Formation Deployment & Separation Simulation of Multi-Satellite Scenarios using SatLauncher

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Abstract—Satellite constellation deployment for formation flying missions is one of the key areas for consideration when realizing the final constellation with reduced propellant mass requirements on the propulsion system.^{1 2} The use of a single launch vehicle to deploy multiple satellites into a formation is faster and cheaper but there is greater risk of collision. This risk must be managed with the competing desire to establish a relatively tight formation for better inter-satellite communication. The launcher attitude, satellite injection times and velocities are key parameters to safely achieve a given separation distance and distribution.

This paper presents a visual simulator to propagate the satellite trajectories from the launcher using an expanded definition of Hill's equations, and extending to polar relative motion. It is assumed that a simple launcher is used which is incapable of reposition once in orbit. Low injection velocities are exploited to inject large numbers satellites into a stable constellation. Utilizing small tight natural motion formations help to reduce perturbations and the propellant mass required for formation maintenance.

SatLauncher is a new visualization tool for investigating the relative motion and key parameters between satellites in these new missions and applications for multi-satellite launchers without the need for any further industrial tool. The QB50 mission is taken forward as a representative scenario requiring our latest software tool and new methods are presented towards collision free formation deployment.

TABLE OF CONTENTS

1. INTRODUCTION.....	1
2. BACKGROUND	2
3. SATLAUNCHER	2
4. COLLISION FREE MOTION	3
5. QB50 CASE STUDY	5
6. CONCLUSIONS	7
REFERENCES	7
ACKNOWLEDGEMENTS	7
BIOGRAPHY	8
APPENDIX	8

1. INTRODUCTION

There is an increasing interest in clusters, constellations, formations, or 'swarms' of satellites for numerous space missions including planetary exploration, Earth observation, communications, and atmospheric sensing missions. Greater temporal and spatial resolution in these multi-satellite scenarios as a *distributed satellite system* will allow for greater space services and capabilities as well as greater science return [1]. But how these constellations or formations are typically built takes large amounts of time and money. An example would be Iridium which utilizes 66 satellites in a constellation for global communications coverage which took just over 1 year and 15 launches [2].

Figure 1. Integration of Demos/UK (SS1E ©)

Traditionally, secondary satellites are deployed at the convenience of a primary satellite either before or after the deployment of that primary. A typical configuration is shown in Figure 1 with a loaded Dnepr rocket. The launcher itself performs some maneuvering action to separately launch each secondary to provide separation between satellites for safety. Constellation and formation topologies are built with multiple launches and maneuvering satellites. However the concept of launching a large number of similar non-maneuvering satellites from a maneuvering launcher presents an obstacle to the establishment of a stable collision free formation. To make formation establishment as simple as possible, this paper assumes that the primary launcher

will not be able to maneuver but only control its attitude and provide a slow rotation from which satellites can be ejected.

These problems are investigated at Surrey Space Centre using SatLauncher, a satellite deployment visualization tool used for calculating satellite paths and highlighting potential collisions. Typical orbit deployment speeds, launcher rotation rates, and deployment times are used to better understand how satellites can be kept together in a formation. The QB50 project is targeted as a good representative mission of multi-satellite deployment [3]. QB50 aims to launch over 50 x 2 kg nanosatellites based on the CubeSat standard for multipoint atmospheric sensing.

The paper is organized as follows: Section 2 looks into the background motivation in a representative frame, Section 3 introduces SatLauncher and how it is configured to quickly simulate multiple satellites, Section 4 investigates collision free motion, Section 5 looks at the QB50 mission as a case study for the launcher with suggested methods for multi-satellite formation, and finally Section 6 concludes.

2. BACKGROUND

The deployment of secondary satellites is a well developed field. However the relative motion analysis of these deployments has typically not been a concern as constellations such as Iridium required multiple launchers. For deployment of a large group of satellites desiring a stable formation, low velocity relative motion is of great importance.

Relative motion of satellites in proximity to one another has been an active area of interest and research since well before artificial satellites. Some of the most well known equations to describe relative motion are the Hill-Clohessy-Wiltshire HCW equations and their solutions [4] [5]. The HCW equations are given in Equation (1) and the solutions are given in Equation (2). The rotating frame of interest is known as the Local Vertical Local Horizontal (LVLH) frame. The LVLH frame is centered on, and rotates with, the circular reference orbit comprised of the prime $\hat{x}$ axis pointing radially away from the Earth, the $\hat{y}$ axis points in the direction of the velocity vector, and the $\hat{z}$ axis completes the right-handed coordinate frame and points in the direction of the orbital angular momentum vector. The origin of the LVLH frame may or may not contain a satellite or the

launcher but is the reference orbit upon which the relative motion is defined.

