Active elimination of radio frequency interference for improved signal-to-noise ratio
for *in-situ* NMR experiments in strong magnetic field gradients

M. Ibrahim ^{a,*}, C. I. Pardi ^a, T. W. C. Brown ^b, P. J. McDonald ^a

^a *Department of Physics, Faculty of Engineering and Physical Sciences, University of Surrey,
Guildford, Surrey, GU2 7XH, UK*

^b *Centre for Communication System Research, Department of Electronic Engineering, University of
Surrey, Guildford, Surrey, GU2 7XH, UK*

* *Corresponding author (mona.ibrahim@surrey.ac.uk)*

12 **Abstract**

13 Improvement in the signal-to-noise ratio of Nuclear Magnetic Resonance (NMR) systems may be
14 achieved either by increasing the signal amplitude or by decreasing the noise. The noise has multiple
15 origins – not all of which are strictly “noise”: incoherent thermal noise originating in the probe and
16 pre-amplifiers, probe ring down or acoustic noise and coherent externally broadcast radio frequency
17 transmissions. The last cannot always be shielded in open access experiments. In this paper, we
18 show that pulsed, low radio-frequency data communications are a significant source of broadcast
19 interference. We explore two signal processing methods of de-noising short T_2^* NMR experiments
20 corrupted by these communications: Linear Predictive Coding (LPC) and the Discrete Wavelet
21 Transform (DWT). Results are shown for numerical simulations and experiments conducted under
22 controlled conditions with *pseudo* radio frequency interference. We show that both the LPC and
23 DWT methods have merit.

24

25 **Keywords:**

26 NMR, signal-processing, linear-predictive-coding, discrete-wavelet-transform, de-noising.

27 **1. Introduction**

28 Nuclear magnetic resonance (NMR) offers enormous opportunity for materials detection and
29 characterisation in the "real-world": that is for open-access, electromagnetically unshielded
30 applications outside of the laboratory where it is necessary to assess the quality, state or presence of
31 materials. These applications include: the minimisation of delays to allow concrete to cure during
32 construction or the assessment of building degradation in the built environment [1]; managed
33 forestry in order to decide which trees to fell or to minimise energy consumption from timber drying
34 during processing [2]; illicit materials detection at secure locations such as airports [3]; and down
35 bore-hole logging for oil and gas well exploration [4]. However, save oil and gas exploration, where
36 the earth's crust provides a natural electromagnetic shield, externally broadcast radio frequency
37 interference (RFI) pick-up restricts materials detection limits using affordable and practical light-
38 weight, low-field strength magnets and hence inhibits the technology being taken up widely by
39 industry. The problem is unwanted pick-up of radio signals such as from aeronautical
40 communications and amateur radio. The pick-up dominates and masks the weak NMR signals
41 coming from materials to be inspected in open-access detectors.

42 "Noise" in NMR experiments can be categorised into different types, Fig. 1. The first is incoherent,
43 thermal white noise originating principally in the NMR coil and receiver pre-amplifier. Most
44 normally it is dealt with by signal averaging, but strategies that fundamentally improve the basic
45 electronics are possible. Filtering and digital signal processing are also widely used.

46

Fig. 1. Categorisation of “noise” affecting the experimental NMR signal to noise ratio. This work addresses RFI reduction.

48 The second derives from, for instance, instrument acoustic ringing in response to the excitation
 49 pulses. It is coherent and instrument specific and may be dealt with by strategies such as phase-
 50 cycling or exploitation of spin-lattice relaxation time, T_1 , contrast [5]. The third is radio frequency
 51 interference (RFI) originating from external broadcasters. Most normally this is dealt with through
 52 Faraday shielding. However, for many *in-situ* applications this is difficult or impossible. As a result,
 53 RFI may be the most challenging “noise” to deal with in *in-situ* applications. Although RFI may
 54 have a large coherent component, its occurrence can be undetermined and the specific frequencies
 55 are uncontrolled by the NMR operator. Typical sources for low frequency NMR include aircraft
 56 navigation communications and amateur / citizens’ band radio transmissions [6].

57 Both hardware and software (digital signal processing) methods have been used to try and overcome
58 the problem of RFI in low-frequency, unshielded type applications. Hardware approaches include the
59 use of auxiliary coils designed to pick up the RFI but not the NMR signal and the use of “figure of
60 8” coils whereby the NMR is detected solely in in one half of the coil but the RFI in both halves, but
61 in anti-phase [7, 8, 9, 10]. This work deals exclusively with software approaches.

62 Numerous authors have used digital signal processing strategies to remove “noise” in the widest
63 sense from NMR signals. NMR broadly comes in two flavours: relaxometry and spectroscopy, or
64 time and frequency domain respectively. Correspondingly, digital signal processing has been
65 attempted in both domains and indeed between them.

66 For the mitigation of incoherent thermal noise in time domain NMR, Lu *et al.* [11] proposed using
67 the generalized Gabor expansion for noise reduction of free induction decay (FID) signals, and Gu *et*
68 *al.* [12], further proposed an efficient way to reduce the noise in FIDs based on the real value discrete
69 Gabor transform. A low-pass filtering technique was used by Dabek *et al.* [13] to improve the
70 quality of FIDs by reducing the noise. The application of discrete and continuous wavelet transforms
71 (DWT and CWT respectively) is a popular option. Ma *et al.* [14] used the DWT method together
72 with a wavelet de-noising algorithm to improve the signal-to-noise ratio (SNR) of FIDs subject to
73 incoherent thermal noise. Subsequently Xie *et al.* [15] used the wavelet transform method for noise
74 reduction in Carr-Purcell-Meiboom-Gill (CPMG) experiments. These authors found that the wavelet
75 threshold method had a good de-noising effect on NMR echoes, leading to improved SNR of echo

76 data. Ge *et al.* [16] reported improvements based on using the wavelet transform with an
77 exponentially weighted moving average.

78 In spectroscopy applications, the wavelet transform method was used by Kim *et al.* [17]. These
79 authors use both DWT and CWT respectively for noise reduction. Zheng *et al.* [18] applied a
80 stationary wavelet transform for noise reduction in low NMR field data.

81 Other digital signal processing techniques for “noise” reduction in NMR spectroscopy have been
82 investigated. These include linear predictive coding (LPC) techniques. In [19], Led and Koehl use
83 both forward and backward LPC before data Fourier Transformation to remove systematic artifacts
84 from FID data in spectroscopic applications. Koehl [20] also used linear prediction for improved
85 NMR spectral analysis.

