Multiuser Scheduler and FDE Design for SC-FDMA MIMO Systems

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Abstract—This paper presents a novel spatial frequency domain packet scheduling and frequency domain equalization (FDE) algorithm for uplink Single Carrier (SC) Frequency Division Multiple Access (FDMA) multiuser MIMO systems. The multiuser scheduler is designed to exploit the available multiuser diversity in time, frequency and spatial domains. Our analysis model is confined to 3GPP uplink SC-FDMA transmission with Multi-user (MU) Spatial Division Multiplexing (SDM). The results show that the proposed MU-MIMO scheduler in conjunction with the new FDE significantly increases the maximum achievable rate and improves the bit error rate (BER) performance for the system under consideration.

Keywords— SC-FDMA, Multiple-Input Multiple-Output (MIMO), packet scheduling, FDE, 3GPP LTE uplink, multiuser diversity, fading channel, multiuser channel.

I. INTRODUCTION

Single Carrier (SC) Frequency Division Multiple Access (FDMA) techniques for uplink transmission have attracted appreciable attention because of its low Peak to Average Power Ratio (PAPR) property compared with competitive Orthogonal FDMA (OFDMA) techniques [1–3]. In 3GPP Long Term Evolution (LTE), SC-FDMA has been adopted for uplink transmission, whereas the OFDMA signaling format has been exploited for the downlink transmission [4]. The SC-FDMA signal can be obtained by applying Discrete Fourier Transform (DFT) to convert time domain input data symbols to the frequency domain before feeding them into an OFDMA modulator.

From user capacity point of view, MIMO technique is preferred due to its capacity enhancement ability. For wide band wireless transmission systems, e.g., LTE OFDMA downlink and SC-FDMA uplink [5, 6], to simply scheduling task, several consecutive subcarriers are usually grouped together for scheduling. A basic scheduling unit is called a Resource Block (RB). The scheduler in a Base Station (BS) may assign single or multiple RBs to a Mobile Station (MS).

Two MIMO schemes for SC-FDMA uplink transmission are being investigated under 3GPP LTE, namely, multi-user MIMO and single user MIMO. For a single user MIMO, the BS only schedule one single user into one RB. For a multi-user MIMO, multiple MSs are allowed to transmit simultaneously on a RB. This paper investigates space frequency domain scheduling for a SC-FDMA based uplink in a multi-user MIMO system. The novelties of this paper are the derivation of the received Signal to Interference plus Noise Ratio (SINR) and the proposal of a scheduling algorithm in space and frequency domain.

The use of frequency domain linear equalization (FD-LE) [7] is a common assumption in the SC-FDMA based LTE uplink

systems due to its simplicity, and is also the focus of our work. We show that the conventional frequency domain equalizers are suboptimal for improper signals, for which the performance can be greatly improved by taking into consideration the complete second order description of the received signal in frequency domain.

Notations: we use upper bold-face letters to represent matrices and vectors. The (n, k) element of a matrix $\mathbf{A}$ is represented by $[\mathbf{A}]_{n,k}$ and the n th element of a vector $\mathbf{b}$ is denoted by $[\mathbf{b}]_n$. Superscripts $(\cdot)^H$, $(\cdot)^T$ denote the Hermitian transpose and transpose, respectively, $(\cdot)^*$ denotes conjugate.

II. SYSTEM MODEL

The cellular multiple access system under study has n_R receive antennas at the BS and a single transmit antenna at the i th user terminal, $i = 1, 2, \dots, K_T$ where K_T is the total number of users in the system. We consider the multi-user MIMO case with K ($K < K_T$) users being served at each time slot and $K = n_R$. On the transmitter side, the user data block containing N symbols is firstly transformed by a N point DFT to a frequency domain representation. The outputs are then mapped to M ($M > N$) orthogonal subcarriers followed by a M point Inverse Fast Fourier Transform (IFFT) to convert to a time domain complex signal sequence. A Circle Prefix (CP) is inserted into the signal sequence before it is passed to the Radio Frequency (RF) module. On the receiver side, the opposite operating procedures are performed after the noisy signals are received by the receive antennas.

A MIMO Frequency Domain Equalizer (FDE) is applied to the frequency domain signals after subcarrier demapping. For simplicity, we employ a linear Minimum Mean Squared Error (MMSE) receiver, which provides a good tradeoff between the noise enhancement and the multiple stream interference mitigation [8].