This paper focuses on the process of eliminating collisions during the establishment of a CubeSat formation, which can be modeled within HCW dynamics previously described. It is assumed that mission will require that the satellites of the formation maintain a stable formation with respect to one another. A stable formation in this paper is considered one that does not drift apart over time in the HCW frame. The formation of satellites will however drift away from the original launch vehicle, but must do so in a way that eliminates collision risk. In order to launch a large number of CubeSats with no self maneuvering ability into a large stable formation, like the QB50 mission concept [3], a new tool which exploits natural motion dynamics for large scale collision-free satellite distribution is needed.

3. SATLAUNCHER

Figure 2. SatLauncher

SatLauncher is a Java-based orbit propagation and visualization tool designed for investigating deployment of satellite formations or clusters. The simulator allows the input launcher settings: altitude, radius, rotation rates, and satellite and deployment settings: initial size, position on the launcher, deployment time, and velocity direction (in Cartesian or Epicycle coordinates). A typical simulator window can be found in Figure 2.

A satellite can be inputted via on-screen entry or via a parsed text file. An example is given below of the file format:

$$\begin{aligned} \ddot{x} - 2\Omega\dot{y} - 3\Omega^2x &= 0 \\ \ddot{y} + 2\Omega\dot{x} &= 0 \\ \ddot{z} - \Omega^2z &= 0 \end{aligned} \quad (1)$$

$$\begin{bmatrix} \Omega x(t_2) \\ \dot{x}(t_2) \\ \Omega y(t_2) \\ \dot{y}(t_2) \end{bmatrix} = \begin{bmatrix} 4 - 3\cos(\tau) & \sin(\tau) & 0 & 2(1 - \cos(\tau)) \\ 3\sin(\tau) & \cos(\tau) & 0 & 2\sin(\tau) \\ -6(\tau - \sin(\tau)) & -2(1 - \cos(\tau)) & 1 & -(3\tau - 4\sin(\tau)) \\ -6(1 - \cos(\tau)) & -2\sin(\tau) & 0 & -(3 - 4\cos(\tau)) \end{bmatrix} \begin{bmatrix} \Omega x(t_1) \\ \dot{x}(t_1) \\ \Omega y(t_1) \\ \dot{y}(t_1) \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \Omega z(t_2) \\ \dot{z}(t_2) \end{bmatrix} = \begin{bmatrix} \cos(\tau) & \sin(\tau) \\ -\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \Omega z(t_1) \\ \dot{z}(t_1) \end{bmatrix}, \text{ where } \tau = \Omega(t_2 - t_1)$$

Satellite 0:
 Satellite position in terms of launcher
 coordinates - Theta (degrees): 0; z (meters): -
 0.1; Velocities (meters/second) - x1: 0.0; y1:
 0.0; z1: -0.0100000000 ; Separation Time
 (seconds): 10;

These values are determined using the satellite launch angle in the $\mathbf{xz}$ -plane, φ , as measured from the (+) $\mathbf{x}$ axis and vertical kick off velocity from launch vehicle surface, ΔV_S . The velocity components are calculated using the following equations (based on the SatLauncher reference frames):

$$\begin{aligned}\Delta V_x &= \Delta V_S \cos(\varphi) - n R \sin(\varphi) \\ \text{and } \Delta V_z &= \Delta V_S \sin(\varphi) + n R \cos(\varphi)\end{aligned}\quad (3)$$

In Equation (3) R is the radius of the launcher, and n is the rotation rate of the launcher about the y axis. Motion in the HCW frame is subject to several assumptions and limitations. Some of these include the absence of perturbations like J_2 and drag which can impact the cohesion of satellite formations. For the formations discussed here satellites are considered to be of similar mass and volume using the CubeSat standard. This similarity and the small relative distances of the formation will help to reduce the relative perturbation differences between vehicles which would normally cause the formation to fall apart. As such the goal is to create a constellation which is relatively tight however with room enough for individual satellites to maintain a safe distance. The validity of the linearization of the HCW frame limits the ability to model this formation over large separation distances. A polar model of relative motion, introduced below, allows for an accurate assessment of motion beyond the separation limits of the HCW frame.