86 Digital signal processing has been attempted for the elimination of RFI from data in a range of other
87 sciences and technology areas including: radio astronomy [21], speech processing [22] and image
88 processing [23]. The problem has been addressed more generally. For instance, Ksibi *et al.* [24]
89 proposed a notch filtering method of RFI cancellation in applications with a low interference to
90 signal amplitude ratio. This technique was found to be adequate for that specific purpose. However,
91 an accurate estimation of the RFI is required for real word data, where complex interference with
92 varying frequencies can occur within the signal. Other approaches such as Fourier Transform with
93 cross-band filtering [25] and adaptive algorithms [26, 27] have also been used for elimination of RFI.

94 Surprisingly, there are relatively few studies reported in the literature that seek to use digital signal
95 processing to overcome the problem of RFI elimination from NMR signals. Amongst the few is the
96 work of Chen *et al.* [28]. These authors propose an RFI cancellation method for FID NMR signals
97 based on the singular value decomposition (SVD) algorithm. By removing the singular values
98 corresponding to the RFI and noise, the amplitude of interference peaks is suppressed by 10 dB.

99 In this paper, we further explore the opportunity to reduce the impact of RFI on low field CPMG
100 NMR data acquired in the strong magnetic field gradients typical of some types of *in-situ* NMR
101 systems. The basic premise of the RFI mitigation analyses used in this work, evidenced by the
102 experimental data, is that RFI tends to originate from data communications and therefore occurs in
103 ‘burst mode’: it is either present or it is not. Moreover, the duration of these bursts is long compared
104 to a typical echo width encountered in *in-situ* NMR experiments using a magnet with a strongly
105 inhomogeneous field, such as Surface GARField [1]. It might be thought that it is most easy to
106 simply discard and re-acquire data collected during RFI bursts. However, the frequency and
107 amplitude of these bursts is such as to severely contaminate at least some echoes in most echo trains.
108 Therefore simple averaging merely results in an overall worse signal-to-“noise” ratio. Moreover,
109 there are some applications – such as in security or on production lines – where immediate
110 acquisition is an imperative and re-acquisition is not a realistic option.

111 This kind of application is the primary focus of our work and indeed, for the most part it is merely
112 necessary to identify the presence (or not) of a signal. The detailed characteristics of that signal are

113 less important. To that end, although in this paper we do address recovery of NMR parameters such
114 as T2 through numerical simulations, this is not our primary purpose.

115 We explore two different strategies. The first is based on the LPC. The idea for LPC is to predict the
116 RFI during the actual echo occurrence from data acquired on either side of the echo: before and after.

117 The prediction is then subtracted from the RFI corrupted recorded echo data to yield just the echo.

118 The second is DWT which is used to separate the recoded data into multiple levels of detail and

119 approximation signal components dependent on the chosen wavelet. By careful choice of

120 decomposition level and wavelet, wanted signals occur in one subset of components, unwanted

121 signals in another. The filtered signal is obtained by reconstructing only the former set.

122 The following sections of this article are organized as follows: the next section presents the

123 theoretical description of the LPC and DWT methods; section 3 details a series of numerical

124 simulations that test the ideas and provide some “ground truth” NONE data under varying conditions

125 of continuous RFI and white noise for the comparison with output from LPC and DWT; section 4

126 describes actual RFI characterisation and experimental methods; section 5 presents experimental

127 results and discussion; finally section 6 offers a conclusion and future outlook.

128 **2. Mathematical models**

129 In this section, we describe the mathematical methods used to address the problem of RFI reduction
130 in NMR data.

131 *2.1 Linear Predictive Coding*

132 Linear predictive coding (LPC) methods are among the most used in signal coding, signal
 133 recognition and signal prediction in multiple areas as diverse as speech recognition and finance.
 134 They can give extremely accurate estimates of signal parameters [29]. The basic assumption of LPC
 135 is that the current value of the signal can be approximated as a linear combination of a finite number
 136 of past values. Given a time series of data $x = \{\dots, x(0), x(1), x(2), \dots, x(n), \dots\}$, the general LPC
 137 model is written as:

$$138 \quad \hat{x}(n) = \sum_{i=1}^p a_i x(n-i) \quad (1)$$

139 where $\hat{x}(n)$ is the prediction of the actual signal value $x(n)$, a_i are model coefficients to be
 140 determined and p is the length of known data series upon which the prediction is based. The
 141 prediction error $e(n)$ is defined as the difference between the actual and the predicted values so that
 142 a signal modelled by LPC can be written as:

$$143 \quad x(n) = \sum_{i=1}^p a_i x(n-i) + e(n) \quad (2)$$

144 The predictor coefficients a_i are determined by minimizing the sum of squared differences $e(n)$ over
 145 a finite interval of known data. The autocorrelation method and the autocovariance method [30] are
 146 well known and robust means to find the coefficients a_i . In this work the autocorrelation method was
 147 used. Once these coefficients are found, it is then possible to use them to predict future values of
 148 unknown data points or to estimate missing data points, as in the current example where NMR data is
 149 masked by RFI. Clearly predictive calculation of data point $\hat{x}(n+1)$ requires $\hat{x}(n)$ as input; while

150 $\hat{x}(n + 2)$ requires both $\hat{x}(n + 1)$ and $\hat{x}(n)$ as input and so forth. Therefore, the quality of prediction
 151 deteriorates the further into the unknown the prediction is made.

152 In order to incorporate LPC removal of RFI into the analysis of NMR echo data, three time periods
 153 of echo data are considered as shown in Fig. 2.

Fig. 2. A schematic showing the three time periods of data used in LPC. Region 1 is centrally located in the expectation window of the short T_2^* echo. Regions 2 and 3 are symmetric and occur before and after the echo respectively. Note that region 2 deliberately avoids the NMR pulse ring down, shown in grey, that affects real data.

154

155 The first region spans the expected location and width of the echo, in the middle of two RF
 156 refocussing pulses. The second region precedes this but avoids the first pulse ring down. The third
 157 region follows the echo and is symmetric with the second. With the width of these regions

158 appropriately defined, it is a reasonable assumption that the second and third intervals comprise only
159 RFI and thermal white noise, whereas the first comprises of RFI, white noise and the NMR echo.
160 The RFI contribution to the central region is estimated by forward and back extrapolation of the
161 signal occurring in the before-echo and after-echo periods respectively. This prediction is then
162 subtracted from the central region to reveal the “clean” echo signal. The LPC is applied to the raw
163 radio frequency signal of each scan, before demodulation and averaging according to the phase cycle
164 in order to obtain the final echo train.