In the following, we let $\mathbf{D}_{\mathbf{F}_M} = \mathbf{I}_K \otimes \mathbf{F}_M$ and denote by $\mathbf{F}_M$ the $M \times M$ Fourier matrix with the element $[\mathbf{F}_M]_{m,k} = \exp(-j\frac{2\pi}{M}(m-1)(k-1))$ where $k, m \in \{1, \dots, M\}$ is the sample number and the frequency tone number, respectively. Here $\otimes$ is the Kronecker product, $\mathbf{I}_K$ is the K dimension identity matrix. We denote $\mathbf{D}_{\mathbf{F}_M}^{-1} = \mathbf{I}_K \otimes \mathbf{F}_M^{-1}$ as the $KM \times KM$ dimension inverse Fourier matrix, and $\mathbf{F}_M^{-1}$ is the $M \times M$ inverse Fourier matrix with the element $[\mathbf{F}_M^{-1}]_{m,k} = \frac{1}{M} \exp(j\frac{2\pi}{M}(m-1)(k-1))$. $\mathbf{D}_{\mathbf{F}_N}$ and $\mathbf{D}_{\mathbf{F}_N}^{-1}$ are defined in the similar way as

$\mathbf{D}_{\mathbf{F}_M}$ and $\mathbf{D}_{\mathbf{F}_M}^{-1}$ with the only difference in the matrix size. Furthermore, we let F_n represent the subcarrier mapping matrix of size $M \times N$ and F_n^{-1} is the subcarrier demapping matrix of size $N \times M$.

The received signal after the RF module and CP removal becomes $\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\mathbf{D}_{\mathbf{F}_M}^{-1}(\mathbf{I}_K \otimes F_n)\mathbf{D}_{\mathbf{F}_N}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}$, where $\tilde{\mathbf{x}} = [\tilde{\mathbf{x}}_1^T, \dots, \tilde{\mathbf{x}}_K^T]^T \in \mathbb{C}^{KN \times 1}$ is the data sequence of all K users, and $\tilde{\mathbf{x}}_i \in \mathbb{C}^{N \times 1}$, $i \in \{1, \dots, K\}$, is the transmitted user data block for the i th user; $\tilde{\mathbf{w}} \in \mathbb{C}^{Mn_R \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{Mn_R \times Mn_R}$, i.e., $\tilde{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$; $\tilde{\mathbf{H}}$ is an $n_RM \times KM$ channel matrix.

III. FREQUENCY DOMAIN EQUALIZATION (FDE)

With the MIMO FDE, the time domain signal output is given by

$$\begin{aligned}\tilde{\mathbf{z}} &= \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{A}^H(\mathbf{I}_K \otimes F_n^{-1})\mathbf{D}_{\mathbf{F}_M}\tilde{\mathbf{r}} \\ &= \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{A}^H(\mathbf{I}_K \otimes F_n^{-1})\mathbf{D}_{\mathbf{F}_M}(\tilde{\mathbf{H}}\mathbf{D}_{\mathbf{F}_M}^{-1}(\mathbf{I}_K \otimes F_n)\mathbf{D}_{\mathbf{F}_N}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}) \\ &= \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{A}^H(\mathbf{H}\mathbf{D}_{\mathbf{F}_N}\tilde{\mathbf{x}} + \mathbf{w}) = \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{z},\end{aligned}$$

(1) where $\mathbf{y} = [\mathbf{r} \quad \mathbf{r}^*]^T$, and

where $\mathbf{A}$ is a $KN \times KN$ equalization matrix and $\mathbf{H} = (\mathbf{I}_K \otimes F_n^{-1})\mathbf{D}_{\mathbf{F}_M}\tilde{\mathbf{H}}\mathbf{D}_{\mathbf{F}_M}^{-1}(\mathbf{I}_K \otimes F_n) \in \mathbb{C}^{KN \times KN}$; $\mathbf{w} \in \mathbb{C}^{n_RN \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0\mathbf{I} \in \mathbb{R}^{n_RN \times n_RN}$, i.e., $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$.

In the frequency domain, $\mathbf{z} = \mathbf{A}^H[\mathbf{H}\mathbf{D}_{\mathbf{F}_N}\tilde{\mathbf{x}} + \mathbf{w}]$. The vector $\tilde{\mathbf{x}}$ can be expressed as $\tilde{\mathbf{x}} = \mathbf{P} \cdot \tilde{\mathbf{s}}$, where $\tilde{\mathbf{s}} = [\tilde{\mathbf{s}}_1^T \dots \tilde{\mathbf{s}}_K^T]^T$ and $\tilde{\mathbf{s}}_i \in \mathbb{C}^{N \times 1}$, $i \in \{1, 2, \dots, K\}$, is the user data block for the i th user, and $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H] = \mathbf{I}_N$. The power loading matrix $\mathbf{P} \in \mathbb{R}^{KN \times KN}$ is a block diagonal matrix with its i th sub-matrix defined as $\mathbf{P}_i = \text{diag}\{\sqrt{p_{i,1}}, \sqrt{p_{i,2}}, \dots, \sqrt{p_{i,N}}\} \in \mathbb{R}^{N \times N}$ and $p_{i,n}$ ($i \in \{1, 2, \dots, K\}$) is the transmitted power for the i th user at the n th subcarrier; $\tilde{\mathbf{s}} \in \mathbb{C}^{KN \times 1}$ represents the transmitted data symbol vector from different users with $E[\tilde{\mathbf{s}}\tilde{\mathbf{s}}^H] = \mathbf{I}_{KN}$.