4. COLLISION FREE MOTION

Establishing a formation that is collision-free is essential to formation flying missions which do not have independent orbital maneuvering capability. The physical geometry of a formation is not in the scope of this research. However using the collision identification approaches, certain design techniques can be imparted to help ensure formation stability and safety. Additionally, these techniques can also be used to define mission parameters like the minimum kick-off velocity needed to be collision free.

P.L. Palmer developed an approach to decouple the dynamics of relative motion via linear combinations of the state dynamics [6]. Palmer uses a transform relationship of Equation (4) along with the $\mathcal{P}$ matrix of Equation (5) below to transform the in-plane coordinates, $\bar{\mathbf{x}}$, to a set of phase-space, or commonly known as Palmer space, coordinates, $\bar{\mathbf{p}}$. The phase-space state transition matrix defined in Equation (6). From this equation it is apparent that the first two states, $\bar{\mathbf{p}}_{1-2}$, are decoupled from the remaining two states, $\bar{\mathbf{p}}_{3-4}$. These phase space coordinates provide an intuitive understanding of the relative motion differences caused by difference in the orbital elements of the satellites.

Eccentricity differences create the oscillation observed in the dynamics of the $\bar{\mathbf{p}}_{1-2}$ states. Semi-major axis differences create the dynamics observed in the final states including a secular relative drift. Note that if one eliminates the differences in semi-major axis, i.e. $\bar{\mathbf{p}}_{3-4} = \mathbf{0}$, a closed stable relative-orbit can be achieved solely from the oscillations in eccentricity and the coupled dynamics (also known as a safety ellipse). However in this paper we will create a constellation which has no relative drift from the other satellites in the constellation and a constant drift rate from the launcher to allow for a constant separation distance between satellites in the formation.

$$\bar{\mathbf{p}} = \mathcal{P}\bar{\mathbf{x}} = [p_1 \ p_2 \ p_3 \ p_4]^T \quad (4)$$

$$\mathcal{P} = \begin{bmatrix} -3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$\mathcal{P}\Phi(\tau)\mathcal{P}^{-1} = \begin{bmatrix} \cos(\tau) & \sin(\tau) & 0 & 0 \\ -\sin(\tau) & \cos(\tau) & 0 & 0 \\ 0 & 0 & 1 & 3\tau \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Where τ is the same as in Equation (2)

For formation generation from an instantaneous ΔV , similar to the kick-off provided by an upper stage vehicle, the effects of the equivalent ΔV must be added to the natural motion dynamics. The phase-space representation of the motion in terms of the relative motion states and ΔV are given in Equation (7). The launch angle is set by the desired y axis separation distance of members within the formation as shown in Equation (8), where δY is the desired separation distance, and L is the radius of the launcher. Note that without a relative velocity in the y axis the drift rate of the released satellites is only dependent on the release height in the x axis. The desired rotation rate of the launcher, Equation (9) is given by the angular separation of the satellites around the launcher, ϕ_{Sat} , and the number of satellites to launcher per orbital period, N_{Sat} . This rotation rate is dependent on the number of satellite one wishes to launch per orbit, m_{Sat} . In Equations (10) the security distances are given by D_x , D_y and D_z for each axis of motion.

$$\begin{aligned}p_1 &= -3\Omega x - 2(\dot{y} + \Delta V_y) \\ p_2 &= \dot{x} + \Delta V_x\end{aligned} \quad (7)$$

$$\begin{aligned}p_3 &= 2\dot{x} + 2\Delta V_x - \Omega y \\ p_4 &= 2\Omega x + \dot{y} + \Delta V_y\end{aligned}$$

$$\varphi = \cos^{-1}\left(\frac{\delta Y}{12\pi L}\right) \quad (8)$$

$$n = \left(\frac{\phi_{\text{Sat}} N_{\text{Sat}} m_{\text{Sat}}}{2\pi}\right) \quad (9)$$

$$\begin{aligned}\Omega D_x &= 2p_{4_o} + p_{1_o} \cos(\Omega t) + p_{2_o} \sin(\Omega t) \\ \Omega D_y &= p_{3_o} + 3\Omega t p_{4_o} - 2p_{1_o} \cos(\Omega t) \\ &\quad + 2p_{2_o} \sin(\Omega t) \\ \Omega D_z &= \Omega z_o \cos(\Omega t) + \dot{z}_o \sin(\Omega t)\end{aligned} \quad (10)$$

For this analysis, we will consider the launcher to lie at the origin of the HCW frame. For our launch purposes it will be oriented as shown in Figure 3 with nose of the launcher pointed into the velocity vector, y axis.

