \right) \right| \quad (21)$$

III. NUMERICAL ANALYSIS

For the electro-optic sensor in Figure 1, we used 10 mm long crystals of Lithium Niobate and Bismuth Germanate, both operating at wavelength of 1310 nm. Since the bandgap for Lithium Niobate is 350 nm and for Bismuth Germanate is 394 nm, 1310 nm of operating wavelength is a safe choice to eliminate photoconductive effect, as indicated in Section II, even for the crystal samples with higher level of impurities. Also, given the dielectric constant $\epsilon_r = 85$ and conductivity $G = 10^{-16} \text{ S/m}$ for Lithium Niobate and $\epsilon_r = 56$, $G = 2 \cdot 10^{-12} \text{ S/m}$ for Bismuth Germanate, and taking into account that according to the Laplace transformation of the system characterised by the time constant τ , the limit frequency $f_{3DB} (=1/2\pi\tau)$ define 3DB frequency response, the sensor with Lithium Niobate sensing crystal will successfully operate down to a frequency $2 \cdot 10^{-8} \text{ Hz}$ and the one with Bismuth Germanate down to $6 \cdot 10^{-4} \text{ Hz}$, which is practically a DC field for both cases.

d) Measurement In The Presence Of Space Charge

Measurement in the bipolar environment, where both positive and negative ions are present is, according to the equation (15), easier as the measurement results are not effected by the dielectric property of the probe and also the presence of ions improves the measurement sensitivity. However, the equation (16) suggests that in unipolar environment the electric field distribution inside the probe is nonuniform. To determine the constant C we used step-by-step layers of the deposition of negatively charged particles. In each iteration the charge was deposited only in the areas with positive field intensities, where the ions are attracted to the surface. This means that the highest deposition rate is in the places where E_r is positive and its absolute value has a maximum. There is no charge deposition allowed if $E_r \leq 0$. In the case of the probe with relative dielectric constant of $\epsilon_r = 4$ (Bismuth Germanate) we found that the internal field varies between $1.1 \cdot E_0$ near the deposition surface to $0.7 \cdot E_0$ on the opposite side, with the value in the middle of the sphere of $E(0,0) = 0.81 \cdot E_0$. These values change with the dielectric properties of the probe ϵ_r . The Table 1 shows the values of the electric field in the middle of the probe exposed to a uniform electric field of 1 V/m. The electric field intensity in the centre of the probe $E(0,0)$ was calculated for two different permittivities of the probe: $\epsilon_r = 4$ (Bismuth Germanate) and $\epsilon_r = 85$ (Lithium Niobate). The values of $E(0,0)$ in an environment without space charge and in bipolar environment are shown for comparison. Method 1 refers to the simulation in which the already deposited charge is not allowed to leave the surface of the probe, whereas the method 2 allows the charge detachment. Both methods gave very similar results showing that the constant $C(r, \theta)$ is a function of the relative permittivity ϵ_r , in contrast to the bipolar environment where it depends only on the external electric field. In Table 1, E_{in0} is the internal electric field without any space charge and E_{bi} is the internal electric field in a bipolar environment.

Table 1 The calculated values of the internal electric field $E(0,0)$ in the centre of the dielectric probe

ϵ_r	$E(0,0)$ Meth od 1	$E(0,0)$ Meth od 2	E_{in0} No deposition	E_{bi} / E_{in0}
4	-0.81	0.805	0.5	3
85	-0.106	-0.101	0.034	44

e) Temperature Stability

Bismuth Germanate is a cubic crystal of $\bar{4}3m$ point symmetry. As such it does not have induced natural birefringence and, consequently, it is a crystal with very high temperature stability. Even if the most stable zero-order quarter wave plate is used, with typical value $\partial\gamma/\partial T = 2 \cdot 10^{-4} \text{ rad/K}$, the changes in the phase retardation of the crystal $\partial\Gamma/\partial T$ are still several orders of the magnitude lower. Hence, from equation (21) and with $\Gamma \ll 1$, $\gamma \approx \pi/2$ and $\partial\Gamma/\partial T \ll \partial\gamma/\partial T$, the overall temperature changes of the output of the sensor with cubic crystal is

$$\alpha_T \approx \left| \frac{\partial\gamma}{\partial T} \right| \quad (22)$$

It becomes apparent from equation (20) that the temperature stability of the sensor with cubic crystal is dominated by the temperature stability of the quarter wave-plate. However, we have found that with a proper positioning of the optical components [10], the temperature effects of the quarter wave-plate can be minimised. More precisely, if the analyser is aligned along the fast axis of the quarter wave-plate, when $\alpha = \pi/4$ and $\zeta = \pi/4$, as shown in Figure 1, the intensity of the light at the photodetector is found to be

$$I = \frac{1}{2} \cos^2\left(\frac{\gamma - \Gamma}{2}\right) + \frac{1}{2} \sin^2\left(\frac{\gamma + \Gamma}{2}\right) \quad (23)$$