In the frequency domain, the received signal can be expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w} = \mathbf{H}\mathbf{P}\mathbf{D}_F\tilde{\mathbf{s}} + \mathbf{w}, \quad (2)$$

where $\mathbf{s} = \mathbf{D}_{F_N}\tilde{\mathbf{s}}$ is the transmitted signal in the frequency domain.

We apply the FDE matrix $\mathbf{A}$ on $\mathbf{r}$ to obtain the equalized signal $\mathbf{z} = \mathbf{A}^H\mathbf{r}$, where $\mathbf{A}$ in the conventional system is derived by minimizing the cost function $e = E[\|\mathbf{z} - \mathbf{s}\|^2] = E[\|\mathbf{A}^H\mathbf{r} - \mathbf{s}\|^2]$, leading to the optimal FDE matrix $\mathbf{A} = \mathbf{R}_r^{-1}\mathbf{T}_r$ [9], where the autocorrelation matrix $\mathbf{R}_r = E[\mathbf{r}\mathbf{r}^H] = \mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H + N_0\mathbf{I}$, and the cross-correlation matrix $\mathbf{T}_r = E[\mathbf{r}\mathbf{s}^H] = \mathbf{H}\mathbf{P}$. This solution is optimum for systems with modulations schemes, such as M -QAM and M -PSK (for which $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H] = \mathbf{0}$). However, for the PAM signals (for which $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H] = \mathbf{I} \neq \mathbf{0}$), the conventional solution becomes suboptimum as will become evident later on. In (2), let us assume $\tilde{\mathbf{s}}_i \in \mathbb{C}^{N \times 1}$ is a PAM signal vector, satisfying the condition $E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H] = E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H] = \mathbf{I}_N$. Since $E[\mathbf{s}_i\mathbf{s}_i^H] = \mathbf{D}_F E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H]\mathbf{D}_F^T = \mathbf{D}_F\mathbf{D}_F^T = \mathbf{I}_N$, $E[\mathbf{s}_i\mathbf{s}_i^H] = \mathbf{D}_F E[\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_i^H]\mathbf{D}_F^T = \mathbf{D}_F\mathbf{D}_F^T \neq \mathbf{0}$, we can conclude that $\mathbf{s}_i$ is an improper signal vector, for which pseudo-autocorrelation $\tilde{\mathbf{R}}_r = E[\mathbf{r}\mathbf{r}^T] \neq \mathbf{0}$ and pseudo-cross correlation $\tilde{\mathbf{T}}_r = E[\mathbf{r}\mathbf{s}^T] \neq \mathbf{0}$

since

$$\begin{aligned}\tilde{\mathbf{R}}_r &= E\{\mathbf{r}\mathbf{r}^T\} = E\{(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w})(\mathbf{s}^T\mathbf{P}^T\mathbf{H}^T + \mathbf{w}^T)\} \\ &= \mathbf{H}\mathbf{P} E[\mathbf{s}\mathbf{s}^T]\mathbf{P}^T\mathbf{H}^T = \mathbf{H}\mathbf{P}\mathbf{D}_F\mathbf{D}_F^T\mathbf{P}^T\mathbf{H}^T, \\ \tilde{\mathbf{T}}_r &= E\{\mathbf{r}\mathbf{s}^T\} = \mathbf{H}\mathbf{P} E[\mathbf{s}\mathbf{s}^T] = \mathbf{H}\mathbf{P}(\mathbf{D}_F\mathbf{D}_F^T)\end{aligned}$$

In the conventional FDE, the coefficient matrix $\mathbf{A}$ is calculated using only the autocorrelation of the observation $\mathbf{R}_r$ and the cross correlation $\mathbf{T}_r$, while $\tilde{\mathbf{R}}_r$ and $\tilde{\mathbf{T}}_r$ are implicitly assumed to be zero, leading to sub-optimum solutions. For improper signals, $\mathbf{R}_r$, $\tilde{\mathbf{R}}_r$, $\mathbf{T}_r$, $\tilde{\mathbf{T}}_r$ give a complete second order description of the received signal, therefore, all of them should be used for deriving the filter coefficient matrix [10, 11]. In order to utilize $\tilde{\mathbf{R}}_r$ and $\tilde{\mathbf{T}}_r$, we redefine the MMSE criterion as follows

$$\epsilon = E[\|\Gamma^H\mathbf{y} - \mathbf{s}\|^2] = \Gamma^H\mathbf{R}_y\Gamma - \Gamma^H\mathbf{T}_y - \mathbf{T}_y^H\Gamma + \mathbf{I}_N, \quad (3)$$

$$\begin{aligned}\mathbf{R}_y &= E\{\mathbf{y}\mathbf{y}^H\} = \begin{bmatrix} \mathbf{R}_r & \tilde{\mathbf{R}}_r \\ \tilde{\mathbf{R}}_r^* & \mathbf{R}_r^* \end{bmatrix}; \\ \mathbf{T}_y &= E\{\mathbf{y}\mathbf{s}^H\} = E\left\{\begin{bmatrix} \mathbf{r} \\ \mathbf{r}^* \end{bmatrix} \mathbf{s}^H\right\} = \begin{bmatrix} \mathbf{T}_r \\ \tilde{\mathbf{T}}_r^* \end{bmatrix}\end{aligned} \quad (4)$$