Figure 3. SatLauncher Coordinate Frames

The initial kick-off, ΔV_S , will be in the x and z axis's, because the geometry of the launcher prevents motion in the y axis. Additionally, a ΔV in the y axis introduces changes to P_4 which would necessitate an exact replication by all satellites in the constellation. The launcher and all satellites will be assumed to have some security distance, a safe keep-out zone around each vehicle, to act as buffer to account for real world uncertainties.

Using Equations (10) which represent the relative separation distances in each axis between satellites in the formation, one can solve for the ΔV_S needed to ensure motion of one satellite remains outside of the keep out zones of the other satellites in the formation. From these equations one can define the minimum ΔV_S needed for collision free motion and then the spring size of a CubeSat launcher.

4.1. Polar Description of Relative Motion

As satellites are deployed over many orbits, their collision-free trajectories drift away from the origin and out of the HCW frame. We must now determine the relative motion within such a wide spread CubeSat formation which goes beyond the local limits of the HCW frame. Also there is a chance of a collision once the formation and launcher have drifted around the orbit and back together. The HCW frame is a

linear approximation of a curved trajectory. The actual curvature of the y axis creates radial errors, in x , when approximating motion with any horizontal displacement, the HCW frame assumes small horizontal displacements and zero radial errors. However actual radial errors can become large even when the angular separation of any two satellites with respect to the Earth remains relatively small.

This limitation of HCW is overcome by reformulating the in-plane equations of motion in polar form, i.e. in terms of a radial displacement from a reference orbit and an angular separation from the host satellite about the centre of the Earth, assuming a circular reference orbit. Relative motion in terms of the new reference coordinates is illustrated in Figure 4. By linearizing about radial displacement and using conservation of angular momentum, equations of relative motion are derived which allow for arbitrary relative separation angles with respect to the Earth. The reformulation of the equations of relative motion can be seen in Equation (11), where the angular momentum is defined as $\mathfrak{J} = (R + \delta s)^2(\dot{\theta} + \Omega)$, δs is the radial displacement from the reference orbit, θ is the relative separation angle, Ω is the reference mean motion, and R is the reference orbit radius. By assuming a small angle approximation for θ , the Equations of (11) simplify to the HCW solutions of Equation (2). The derivation of these equations can be found in the Appendix.

Figure 4. Polar Relative Motion Reference Frame

Via the process for collision identification above, these results allow for the first description of relative motion and collision identification for large separation differences, well beyond the linear range of the HCW equations. These results are still subject to some of the same assumptions as the HCW solutions with regards to a circular reference orbit and no additional perturbations. The greatest errors between these polar equations of motion and the true trajectories will be due to the large potential differences at large separation angles and drift times cause by J_2 . Future papers plan to

$$\begin{bmatrix} bs(t_2) \\ \dot{s}(t_2) \\ b\theta(t_2) \\ \dot{\theta}(t_2) \end{bmatrix} = \begin{bmatrix} \cos(\tau) & \sin(\tau) & 0 & 0 \\ -\sin(\tau) & \cos(\tau) & 0 & 0 \\ \epsilon_2 \sin(\tau) & \epsilon_2(1 - \cos(\tau)) & 1 & 0 \\ \epsilon_2 \cos(\tau) & \epsilon_2 \sin(\tau) & 0 & 0 \end{bmatrix} \begin{bmatrix} bs(t_1) \\ \dot{s}(t_1) \\ b\theta(t_1) \\ \dot{\theta}(t_1) \end{bmatrix} + \begin{bmatrix} bL(1 - \cos(\tau)) \\ bL\sin(\tau) \\ \epsilon_1\tau - \epsilon_2bL\sin(\tau) \\ \epsilon_1 - \epsilon_2bL\cos(\tau) \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \Omega z(t_2) \\ \dot{z}(t_2) \end{bmatrix} = \begin{bmatrix} \cos(\tau) & \sin(\tau) \\ -\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \Omega z(t_1) \\ \dot{z}(t_1) \end{bmatrix}$$

$$\text{Where } \tau = b(t_2 - t_1); b = \frac{3\mathfrak{J}^2}{a^4} - \frac{2\Omega^2}{a}, L = \frac{a\mathfrak{J}^2 - a^5\Omega^2}{3\mathfrak{J}^2 - 2a^2\Omega^4}, \epsilon_1 = \frac{\mathfrak{J}}{R^2} - \Omega + \epsilon_2bL; \epsilon_2 = \frac{-2\mathfrak{J}}{R^3b}$$

address the introduction of J_2 and other geo-potential perturbations into this definition to allow for a more realistic assessment of relative motion at great separation distances.