Using equation (17) and with $\Gamma \ll 1$ and $\gamma \approx \pi/2$, the temperature coefficient α_T is found to be

$$\alpha_T = \left| \frac{1}{2} \frac{\partial\gamma}{\partial T} (\sin(\gamma + \Gamma) - \sin(\gamma - \Gamma)) + \frac{1}{2} \frac{\partial\Gamma}{\partial T} (\sin(\gamma + \Gamma) + \sin(\gamma - \Gamma)) \right| \approx \left| \frac{\partial\Gamma}{\partial T} \right| \quad (24)$$

It is apparent from equation (24) that the temperature stability of the sensor with cubic crystal and properly aligned polariser (along the fast axis of the quarter wave-plate) is dominated by the temperature dependence of the phase retardation Γ in the crystal, which is much smaller than that of the quarter wave-plate.

On the other side, Lithium Niobate (LiNbO_3) is an anisotropic crystal of the point symmetry $3m$ and its temperature stability is dominated by the temperature dependence of the associated natural birefringence. Additionally, this crystal also exhibits a pyroelectric effect, which further compounds temperature stability. Both of these effects can be minimised, even completely eliminated, by a perfect alignment of the beam along

the optical axis of the crystal. Any misalignment, however, could increase the temperature instability of the sensor.

From the geometry of the index ellipsoid it follows that when the light beam is at an angle ϑ with the optical axis, the natural birefringence in Lithium Niobate can be expressed as

$$\Delta n = \frac{n_o n_e}{\sqrt{n_e^2 \cos^2(\vartheta) + n_o^2 \sin^2(\vartheta)}} - n_o \quad (25)$$

where n_e , n_o are extraordinary and ordinary refractive indices of the crystal, respectively. Following the same procedure as in equation (18), it is not difficult to show that the output light intensity at the detector I of the sensor without electric field is

$$I = I_0 \left(1 + \sin \left(2\pi \frac{l \Delta n}{\lambda} \right) \right) \quad (26)$$

where l is the length of the crystal traversed by the light and λ is wavelength of the light wave in vacuum. Figure 4 shows the relative change of the sensor output as the function of misalignment angle ϑ . It is apparent that relatively small misalignment causes a large change in the output of the sensor. For example, the deviation of only 1 degree at a wavelength of 850 nm shifts the working point of the sensor to its minimum in the nonlinear region, hence severely reducing the sensor sensitivity for small field intensities.

Figure 4 Normalised change of the sensor output with Lithium Niobate sensing crystal with misalignment angle ϑ

However, the most important effect of natural birefringence is its temperature dependence. Applying (17), the temperature stability of the sensor output due to the natural birefringence is then found to be

$$\alpha_T = \frac{2\pi}{\lambda} \left(\Delta n \frac{dl}{dT} + l \frac{d\Delta n}{dT} \right) \cos \left(2\pi \frac{l \Delta n}{\lambda} \right) \approx \left| \frac{2\pi}{\lambda} \left(\Delta n \frac{dl}{dT} + l \frac{d\Delta n}{dT} \right) \right| \quad (27)$$

It is apparent from equation (27) that the temperature coefficient α_T decreases with the wavelength of the light, but also that the natural birefringence can cause a very strong temperature instability of the sensor. For example, a misalignment of the crystal optical axis of only 0.1° at a wavelength of 1310 for Lithium Niobate provides $\alpha_T = 10^{-7} \text{ rad/K}$. To avoid effect of natural birefringence, the crystal of Lithium Niobate has to be aligned along the optical axis with a precision better than 0.01° .

Lithium Niobate also exhibits a very strong pyroelectric effect [10]. A temperature change of only 1°C can induce the internal electric field of magnitude 10^5 V/m in the direction of optical axis. This, however, does not effect the sensor output if the alignment of the light beam is perfect. We have experimentally established that perpendicular component of pyroelectric effect is always present, at least in several samples of Lithium Niobate crystals that we tested, which is probably the result of structural inhomogeneity in the crystal. It is always possible, to suppress that component of pyroelectric effect by adjusting the analyser and to completely remove this effect. This has to be done experimentally for each crystal sample.