Differentiating ϵ in (3) with respect to Γ and setting it to zero yields the optimum vector of Γ

$$\begin{aligned}\Gamma &= \mathbf{R}_y^{-1}\mathbf{T}_y \\ &= \begin{bmatrix} \mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H + N_0\mathbf{I} & \mathbf{H}\mathbf{P}\mathbf{D}_F\mathbf{D}_F^T\mathbf{P}^T\mathbf{H}^T \\ \mathbf{H}^*\mathbf{P}^*\mathbf{D}_F^*\mathbf{D}_F^H\mathbf{P}^H\mathbf{H}^H & \mathbf{H}^*\mathbf{P}^*\mathbf{P}^T\mathbf{H}^T + N_0\mathbf{I} \end{bmatrix}^{-1} \\ &\quad \cdot \begin{bmatrix} \mathbf{H}\mathbf{P} \\ \mathbf{H}^*\mathbf{P}(\mathbf{D}_F\mathbf{D}_F^T)^* \end{bmatrix}.\end{aligned} \quad (5)$$

IV. SINR AND MI EXPRESSIONS

In this section, we will derive the mathematical expressions of the SINR for SC FDMA uplink MIMO systems with improved FDE. The calculated SINR values are used to map the mutual information (MI) of the studied systems. The maximum mutual information [12, 13] represents the maximum achievable rate of systems with finite signal constellation constrained signaling [14, 15], e.g., QPSK, PAM, etc.

A. SINR expression for improved FDE

With the improved FDE, the frequency domain signal is given by $\mathbf{z} = \zeta\mathbf{r} + \eta\mathbf{r}^*$. The corresponding time domain representation is

$$\begin{aligned}\tilde{\mathbf{z}} &= \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{z} = \mathbf{D}_{\mathbf{F}_N}^{-1}(\zeta(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w}) + \eta(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{w})^*) \\ &= \mathbf{D}_{\mathbf{F}_N}^{-1}(\zeta(\mathbf{H}\mathbf{P}\mathbf{D}_F\tilde{\mathbf{s}} + \mathbf{w}) + \eta(\mathbf{H}\mathbf{P}\mathbf{D}_F\tilde{\mathbf{s}} + \mathbf{w})^*)\end{aligned} \quad (6)$$

Let $\mathbf{C} = \zeta\mathbf{H}\mathbf{P}$ and $\mathbf{Q} = \eta\mathbf{H}^*\mathbf{P}$ and decompose $\mathbf{C}$ and $\mathbf{Q}$ into the block matrices $\mathbf{C}_{ij}$ and $\mathbf{Q}_{ij}$, respectively, in the similar way as for decomposing matrix $\mathbf{A}$. The time domain received

signal for the i th user is then

$$\begin{aligned}\tilde{\mathbf{z}}_i &= \sum_{j=1, j \neq i}^K \mathbf{F}_N^{-1} \mathbf{C}_{ij} \mathbf{F}_N \tilde{\mathbf{s}}_j + \mathbf{F}_N^{-1} \mathbf{C}_{ii} \mathbf{F}_N \tilde{\mathbf{s}}_i + \sum_{j=1}^K \mathbf{F}_N^{-1} \zeta_{ij} \mathbf{w}_j \\ &+ \sum_{j=1, j \neq i}^K \mathbf{F}_N^{-1} \mathbf{Q}_{ij} \mathbf{F}_N^* \tilde{\mathbf{s}}_j^* + \mathbf{F}_N^{-1} \mathbf{Q}_{ii} \mathbf{F}_N^* \tilde{\mathbf{s}}_i^* + \sum_{j=1}^K \mathbf{F}_N^{-1} \boldsymbol{\eta}_{ij} \mathbf{w}_j^*\end{aligned}$$

The k th symbol of $\tilde{\mathbf{z}}_i$ can be expressed as

$$\begin{aligned}\tilde{z}_i(k) &= \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ii} \mathbf{F}_N(:, k) \tilde{s}_i(k) + \mathbf{Q}_{ii} \mathbf{F}_N(:, k)^* \tilde{s}_i(k)^*) \\ &+ \sum_{j=1, j \neq i}^N \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ij} \mathbf{F}_N(:, j) \tilde{s}_i(j) + \mathbf{Q}_{ij} \mathbf{F}_N(:, j)^* \tilde{s}_i(j)^*) \\ &+ \sum_{j=1, j \neq i}^K \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ij} \mathbf{F}_N \tilde{s}_j + \mathbf{Q}_{ij} \mathbf{F}_N^* \tilde{s}_j^*) \\ &+ \sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\zeta_{ij} \mathbf{w}_j + \boldsymbol{\eta}_{ij} \mathbf{w}_j^*)\end{aligned}\quad (8)$$