165

166

167 *2.2 Discrete wavelet transform*

168 The Discrete Wavelet Transform (DWT) is widely used for time-frequency analysis in different
169 domains [31, 32, 33], and is a very powerful technique for non-stationary signals analysis, which is
170 the case considered in this study. The DWT convolutes the data with a set of basis functions that are
171 time stretched and shifted versions of a wavelet that, in an ideal case, well approximates the expected
172 data function so as to find occurrences of that function within the data set. The wavelet φ must be an
173 oscillating, well-localized function having a finite vanishing moment. Using this function φ , which is
174 normally called the “mother” wavelet, a second wavelet ω , the “father” wavelet, can be constructed
175 [33]. From both φ and ω , an orthogonal basis of L_2 the set of signals with finite energy, can be
176 constructed. Using these two functions, the wavelet basis is defined by:

$$WB = \{\varphi_{j,k}(t) = \varphi(2^{-j}t - k), \omega_{j,k}(t) = \omega(2^{-j}t - k), j, k \in \mathbb{Z}\} \quad (3)$$

where k is the translation parameter (in time) and j is the scale or level of decomposition. The scale is an integer indicating how the wavelet is stretched or compressed; see Fig. 3. The projection of the signal x over the basis WB is called the discrete wavelet transform. For a given scale j , the DWT of x can be written as follows:

$$x(t) = \sum_{k \in \mathbb{Z}} 2^{-\frac{j}{2}} c_j(k) \varphi(2^{-j}t - k) + \sum_{i=1}^j \sum_{k \in \mathbb{Z}} 2^{-\frac{j}{2}} d_i(k) \omega(2^{-j}t - k) \quad (4)$$

The first series $c_j(k)$ is known as the approximation coefficients and represent the trend of x , while $d_j(k)$ are known as the detail coefficients, and can represent the fast variations in x . In principle, a very large number of levels of decomposition, j , are possible, but in practice only a small number is usually required. Careful inspection is used to identify the components in which the required signals occur and those components in which the unwanted signals occur. The former set are retained and the latter discarded. The retained set is reconstructed to reveal the filtered signal.

There is no universal method for choosing the mother wavelet and the decomposition level j .

Generally, users select wavelets whose shapes best approximate the expected data. Therefore, a maximum correlation is obtained. As for the decomposition level, it has been found in [32] that j can be selected according to the frequency range presented in the data. In what follows, we apply DWT with 2 levels of decomposition and the *bior6.8* [33] mother wavelet. We find that the useful information is in the first and second detail parts, denoted D1 and D2. These are retained with the

195 remaining approximation part discarded. D1 and D2 alone are reconstructed to make the filtered
 196 signal. DWT is applied individually to all scans of the experiment and the results demodulated and
 197 averaged over its entire duration including the pulses. The choice of *bior6.8* and the retention of D1
 198 and D2 is discussed further in section 3.

199

Fig. 3. Translation and scaling of the wavelet ‘bior6.8’: (a) the basic wavelet; (b) shifted by 60 points; and (c) scaled by a factor $j = 5$.

200

201 **3. Numerical Simulations**

202 *3.1 Methodology.*

Figure 4. (a) The basic echo shape. Data is created and used in the range 50 to 150 μs although for clarity only 80 to 120 μs is shown. (b) The echo shape modulated at the NMR frequency and corrupted by continuous RFI. (c) The recovered echo after application of LPC (red line), DWT (blue line) and “NONE” (black line) mitigation strategies. The recovered echo in the absence of RFI with “NONE” is shown in green as a control. (d) A repeat of (c) except that the pre-processing echo trace has added white noise.

$$E_i = E^0 \frac{\sin\left(\frac{2\pi(t_i - t_c)}{t_w}\right)}{\left(\frac{2\pi(t_i - t_c)}{t_w}\right)} \times \exp\left(-\left(\frac{t_i - t_c}{0.85 t_w}\right)^2\right) \quad (5)$$

206 with $t_c = 100 \mu s$ and $t_w = 5 \mu s$. The amplitude of the echo was held constant at $E^0 = 1$. The echo
 207 shape is shown in Figure 4a. The echo was modulated at the NMR frequency, 3 MHz, so as to
 208 represent the signal at NMR pre-amplifier level.

209 Continuous RFI of the form

$$I_i = I^0 \cos(2\pi f_{RFI}(t_i - t_c) + \varphi_{RFI}) \times (1 + \cos(2\pi f_{SB}(t_i - t_c))) / 2 \quad (6)$$

210

211 was added to echo. The initial RFI carrier frequency was set to $f_{RFI} = 2.8$ MHz , the side band
 212 frequency to $f_{SB} = 0.1$ MHz, initial phase to $\varphi_{RFI} = 0$ and initial amplitude to equal the echo
 213 amplitude, so $I^0 = 1$. The signal was sampled at 12.5 MHz: $t_i = [50:0.08:150] \mu s$. The modulated
 214 echo with RFI is shown in Figure 4b.

215 Figure 4c shows the real part of the demodulated, recovered, echo plus RFI shape without
 216 application of any RFI mitigation strategy (“NONE”) and with LPC and DWT as described in
 217 section 2 and also the recovered shape with “NONE” in the absence of RFI. The LPC algorithm was
 218 applied with forward and backward prediction windows of length $40 \mu s$ (500 points, regions 2 and 3
 219 in Figure 2) and an echo width of $9.6 \mu s$ was predicted (120 region 1). DWT was applied using
 220 *bior6.8* and retaining D1 and D2.

221 The basic echo shape without RFI (green trace) is recovered “perfectly” indicating that the
 222 demodulation works. This trace provides a reference shape for the others. With RFI, “NONE” results
 223 in a highly distorted echo from which little meaningful information can be extracted, (black trace).

Both LPC and DWT improve the situation significantly. The LPC results in an echo of comparable width and intensity to the original although detail is lost in the wings. The negative sinc lobes are not seen. This is a subtlety that occurs in the absence of white noise and is addressed in sections below. The DWT tends to narrow the echo and it has a greater central intensity. However, the integrated area is comparable to the reference.

