Similitude relationships for physical modelling of monopile-supported offshore wind turbines

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Offshore wind turbines are considered as an important element of the future energy infrastructure. There is currently a surge in the construction of such facilities in Europe, yet there is no track record of long-term performance of these structures. Offshore wind turbines are dynamically sensitive structures because of the very nature of the structural form (tall and slender) and the different types of dynamic and cyclic loading imposed on them. Lack of data concerning long-term performance indicates a need for detailed investigation to predict the future performance of such structures. Arguably this can be best carried out through small-scale well-controlled laboratory experimental investigation. In this paper, scaling laws are derived for the design of such model tests for studying the long-term performance. Non-dimensional groups that need to be preserved are identified while carrying out these tests. The effectiveness of these chosen non-dimensional groups is investigated by carrying out controlled tests on a 1:100 scale offshore wind turbine. Typical experimental data are presented.

Notation

CSR	cyclic stress ratio
D	pile diameter
E	pile Young's modulus
e_p	strain in pile wall thickness
e_s	average strain in soil
f_f	forcing frequency
f_n	natural frequency
G	soil shear modulus
K_L	stiffness of transverse spring
K_R	stiffness of rotational spring
K_V	stiffness of vertical spring
k_h	horizontal coefficient of soil permeability
M	external moment acting at the pile head
N	number of cycles of loading
P	horizontal load
t	time
t_w	pile wall thickness
y	distance between foundation level and application of P
γ_{tl}	linear shear strain threshold
δ	pile head lateral deflection
ξ	damping ratio
σ'_v	effective vertical stress on the soil at the same depth as above
σ_y	pile yield stress

τ_{cyc}

cyclic shear stress imposed by the pile on the soil at a particular depth

1. Introduction

1.1 Loading on offshore wind turbines

Offshore wind turbines are dynamically sensitive structures that are placed in adverse environmental conditions (with strong wind and wave loading of a cyclic nature). This makes the design of foundations extremely challenging. Figure 1(a) shows a photograph of a monopile-supported wind turbine and Figure 1(b) schematically shows the typical wind and wave pressure distribution along the length of the tower and the foundation. The tower, above the water, experiences two types of loads:

- the bottom part of the tower, unobstructed by the spinning turbine blade, experiences a nearly constant value of the wind loading
- the top part of the tower, which is periodically obstructed by the spinning of the blades, is subjected to a cyclic loading often called the blade passing effect (2P/3P) or blade shadowing effect or wind shielding effect in the literature.

Figure 1 (a). Typical offshore wind turbine (Cox and Jones, 2010); (b). Schematic representation of the environmental load acting on offshore wind turbine supported on a monopile; (c) equivalent force model showing a force and a moment at the foundation level

The total environmental lateral load acting on the offshore wind turbine in Figure 1(b) can be modelled simplistically as an instantaneous static horizontal load, P , acting at a distance y above the foundation level. Thus P represents the resultant lateral load on the tower that must be resisted by the monopile foundation. Figure 1(b) shows an equivalent force model for the foundation where the lateral load (P) on the tower is replaced by a force (P) and a moment (M) at the pile head.

Figure 2 shows a structural model of a wind turbine currently being used in practice (see for example the software GH Bladed or Samtech) to predict the natural frequency of the system. The lateral vibration of wind turbines is controlled by two foundation springs: K_L (transverse spring) and K_R (rotational spring). The dynamic response of these structures is dependent on the support condition (i.e. the stiffness of the foundation in Figure 2), which relies on the strength and stiffness of the surrounding soil. Under moderate to high cyclic loading most soils change their properties, which may alter the stiffness of

Figure 2. Simplified structural model

the foundation and this may have an adverse effect on the long-term performance. Therefore, the dynamic behaviour and the prediction of the long-term performance of offshore wind turbines require a deeper understanding of various dynamic interactions between the superstructure, foundation, soil and external loads.