The first term on the right hand side of (8) represents the desired signal, the second term is the intersymbol interferences from the same substream, the third term is the interference from the other substreams, and the fourth one is the noise. The power of the received desired signal is then

$$\begin{aligned}P_d^i(k) &= \mathbf{F}_N^{-1}(k, :) \mathbf{C}_{ii} \mathbf{F}_N(:, k) \mathbf{F}_N(:, k)^{\mathcal{H}} \mathbf{C}_{ii}^{\mathcal{H}} \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}} \\ &+ \mathbf{F}_N^{-1}(k, :) \mathbf{Q}_{ii} \mathbf{F}_N(:, k)^* [\mathbf{F}_N(:, k)^*]^{\mathcal{H}} \mathbf{Q}_{ii}^{\mathcal{H}} \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}} \\ &+ \mathbf{F}_N^{-1}(k, :) \mathbf{Q}_{ii} \mathbf{F}_N(:, k)^* \mathbf{F}_N(:, k)^{\mathcal{H}} \mathbf{C}_{ii}^{\mathcal{H}} \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}} \\ &+ \mathbf{F}_N^{-1}(k, :) \mathbf{C}_{ii} \mathbf{F}_N(:, k) \mathbf{F}_N(:, k)^{\mathcal{H}} \mathbf{Q}_{ii}^{\mathcal{H}} \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}}.\end{aligned}\quad (9)$$

Eq. (9) holds since $\mathbb{E}[\tilde{s}_i(k)^* \tilde{s}_i(k)^{\mathcal{H}}] = \mathbb{E}[\tilde{s}_i(k) \tilde{s}_i(k)^{\mathcal{T}}] = \mathbf{I}_N$ for improper signal vector $\tilde{s}_i(k)$.

The total power of the received signal can be expressed as

$$\begin{aligned}P_t^i(k) &= \mathbb{E}[\sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ij} \mathbf{F}_N \tilde{s}_j + \mathbf{Q}_{ij} \mathbf{F}_N^* \tilde{s}_j^*) \\ &(\sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ij} \mathbf{F}_N \tilde{s}_j + \mathbf{Q}_{ij} \mathbf{F}_N^* \tilde{s}_j^*))^{\mathcal{H}}] \\ &= \sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\mathbf{C}_{ij} \mathbf{C}_{ij}^{\mathcal{H}} + \mathbf{Q}_{ij} \mathbf{Q}_{ij}^{\mathcal{H}} \\ &+ \mathbf{Q}_{ij} \mathbf{F}_N^* \mathbf{F}_N \mathbf{C}_{ij}^{\mathcal{H}} + \mathbf{C}_{ij} \mathbf{F}_N \mathbf{F}_N^{\mathcal{T}} \mathbf{Q}_{ij}^{\mathcal{H}}) \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}}.\end{aligned}\quad (10)$$

The power of the noise is

$$\begin{aligned}P_n^i(k) &= N_0 \sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\zeta_{ij} \zeta_{ij}^{\mathcal{H}} + \boldsymbol{\eta}_{ij} \boldsymbol{\eta}_{ij}^{\mathcal{H}}) \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}} \\ &+ \sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\boldsymbol{\eta}_{ij} \mathbb{E}[\mathbf{w}_j^* \mathbf{w}_j^{\mathcal{H}}] \zeta_{ij}^{\mathcal{H}} \\ &+ \zeta_{ij} \mathbb{E}[\mathbf{w}_j \mathbf{w}_j^{\mathcal{T}}] \boldsymbol{\eta}_{ij}^{\mathcal{H}}) \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}} \\ &= N_0 \sum_{j=1}^K \mathbf{F}_N^{-1}(k, :) (\zeta_{ij} \zeta_{ij}^{\mathcal{H}} + \boldsymbol{\eta}_{ij} \boldsymbol{\eta}_{ij}^{\mathcal{H}}) \mathbf{F}_N^{-1}(k, :)^{\mathcal{H}}.\end{aligned}\quad (11)$$

The second equality in (11) follows from the fact that $\mathbb{E}[\mathbf{w}_j^* \mathbf{w}_j^{\mathcal{H}}] = \mathbb{E}[\mathbf{w}_j \mathbf{w}_j^{\mathcal{T}}] = \mathbf{0}$. The received SINR for the i th symbol at time interval k is then

$$\gamma_{imp}^i(k) = \left[\frac{P_t^i(k) + P_n^i(k)}{P_d^i(k)} - 1 \right]^{-1}, \quad (12)$$

where $P_d^i(k)$, $P_t^i(k)$ and $P_n^i(k)$ are given by (9), (10) and (11), respectively.