3.2 *Effect of white noise*

Gaussian white noise of standard deviation was $N^{RMS} = 0.2$ (implying a conventional signal-to-noise ratio of 5) was added to the simulated signal. The recovered echoes and a reference signal are shown in Figure 4d. In the presence of noise, the LPC and DWT continue to aid recovery of the echo. The basic shapes are unchanged compared to Figure 4c, although, of course, the traces are noisier. Notice that the LPC algorithm in particular is better at recovering the negative lobes of the sinc function in the presence of noise compared to without. In the absence of noise, LPC suffers from the fact that it accurately fits the tail of the echo shape and hence this detail is subtracted out.

3.3 *Recovery of NMR parameters.*

In time domain NMR, the experimenter is usually seeking to recover the initial signal amplitude and T_2 relaxation time from CPMG data perhaps for pore size discrimination, or to recover the echo shape for Fourier transform in imaging applications.

In order to evaluate the efficacy of the methods for relaxometry analysis, a CPMG echo train of 16 echoes with T_2 equal to 5 times the echo spacing was created. RFI as described in Eq. 6 was added

243 save that a uniformly distributed random phase was introduced into the carrier waveform for every
 244 echo. Gaussian white noise was also added. Then each of the “NONE” control, LPC and DWT
 245 algorithms were applied. The integrated intensity of the central lobe of each recovered echo was
 246 evaluated and used to calculate the average T_2 and initial signal amplitude over 32 repeats of the
 247 simulation, each repeat using different random RFI phases and white noise. The 32 average
 248 simulation was performed as a function of the signal to RFI intensity ratio and of the signal to white
 249 noise intensity ratio. Exactly the same time dependence of RFI and white noise were used for each
 250 set of 32 averages: only the relative amplitudes were changed.

Figure 5. (a). The mean T_2 (expectation = 5 echo spacing) for simulated data as a function of RFI amplitude for a fixed Gaussian noise level of 0.15 and signal level of unity. The RFI levels are all from the set $\{0.1, 0.2, 0.5, 1, 2, 5, 10, 20\}$ but the data sets are shown slightly displaced to enable the standard error in the mean uncertainty bars to be clearly seen. Black symbols are for “NONE”;

red for LPC; and blue for DWT. No data point implies the result is off-scale. (b). Mean T_2 similar to (a) but as a function of signal to white noise ratio for a fixed RFI level of 2. The noise levels are all from the set $\{0: 0.05: 0.5\}$ again shown slightly displaced. (c) and (d) are calculated signal amplitudes corresponding to (a) and (b) respectively. The expectation amplitude is 3.25.

251

252

253 Figure 5a shows the mean calculated, T_2 for each of the 3 considered scenarios: NONE, LPC and
 254 DWT as a function of RFI amplitude for signal to white noise ratio 0.15. No meaningful results are
 255 obtained with “NONE” for RFI to signal ratio equal to 1 or greater. LPC provides a much better
 256 estimate of the T_2 across the board. Even for RFI to signal ratio of 20, the calculated T_2 is within 25%
 257 of expectation; 15% at ratio 10. DWT is not as good as LPC across the full range. Indeed, it makes
 258 the situation worse than doing nothing at the higher levels. However, for the lower levels of RFI
 259 where improvements are marginal, it is better than LPC.

260 Figure 5b shows a data cut in the orthogonal direction. It is the mean T_2 for constant signal to RFI
 261 ratio, $E^0/I^0 = 0.5$ (that is RFI twice as large as the echo) as a function of signal to white noise ratio.
 262 Here LPC routinely outperforms both “NONE” and DWT. However, notice that LPC is actually
 263 worse when there is no noise compared to when there is some noise as previously discussed.

264 In general, LPC recovers the amplitude well and better than “NONE” within similar constraints to
 265 the T_2 recovery. DWT is not so good although the low scatter in the data and the smaller uncertainty

266 bars at low to medium RFI levels compared to “NONE” suggests that perhaps this is due to a
 267 systematic attenuation of the signal. However, this has not been investigated in detail.

Figure 6. The Fourier transform of the recovered echo for: (a) $(I^0, N^{RMS}) = (0,0)$; (b) $(0,0.2)$; (c) $(1,0)$; and (d) $(1,0.2)$ for the algorithms: “NONE” - black line; LPC – red line; and DWT – blue line.

268

269 Imaging places different performance requirements on the algorithms. A continuous sinc function of
 270 infinite extent Fourier transforms to a perfect “top-hat” function with infinitely sharp resolution. The
 271 Fourier transform of the echo defined in Eq. 5 after modulation and demodulation is shown as the
 272 reference “NONE” trace in Figure 6a. It is a broadened “top-hat”. The first echoes of the same data
 273 set as created for the relaxation analysis were Fourier transformed. The results are plotted for four
 274 exemplar cases for each of the NONE, LPC and DWT algorithms in other parts of Figure 6. RFI

introduces spikes into the profile in the case of “NONE” as might be expected. LPC and DWT lessen these spikes, but overall LPC is much better. In the absence of LPC and RFI, LPC has minimal effect. However, the profile is slightly broadened compared to the reference. Partly this is because LPC truncates the echo in the time domain. The failure to detect sinc lobes in the recovered echo in the absence of noise (see Figure 4c) introduces small wiggles into the wings of the calculated profile. The consequence is a loss of resolution in the frequency domain.

3.4 *The effect of RFI frequency*

Simulations were carried out as a function of RFI carrier frequency for $I^0 = 1$ and noise level 0.1. Under these circumstances for carrier frequency 2.8 MHz both algorithms improve T_2 estimation. The sideband frequency is held constant at 0.1 MHz. Figure 7 shows plots of recovered T_2 and amplitude. As the RFI frequency moves further from the NMR frequency, so the echo recovery improves, especially for the DWT algorithm. However, what is most noticeable is that LPC is useful across the full frequency range including frequencies at which the RFI carrier and sidebands overlap the NMR frequency whereas DWT fails in this central region. DWT is acting as a sophisticated filter, and like other filters struggles to separate signals of equal frequency; LPC predicts signals in the time domain and can separate signals of different temporal behaviour even if they are the same frequency.

Figure 7. (a) Plot of T_2 and (b) amplitude against RFI carrier frequency. Black diamonds are for “NONE”; red circles are for LPC; and blue squares are for DWT.

291

292 3.5 The effect of RFI saturation

293 Simulations so far have assumed sinusoidal RFI. In a real experiment, at sufficiently high amplitude,
 294 the RFI saturates the NMR receiver. As a result, the waveform is distorted. If the receiver slew rate is
 295 sufficiently fast, then it tends towards a square wave. This introduces discontinuities that degrade the
 296 performance of the LPC algorithm in particular.