1.2 Frequency of the loading on wind turbine foundations:

Figure 3 shows the main frequencies for a three-bladed 3 MW Vestas V90 wind turbine with an operational interval of 8.6 to 18.4 revolutions/min: the rotor frequency (often termed as 1P) lies in the range 0.14–0.3 Hz and the corresponding ‘blade passing frequency’ for a three-bladed turbine lies in the range 0.42–0.9 Hz. The figure also shows a typical frequency distribution for wind and wave loading. The peak frequency of offshore waves is about 0.1 Hz. It is clear from the frequency content of the applied loads that the designer of the turbine and foundation has to select a system frequency (the global frequency of the overall wind turbine–foundation system) which lies outside

Figure 3. Main forcing frequencies acting on a Vestas V90 3 MW offshore wind turbine

these in order to avoid system resonances. The usual choice would lie between turbine and blade passing frequencies (so-called ‘soft–stiff’, option 2 in Figure 3). There are two challenges

- (a) the foundation stiffness must be estimated very accurately from the available soil data
- (b) the potential for change in foundation stiffness with time as a result of the cyclic loading must be understood so that the risks of the system frequency coinciding with a loading frequency can be avoided.

1.3 Purpose and scope of the paper

Offshore wind turbines are relatively new structures and there is no track record of long-term performance (say 20 to 30 years). Under these circumstances, experimental investigations on physical wind turbine models can provide valuable information for understanding the dynamic behaviour and long-term performance of these relatively new structures. The design and interpretation of any test carried out on a small-scale model require the assessment of a set of laws of similitude that relate the model to the prototype structure. The paper therefore has the following aims:

- (a) to derive the scaling laws necessary to study dynamic soil–structure interaction and also to identify appropriate controlling dimensionless groups
- (b) to present typical test results from a study in which a 1:100 scale wind turbine model was tested in the Bristol Laboratory for Advanced Dynamics Engineering (BLADE) and then to comment on the usefulness of the dimensionless groups derived in this paper.

It must be mentioned that there are three design limit states that may govern the design of offshore wind turbines foundations:

- (a) ultimate limit state, that is failure of the foundations
- (b) serviceability limit state, that is the deformations or the tilting of the wind turbine tower or the foundation is more than acceptable
- (c) fatigue limit state, that is the material is subjected to large number of cycles which may be close to its fatigue life.

The focus of the paper is not on the ultimate limit state but on serviceability limit state.

1.4 A brief summary of the available literature in the public domain

Long-term performance of wind turbines is currently a topic of research as monitoring of limited installed monopile offshore wind turbines has indicated a gradual degradation of the overall system structural dynamics from design requirements; see for example the data from Lely wind farm reported by Zaaier (2006) and Kuhn (2002). Research on pile–soil interactions without the effects of superstructure has recently been carried out by Li *et al.* (2010) and Leblanc *et al.* (2010). Research on the effects of large numbers of cycles of loading on a soil sample has been investigated by Wichtmann (2007). An analytical solution for estimating the first natural frequency of the system taking soil–structure interaction into account has been carried out by Adhikari and Bhattacharya (2010, 2011). In contrast to these reported researches, the work presented in this paper is unique in considering the dynamics of the overall system.

2. Scaling and modelling

Derivation of the correct scaling laws constitutes the first step in an experimental study. These are necessary to interpret the model test results in order to scale up the results for prediction of prototype consequences. Every physical process can be expressed in terms of non-dimensional groups and the fundamental aspects of physics must be preserved in the design of model tests. The necessary steps associated with designing such a model, to be implemented either in one-*g* or a multi-*g* testing environment, can be stated as follows:

- (a) to deduce the relevant non-dimensional groups by thinking of the mechanisms that govern the particular behaviour of interest both at model and prototype scale
- (b) to ensure that a set of crucial scaling laws are simultaneously conserved between model and prototype through pertinent similitude relationships
- (c) to identify scaling laws which are approximately satisfied, and those which are violated and which therefore require special consideration.

Examples of derivation of scaling laws for general dynamic problems can be found in the literature, see for example Iai (1989) and Muir Wood *et al.* (2002).

2.1 Identifying non-dimensional groups

Offshore wind turbines are dynamically sensitive structures because of the multiple frequencies which contribute to the complexity of the interaction of the foundation with the supporting soil. The study of the dynamics as well as the assessment of the long-term performance of offshore wind turbines involves the following issues.