B. Mutual Information and maximum achievable rate

Based on Gaussian signaling and Shannon's capacity theorem, the maximum achievable spectrum efficiency in bits/second/Hz for the i th user can be expressed as

$$r_i = \log_2(1 + \gamma_i), \quad (13)$$

where γ_i is the received SINR for the i th user, it is given by (12) for the improved equalizer. Due to the space limit, we omitted the SINR expression for the conventional equalizer.

For broadband wireless communication systems, e.g., 3GPP LTE uplink, the total bandwidth B is usually divided into a number of M subcarriers. Among M subcarriers, N subcarriers ($N < M$) are allocated for data transmission. L contiguous subcarriers form a scheduling RB. Let $I_{sub,i}$ and $|I_{sub,i}|$ be the index set of subcarriers assigned to user i and the length of the set $I_{sub,i}$, respectively. Denote by P_t^i the total transmitted power of user i . Assuming that the power is equally allocated over $I_{sub,i}$, then $p_{n,i} = P_t^i / |I_{sub,i}|$. The maximum achievable rate in bits per second for the i th user can then be written as

$$C_i = \frac{B|I_{sub,i}|}{M} \log_2(1 + \gamma_{k,i}). \quad (14)$$

So far, we discussed the maximum achievable rate by considering the channel input using Gaussian signaling. In real LTE systems, discrete time finite size signal constellations, e.g., MQAM, are employed. The maximum achievable rate approach based on Gaussian signaling, e.g., (13) and (14), are therefore likely to be too optimistic for estimating the real achievable rate. In this work, we consider to employ the mutual information between the discrete channel input $\mathbf{u}$ and the channel output $\mathbf{v}$. For a MIMO channel $\mathbf{\Lambda}$ with n_T transmit antennas and n_R receive antennas, we have $\mathbf{v} = \mathbf{\Lambda} \mathbf{u} + \mathbf{v}$. Here $\mathbf{v} \in \mathbb{C}^{n_R \times 1}$ is the white Gaussian noise, with $\mathbb{E}[\mathbf{v} \mathbf{v}^{\mathcal{H}}] = \mathbf{I}_{n_R} \sigma_v^2$. The mutual information can be calculated by

$$\Psi(\mathbf{u}; \mathbf{v}) = H(\mathbf{v}) - H(\mathbf{v}|\mathbf{u}), \quad (15)$$

where $H(\cdot) = -\mathbb{E}[\log_2(p(\cdot))]$ is the entropy function, and $p(\cdot)$ represents the Probability Density Function (PDF). The mutual information can be calculated by [16],

$$\begin{aligned}\Psi(\mathbf{u}; \mathbf{v}) &= -\mathbb{E} \left\{ \log_2 \left(\frac{1}{2^{M n_T}} \frac{1}{(2\pi \sigma_v^2)^{n_R}} \right. \right. \\ &\left. \left. \sum_{\mathbf{u} \in \mathbb{S}} \exp \left[-\frac{\|\mathbf{v} - \mathbf{\Lambda} \mathbf{u}\|^2}{2\sigma_v^2} \right] \right) \right\} - n_R \log_2(2\pi e \sigma_v^2),\end{aligned}\quad (16)$$

In general Eq. (16) cannot be expressed in a closed form. Nevertheless, it can be evaluated by using Monte-Carlo simulations. The mutual information is a function of the received SNR

Fig. 1. Mutual Information versus received SINR for uncoded 1×1 and 2×2 MIMO systems with QPSK signaling and with 4PAM signaling.

at the receiver antennas. The mutual information for uncoded 1×1 and 2×2 MIMO systems with QPSK signaling and the system with 4PAM signaling are shown in Fig. 1. The x-axis is the received SINR in dB. Both the simulation results and the curve fitting results are given.

V. SPATIAL FREQUENCY MULTIUSER SCHEDULING

For localized FDMA uplink multiuser MIMO transmission¹, each SC-FDMA uplink transmission sub-frame can be partitioned into several RBs for the convenience of multiple user channel aware packet scheduling [4, 5]. Let $I_{RB,i}$ be the index set of RBs assigned to user i within one sub-frame and $|I_{RB,i}|$ be the length, the number of total RBs in one sub-frame is $|I_{RB}|$. Then $|I_{RB,i}|L = |I_{sub,i}|$. Multiple contiguous RBs can be assigned to one user within one sub-frame.

Denote by ϕ_j the j th set of K users which are selected from the total K_T users in the system and let Φ be the whole set of K users chosen from total K_T users, $\phi_j \in \Phi, \forall j \in \{1, 2, \dots, |\Phi|\}$, where $|\Phi|$ is the size of Φ , and

$$|\Phi| = \binom{K_T}{K}.$$

Let us define $U_j(\phi)$ as the utility function for the j th RB. As will show later, $U_j(\phi)$ is a function of the MIs of the scheduled users. The objective is to maximize the utility function by selecting the users group with appropriate channel condition and optimizing the set of RBs assigned to each user within one sub-frame. The optimization problem can be described as