297 To investigate this effect, the cosine functions in Eq. 6 for the RFI were replaced by square wave
 298 generator functions. Results are shown in Figure 8 where “saturated” RFI is compared to
 299 “sinusoidal” RFI. The RFI amplitude is $I^0 = 0.3$ and the noise is $N^{RMS} = 0.1$, values that give good
 300 improvements for “sinusoidal” RFI for both LPC and DWT. The RFI carrier frequency is 2.8 MHz.

301 Although the two plots are obviously different in detail, they are similar in broad scope indicating
 302 that both LPC and DWT can be used – in principle - for saturated RFI. There is no fundamental
 303 reason why saturated RFI does not work although similar limits to relative amplitudes as already
 304 discussed continue to apply. This is a further example of where these methods are to be preferred
 305 over direct band pass filtering. The saturated RFI has a very broad frequency spectrum not easily
 306 filtered out.

Figure 8.(a): The recovered echo shapes for “saturated” RFI where the cosine function is replaced by a square wave generator compared to, (b), the original “sinusoidal” RFI. The red trace is LPC and blue is DWT. Although the traces are different, both LPC and DWT are comparably useful for “saturated” and “sinusoidal” RFI.

307

308 3.6 Discussion

309 A summary of the simulation data sets is that for T_2 estimation, LPC provides good results compared
310 to “NONE” and generally outperforms DWT across a wide spectrum of RFI levels, including where
311 the RFI is 10 times the signal level. Perversely, however, it can fail to improve the situation if the
312 white noise is extremely low due to the resolved tails of the echo in the time domain. DWT
313 outperforms LPC only in conditions of greater white noise, and relatively low RFI levels. In the case
314 of imaging, the echo shape is important. LPC is limited as it impairs the resolution but DWT is not
315 although it only works well for low RFI levels. LPC has the clear advantage of always outperforming
316 DWT and “NONE” when RFI is present and the RFI and NMR frequencies are well matched. This is
317 a scenario where any traditional band pass filtering algorithm will fail.

318 Two final points in this section are worthy of mention. In the case of LPC, we find that the method
319 works optimally with symmetric-length, long-duration windows of data to extrapolate forwards and
320 backwards, before and after the echo respectively as in Figure 2. However, this is not strictly
321 required and reasonable improvements in RFI to signal ratio can be made from uni-directional
322 extrapolation. This is most likely to occur if the user seeks a minimum duration echo time where the
323 echo occurs immediately after the refocussing pulse ringdown. Such scenarios are not reported here.
324 In the case of DWT, the choice of mother wavelet does not seem to be too critical. There are very
325 many in the literature. That which we chose works well – but so do others. The optimisation of the
326 number of decomposition levels required and which components of each decomposition to retain is
327 subtler. We conducted simulations that showed a key factor determining this is the ratio of the
328 sampling to NMR frequencies (which is of the order 4 in the work reported here). As this ratio

329 changes, so the desired signal to be retained moves between components. Testing different levels and
330 retention schemes, while system dependent, is not onerous. Moreover, if these frequencies stay fixed
331 optimisation it is unlikely to need repeating.

332

333 **4. Experimental**

334 *4.1 Radio frequency interference spectrum*

335 Radio Frequency Interference was detected and amplified using a wideband receiver coil tuned to
336 3MHz and pre-amplifier in the open laboratory. The amplified signal was directly recorded at a
337 sampling frequency of 10 MHz for time periods of 900 ms using a Rohde and Schwarz RTO
338 Oscilloscope. In addition, the envelope of the same signal was recorded for multiple, longer periods
339 of several seconds in parallel with pseudo-continuous inspection of a spectrum analyser to check that
340 the average underlying frequency spectrum was approximately constant and that issues of aliasing
341 were avoided. The general characteristics of the detected RFI varied little from day-to-day. Three
342 principal components were identified. First was white noise that was confirmed as emanating from
343 the receiver preamplifier. The second was continuous radio broadcast at specific frequencies starting
344 from 6 MHz. The third and largest components were pulsed RFI at frequencies closer to our Surface
345 GARfield and Tree Hugger NMR frequencies (3 and 1 MHz respectively) believed to be due to
346 aeronautical and amateur data communications (it is noted that Guildford is within 150 km of 7
347 international airports the closest of which are large - London Heathrow and London Gatwick. See
348 also Ref. [6].). An example 0.9 s, 10 MHz recording is shown in Fig. 9.(a). Figures 9(b) and 9(c)

349 zoom into regions of the data with time axis magnifications of $100\times$ and $10,000\times$. These plots
 350 emphasise the pulsed nature of the transmission and how in this case the dominant component is at
 351 about 4.8 MHz with a sideband frequency of about 500 kHz, confirmed by spectrum analysis and
 352 Fourier Transformation of the signal.

353 In order to have a “controlled” source of RFI with temporal characteristics analogous to “real” RFI,
 354 but with opportunity to vary the frequency and without incorporation of white noise, the modulus
 355 moving average of the recorded signal was subject to a threshold to locate the pulses of the largest
 356 amplitude RFI transmissions. In this instance, these occur at a frequency of 4.8 MHz. The resultant
 357 binary function $T(t)$ is shown as the offset line in each of Figs. 9 (a), (b) and (c). The moving
 358 average filter width is 100 kHz, and the threshold is set to retain $\text{RFI} > 0.04 \text{ V}$. The mean value of
 359 the binary function, $\langle T \rangle$, is 0.166 indicating that the RFI is “on” about 16% of the time and the
 360 mean pulse duration is $24 \mu\text{s}$ a little more than the typical echo width in a GARField experiment. The
 361 normalised auto-correlation function $\langle T(t) T(t + \tau) \rangle$, shown in Figure 10, is highly structured
 362 and shows that the typical RFI pulse repetition time is $100 \mu\text{s}$ while the duration of a typical pulse
 363 burst, presumably corresponding to a data transmission, is of the order of 4 ms – this being the time
 364 constant of an exponential fit to the front end envelope of the auto-correlation function.