- (a) Vibration of the pile owing to the environmental loads will induce cyclic strains in the soil in the vicinity of the pile. In order to study the changes in soil stiffness owing to these cyclic strains, the developing soil strain around the field must be monitored. These soils are saturated and therefore pore pressures are likely to develop as a result of these cyclic strains. The pore pressure developed may dissipate to the surrounding soil, depending on the frequency of the loading.
- (b) Changes in the soil stiffness owing to the cyclic loading may lead to changes in foundation stiffness (K_L , K_R in Figure 2), which in turn will alter the natural frequency of the system. Therefore the relationship between the foundation characteristics and the overall system dynamics, in other words, the soil–structure interaction, is important for overall system performance.
- (c) Repeated cyclic stresses will be generated in the pile owing to cyclic loading. Therefore foundation fatigue is also a design issue.

Based on the above discussion, the following physical mechanisms are considered important in order to develop the non-dimensional groups.

- (a) the strain field in the soil around a laterally loaded pile which will control the degradation of soil stiffness
- (b) the cyclic stress ratio in the soil in the shear zone
- (c) the rate of soil loading which will influence the dissipation of pore water pressure
- (d) the system dynamics, the relative spacing of the system frequency and the loading frequency
- (e) bending strain in the monopile foundation for considering the non-linearity in the material of the pile
- (f) fatigue in the monopile foundation.

2.2 Strain field in the soil around the laterally loaded pile

Repeated shear strain may reduce the stiffness of saturated soils. Assuming that the changes in soil stiffness drive the long-term performance, the average strain next to a pile is a governing criterion and must be preserved in order to ensure similar stiffness degradation in both model and prototype. The relevant non-dimensional group can be derived by considering that the average shear strain field around a laterally moving pile can be expressed as a function of pile head deflection (δ) and pile outer diameter (D)

$$1. \quad \varepsilon_s \propto \frac{\delta}{D}$$

Klar (2008) suggested a value of 2.6 for the coefficient of proportionality between the average strain in the soil and the ratio of head deflection and pile diameter. However, there is a lack of consensus in the literature on this proportionality coefficient. The pile head deflection is a function of the external load, P , the shear modulus of the soil, G , and the pile diameter, D . Therefore, the average strain field in the soil around a pile can be expressed as a function of only three parameters

$$2. \quad \varepsilon_s = f(P, D, G)$$

The parameters in Equation 2 can be used to obtain a dimensionless group as follows

$$3. \quad \varepsilon_s = f\left(\frac{P}{GD^2}\right) \frac{[F]}{[FL^{-2}][L]^2}$$

Equation 3 describes the non-dimensional group that takes into account the strain field in the soil generated by a lateral loaded pile. Equation 3 shows that the strain in the soil is directly proportional to the horizontal load applied at the pile

head, inversely proportional to the soil stiffness and inversely proportional to the square of the pile diameter.

2.3 Cyclic stress ratio (CSR) in the soil in the shear zone

In geotechnical earthquake engineering, it is well established that degradation of a soil owing to liquefaction-type failure is a function of cyclic stress ratio (CSR), which is defined as the ratio of the shear stress to the effective vertical stress at a particular depth, defined by Equation 4, (see, for example, Seed and Idriss (1971)). The CSR can be expressed by Equation 4

$$4. \quad \text{CSR} = \frac{\tau_{\text{cyc}}}{\sigma'_v}$$

$$5. \quad \tau_{\text{cyc}} \propto \frac{P}{D^2} \quad \frac{[F]}{[L]^2}$$

where

τ_{cyc} = cyclic shear stress imposed by the pile on the soil at a particular depth

σ'_v = effective vertical stress on the soil at the same depth

The vertical effective stress can be related to the shear modulus (G) of the soil

$$6. \quad \sigma'_v \propto G \frac{[F]}{[L]^2}$$

It is usually found that G is proportional to $\sigma_v'^n$ where the value of n depends on the type of soil. The value of n varies between 0.435 to 0.765 for sandy soil (Wroth *et al.*, 1979) but a value of 0.5 is commonly used. For clayey soil, the value of n is generally taken as 1.