$$\begin{aligned} \max_{\forall \phi \in \Phi; \phi: I_{RB,i}, P_t^i, \forall i \in \phi} & \sum_{j=1}^{|I_{RB}|} U_j(\phi), \\ \text{s.t.1:} & \bigcup_{i \in \phi} |I_{RB,i}|L = N, \\ \text{s.t.2:} & I_{sub,i}^{k+1} - I_{sub,i}^k = 1, \\ & \forall k \in \{1, 2, \dots, |I_{sub,i}| - 1\}, \end{aligned} \quad (17)$$

where $I_{sub,i}^k$ is the k th element in the set $I_{sub,i}$. The subconstraint 1 ensures that all the available RBs are assigned to the users in ϕ . The subconstraint 2 corresponds to the localized FDMA transmission, i.e., the user data is transmitted by a group of consecutive subcarriers. Note that the above optimization problem is not equivalent to maximize the utility function for each RB subject to the user's power constraint. This is because if we consider each RB independently, the channel frequency selective fading property cannot be exploited. The frequency selective fading channel property may make some users experience excellent channel condition for two or more consecutive RBs. Since the power of the i th user is constrained to be P_t^i , and the power for each subcarrier is obtained by $p_{n,i} = P_t^i / |I_{sub,i}|$, in the case of multiple RBs assigned for the i th user, the power for each subcarrier $p_{n,i}$ of that user is reduced.

For the multiuser MIMO scheduling scheme, the set of the RBs should be optimized for each user within each transmitted sub-frame. The optimization problem can be summarized as: among K_T users, we choose K of them and allocate these K users to the available RBs to maximize the utility function (17). The optimal solution to the optimization problem involves a high computational complexity. Therefore, low complexity suboptimal algorithms are needed for practical implementation. In what follows, we propose a greedy algorithm to solve the above optimization problem. The algorithm is performed in two steps: the first step is to schedule users for each RB, i.e., find users group or paired users for each RB to optimize the utility function. The second step is to assign available RBs for the paired users.

For the first step, we need to find the best users group for each RB. At this stage, we can maximize the utility function for each RB. We can define $U(\phi) = \sum_{i \in \phi} \Psi_i$, where Ψ_i is the MI for user i , which is defined by Eq. 16. Maximization of this utility function is equivalent to optimization of the maximum achievable rate for systems with a finite alphabet constrained signal constellation. This may result in an unfair situation, i.e., only the users with good channel conditions get resources.

To tackle this problem, we propose a resource fair allocation algorithm for each RB based utility function maximization. The key idea of the proposed fair resource allocation algorithm is to limit the users with more RBs used in a past certain period T_{win} , and give priority to those users with less transmissions in the period T_{win} . The algorithm works as follow: Let α_i be the moving average of used RBs by the i th user in the past T_{win} at interval k and $\alpha_i^k = (1 - \frac{1}{T_{win}})\alpha_i^{(k-1)} + \frac{1}{T_{win}}\delta$, where $\delta = 1$ if the user i gets scheduled, otherwise $\delta = 0$. We define the utility function at the k th interval as $U_k(\phi) = \sum_{i \in \phi} f(\alpha_i^k, \Psi_i)$, where $f(\alpha_i^k, \Psi_i)$ is a function of α_i^k and Ψ_i . The per RB based

¹In the localized FDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, while for the distributed FDMA transmission scheme, the user's data is transmitted by distributed subcarriers [4].

scheduling problem then becomes

$$\phi^* = \arg \max_{\phi \in \Phi} \sum_{i \in \phi} f(\alpha_i^k, \Psi_i). \quad (18)$$

For comparison purpose, the Proportional Fair (PF) scheduling algorithm [6, 17] is also investigated in this work.

After the best user groups for each RB are determined, we are ready for the second step. Let $I_{RB,i}^q$ denote the set of RBs allocated to user group i with an additional adjacent RB q added into the set $I_{RB,i}$ within one sub-frame. Let $\Lambda_{k,i}^q$ be the rate increment at time interval k when the i th user group is allocated $I_{RB,i}^q$ instead of $I_{RB,i}$. Then the greedy RB allocation based on rate increment can be described as follows. First, we pick the Q best user groups for each RB in relation to the rate increment, and add them into the available user group set S_{ugp} .

Step 1. Add all available Q RBs into a set $S_{RB} = \{1, 2, \dots, Q\}$.

Step 2. For each RB, find the best user group in terms of the highest rate increment. That is, for each user group i^* and RB q^* , find the pair

$$[i^*, q_i^*] = \arg \max_{q \in S_{RB}, i \in S_{ugp}} \Lambda_{k,i}^q \quad (19)$$

Step 3. For each user group j^* , find an adjacent RB q_o^* of q_j^* from the set $\{S_{RB} - q_j^*\}$ which has the maximum rate increment $\Lambda_{k,j^*}^{q_o^*}$.

Step 4. Choose the user group and the additional adjacent RB pair $[h^*, q_t^*]$, which has the maximum rate increment $\Lambda_{k,h^*}^{q_t^*}$ among all the available user groups and RBs.