365 In experiments, the function $T(t)$ was used to modulate on and off a coherent oscillator programmed
 366 to output a *pseudo* RFI signal $C(t) = T(t) A \sin(\omega_c) (1 + \sin \omega_{sb} t)$ where A is an amplitude
 367 factor and ω_c and ω_{sb} are the carrier and sideband frequencies respectively. An example of the
 368 function $C(t)$ is illustrated in Fig. 9. (d), (e) for the same time windows as in Fig. 9 (b) and (c)

369 respectively. In this example, $\omega_c = 4.8$ MHz and $\omega_{sb} = 500$ kHz. The *pseudo*-RFI source was
370 broadcast in the vicinity of the NMR spectrometer located within a Faraday cage. The carrier
371 frequency was deliberately changed so as to be close to the NMR frequency. The power level was
372 such that, for a carrier frequency 200 kHz off NMR resonance, a *pseudo*-RFI signal of amplitude -40
373 dBm, when picked up by the NMR spectrometer, was approximately 50% of the saturation level of
374 the NMR receiver.

375 4.2 NMR data

376 Carr Purcell Meiboom Gill (CPMG) experiments were carried out on a GARField NMR magnet
377 operating at a NMR frequency of 22.55 MHz with a field gradient strength of 8.7 T/m. This system
378 was chosen for preliminary work as (i) it is within a large Faraday cage, (ii) the normal signal to
379 noise ratio (SNR) for a rubber sample is sufficient to see echoes with only a single scan and (iii) the
380 strong gradient is typical of GARField systems ensuring a very short T_2^* NMR relaxation time.
381 These factors facilitated refinement of the methodology. Later experiments using a Surface
382 GARfield operating at 3 MHz and with much lower SNR will be reported elsewhere. The magnet
383 was operated in conjunction with a Magritek KEA spectrometer. The NMR experimental parameters
384 were kept constant: the 90° and 180° pulse lengths are both 5 μ s; the 90° to 180° pulse gap is 100 μ s;
385 16 echoes are recorded with 64 points per echo and a dwell time of 1 μ s; 32 scans are averaged with
386 a repetition time of 1 s. In every case the sample was a block of white rubber SE2005, CA28 (ACC
387 Silicones Europe).

388 In addition to recording, demodulating and averaging the echo train signal using the KEA
 389 spectrometer in conventional mode, the low level RF pulses from the KEA transmitter and raw RF
 390 signal output from the NMR receiver pre-amplifier were recorded separately (without averaging) for
 391 every echo train using a National Instruments PCI bus extension (NI PXIE) oscilloscope at a
 392 sampling rate almost 4 times the NMR frequency, equal to 83.33 MHz.

393 Echo trains were recorded without *pseudo*-RFI transmission and also with *pseudo*-RFI set at power
 394 levels of -50, -45, -40, -35, and -30 dBm. Data was recorded for *pseudo*-RFI carrier frequencies in
 395 the range 20 MHz to 25 MHz – a range that more than spanned the full bandwidth of the NMR
 396 sensor tank circuit. The *pseudo*-RFI sideband frequency was held constant at 200 kHz. The *pseudo*-
 397 RFI signal of duration 900 ms was looped continuously during the NMR data acquisition. The time
 398 origin of the *pseudo*-RFI signal was not synchronous with the NMR experiment and varied from
 399 experiment to experiment.

400 Individual echo trains (i.e. before averaging) recorded by the NI PXIE oscilloscope were analysed in
 401 MATLAB®. The NI PXIE oscilloscope time base was not phase locked to the NMR spectrometer.
 402 Therefore, the frequency and phase of the spectrometer reference signal, $S_{ref}(t)$ were reconstructed
 403 by fitting a sine wave to the train of recorded RF transmitter pulses. Care was taken to ensure that
 404 pulse edge artefacts were not included in the fitting and that signal aliasing across inter-pulse
 405 windows did not occur. RFI filtering by each of the LPC and DWT methods or indeed no filtering
 406 was performed on the raw echo train traces $S_{nmr}(t)$ before demodulation and averaging. Filtered
 407 signals, $S_{nmr}^{filt.}(t)$, were demodulated in the normal manner. First the point-by-point product

408 $S_{ref}(t) \times S_{nmr}^{filt.}(t)$ was formed. The product was Fourier transformed and a low-pass Gaussian filter
 409 applied to extract the low frequency signal. The signal was then back transformed and the echo trains
 410 from multiple scans combined (averaged) according to the NMR phase cycle. For the system at
 411 22.55 MHz, and in the absence of RFI, echoes were discernible from single scans.

412

Fig. 9. (a) An example of RFI recorded at 10 MHz for a total of 0.9 s with small regions of length (b) 9 ms and (c) 90 μ s shown zoomed. The line above each plot shows the binary function $T(t)$ used to identify the presence of the largest RFI transmissions. (d-e) *Pseudo*-RFI as described in the text shown for the same zoomed time windows as in parts (b-c) respectively.

413

414

Figure 10. The short time front end of the normalised autocorrelation function of the RFI envelope,

$T(t)$. The early structure confirms and describes the pulsed nature of the broadcast. The inset

shows the first 8 ms from which the envelope decay constant of 4 ms.

416 5. RFI mitigation: results and discussion

417 In total, more than 300 experiments were carried out under different conditions of *pseudo*-RFI and
 418 tested using the different RFI reduction methods. Exemplar results and a summary is presented here.

419 5.1 Data capture and demodulation validation

420 Fig. 11 (a) shows NMR echoes in the absence of RFI as recorded and demodulated by the KEA
 421 spectrometer. For clarity, just the first 4 echoes of the 16 recorded are shown. The echoes are clearly
 422 seen with a long T_2 decay constant. It is well established that in high field gradient experiments, the

423 first echo is systematically smaller than all the rest [34]. Fig. 11 (b) shows the same data as captured
 424 by the NI PXIE oscilloscope and reconstructed in MATLAB® as detailed in the previous section.
 425 The filter width is 0.4 MHz. The signal is recorded and demodulated throughout the NMR
 426 experiment. The large saturating peaks in Fig. 11 (b) are the filtered RF pulses and the echoes occur
 427 midway between them. Apart from this obvious difference and an arbitrary amplitude factor, the two
 428 traces are very similar. They are not exactly the same as the precise details of the internal filtering of
 429 the KEA spectrometer are unknown. Taken together, the data in Fig. 11 give us confidence that, all
 430 else being equal, we can tap into the KEA data stream post pre-amplifier and interpret it off line
 431 equally as well as the KEA spectrometer. From here on, comparisons are always made to and
 432 between reconstructed NI PXIE data.

Fig. 11. (a) NMR echoes without RFI as recorded and demodulated by the KEA spectrometer. The 180-180 pulse gap is 200 μ s. (b) The same data as recorded and demodulated by the NI PXIE oscilloscope.