5. QB50 CASE STUDY

As an example for a multi-satellite deployment scenario, the QB50 concept is chosen for simulation from the von Karman Institute for Fluid Dynamics [7]. This mission investigates the lower thermosphere (90-320 km) using 2 kg picosatellites based on the CubeSat standard. At an altitude of 350 km, atmospheric drag will be the key perturbation which limits the mission to around 3 months. This is highly dependent on orbit simulation tool used [8] and the worst case scenario should be taken forward in missions where there are this many unknowns. There are two proposed deployment schemes: 1) with the velocity vector in the y -plane and 2) radially away from the velocity vector in the xz -plane. These two schemes are investigated using SatLauncher and are visualized in Figure 5 and Figure 6 depicting the launcher with satellites and the paths they take when deployed. For these two examples, two text files with the launcher and satellite constellation settings were parsed to quickly enter the initial conditions as follows:

- Each simulation has four rows of 13 equally spaced and angled satellites around the launcher body.
- To simulate a 3U CubeSat deployer, there is a 5 second delay between each satellite, and again with each row (i.e. Sat0 at 0, Sat 1 at 5 ... Sat13 at 5).
- The maximum time for this simulation is based on the highest allowable error in the HCW frame before the semi-major axis is affected; set to 1 km.

Figure 5. QB50 – In-plane Deployment

Figure 6. QB50 – Radial Deployment

From the in-plane simulation, there are multiple collisions unless each satellite is timed so that they follow each other, as a typical string of pearls formation, shown in Figure 7. The radial deployment creates a ‘corkscrew’ or ‘helix’ formation with each cluster of satellites, shown in Figure 10. The radial deployment shows much slower separation than in-plane deployment as shown in Figures 8 and 9.

Figure 7. Relative Distance for In-plane Deployment (Launcher to all satellites)

Figure 8. Relative Distance for Radial Deployment (Launcher to all satellites)

Figure 9. Relative Distance for Radial Deployment within a sub-cluster (from a CubeSat deployer)

It can be seen in Figure 9 that with arbitrarily chosen deployment times, the CubeSats in a sub cluster almost collide with each other at a minimum range of 0.3 m with a 5 second separation time. Greater consideration for the deployment angles and timing is required to accurately find collision free deployment.

5.1. Collision-free Radial Deployment

From the previous simulations, the radial deployment method can be analyzed further by choosing keep-out zones and separation times. As an example, the following parameters are chosen:

- Each simulation has four rows of 13 equally spaced and angled satellites around the launcher body with

an angle, $\varphi = 27.69$ deg

- The launcher radius, L , from where the CubeSats are launched is 0.6 meters.
- There is a 1373.07 second delay between each satellite as the launcher rotates at a rate of $n = 0.02$ deg/sec. This allows four satellites to be launched per orbit, and the formation is established over 13 orbits.
- It is desired in this simulation to keep the drift between every fourth satellite to less than 10 meters to allow for intersatellite communication.
- The minimum kick off velocity, $\Delta V_s = 0.00555$ was solved to allow collision free motion assuming a 1x1x1 metre security distance around each CubeSat while keeping the formation as tight as possible to minimize relative perturbation and reduce communication distances.

This constellation produces a stretched helix of satellites, with one spacecraft at each quarter of the helix, all with a short relative separation distances to many other satellites and collision free motion within the formation. The sizes of the constellation can be easily scaled with the Equations (8)-(10). From the polar representations of motion, we can also see the relative separation distances of each satellite in the constellation. These distances can be seen in Figure 11 below over 8 orbital periods with a final controlled formation in Figure 12.