Step 5. Delete the RB q_t^* from the available RB set S_{RB} and its corresponding user group t^* in the available user group set S_{ugp} . Repeat Step 3, 4 and 5 until all RBs get assigned.

This greedy algorithm is a suboptimal solution for allocation of the RBs among users. The overall performance is suboptimal when it is combined with the maximum rate sum algorithms. In terms of the capacity performance, the maximum rate sum algorithm achieves the largest capacity when coupled with the greedy algorithm when compared with other scheduling schemes. The fairness among users can be provided by the resource fair scheduling scheme.

VI. NUMERICAL AND SIMULATION RESULTS

For the results presented in this section, we consider the 3GPP LTE baseline antenna configuration with two receive antennas at the BS and one transmit antenna at the MS [18]. Two MSs are grouped together and synchronized to form a virtual MIMO channel between BS and MSs. We consider a frequency selective Rayleigh fading channel model with six independent paths. The system bandwidth is set to 900 kHz with a subcarrier spacing of 15 kHz. Hence there are 60 occupied subcarriers for full band transmission. We further assume these 60 subcarriers are arranged in 5 consecutive RBs per sub-frame, so that each RB contains 12 subcarriers. At each Monte-Carlo run, 100 sub-frames are used for data transmission and the power of each user is randomly generated to simulate the fact that users maybe in different locations. The simulation results are averaged over 50 Monte-Carlo run.

Fig. 2 shows the simulation results for the maximum achievable rate in bits/second versus the number of available users for

Fig. 2. Sum mutual information for SC-FDMA uplink 2 by 2 MIMO system with conventional FDE and the improved FDE, the transmitted SNR is 15 dB.

the uplink MIMO systems with different equalizers and with various scheduling algorithms. The transmitted symbols are selected from a signal constellation for 4PAM and the transmitted SNR is 15 dB. Random user Pairing Scheduling (RPS) algorithm described in [19] is also investigated for a baseline comparison. For random pairing scheduling, the first user is selected in a round robin fashion, while the second user is randomly selected from the rest of the users in the system. It can be seen that as the number of users increases, the multiuser diversity gain can be achieved for all the investigated systems except the one with the RPS algorithm. This observation is valid for both the system with the improved FDE and the one with the conventional FDE. The reason is that those non-random pairing schedulers have more freedom to choose the MSs with good channel condition and multiuser diversity can thus be exploited. Compared with the conventional MMSE equalizer, the proposed equalizer can significantly increase the maximum achievable rate.

In terms of the fairness, we can look at the outage probabilities of these different algorithms. Here, the outage probability is defined as the probability that the user data rate is less than a certain value. It is obtained by computing the ratio of the number of users whose data rates are lower than a certain value divided by the total number of users in the system. Throughout the simulations, the predetermined threshold is set to 0.01. The simulation results are shown in Fig. 3 for the systems with different equalizers. In our simulations, zero outage probabilities for both of the PF scheduling and the proposed resource fair scheduling algorithms have been observed, which means that the PF and resource fair scheduling provide fairness for both schemes. For both systems, the max sum rate scheduling algorithm has the worst performance in terms of the outage probability. This is because the maximum rate sum algorithm always chooses the users with the good channel conditions, the users with poor channel quality have less opportunities to be scheduled. One can see from Fig. 3 that the system with the improved equalizer has better outage probability performance compared with the conventional linear MMSE receiver.

Fig. 3. Outage probability for SC-FDMA uplink 2 by 2 MIMO system with conventional FDE and the improved FDE, the transmitted SNR is 15 dB. Note that the PF scheduling and the proposed resource fair scheduling algorithms have zero outage probability, therefore are not shown in the figure. The curves in the figure correspond to the max sum rate scheduling scheme.

VII. CONCLUSION

In this paper, we introduced an improved spatial frequency domain scheduling and frequency domain equalization algorithm for PAM signals in the SC-FDMA based LTE uplink MIMO system. Mathematical expressions of the received SINR for the studied MIMO systems have been derived. In the current LTE standard, QAM/QPSK modulation schemes are used for the uplink transmission. However, our study reveals that the improperness property of the signal is preserved in the frequency domain for SC-FDMA systems with PAM modulation schemes, and the proposed FDE scheme utilizing the complete second order statistics of the improper signal in frequency domain significantly outperforms the conventional SC-FDMA system with proper modulation schemes, such as QPSK. The comparison has been made under the condition that the two systems have the same transmission rate or spectral efficiency. The investigated scheduling algorithms can achieve a multiuser diversity gain relative to the RPS algorithm, and are able to exploit the available multiuser diversity in time, frequency and spatial domains. When coupled with the improved FDE scheme, the proposed MU-MIMO scheduler significantly increases the maximum achievable rate for the uplink SC-FDMA system under question. This paper addresses fundamental issues in relation to the 4G/LTE standard and provides a useful source of information for the implementation of the practical 4G wireless systems.

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