433

434 Data was recorded with *pseudo*-RFI transmission on for RFI with a carrier frequency in the range 19
435 to 25 MHz. The sideband frequency was 200 kHz. As an example, Fig. 12 shows the raw radio
436 frequency NMR signal as recorded by the NI PXIE oscilloscope for a *single* scan corrupted with
437 *pseudo*-RFI. No demodulation has been attempted and no echoes are apparent to the eye. The NMR
438 pulses give rise to the uniformly spaced square pulses in the trace with amplitude about 0.55 V
439 (receiver saturation level). A pulse of RFI is seen as the strong beat pattern signal with amplitude
440 about 0.4 V between about 2800 and 3400 μ s. In this instance, the RFI transmit amplitude is -40
441 dBm and the carrier frequency 22.35 MHz. Interestingly there is an additional intermittent signal of
442 about 0.1 V amplitude. This is “genuine”-RFI that we believe leaked into the system through the
443 cables on the *pseudo*-RFI transmit.

Fig. 12. A typical single (not averaged) scan as recorded by the NI PXIE. It is corrupted with *pseudo*-RFI between about 2800 and 3400 μs . RF pulses are seen as the narrow rectangles. There is additional “real” RFI spanning much of the remainder of the scan with an amplitude of about 0.1

V.

Fig. 13. The result of demodulation and averaging of 32 RFI affected scans of which that in Fig.

12 is one. Demodulated RFI dominates the underlying echoes that cannot be discerned.

444

445 Fig. 13 exemplifies what happens when 32 scans of which that in Fig. 12 is one are averaged and
446 demodulated using the same reconstruction algorithm as previously. The RFI is different in every
447 case. It is not synchronous with the NMR experiment. Indeed, the total back-to-back length of
448 *pseudo*-RFI data is significantly greater than the total length the 32 NMR data scans. The phase cycle
449 ensures that the 180° pulses do not appear as strongly in this figure as in Fig. 12, (note the change in
450 vertical scale). Nonetheless there are edge effects and the strong vertical lines enable their position to
451 be visualised easily. There are no obvious echoes between. They are obliterated by the *pseudo*-RFI
452 bursts. Statistically, we expect the RFI to be “on” 16.6% of the time. Therefore we expect each
453 region of the echo train to be affected by about 5.3 bursts on average. Given that the bursts are not
454 phase coherent between scans, the observed amplitude of 0.02 to 0.03 V seems consistent with

455 expectation: $\frac{1}{32} \times \sqrt{\frac{5.3\pi}{4}} \times 0.4 \text{ V} = 0.025 \text{ V}.$

456

457 5.2 LPC algorithm

458 Results of applying LPC to the data are shown in Fig. 14 and Fig. 15. In Fig. 14 results are shown for
459 a fixed RFI carrier frequency of 22.75 MHz and three different RFI amplitudes: -30, -40 and -50

460 dBm, whereas in Fig. 15 results are shown for a fixed RFI amplitude of -40dBm and three different
 461 frequencies: 22.65, 22.85 and 23.15 MHz.

Fig. 14. Complete echo trains (left) with LPC RFI reduction for a RFI fixed carrier frequency of 22.75 MHz. The RFI amplitude is (a) -30 dBm, (b) -40 dBm and (c) -50 dBm. The zooms to the right focus on four central echoes where the RFI impact is worst. The real and imaginary quadrature signals are shown in blue and red respectively.

462

463 LPC was applied using $p = 1400$ points, equivalent to $16.8 \mu s$ at 83.3 MHz for $12 \mu s$ each forward
 464 or backward to predict an echo of $24 \mu s$. The forward and back predictions were averaged at the
 465 echo centre. Cursory inspection of Fig. 14 and Fig. 15 reveals that LPC improves the echo recovery
 466 but that the performance diminishes with increasing RFI amplitude, as expected from the
 467 simulations, (Fig. 14). It also diminishes with decreasing RFI carrier offset from the NMR frequency
 468 (Fig. 15), not as expected.

Fig. 15. Complete echo trains (left) with LPC RFI reduction for a RFI fixed amplitude of -40 dBm. The RFI carrier frequency offsets from the NMR frequency are (a) 0.1 Mz, (b) 0.3 MHz and (c) 0.5 MHz. The zooms to the right focus on four first echoes where the RFI impact is worst in every case. The real and imaginary quadrature signals are shown in blue and red respectively.

470 It is not possible to directly compare like-with-like since the exact same echoes cannot be recorded
 471 with and without RFI. Nonetheless, if “clean” echo data, E_{REF} , recorded in the absence of RFI is
 472 taken as a reference, then a figure of merit, f^{filt} , can be defined as the root mean square residual of
 473 filtered RFI affected data, E_{RFI}^{filt} , compared to the “clean” data.

474

$$f^{filt} = \frac{\sqrt{\sum_{i=1}^n (E_{RFI}^{filt} - E_{REF})^2}}{\sqrt{\sum_{i=1}^n (E_{REF2} - E_{REF})^2}} \quad (7)$$

where f_{ilt} is either LPC or DWT or, indeed, no filtering, “NONE”. This figure of merit makes a point by point comparison of the echo shape with expectation based on an “ideal experiment” and as such is designed to capture both shape and intensity changes.

Fig. 16. (a) f^{LPC} and (b) f^{DWT} as a function of RFI carrier frequency offset from the NMR frequency for different RFI amplitudes. (c) The average value of f^{filt} in the central 2 MHz region for $filt = LPC$ (open squares), DWT (closed squares) and “NONE” circles as a function of RFI amplitude.

The figure of merit is normalised by a measure of the standard deviation of white noise in “clean” data using, for comparison a repeat, second, reference measurement in the denominator of equation 7. The figure of merit can be meaningfully calculated only in the central data region. Hence, since the echo width is $24 \mu s$ and the sampling rate is 83.33 MHz , $n = 2000$ in Eqn 5. The figure of merit f^{LPC} is plotted in Fig. 16 (a) as a function of RFI carrier frequency offset for each of the different

RFI amplitudes studied. Except when the RFI bandwidth straddles the NMR frequency, $\approx \omega_c \pm \omega_{SB}$, the figure of merit is of the order of 10 - 20. It rises to a few hundred at the centre for the highest RFI levels studied. Average values as a function of RFI amplitude over the central frequency bandwidth ($\approx \omega_c = 22.55 \pm 1.0$ MHz) are presented in Figure 16 c as open squares. By way of comparison, the figure of merit for unfiltered RFI but affected data, f^{NONE} , has also been calculated. As a specific example, $f^{NONE} = 96$ for RFI of frequency and amplitude 22.45 MHz (RFI 100 kHz offset to NMR) and -40 dBm respectively; about 3 times greater than with LPC filtering, 28, giving some quantification for the improvement gained. Average values are shown in Fig. 16 c. Across the whole range of RFI amplitudes, they are typically 50% greater than with LPC filtering. Equally, the figure of merit for LPC filtered “clean” data – that is substituting E_{REF2} for E_{RFI}^{LPC} in Eqn. 5 – is about 2. Ideally, a value of 1 is expected suggesting that, for sufficiently low level RFI, the LPC filtering actually makes the situation worse. This is expected since white noise is used to predict other white noise which is then (negatively) added into the data. The implications of these numbers are considered further in section 5.4.