Figure 11. Relative Distances (Launcher to all satellites)

Comparing Figures 10 and 12, SatLauncher and the

formation design equations presented here have resulted in a controlled radial deployment of 50 picosatellites in a collision free formation in a total of 140 m. This demonstrates how valuable SatLauncher can be in the future exploration of multi-satellite deployment from a single launcher rocket body, as targeted by the QB50 project. The numbers shown in this paper can be increased for larger groups with various deployment configurations.

6. CONCLUSIONS

This paper has overviewed SatLauncher, a new simulation tool for investigation multi-satellite deployment from a single non-maneuvering launcher. The HCW and polar coordinate frame equations are presented towards collision free relative motions which are implemented in the simulator with the aim of introducing perturbation effects for higher accuracy modeling of new constellations.

The QB50 mission is taken forward with two proposed deployment schemes: in-plane and radial orbit injection to find that radial deployment had the potential to keep satellites in a closer formation than the in-plane scheme. This is then analyzed further by choosing a separation distance and determining picosatellite deployer injection velocities and launcher spin rates. An example deployment of 52 CubeSats within a 140 m range in a collision free formation shows how new multi-satellite formation or clustering missions can be developed using a single launcher for a number of new and exotic applications. We hope the visualization of these constellations will prove to be invaluable in multi-satellite cluster and formation missions.

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BIOGRAPHY

Dr Christopher P. Bridges is a Research Fellow in the Astrodynamics Group at Surrey Space Centre. He gained his BEng in Electronic Engineering from the University of Greenwich and completed his PhD in 'Agent Computing for Distributed Satellite Systems' at the University of Surrey in 2009. He won the PhD+

scholarship to continue work on CubeSat computing payloads and Java technologies and has since started in the Astrodynamics Group on mission analysis and a visual inspection payload.

Luke Sauter is a Captain in the U.S. Air Force currently on assignment to AFIT/CI at the University of Surrey, UK. Here he is a PhD researcher working with the Astrodynamics Group. Prior to this assignment he has held positions as an instructor of Astronautics at the U.S. Air Force

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Dr Phil Palmer is a Reader in astrodynamics at the Surrey Space Centre. He holds degrees in Astrophysics from Leicester University and Mathematics from Cambridge University, and completed his PhD at the Institute of Astronomy, Cambridge. Phil was appointed Deputy Director of

the Surrey Space Centre (2000-2005) and currently is head of the Astrodynamics team. He has established a research group in Spacecraft Autonomy, supports the developments of the Space Propulsion group, and has close links with the Mathematics department at Surrey. Phil has published widely in the areas of Astronomy, Computer Vision, Pattern Recognition, Remote Sensing and Astronautics. He is a member of the AIAA and a fellow of the IET.

APPENDIX

The following is a description of the derivation for polar equations of relative motion. Referencing Figure 4, the force of gravity can be directly related to the mass times acceleration, assuming a constant angular rotation rate for some point in the orbit, $\dot{f}$, expressed in polar form:

$$\frac{-\mu}{R^2} = \ddot{R} - R\dot{f}^2$$

Where, μ is the gravitational parameter

Note that this motion is only defined in the orbital plane. The out of plane motion remains decoupled and shares the same solution as the HCW equations. The angular separation of the satellite from some stationary point on an orbit, f , can be related to the relative separation angle, θ , from a point moving on the reference orbit with a constant mean motion, n , is: $f = \alpha + \theta$, where α is the separation of the moving reference point with respect to that same stationary point on the orbit. Defining angular momentum squared, $\mathfrak{J}^2$, and multiplying the above equation by R , one can directly replace variables for the constant of orbital motion.

$$\mathfrak{J}^2 = R^4 \dot{f}^2$$

$$-R\mu = R^3 \ddot{R} - \mathfrak{J}^2$$

Refining our definition for the position of the secondary satellite with respect to the host reference orbit, R , and again assuming a circular reference orbit: $R = a + s$. Where, a is the semi-major axis of the reference orbit and s is the radial displacement as shown in Figure 4. Using Kepler's 3rd law, an expression is developed for the relative radial distance as a function of angular momentum and the mean motion of the reference orbit. Additionally the expression for the angular separation velocity can also be represented in terms of s and $\mathfrak{J}$.