Combining Equations 4–6, one can see that the non-dimensional group expressed by Equation 3 can also guarantee similarity of cyclic stress ratio. This leads us to a non-dimensional group (Equation 7) that must be satisfied

$$7. \quad \left(\frac{P}{GD^2} \right)_{\text{model}} = \left(\frac{P}{GD^2} \right)_{\text{prototype}}$$

It is interesting to note that using two different approaches based on average strain in the soil and on the CSR dimensional analysis leads to a unique non-dimensional group given by Equation 7.

2.4 Rate of soil loading

Pore pressure generation and subsequent dissipation is a function of the frequency of loading exerted on soil. The time, t , in which the pore pressure dissipates will be directly proportional to the soil permeability, k_h , and inversely proportional to characteristic length, for example monopile diameter, D)

$$8. \quad t \propto \frac{k_h}{D}$$

Considering the variables in Equation 8, one can obtain the only possible dimensionless group

$$9. \quad \left(\frac{k_h t}{D} \right) \propto \frac{[LT^{-1}][T]}{[L]}$$

Replacing time (t) by forcing frequency (f_f), the pore pressure dissipation is therefore correctly modelled when

$$10. \quad \left(\frac{k_h}{f_f D} \right)_{\text{model}} = \left(\frac{k_h}{f_f D} \right)_{\text{prototype}}$$

2.5 System dynamics

In order correctly to simulate the system dynamics resulting from the interaction between the external loads (i.e. forcing frequency) and the wind turbine (i.e. natural frequency), the ratio between the forcing frequency and the natural frequency of the turbine should be of the same order in the physical model and prototype. The main forcing frequencies are outlined below (see also Figure 3).

- (a) Environmental loading: the predominant wave and wind forcing frequency is typically around 0.1 Hz.
- (b) Rotor frequency: it represents the frequency of the blade rotation, and is generally indicated by 1P. As shown in Figure 3, typical values are in the range 0.14–0.3 Hz.
- (c) Wind shielding effects of the blade on the tower: when the blade passes the tower the shadowing effect of the wind load causes a cyclic load on the tower. This frequency is indicated by 3P (or 2P for a two-bladed rotor). Evidently the blade passing frequency is the product of the rotor frequency and the number of blades.

It is considered useful to review the relevant codes of practice. DNV guidelines (2002) suggest that the natural frequency of the wind turbine should not come close to the forcing frequency arising from the imposed environmental loads. The guidelines also specify that the global frequency of the system should be at least $\pm 10\%$ away from the 1P and 2P/3P frequencies indicated by dotted lines in Figure 3. Depending on the value of the

natural frequency of the wind turbine in relation to the loading frequencies, three different design regimes are possible: soft–soft, soft–stiff and stiff–stiff (as indicated in Figure 3). From Figure 3 it may be noted that for any of these three design approaches, the ratio between the forcing and the natural frequency is in fact close to 1. This aspect must be preserved also in the physical model. Therefore the non-dimensional groups for the correct modelling of the dynamics of the system can be expressed as a ratio between the forcing and natural frequency

$$11. \left(\frac{f_f}{f_n}\right)_{\text{model}} = \left(\frac{f_f}{f_n}\right)_{\text{prototype}}$$

2.6 Bending strain in the monopile

The strain in the monopile is a function of its mechanical properties and the characteristics of the external loads. As shown in Figure 1(b), the external loads owing to the wind and the waves can be conveniently modelled as a horizontal force (P) acting at the distance y above the foundation level. Making the assumption that the monopiles remain elastic, the strain in the pile wall will be a function given by Equation 12

$$12. \varepsilon_p = f(P, y, D, E, t_w)$$

where

t_w is the pile wall thickness

E is Young's modulus of the pile

The parameters in Equation 12 can be combined to obtain a dimensionless group

$$13. \varepsilon_p = f\left(\frac{Py}{ED^2t_w}\right) \frac{[F][L]}{[FL^{-2}][L]^2}[L]$$

In order to model the material non-linearity of the pile correctly, the non-dimensional group expressed by Equation 14 must be preserved in the model and the prototype

$$14. \left(\frac{Py}{ED^2t_w}\right)_{\text{model}} = \left(\frac{Py}{ED^2t_w}\right)_{\text{prototype}}$$

2.7 Fatigue in the monopile

The stress in the monopile can be an important parameter influencing the fatigue phenomena. Fatigue relates to the degradation of the material after a large number of load cycles ($>10^7$ cycles). The design of an offshore wind turbine must ensure that the fatigue limit state (FLS) is satisfied (DNV, 2007).