5.3 DWT algorithm

Figs. 17 and 18 show the results of applying the DWT filtering to the exact same raw data corrupted by *pseudo*-RFI as was used for LPC. Fig. 17 shows results with a fixed frequency of 22.75 MHz and three different amplitudes, and Fig. 18 with a fixed amplitude of -40 dBm and three different frequencies. From Fig. 17 it is seen that, as with LPC, RFI elimination with DWT enhances as the amplitude of RFI decreases. Similarly, from Fig. 18 it is seen that increased frequency offset also

503 improves RFI elimination. Fig. 16 (b) shows the figure of merit for the DWT data. There is some
 504 indication that DWT works better than LPC for larger frequency offsets, but less well when the
 505 NMR and RFI carrier frequencies are well matched. However, in experiment and unlike the
 506 simulations, overall the two methods are comparably good. We suspect that, in experiment, LPC is
 507 under-performing rather than DWT over-performing compared to the simulations.

Fig. 17. Complete echo trains (left) with DWT RFI reduction for a RFI fixed carrier frequency of
 22.75 MHz. The RFI amplitude is (a) -30 dBm, (b) -40 dBm and (c) -50 dBm. The zooms to the
 right focus on four central echoes where the RFI impact is worst. The real and imaginary
 quadrature signals are shown in blue and red respectively.

508

509

Fig. 18. Complete echo trains (left) with DWT RFI reduction for a RFI fixed amplitude of -40 dBm. The RFI carrier frequency offsets from the NMR frequency are (a) 0.1 Mz, (b) 0.3 MHz and (c) 0.5 MHz. The zooms to the right focus on four first echoes where the RFI impact is worst. The real and imaginary quadrature signals are shown in blue and red respectively.

510

511 5.4 Further discussion and future perspective

512 As was shown in sections 3 and 5.2, RFI filtering actually makes “clean” data worse. For the

513 experiment, the “improvement” in the average LPC figure of merit $\langle f^{LPC} \rangle$ without RFI is about

514 $1/2$ (*i.e.* less than 1) whereas with RFI it is $3/2$. Recognising that only 16% of the data is actually

515 affected by RFI, then the actual improvement in RFI affected echoes is closer to a factor of

516 $\left(\frac{3}{2} - \frac{0.84}{2}\right) / 0.16 = 6.7$. These figures imply that the brute-force, across-the-board, filtering adopted

517 here is of benefit when as few as 8% of all echoes are RFI affected. They also suggest that

518 “intelligent” filtering that only applies LPC (or indeed DWT) to RFI affected echoes would be of

519 benefit. Such an approach is being tested in our laboratory and will be discussed in a future
520 communication.

521 This study has been conducted in strong magnetic field gradients such as are encountered in open
522 access Surface GARField experiments in the built environment. In these circumstances the RFI data
523 pulse tends to be longer than the NMR T_2^* . The reverse situation is easily envisaged where T_2^* is
524 longer than the RFI data pulse. Following the work of Led and Gesmar [19] and Koehl [20], we
525 anticipate that it will still be possibly to use both LPC and DWT methods save that, in this case, for
526 LPC the NMR echo signal outside of the RFI pulse is used to predict the signal within it. This may
527 be the more common scenario in NMR generally, but for *in-situ* work where one sided magnets are
528 very frequently used this is not necessarily the case.

529 A more difficult scenario may arise when the RFI and echo are of matched length. In this case, there
530 is a high probability of a burst starting or ending in the middle of the echo. Now LPC is expected to
531 fail. For the future, one can imagine introducing “intelligent” processing to seek to identify the
532 transitions from the (real time) step change in average signal level. This scenario may come about
533 when multiple NMR spectrometers work in close proximity where the transmission pulses of one are
534 detected by another. For this special case there are two alternate options: the spectrometers are made
535 to work at sufficiently different frequencies or use of software-scheduling. Further processing might
536 then be reserved for truly “external” RFI.

537 This study focussed entirely on amplitude modulated RFI. We have not considered whether the
538 mitigation schemes would work similarly for frequency or phase modulated signals. The reason is

539 that at low frequencies amplitude modulation is of most practical importance. However, this is
540 clearly an area for future testing.

541 Finally we note that in further studies not reported here we explored the use of Haar filtering (which
542 is a special case of DWT). Except in very special circumstances where the NMR and RFI carrier and
543 data capture frequencies were in especially advantageous relationship, this filtering offered little
544 additional improvement. However, where such relationship occurred, there were very significant
545 advantages. The fact that the RFI carrier frequency is variable means, however, that the method
546 appears to be of little practical use.

547 **6. Conclusion**

548 This paper has explored the possibility of reducing the deleterious effects of semi-coherent broadcast
549 radio frequency interference in NMR experiments. The specific case of RFI from low frequency
550 pulsed data communications has been considered in the context of low magnetic field experiments
551 performed in strong magnetic field gradients. Two signal processing approaches have been tested
552 with numerical simulations and experimentally under partially controlled conditions. The first is
553 based on linear predictive coding that seeks to forecast the RFI signal during the data acquisition
554 given knowledge of the RFI immediately before and after the NMR echo. The second uses the
555 discrete wavelet transform method to filter the signal. In simulation, LPC is clearly better but in
556 experiment both methods achieve a reasonable degree of success. LPC tends to work better when the
557 RFI carrier frequency is closer to the NMR frequency; DWT when it is further removed. This is
558 perhaps not surprising as it is difficult to filter out one signal when both are at the same frequency.

559 However, when the RFI is of very low amplitude or the frequency is even more distant, then the
560 effects of both LPC in particular can make the signal to noise ratio worse.

561

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