$$-a^3 n^2 (a + s) = (a + s)^3 \ddot{s} - \mathfrak{J}^2$$

$$\dot{f} a^2 = \frac{\mathfrak{J} a^2}{(s + a)^2}$$

Using binomial expansion and linearizing about s , i.e. s^2 terms are essentially zero, a second order differential equation for radial displacement and an equation for the angular velocity arises. If one assumes that $(a + s)^3 \approx a^3$ the b term simplifies to the mean motion of the circular reference orbit, n , and the n^2 term multiples by a . This equation and the standard form of the solution for the radial displacement about a circular reference orbit are given below:

$$\ddot{s} + b^2 s = \frac{\mathfrak{J}^2}{a^3} - n^2$$

$$\text{Where, } b = \frac{3\mathfrak{J}^2}{a^4} - \frac{2n^2}{a}$$

$$\dot{f} = \frac{\mathfrak{J}}{a^2} \left(1 - \frac{2s}{a} \right)$$

The solutions for the second order differential equation of radial displacement are given below for radial displacement and rate of change of that displacement in terms of the initial conditions.

$$s(t) = D_1 \cos(bt) + D_2 \sin(bt) + L$$

$$\dot{s}(t) = -D_1 n \sin(bt) + D_2 n \cos(bt)$$

$$\text{Where, } D_1 = s(t_0) - L, D_2 = \frac{\dot{s}(t_0)}{b}, \text{ and } L = \frac{a\mathfrak{J}^2 - a^5 n^2}{3\mathfrak{J}^2 - 2a^2 n^4}$$

Substituting the equation for radial displacement into the angular separation velocity and integrating gives an equation for angular separation from a stationary point. Angular separation velocity with respect to the moving reference frame by the following relationship: $\dot{f} = \dot{\theta} + n$, because

$\dot{\alpha} = n$ for circular orbits. Integrating, and solving for initial conditions results in the following equations for the angular separation and separation velocity as a function of time and initial conditions.

$$\begin{aligned}\theta(t) &= s(t_0)G_1 \sin(bt) + \frac{\dot{s}(t_0)G_1}{b}(1 - \cos(bt)) + \theta(t_0) \\ &\quad - \frac{\dot{s}(t_0)2\mathfrak{I}}{a^3b^2} + G_2t - G_1L \sin(bt) \\ \dot{\theta}(t) &= s(t_0)G_1 \cos(bt) + \dot{s}(t_0)G_1 \sin(bt) + G_2 \\ &\quad - G_1bL \cos(bt)\end{aligned}$$

Where, $G_1 = \frac{-2\mathfrak{I}}{a^3b}$, and $G_2 = \frac{\mathfrak{I}}{a^2} - n + bG_1L$

This completes the derivation of the polar form of relative motion allowing for large separation angles and only linearizing about the radial displacement. Note that $R = a$ for the reference orbit as well as $\Omega = n$. These equations can be written more compactly as illustrated in Equation (11) where the decoupled out of plane motion is also given. If the earlier noted assumption, $(a + s)^3 \approx a^3$, is made and carried through, the polar equations of relative motion reduce to the following with minor errors compared with the above solution.

$$\begin{bmatrix} \Omega s(t_2) \\ \dot{s}(t_2) \\ \Omega \theta(t_2) \\ \dot{\theta}(t_2) \end{bmatrix} = \begin{bmatrix} \cos(\tau) & \sin(\tau) & 0 & 0 \\ -\sin(\tau) & \cos(\tau) & 0 & 0 \\ \epsilon_2 \sin(\tau) & \epsilon_2(1 - \cos(\tau)) & 1 & 0 \\ \epsilon_2 \cos(\tau) & \epsilon_2 \sin(\tau) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Omega s(t_1) \\ \dot{s}(t_1) \\ \Omega \theta(t_1) \\ \dot{\theta}(t_1) \end{bmatrix} + \begin{bmatrix} \epsilon_3(1 - \cos(\tau)) \\ \epsilon_3 \sin(\tau) \\ \epsilon_1 \tau + \epsilon_4 \sin(\tau) \\ \epsilon_1 + \epsilon_4 \cos(\tau) \end{bmatrix}$$

Where $\tau = \Omega(t_2 - t_1)$; $\epsilon_1 = \frac{3\mathfrak{I}}{R^2} - \Omega - \frac{2\mathfrak{I}^3}{R^6\Omega^2}$; $\epsilon_2 = \frac{-2\mathfrak{I}}{R^3\Omega}$; $\epsilon_3 = \left(\frac{\mathfrak{I}^2}{R^3\Omega} - R\Omega\right)$; $\epsilon_4 = -\epsilon_2\epsilon_3$