The fatigue in the monopile can be expressed as a function of the external load P , pile diameter, D , pile wall thickness, t_w , vertical distance y between the application point of the load P and the pile head, and the yield stress of the material, σ_y

$$15. f(P, D, t_w, y, \sigma_y)$$

The parameters in Equation 15 can be combined to obtain a dimensionless group

$$16. f\left(\frac{Py}{\sigma_y D^2 t_w}\right) \frac{[F][L]}{[FL^{-2}][L]^2}[L]$$

Equation 16 represents the non-dimensional group required to take account of the fatigue phenomenon. This can be correctly modelled when Equation 17 is satisfied

$$17. \left(\frac{Py}{\sigma_y D^2 t_w}\right)_{\text{model}} = \left(\frac{Py}{\sigma_y D^2 t_w}\right)_{\text{prototype}}$$

The next section of the paper describes typical results of high-quality small-scale experiments to explore the usefulness of these dimensionless groups.

3. 1-g tests on model offshore wind turbine

Pilot experiments have been carried out at the University of Bristol in order to study the long-term performance of offshore wind turbines.

3.1 Details of test set-up

An experimental investigation has been carried out in the Bristol Laboratory for Advanced Dynamics Engineering (BLADE). Nineteen tests were carried out on a small-scale offshore wind turbine, with a Vestas V90 3MW turbine being taken as a notional prototype. The experimental arrangement is shown in Figure 4 and more details are given by Lombardi (2010). Table 1 shows the relevant parameters for the prototype and model wind turbines. The soil conditions at the Vestas installation in the Kentish Flat wind farm are considered as the k_h prototype. The soil at the site is layered with seabed sand underlain by soft to firm clay on top of the London Clay formation. The average soil shear modulus is taken to be in the order of 80 MPa. Based on the method developed by Adhikari and Bhattacharya (2010), Lombardi (2010) estimated the natural frequency of the entire system at the Kentish Flat location to be around 0.38 Hz. A homogeneous soil profile of soft speswhite kaolin clay was prepared for the model tests. From bender element tests, the shear modulus was estimated to be 6 MPa.

Figure 4. Physical model of a small-scale offshore wind turbine

Modelling of the environmental dynamic loads which act on a prototype offshore wind turbine was achieved using an electrodynamic actuator (Figure 4) fixed to the laboratory strong wall and connected to the model wind turbine tower. The force applied to the wind turbine could be constantly monitored by a force sensor. The blades were rotated by a DC electric motor to model the 1P loading and also to give some aerodynamic damping to the system. Precise measurement of displacements, velocities and accelerations was performed using LVDTs (linearly varying differential transformer) and piezoelectric accelerometers. Table 2 shows the non-dimensional groups for the model and a typical prototype (3MW Vestas V90), while the meaning of the symbols is illustrated in Figure 5.

The data recorded during the tests (i.e. acceleration response) were analysed in the frequency domain for the assessment of the natural frequency and damping of the model wind turbine after different numbers of cycles of loading.

3.2 Experimental procedure

A typical test consists of the application of the cyclic loading for a particular time interval (or certain number of cycles) and then measuring the frequency and damping of the system by a free vibration test. In the free vibration test (also known as a ‘snap back’ test in the literature), the actuator was disconnected from the tower and the tower was given a small-amplitude vibration and the acceleration of the system was recorded.