IV. EXPERIMENTAL MEASUREMENTS

The experimental work aimed to confirm findings in Section II and III and in particular to further investigate the possibility of using the sensor in the space charge environment. The experimental set-up is shown in Figure 1. The electric field was generated by two parallel plates wide enough to provide a uniform electric field across the crystal. The positive ions were generated by a sharp needle maintained at a positive potential and placed in the middle of the aperture of the ground plate. The negative ions were generated by the negative corona discharge at the needle placed in the middle of the aperture of the negative high voltage electrode. The electric field contribution due to the high potential of the needle was considered to be negligible.

Figure 5 The experimental set-up for measurement in space charge environment

One of the main problems experienced during the measurement was drift of the zero output level of the sensor. This drift was caused mostly by an uncorrelated long-term drift of the photodetector, and, to a lesser extends, by laser power fluctuations. It was, therefore, necessary to define the zero level before each measurement. Direct light output to the photodetector 2 was used for this purpose. In the case of space

charge measurements, the zero level was defined always after neutralizing the sensing crystal by an AC corona discharge and waiting until the output of the sensor was stable.

Figure 6 shows the normalised output of the sensor with Lithium Niobate in both the unipolar environment and the environment without space charge. It is apparent that in both cases the sensor output is linear and proportional to the applied electric field. However, while the field was decreased, the sensor output did not follow expected values due to accumulated charge on the crystal.

Figure 6 Normalised output as a function of applied electric field for the DC field sensor with Lithium Niobate crystal in both unipolar and environment without space charge

Table 2 shows the slope of the transfer characteristic for different corona currents. The space charge density was assumed to be proportional to the current I_{SPCest} between the electrodes. The slopes are found to be equal within the experimental error. This leads to the conclusion that in the performed experiments the sensor response was not influenced by the space charge density, as suggested by the theoretical analysis.

Table 2 Measured slopes of transfer characteristics for different space charge density

I_C [μA]	2	4	8	14
I_{SPCest} [μA]	0.23	0.3	0.34	0.43
Slope [m/kV]	0.338	0.314	0.352	0.33

The experimental results in bipolar environment are shown in Figure 7. The experiments were carried out by switching on the DC field, then the positive ion generator, and finally the negative ion generator. It is apparent from Figure 7 that the results do not correspond to the prediction fully. The measured ratio of the

internal electric field in the bipolar environment to that in the environment without the space charge was 38. The difference to that given in Figure 7 was attributed to the rectangular shape of the crystal, while the theoretical results were obtained for a ball. Consequently, in the charge equilibrium the surface charge density exceeds predicted equilibrium. Additionally, the sensor output was not very stable during the experiment due to the fact that the bipolar charging was preceded by unipolar charging and concomitant release of excessive charge deposited during the first stage of the unipolar charging. Still, the experiments show that the sensor transfer characteristic is linear and independent on the space charge density. As the mentioned difference is attributed by the type and shape of the sensing crystal, in practice this difference can be easily solved by the sensor calibration, as long as the charge density is low enough not to influence the measured field. Achieved measurement resolution in both cases was 100 V/m.

Figure 7 Normalised output as a function of applied electric field for the DC field sensor with in bipolar, unipolar and environment without space charge

A temperature controlled chamber was used to confirm the temperature dependency using the same arrangement as in Figure 5. With a proper alignment of the optical components, as suggested in Section IV, the temperature stability in both environment with unipolar and bipolar space charge and in the temperature range between 10 and 45°C was within 1%.

V. CONCLUSION

The theoretical analysis and the experiments with Bismuth Germanate and Lithium Niobate electro-optic crystals in unipolar environment confirmed that the sensor output is a linear function of the measured electric field and independent on the space charge density. It is therefore possible to measure electric field in the unipolar environment once the probe is calibrated and the space charge density is low enough not to influence the measured field. The measurement results in the bipolar environment were slightly different and this was attributed to different crystal shape used for the experiments. Appropriate alignment of optical components enable the sensor temperature stability within 1%.

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Figure Legend

Figure 1 Schematic diagram of the electro-optic sensor

Figure 2 Charge relaxation time constant τ of selected crystals
Figure 3 polar coordinate system for the ball-shaped dielectric probe

Figure 4 Normalised change of the sensor output with Lithium Niobate sensing crystal with misalignment angle ϑ

Figure 5 The experimental set-up for measurement in space charge environment

Figure 6 Normalised output as a function of applied electric field for the DC field sensor with Lithium Niobate crystal in both unipolar and environment without space charge

Figure 7 Normalised output as a function of applied electric field for the DC field sensor with in bipolar, unipolar and environment without space charge

Table Legend

Table 1 The calculated values of the internal electric field $E(0,0)$ in the centre of the dielectric probe

Table 2 Measured slopes of transfer characteristics for different space charge density