Parameter	Prototype turbine	Model turbine
Lateral load (P)	1.16 MN (Typical value)	2N
Soil stiffness (G)	80 MPa	6 MPa
Diameter of the pile (D)	4300 mm	22 mm
Soil permeability (k_h)	10^{-9} m/s	10^{-9} m/s
Natural frequency (f_n) of the system	0.38 Hz Lombardi (2010)	2.96 Hz (measured in the lab)
Forcing frequency (f_f) *	0.14–0.3 (1P) 0.42–0.9 (3P)	0.66 (1P) 2 Hz (3P) 20 Hz (3P) 125 Hz (3P)
Distance of the lever arm of total lateral load (P), i.e. y	40 000 mm	600 mm
Thickness of the pile wall (t_w)	45 mm	1.3 mm
Young’s modulus of the pile material (E)	Steel GPa	Dural alloy 70 GPa

*For model turbine, tests were carried out with three different forcing frequencies of 2 Hz, 20 Hz and 125 Hz applied by electrodynamic actuator. The 1P frequency of 0.66 Hz represents the rotational rotor speed was applied by an electric motor powered by a DC supply

Table 1. Values for the prototype turbine in Kentish Flat wind farm and model wind turbine

Physical mechanism	Non-dimensional group	Model	Prototype
Strain field in the soil and cyclic stress ratio (CSR)	$\left(\frac{P}{GD^2}\right)$	6.89×10^{-4}	7.84×10^{-4}
Rate of loading*	$\left(\frac{k_h}{f_f D}\right)$	2.27×10^{-8} ($f_f = 2$ Hz) 2.27×10^{-9} ($f_f = 20$ Hz) 3.64×10^{-10} ($f_f = 125$ Hz)	3.88×10^{-10} (Here the blade passing frequency of 0.6 Hz corresponding to the operational frequency of 12 revolutions/min is considered)
System dynamics	$\left(\frac{f_f}{f_n}\right)$	0.6–1.0	0.5 (based on 1P) 1.6 (based on 3P) The range: 0.36 to 0.78 (1P) 1.1 to 2.36 (3P)
Strain in the monopile	$\left(\frac{Py}{ED^2 t_w}\right)$	2.72×10^{-5}	2.66×10^{-4}
Stress in the monopile	$\left(\frac{Py}{\sigma_y D^2 t_w}\right)$	7.63×10^{-3}	0.10

*The non-dimensional value related to rate of loading is assessed in the model for three different force frequencies (2, 20 and 125 Hz)

Table 2. Values of the non-dimensional groups for both model and prototype

The cyclic lateral loading was applied at three different frequencies (2 Hz, 20 Hz and 125 Hz) and for different lateral load magnitudes. This set of tests created a database of change of frequency and damping of the wind turbine system for different values of

- (a) strain field in the soil, that is various values of (P/GD^2)
- (b) forcing frequency (f_f)
- (c) number of cycles of loading (N).

The next section of the paper presents typical test results in order to illustrate the usefulness of the non-dimensional groups.

3.3 Typical results and discussion

Figure 6 shows the variation of the normalised frequency of the system with respect to the number of cycles of loading for various strain levels in the soil. As expected, higher strain levels lead to higher reduction in natural frequency of the model. It is interesting to note that for a low value of (P/GD^2) there is practically no degradation in the natural frequency. Figure 7 shows the variation of secant stiffness with strain for fully saturated cohesive soil adapted from Vucetic (1994). It is clear from Figure 7 that there exists a strain level γ_{tl} for which there is no stiffness degradation. This reference shear strain is often termed as ‘threshold linear (tl) shear strain’. The experimental results presented in Figure 6 indicate that the non-dimensional group (P/GD^2) can be mapped to γ_{tl} , which can be considered

as a fundamental property of a soil that can be obtained from routine element tests. However, more study is necessary to prove or disprove the hypothesis. As soil stiffness decreases, the frequency ratio (f_f/f_n) also changes during the test in the range 0.6–0.9.

Figure 8 shows, as expected, the increase in damping of the model with the number of cycles. Higher damping variations are recorded for larger strain fields in the soil. Figure 9 shows that for a particular strain level, $P/GD^2 = 0.34\%$, the drop in frequency is higher if the forcing frequency is close to the natural frequency of the system. This is expected because of resonance type mechanisms. Figure 10 presents the corresponding damping values. Figure 11 shows the dynamic response for the entire set of tests.

The test results indicate that the non-dimensional groups identified can capture the stiffness degradation of the system. The test result justifies the current trend of large-diameter piles being used to support offshore wind turbines since average strain decreases with square of the diameter of the pile.

There are a few points that should be noted.

- (a) It may be observed from Table 2 that, in order to maintain the similitude of rate of loading, that is satisfy Equation 10, the forcing frequency should be 125 Hz.

Figure 5. Schematic representation of the parameter that characterised the strain field around the pile

This would ensure that the group $(k_h/f_i D)$ is of the same order of magnitude for both the model and the prototype. However, this will not model the system dynamics correctly because the resulting ratio f_i/f_n is about 42. On the other hand, if a forcing frequency of 2 Hz is used to maintain the similitude of system dynamics, the dissipation of pore pressure is not modelled accurately as the $(k_h/f_i D)$ group is two orders of magnitudes less than the prototype. Therefore, if the system dynamics parameter is

Figure 6. Change in frequency with the number of cycles for different amplitudes of strain field in the soil

Figure 7. Secant shear modulus reduction curve for fully saturated cohesive soils cyclically sheared in undrained condition (after Vucetic, 1994)

used for designing the experiments, the model stiffness degradation (the drop in system frequency) will be slower than the prototype. This must be considered when up-scaling the tests to predict the prototype consequences.

- (b) The 'rate of loading parameter' $(k_h/f_i D)$ and 'system dynamics' (f_i/f_n) cannot be simultaneously satisfied with the same pore fluid in model and prototype. It is obvious that k_h (coefficient of horizontal permeability) together with the diameter of the pile (D) takes into account the drainage condition and the type of soil. Modelling both the non-dimensional groups together can be conveniently accommodated through the use of a different pore fluid

Figure 8. Change in damping with the number of cycles for different amplitudes of strain field in the soil

Figure 9. Change in frequency with the number of cycles for different ratio between forcing and natural frequencies

Figure 10. Change in damping with the number of cycles for different ratio between forcing and natural frequencies

Figure 11. Deformation response for different ratios of forcing-natural frequency

such as silicone oil or methyl cellulose solution for the model tests.

- (c) It may be noted that soil strength (S_u) is not accounted for in any dimensionless group. This is because the focus of the current paper is on serviceability limit state, which requires stiffness of the soil (G) at its core to carry out any calculations.
- (d) Table 2 also shows that the stresses and the strains in the model pile are much lower than in the prototype. In particular, the strains in the model pile are one order of magnitude less than the prototype. In practice, monopiles are rigid and are designed to remain elastic in the design life. Therefore this low strain in the model piles would not affect the test results and the conclusions reached in the paper. However, if a thin-walled flexible pile is to be modelled then the material non-linearity of the pile has to be considered and then the dimensionless group given by Equation 14 has to be satisfied. On the other hand, the stresses in the model pile are two orders of magnitude lower than the prototype. This would ensure minimum reduction of fatigue life of the pile and the test results are unaffected by the fatigue phenomenon. Also, if the fatigue of the monopile is to be modelled, Equation 17 needs to be satisfied.

4. Conclusions

The paper shows that small-scale experimental studies can be carried out to study complex dynamic soil-structure interaction problems where there is no prior information. The long-term performance of offshore wind turbines has been studied because there is a real concern regarding the effect on performance of changes in the foundation stiffness.

Mechanics-based non-dimensional groups have been derived in order to study the various aspects of this problem and their validity has been verified using physical modelling. Under certain loading conditions, the natural frequency of the system decreases as a result of the effects of cyclic loading. These critical conditions relate to: (a) the strain level in the soil; and (b) the relative position of the system frequency in comparison to the forcing frequency.

The non-dimensional group representing the strain level in the soil is given by (P/GD^2) , which indicates that strain level in the soil adjacent to the pile decreases with square of the diameter. These results therefore justify the use of large-diameter monopiles as foundations for modern offshore wind turbines. Presently the majority of UK offshore turbines are built with steel monopile foundations, but for the new projects, which are at a greater distance from shore in much deeper water, and with far larger turbines, monopiles may become uneconomical owing to the large size required and the complexity of installation.

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